Homework5

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Question1 1

1.1 1a

1.1.1 1a(i)

 $Pr(Popularity = 'P') = \frac{7}{10} = 0.7$

1.1.2 1a(ii)

 $Pr(Popularity = 'NP') = \frac{3}{10} = 0.3$

1.1.3 1a(iii)

Set event x as Price = '\$', Delivery = 'Yes', Cuisine = 'Korean'

Pr(Popularity = 'P' | Price = '\$') = $\frac{3}{4}$ Pr(Popularity = 'P' | Delivery = 'Yes') = $\frac{5}{6}$

Pr(Popularity = 'P' | Cuisine = 'Korean') = $\frac{2}{3}$

Pr(x) = 0

Pr(x | Popularity = 'P') = Pr(Popularity = 'P' | Price = '\$') Pr(Popularity = 'P' | Delivery = 'Yes')

 $Pr(Popularity = 'P' \mid Cuisine = 'Korean') Pr(x) = 0$

1.1.4 1a(iv)

Set event x as Price = '\$', Delivery = 'Yes', Cuisine = 'Korean'

Pr(Popularity = 'NP' | Price = '\$') = $\frac{1}{4}$

Pr(Popularity = 'NP' | Delivery = 'Yes') = $\frac{1}{6}$

Pr(Popularity = 'NP' | Cuisine = 'Korean') = $\frac{1}{3}$

Pr(x) = 0

Pr(x | Popularity = 'NP') = Pr(Popularity = 'NP' | Price = '\$') Pr(Popularity = 'NP' | Delivery =

'Yes') $Pr(Popularity = 'NP' \mid Cuisine = 'Korean') Pr(x) = 0$

1.2 **1b**

Pr(Popularity = 'P' | x) = Pr(Price = '\$' | Popularity = 'P') Pr(Delivery = 'Yes' | Popularity = 'P') Pr(Cuisine = 'Korean' | Popularity = 'P') Pr(Popularity = 'P') = $\frac{3}{7}$ $\frac{5}{7}$ $\frac{2}{7}$ $\frac{7}{10}$ = $\frac{3}{49}$

1.3 1c

- 1. Various Navie Bayes classifier with different features will be created.
- 2. Ensemble different classifier.
- 3. Boosting method can be used to improve the accuracy. It assigns weights for each training tuple, and use ensemble naive Bayes classifier to iteratively learn. The weights will be adjusted and finally find the best one.

1.4 1d

Precision and sensitive. Precision: $\frac{TP}{TP+FP}$, and Sensitive: $\frac{TP}{P}$. These two metrics only use positive examples, which is good since we have rare positive examples.

2 Question 2

2.1 2a

K = 1, as you can see in the Figure 1, the testing error is 25%

										k=1	
its		Label		Pnts		Label	Pnts		Label	Test	Actual
2.7	2.7	1		2.3	3	-1	2.5	2	1		-1 1
			Distance	0.5			0.728011				
2.5	1	1		2	1.2	1	2.5	2	1		1 1
			Distance	0.538516			1				
1.5	2.5	-1		1.5	2	-1	1.2	1.9	-1		-1 -1
			Distance	0.5			0.67082				
1.2 1	1	-1		0.8	1	-1	1	0.5	1		-1 -1
			Distance	0.4			0.538516				
										Error	0.25
			Distance	0.4			0.550510			Error	

Figure 1: K=1

2.2 2b

For K = 2, we can arbitrarily select the label when two points have different labels. So in this case, testing error is 0

	Actual		
1		1	
1		1	
-1		-1	
-1		-1	
		0	
	1 -1	1 -1	1 1 1 -1 -1 -1

Figure 2: K=2

2.3 2c

Choose a,b,c = 1, -1, 0.1, respectively. The training error is is 0, and the testing error is 50%. $f(x) = x_1 - x_2 - 0.1$, it satisfies all training dataset, and for the test dataset, the predictions for the first and last one are wrong, and the left are correct. Thus, the test error is 50%.

