Digital Logic

CS207 Assignment 1: Theory (Part 1)

Assignment is pledged that you have neither given nor received unauthorized help. All course work should be completed entirely on your own. Students who commit an act of academic dishonesty may receive a zero on the assignment or in the course.

Due on 23:55, Oct. 4, 2022

南方科技大学

SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Assignment Notes

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- Write neatly according to the answer template and submit an e-copy to Sakai on time.
  - You can finish the theory questions on a paper, scan it and paste into the template.
- Do write down all procedures. Only presenting the final answer will lead to a zero, even the answer is correct.
- Do double-check your submitted file. No re-submission is allowed for student reasons, e.g., corrupted file uploaded.
- Box answers when applicable.
- Draw logic diagrams with a pen or any software applicable.
- Turn assignments in early if possible.
- Part 1 of this assignment accounts for 60%, Part 2 the remaining 40%.
- Request to regrade will lead to a complete regrade on all questions in the assignment. Final grades may increase or decrease.

Question 1

- (10 points) Convert the decimal number 234.5 to base 3, base 5, base 6, base 12, and base 16. For repeating decimals, write down two reptends (循环节).

(1) $3\overline{)234}$
$3\overline{)78}$ 0
$3\overline{)21}$ 0
$3\overline{)8}$ 2
2 2

$0.5 \times 3 = 1.5$
$0.5 \times 3 = 1.5$
$(234.5)_{10}$
$= (22200.11)_3$

(2) $5\overline{)234}$
$5\overline{)46}$ 4
$5\overline{)9}$ 1
1 4

$0.5 \times 5 = 2.5$
$0.5 \times 5 = 2.5$
$(234.5)_{10}$
$= (1414.\overline{55})_5$

(3) $6\overline{)234}$
$6\overline{)39}$ 0
$6\overline{)6}$ 3
1 0

$0.5 \times 6 = 3.0$
$(234.5)_{10}$
$= (1030.3)_6$

(4) $12\overline{)234}$
$12\overline{)19}$ 6
1 7

$0.5 \times 12 = 6$
$(234.5)_{10}$
$= (176.6)_{12}$

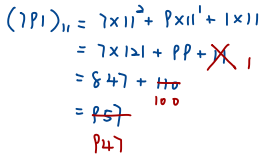
(5) $16\overline{)234}$
14 10

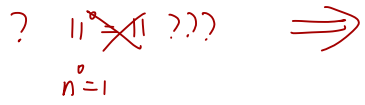
$0.5 \times 16 = 8$
$(234.5)_{10}$
$= (EA.8)_{16}$

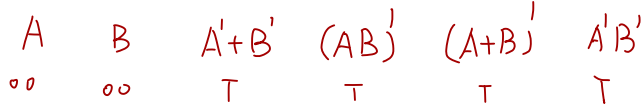
注：求小数的时候取整数部分

Question 2

- (5 points) Find the 10's complement of $(791)_{11}$.





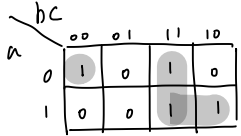


Question 3

use kmap / math is OK

- (15 points) Simplify the following Boolean expressions to a minimum number of literals:
  1. $(a+b+c')(a'b'+c)$, and
  2. $a'b'c+ab'c+abc+a'bc$, and
  3. $(a+c)(a'+b+c)(a'+b'+c)$.

(1)



$(a+b+c')(a'b'+c)$

$= ac+bc+a'b'c'$

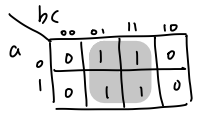
$= a'b'c' + bc + ac$

$\Downarrow$

$(b+a)c$

minimum literal

(2)

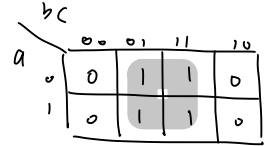


$a'b'c + ab'c + abc + a'bc$

$= b'c + bc$

$= c$

(3)



$(a+c)(a'+b+c)(a'+b'+c)$

$= (a+c)(a'+c+b'b)$

$= (a+c)(a'+c)$

$= c + aa'$

$= c$

Question 3

- (15 points) Simplify the following Boolean expressions to a minimum number of literals:
  1. $(a + b + c')(a'b' + c)$,
  2. $a'b'c + ab'c + abc + a'bc$, and
  3. $(a + c)(a' + b + c)(a' + b' + c)$.

