1-6 Trivial.

7. Recall Euler's totient function as $\phi(n) =$ (#the number of positive integers smaller than $n$ that are coprime to $n$). Prove that if $m, n > 2$, $m, n \in \mathbb{N}$ and $m \mid n$, then $\phi(m) \mid \phi(n)$. Then, prove that $\phi(mn) = m \cdot \phi(n)$.

8. Suppose $n \in \mathbb{N}^+$, and $x_1, \dots, x_n > 0$, then $P(n)$ is true IFF

$$x_1 \cdot x_2 \cdots x_n \leq \left(\frac{x_1 + x_2 + \cdots + x_n}{n}\right)^n$$

is true.

(1) Prove that $\forall n \in \mathbb{N}^+$ and $n > 1$, $P(n)$ implies $P(n-1)$.
(2) Prove that if $P(n)$ and $P(2)$ are true (where $n \geq 2$ and $n \in \mathbb{N}^+$), then $P(2n)$ is true.
(3) Prove that for all $n \in \mathbb{N}^+$, $P(n)$ is true.

9. Suppose there is a non-constant polynomial $f(n) = a_0 + a_1 n^1 + \cdots + a_{t-1} n^{t-1} + n^t$ where $\forall i \in \mathbb{N}$ and $i < t$, $a_i \in \mathbb{Z}$. Denote $c = a_0$.

(1) Prove that $c \mid f(cm)$ for all $m \in \mathbb{Z}$.
(2) Prove that for any $c > 1$, there are infinitely many $n$ such that $f(n)$ is not a prime. (Hint: you may use the fact that $f(n)$ grows unboundedly when $n$ grows.)
(3) Prove that for any non-constant $f(n)$ where all coefficients are integers, there exists at least one $n \in \mathbb{N}$ such that $f(n)$ is not a prime. (Hint: there left only one case.)