2.4 2d

Based on 2a-2c, we can find

KNN:

- 1. Do not assume an explicit form for f(X), providing a more exible approach.
- 2. The number of cluster is pre-defined which sometimes can not ensure to get the most optimized clusters
- 3. Easy to understand and implement.

Linear Classifier:

- 1. They make strong assumptions about the form of f(X).
- 2. Suppose we assume a linear relationship between X and Y but the true relationship is far from linear, then the resulting model will provide a poor t to the data.

3 Question 3

3.1 3a

Based on the calculation, we can find the mean points of two cluster are (1,8333, 2.1667), (5, 4.14) respectively. and index from 1 to 6 belong to cluster 1, the rest belong to cluster 2.

First Step												
Initial	0	3	6	4	Index	x_1	x_2	Clust				
Distance	1		5.09902		1	1	3 +1	1.833333	2.166667	5	4.142857	
		1.414214		5.385165		2	1	2 +1				
		2.828427		5		3	2	1 +1				
		2.236068		4.472136		4	2	2 +1				
		2		4.123106		5	2	3 +1				
		3.162278		3.605551		6	3	2 +1				
		5		1.414214		7	5	3 -1				
		4		2.236068		8	4	3 -1				
		4.472136		2.236068		9	4	5 -1				
		5.09902		1		LO	5	4 -1				
		5.385165		1.414214	1	11	5	5 -1				
		6.082763		0	1	12	6	4 -1				
		6.324555		1		13	6	5 -1				
Second St	ер											
Center	1.833333	2.166667	5	4.142857	Index	x_1	x_2	Clust				
Distance		1.178511		4.160063		1	1	3 +1	1.833333	2.166667	5	4.142857
		0.849837		4.537823		2	1	2 +1				
		1.178511		4.34483		3	2	1 +1				
		0.235702		3.686711		4	2	2 +1				
		0.849837		3.210315		5	2	3 +1				
		1.178511		2.931184		6	3	2 +1				
		3.27448		1.142857		7	5	3 -1				
		2.321398		1.518592		8	4	3 -1				
		3.566822		1.317078		9	4	5 -1				
		3.659083		0.142857	:	10	5	4 -1				
		4.249183		0.857143	1	11	5	5 -1				
		4.552167		1.010153	1	12	6	4 -1				
		5.038739		1.317078		L3	6	5 -1				

Figure 3: K=2

3.2 3b

First step, arbitrarily select a point. We can find it satisfy MinPts = 2 and Eps = 1.5. So we say this point is a core point and a cluster is formed.

Then we select another points. There will be three cases.

- 1. It's a another core point, which means it can form a new cluster.
- 2. It's a density reachable point from last core point.
- 3. It's a directly density-reachable from last core point.

After iteration, we can find all points are either density-reachable or density-connected. Therefore, clusters are integrated, and a big cluster is formed for this entire dataset.