① c' c

$\Downarrow$

$A \oplus B = A'B + AB'$

$\Downarrow$ $\overline{(a+b)}$

$(a+b+c')(a'b'+c)$

$= (a+b) \oplus c$

③ $(a'+b+c)(a'+b'+c)$

$= (a'+c)$

$(A+x')(A+x)$

$x = T \quad A(A+x) = A+A = A$

$x = F \quad A(A+x) = AA = A$

Question 4

- (15 points) Simplify the following three-variable Boolean functions algebraically:

1. $F_1(A, B, C) = \sum(1, 2, 6, 7)$,
2. $F_2(A, B, C) = \sum(0, 1, 2, 3, 5)$, and
3. $F_3(A, B, C) = \sum(3, 5, 6, 7)$.

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

(1) $F_1(A,B,C)$
$= A'B'C + A'BC' + ABC' + ABC$
$= A'B'C + A'(B'C')' + AB(C'+C)$
$= A'[BC + (BC)'] + AB$
$= A' + AB$ $\quad A'B'C + AB + BC'$

$(B'C')' = B + C' \neq BC'$

(2) $F_2(A,B,C)$
$= A'B'C' + A'B'C + A'BC'$
$+ A'BC + AB'C$
$= A'B'(C+C') + A'BC(C+C')$
$+ AB'C$
$= A'B' + A'B + AB'C$
$= A' + AB'C$ $\quad A' + B'C$

(3) $F_3(A,B,C)$
$= A'BC + AB'C$
$+ ABC' + ABC$
$= C(A'B + AB')$
$+ ABC(C+C')$
$= C \oplus AB$
$CB + AB + AC$

Question 4

- (15 points) Simplify the following three-variable Boolean functions algebraically:

1. $F_1(A, B, C) = \sum(1, 2, 6, 7)$,
2. $F_2(A, B, C) = \sum(0, 1, 2, 3, 5)$, and
3. $F_3(A, B, C) = \sum(3, 5, 6, 7)$.

① $F_1(A, B, C)$

$= A'B'C + A'BC' + ABC' + ABC$

$= A'B'C + A'BC' + AB(C' + C)$

$= A'B'C + A'BC' + AB$

① $(B'C)' = B + C'$

② $AB + AC = AB + C$

$\Rightarrow A'B'C + A'BC' + AB$

$= A'B'C + BCA'C' + A)$

③ $\bar{A}B + A = A + B$

$\Rightarrow A'B'C + BC(A'C' + A)$

$= A'B'C + BC(A + C')$

$= A'B'C + AB + BC'$

(1)

$A'B'C + A'BC' + ABC' + ABC$

① $A + A = A$

$= A'B'C + A'BC' + ABC' + ABC$

$+ ABC'$

$= A'B'C + (A'BC' + ABC')$

$+ (ABC + ABC')$

$= A'B'C + BC' + AB$

Question 4

- (15 points) Simplify the following three-variable Boolean functions algebraically:

1. $F_1(A, B, C) = \sum(1, 2, 6, 7)$,
2. $F_2(A, B, C) = \sum(0, 1, 2, 3, 5)$, and
3. $F_3(A, B, C) = \sum(3, 5, 6, 7)$.

② $F_2(A,B,C)$

$= A'B'C' + A'B'C + A'BC' + A'BC + AB'C$

$= A'B' + A'B + AB'C$

$= A' + AB'C$

$A' + AB'C = A' + B'C$

① absorption law

$\bar{A}B + A = A + B$

Proof: $A + \bar{A}B = [A' \cdot (A + B')']' = [A' \cdot (A + B')]' = (A + B')' = \bar{A}B$

③ $F_3(A,B,C)$

$= A'BC + AB'C + ABC' + ABC$

① use kmap to check the answer finally.

$= A'BC + AB + AC$

$= B(A'C + A) + AC$

$= B(A + C) + AC$

$= AB + BC + AC$

② $\times$ $A + \bar{A}B = A + B$ → can lessen the number of literals

& distributive laws

$\Rightarrow A'BC + AB'C + ABC' + ABC$

$= A'BC + AB'C + AB$

$= A'BC + A(B + B'C)$

$= A'BC + A(B + C)$

Question 4

- (15 points) Simplify the following three-variable Boolean functions algebraically:

  1. $F_1(A, B, C) = \sum(1, 2, 6, 7)$,
  2. $F_2(A, B, C) = \sum(0, 1, 2, 3, 5)$, and
  3. $F_3(A, B, C) = \sum(3, 5, 6, 7)$.