3.3 3c

The order is

$$(1,2),3,4,5,6,7,8,9,10,11,12,13\\ (1,2),(3,4),5,6,7,8,9,10,11,12,13\\ (1,2),(3,4),5,6,(7,8),9,10,11,12,13\\ (1,2),(3,4),5,6,(7,8),9,(10,11),12,13\\ (1,2),(3,4),5,6,(7,8),9,(10,11),(12,13)\\ ((1,2),(3,4)),5,6,(7,8),9,(10,11),(12,13)\\ (((1,2),(3,4)),5),6,(7,8),9,(10,11),(12,13)\\ ((((1,2),(3,4)),5),6),(7,8),9,(10,11),(12,13)\\ ((((1,2),(3,4)),5),6),(7,8),9,(10,11),(12,13))\\ ((((1,2),(3,4)),5),6),(7,8),9,((10,11),(12,13))\\ (((((1,2),(3,4)),5),6),((7,8),((10,11),(12,13))),9\\ (((((1,2),(3,4)),5),6),((7,8),((10,11),(12,13)))),9\\ ((((((1,2),(3,4)),5),6),((7,8),((10,11),(12,13)))),9)\\ ((((((1,2),(3,4)),5),6),((7,8),((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),((7,8),((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),((7,8),((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),((7,8),((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),((7,8),((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),(((7,8),((10,11),(12,13)))),9)\\ ((((((((1,2),(3,4)),5),6),(((7,8),((10,11),(12,13)))),9)\\ ((((((((1,2),(3,4)),5),6),(((7,8),((10,11),(12,13)))),9)\\ ((((((((1,2),(3,4)),5),6),(((7,8),((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),(((7,8),((10,11),(12,13)))),9)\\ ((((((((1,2),(3,4)),5),6),((((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),((((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),((((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),((((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),((((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),(((((10,11),(12,13)))),9)\\ (((((((1,2),(3,4)),5),6),(((((10,11),(12,13)))),9)\\ ((((((((1,2),(3,4)),5),6),((((((10,11),(12,13)))),9)))$$

Index	x_1	x_2	_												
1	1	3													
2	1	2													
3	2	1													
4	2	2													
5	2	3													
6	3	2													
7	5	3													
8	4	3													
9	3	5													
10	5	4													
11	5	5													
12	6	4													
13	6	5													
Distance Matrix		1	2	3	4	5	6	7	8	9		10	11	12	13
Distance Highlia	1	0			+ -		-						1 11	12	13
	2	1	0		1								+		+
	3	2.23606797		14 0	+					1			1		+
	4	1.41421356		1	0					İ					+
	5	1	1.4142	_	1	0				İ					+
	6	2.23606797		1.41421		1.414214	0			1					
	7	4		.06 3.60555			2.236068	0		İ					
	8	3		78 2.82842			1.414214	1	0						
	9	2.82842712		51 4.12310			3		2.236068	0)				
	10			.36 4.24264				1	1.414214	2.236	5068	0			
	11	4.47213595		5	4.24264			2	2.236068	2		1	0		
	12	5.09901951	1 5.3851	.65 5	4.47213	5955 4.123106	3.605551	1.414214	2.236068	3.162	2278	1	1.41421	4 0	
	13	5.38516480	7 5.8309	5.65685	4 5	4.472136	4.242641	2.236068	2.828427	3	1.4	14214	1	1	0
First Step	1,2														
	1,2		2												
1.2			3	4	5	6	7	8	9		10		11	12	13
1,2	0		3	4	5	6	7	8	9	1	10		11	12	13
1,2 3	0	562		4	5	6	7	8	9		10		11	12	13
3	0 1.414213	562	0		5	6	7	8	9		10		11	12	13
3	0 1.414213 1	562	0	0		6	7	8	9		10		11	12	13
3 4 5	0 1.414213 1 1		0 1 2	0 1	0		7	8	9		10		11	12	13
3 4 5 6	0 1.414213 1 1 2	1.414	0 1 2 213562	0 1 1	0 1.414214	0		8	9		10		11	12	13
3 4 5 6 7	0 1.414213 1 1 2 4	1.414 3.605	0 1 2 213562 551275	0 1 1 3.162278	0 1.414214 3	0 2.236067977	0		9		10		11	12	13
3 4 5 6 7 8	0 1.414213 1 1 2 4 3	1.414 3.605 2.828	0 1 2 213562 551275 427125	0 1 1 3.162278 2.236068	0 1.414214 3 2	0 2.236067977 1.414213562	0 1	0			10		11	12	13
3 4 5 6 7 8 9	0 1.414213 1 1 2 4 3 2.828427	1.414 3.605 2.828 125 4.123	0 1 2 213562 551275 427125 105626	0 1 1 3.162278 2.236068 3.162278	0 1.414214 3 2 2.236068	0 2.236067977 1.414213562 3	0 1 2.82842	0 7 2.2360	068 0				11	12	13
3 4 5 6 7 8 9	0 1.414213 1 1 2 4 3 2.828427 4.123105	1.414 3.605 2.828 125 4.123 626 4.242	0 1 2 213562 551275 427125 105626 640687	0 1 1 3.162278 2.236068 3.162278 3.605551	0 1.414214 3 2 2.236068 3.162278	0 2.236067977 1.414213562 3 2.828427125	0 1 2.82842	0 7 2.2360 1.4142	068 0	5068	0			12	13
3 4 5 6 7 8 9	0 1.414213 1 1 2 4 3 2.828427	1.414 3.605 2.828 125 4.123 626 4.242	0 1 2 213562 551275 427125 105626	0 1 1 3.162278 2.236068 3.162278	0 1.414214 3 2 2.236068 3.162278	0 2.236067977 1.414213562 3	0 1 2.82842	0 7 2.2360	068 0	5068			0	12	13
3 4 5 6 7 8 9	0 1.414213 1 1 2 4 3 2.828427 4.123105	1.414 3.605 2.828 125 4.123 626 4.242 955	0 1 2 213562 551275 427125 105626 640687	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641	0 1.414214 3 2 2.236068 3.162278	0 2.236067977 1.414213562 3 2.828427125	0 1 2.82842 1 2	0 7 2.2360 1.4142 2.2360	068 0	5068	0	1.4		0	13
3 4 5 6 7 8 9 10	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135	1.414 3.605 2.828 125 4.123 626 4.242 955 514	0 1 2 213562 551275 427125 105626 640687 5	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641	0 1.414214 3 2 2.236068 3.162278 3.605551	0 2.236067977 1.414213562 3 2.828427125 3.605551275	0 1 2.82842 1 2	0 7 2.2360 1.4142 2.2360 4 2.2360	068 0 214 2.236 068 2 068 3.162	0 6068 2278	0 1		0		13
3 4 5 6 7 8 9 10 11	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019	1.414 3.605 2.828 125 4.123 626 4.242 955 514	0 1 2 213562 551275 427125 105626 640687 5	0 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136	0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106	0 2.236067977 1.414213562 3 2.828427125 3.605551275 3.605551275	0 1 2.82842 1 2 1.41421	0 7 2.2360 1.4142 2.2360 4 2.2360	068 0 214 2.236 068 2 068 3.162	0 6068 2278	0 1 1		0	0	