(3) $F_3(A, B, C)$

$= A'BC + AB'C$

$AB C' + ABC$

① $A + A = A$

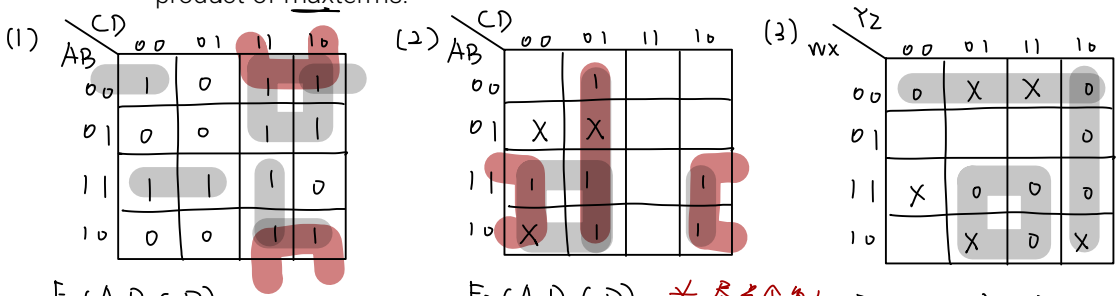
$= A'BC + ABC + AB'C + ABC$

$+ ABC' + ABC$

$= BC + AC + AB$

Question 5

- (15 points) Using a Karnaugh map, simplify the following functions:
  1. $F_1(A, B, C, D) = \sum(0, 2, 3, 6, 7, 10, 11, 12, 13, 15)$ in sum-of-minterms,
  2. $F_2(A, B, C, D) = \sum(1, 9, 10, 12, 13, 14) + d(4, 5, 8)$ in sum-of-minterms, and
  3. $F_3(W, X, Y, Z) = \prod(0, 2, 6, 11, 13, 14, 15) + d(1, 3, 9, 10, 12)$ in product-of-maxterms.



$F_1(A, B, C, D)$

$= A'B'D' + A'C + ABC' + ACD + AB'C$

$F_2(A, B, C, D)$

$= AC' + C'D + ACD'$

$C'D + AD'$

尽量个多个 group 的面积

$F_3(W, X, Y, Z)$

$= WX + Y'Z + W'Z'$

$(W+X) \cdot (Y'+Z)(W'+Z')$

Question 6

- (10 points) With the use of maps, find the simplest sum-of-products form of the function $F = fg$, where $f = abd' + c'd + a'cd' + b'cd'$ and $g = (a + b + d')(b' + c' + d)(a' + c + d')$. ⇒ write 0

ab \ cd	00	01	11	10
00	0	0	0	1
01	0	1	0	0
11	1	0	0	0
10	0	0	0	1

$F = fg$

$= abc'd' + a'b'c'd + b'cd'$

Question 7

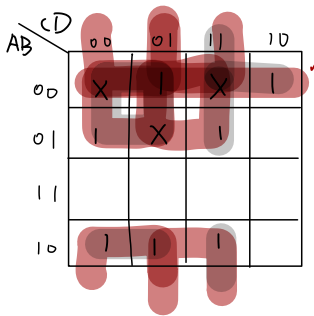
- (15 points) Obtain the sum-of-products expression for $F(A,B,C,D) = \sum(1,2,4,7,8,9,11) + d(0,3,5)$ and implement it with
  1. NAND gates only, and
  2. NOR gates only.

  Draw the two logic diagrams.

SOP

POS

1x4 的大条. 不用 1x2 的



$$F(A,B,C,D) = A'C' + AB'C' + A'CD + A'B'C + B'CD$$

$$A'C' + A'D + A'B' + B'C' + B'D$$

Question 7

- (15 points) Obtain the sum-of-products expression for $F(A, B, C, D) = \sum(1, 2, 4, 7, 8, 9, 11) + d(0, 3, 5)$ and implement it with
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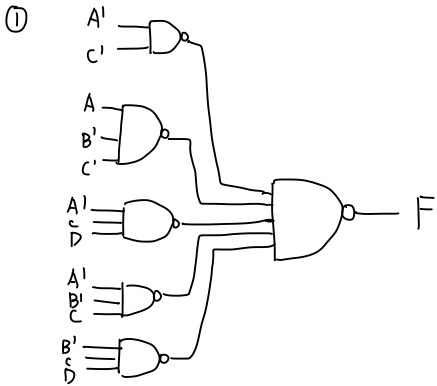
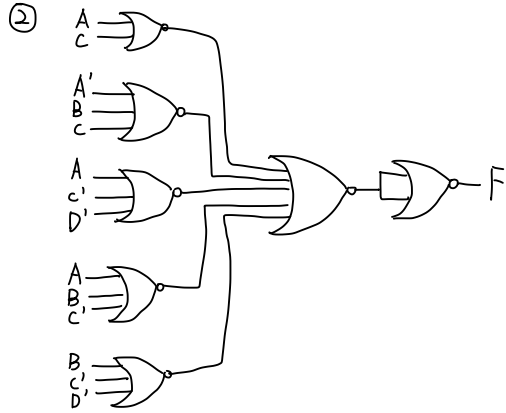
  Draw the two logic diagrams.