3 4 5 6 7 8 9 10 11	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019	1.414 3.605 2.828 125 4.123 626 4.242 955 514	0 1 2 213562 551275 427125 105626 640687 5	0 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136	0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106	0 2.236067977 1.414213562 3 2.828427125 3.605551275 3.605551275	0 1 2.82842 1 2 1.41421	0 7 2.2360 1.4142 2.2360 4 2.2360	068 0 214 2.236 068 2 068 3.162	0 6068 2278	0 1 1		0	0	
3 4 5 6 7 8 9 10 11 12 13	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656	0 1 2 213562 551275 427125 105626 640687 5	0 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136	0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106	0 2.236067977 1.414213562 3 2.828427125 3.605551275 3.605551275	0 1 2.82842 1 2 1.41421	0 7 2.2360 1.4142 2.2360 4 2.2360	068 0 214 2.236 068 2 068 3.162	0 6068 2278	0 1 1		0	0	
3 4 5 6 7 8 9 10 11 12 13	0 1.414213 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656	0 1 2 213562 551275 427125 105626 640687 5 5 8854249	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5	0 1.414214 3 2 2.236068 3.162278 4.123106 4.472136	0 2.236067977 1.414213562 3 2.828427125 3.605551275 3.605551275 4.242640687	0 1 2.82842 1 2 1.41421 2.23606	0 7 2.2360 1.4142 2.2360 4 2.2360 8 2.8284	068 0 214 2.236 668 2 668 3.162 127 3	0 6068 2278	0 1 1 1.41421		0 414214 1	0 1	
3 4 5 6 7 8 9 10 11 12 13	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656	0 1 2 213562 551275 427125 105626 640687 5 5 8854249	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5	0 1.414214 3 2 2.236068 3.162278 4.123106 4.472136	0 2.236067977 1.414213562 3 2.828427125 3.605551275 3.605551275 4.242640687	0 1 2.82842 1 2 1.41421 2.23606	0 7 2.2360 1.4142 2.2360 4 2.2360 8 2.8284	068 0 214 2.236 668 2 668 3.162 127 3	0 6068 2278	0 1 1 1.41421		0 414214 1	0 1	
3 4 5 6 7 8 9 10 11 12 13 Second Step	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164 3,4 1,2	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656	0 1 2 213562 551275 427125 105626 640687 5 5 854249	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5	0 1.414214 3 2 2.236068 3.162278 4.123106 4.472136	0 2.236067977 1.414213562 3 2.828427125 3.605551275 3.605551275 4.242640687	0 1 2.82842 1 2 1.41421 2.23606	0 7 2.2360 1.4142 2.2360 4 2.2360 8 2.8284	068 0 214 2.236 668 2 668 3.162 127 3	0 6068 2278	0 1 1 1.41421		0 414214 1	0 1	
3 4 5 6 7 8 9 10 11 12 13 Second Step	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164 3,4 1,2 0 1	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656	0 1 2 213562 551275 427125 105626 640687 5 5 854249	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5	0 1.414214 3 2 2.236068 3.162278 4.123106 4.472136	0 2.236067977 1.414213562 3 2.828427125 3.605551275 3.605551275 4.242640687	0 1 2.82842 1 2 1.41421 2.23606	0 7 2.2360 1.4142 2.2360 4 2.2360 8 2.8284	068 0 214 2.236 668 2 668 3.162 127 3	0 6068 2278	0 1 1 1.41421		0 414214 1	0 1	
3 4 5 6 7 8 9 10 11 12 13 Second Step 1,2 3,4 5	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164 3,4 1,2 0 1 1 2	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656	0 1 2 2213562 551275 427125 105626 640687 5 5 8854249	0 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5	0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106 4.472136 6	0 2.236067977 1.414213562 3 2.828427125 3.605551275 3.605551275 4.242640687	0 1 2.82842 1 2 1.41421 2.23606	0 7 2.2360 1.4142 2.2360 4 2.2360 8 2.8284	068 0 214 2.236 668 2 668 3.162 127 3	0 6068 2278	0 1 1 1.41421		0 414214 1	0 1	