Question 7

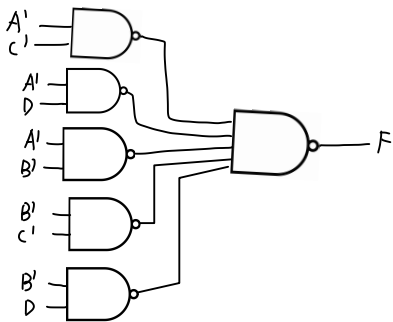
- (15 points) Obtain the sum-of-products expression for $F(A, B, C, D) = \sum(1, 2, 4, 7, 8, 9, 11) + d(0, 3, 5)$ and implement it with
  1. NAND gates only, and
  2. NOR gates only.

  Draw the two logic diagrams.

$A + B = (A' \cdot B')' = A + B$

$A \cdot B = [(A \cdot B)' \cdot (A \cdot B)']'$

$A'C' + A'D + A'B' + B'C' + B'D$

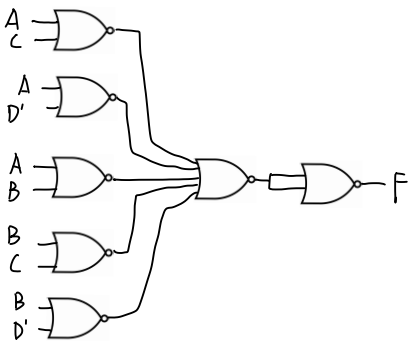


Question 7

A'C' + A'D + A'B' + B'C' + B'D

- (15 points) Obtain the sum-of-products expression for $F(A, B, C, D) = \sum(1, 2, 4, 7, 8, 9, 11) + d(0, 3, 5)$ and implement it with
  1. NAND gates only, and
  2. NOR gates only.

  Draw the two logic diagrams.



Question 8

- (15 points) Find the minimized sum-of-products for the logical product $F = F_1 F_2$ of the following pairs of functions:

  1. $F_1(A, B, C, D) = \sum(1, 3, 5, 7)$
     $F_2(A, B, C, D) = \sum(2, 3, 6, 7)$
  2. $F_1(A, B, C, D) = \sum(1, 3, 5, 6, 8, 10, 11, 12, 13)$
     $F_2(A, B, C, D) = \sum(0, 3, 5, 8, 9, 11, 13, 15)$
  3. $F_1(A, B, C) = \prod(0, 3, 6, 7)$
     $F_2(A, B, C) = \prod(1, 3, 7)$

3 variable

$= \sum(3.5.8.11.13)$

有一个0就0

$\Rightarrow F_1 = \sum(1.2.4.5)$
$F_2 = \sum(0.2.4.5.6)$

$\pi = (0.1.3.6.7)$
$= \sum(2.4.5)$

① CD / AB

AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	0	0
10	0	0	0	0

$F = F_1 \cdot F_2 = A'CD$

② CD / AB

AB \ CD	00	01	11	10
00	0	0	1	0
01	0	1	0	0
11	0	1	0	0
10	1	0	1	0

$F = AB'C'D' + BC'D + B'CD$

BC / A

A \ BC	00	01	11	10
0	0	0	0	1
1	1	1	0	0

$F = A'B' + A'C + \cancel{BC} + AB$

$AB' + A'BC'$

Question 8

- (15 points) Find the minimized sum-of-products for the logical product $F = F_1 F_2$ of the following pairs of functions:
  1. $F_1(A, B, C, D) = \sum(1, 3, 5, 7)$
     $F_2(A, B, C, D) = \sum(2, 3, 6, 7)$
  2. $F_1(A, B, C, D) = \sum(1, 3, 5, 6, 8, 10, 11, 12, 13)$
     $F_2(A, B, C, D) = \sum(0, 3, 5, 8, 9, 11, 13, 15)$
  3. $F_1(A, B, C) = \prod(0, 3, 6, 7)$
     $F_2(A, B, C) = \prod(1, 3, 7)$

(3) ① when checking, regarded as 4 variables.

② find SOP $\Longrightarrow$ find 1 in Kmap

$\Longrightarrow$ $F_1 \cdot F_2 = \Pi \cdots$ is written as POS

$\Longrightarrow$ as long as F_1 OR $F_2 = 0$. $F \neq 1$

Don’t forget that this assignment has a

Part 2: Digital Logic Lab

to complete!

Submit your answers using the template

in the Part 2 document!