3 4 5 6 7 8 9 10 11 12 13 Second Step 1,2 3,4 5 6 7	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164 3,4 1,2 0 1 1 2 4	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656	0 1 2 2 2213562 551275 427125 105626 640687 5 5 8854249 0 1 1 1	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5	0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106 4.472136 6	0 2.236067977 1.414213562 3 2.828427125 3.605551275 3.605551275 4.242640687	0 1 2.82842 1 2 1.41421 2.23606	0 7 2.2360 1.4142 2.2366 4 2.2366 8 2.8284	068 0 214 2.236 668 2 668 3.162 127 3	0 6068 2278	0 1 1 1.41421		0 414214 1	0 1	
3 4 5 6 7 8 9 10 11 12 13 Second Step 1,2 3,4 5 6 7 8	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164 3,4 1,2 0 1 1 2 4	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656	0 1 2 2 2213562 551275 427125 105626 640687 5 5 8854249 8,4	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5	0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106 4.472136 6 0 2.236068 1.414214	0 2.236067977 1.414213562 3 2.828427125 3.605551275 4.242640687 7	0 1 2.82842 1 2 1.41421 2.23606	0 7 2.2360 1.4142 2.2360 4 2.2360 8 2.8284	068 0 214 2.236 668 2 668 3.162 127 3	0 6068 2278	0 1 1 1.41421		0 414214 1	0 1	
3 4 5 6 7 8 9 10 11 12 13 Second Step 1,2 3,4 5 6 7 8 9	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164 3,4 1,2 0 1 1 1 2 4 3 2.828427	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656	0 1 2 2 2 2 2 2 3 5 5 5 2 2 3 5 5 5 5 5 5 8 5 4 2 7 1 2 5 5 1 2 5 5 1 2 5 5 1 2 5 5 1 2 5 5 5 1 2 5 5 5 5	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5 5	0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106 4.472136 6 0 2.236068 1.414214 3	0 2.236067977 1.414213562 3 2.828427125 3.605551275 4.242640687 7	0 1 2.82842 1 2 1.41421 2.23606	0 7 2.2360 1.4142 2.2360 4 2.2360 8 2.8284 9	068 00 214 2.236 168 2 168 3.162 127 3	2278	0 1 1 1.41421		0 414214 1	0 1	
3 4 5 6 7 8 9 10 11 12 13 Second Step 1,2 3,4 5 6 7 8 9 10	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164 3,4 1,2 0 1 1 2 4 3 2.828427 4.123105	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656 3.166 2.236 125 3.166	0 1 2 2 2 2 2 3 5 5 5 2 2 1 2 3 4 2 7 1 5 5 5 5 8 5 4 2 7 1 5 5 5 5 6 4 6 4 0 6 6 6 4 0 6 6 6 6 6 6 6 6 6 6	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5 5 0 1.414214 3 2 2.236068 3.162278	0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106 4.472136 6 0 2.236068 1.414214 3 2.828427	0 2.236067977 1.414213562 3 2.828427125 3.605551275 4.242640687 7 0 1 2.828427125	0 1 2.82842 1 2 1.41421 2.23606 8	0 7 2.2360 1.4142 2.23664 2.23668 2.8284 9 9	068 00 214 2.236 168 2 168 3.162 127 3	2278	0 1 1 1.41421		0 414214 1	0 1	
3 4 5 6 7 8 9 10 11 12 13 Second Step 1,2 3,4 5 6 7 8 9 10 11	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164 3,4 1,2 0 1 1 2 4 3 2.828427 4.123105 4.472135	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656 3.166 2.236 125 3.166 626 3.605 955 4.242	0 1 2 213562 551275 427125 105626 640687 5 5 854249 0 1 1 1 227766 067977 127766 551275 640687	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5 5 0 1.414214 3 2 2.236068 3.162278 3.605551	0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106 4.472136 6 0 2.236068 1.414214 3 2.828427 3.605551	0 2.236067977 1.414213562 3 2.828427125 3.605551275 4.242640687 7 0 1 2.828427125 1 2	0 1 2.82842 1 2 1.41421 2.23606 8 8 0 2.23606 1.41421 2.23606	0 7 2.2360 1.4142 2.2360 4 2.2368 8 2.8284 9 9 8 0 4 2.2360 8 2	068 008 008 008 008 008 008 008 008 008	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 1 1 1.41421 11	4	0 414214 1	0 1	
3 4 5 6 7 8 9 10 11 12 13 Second Step 1,2 3,4 5 6 7 8 9 10	0 1.414213 1 1 2 4 3 2.828427 4.123105 4.472135 5.099019 5.385164 3,4 1,2 0 1 1 2 4 3 2.828427 4.123105	1.414 3.605 2.828 125 4.123 626 4.242 955 514 807 5.656 3.166 2.236 125 3.166 626 3.605 955 4.242 514 4.472	0 1 2 2 2 2 2 3 5 5 5 2 2 1 2 3 4 2 7 1 5 5 5 5 8 5 4 2 7 1 5 5 5 5 6 4 6 4 0 6 6 6 4 0 6 6 6 6 6 6 6 6 6 6	0 1 1 3.162278 2.236068 3.162278 3.605551 4.242641 4.472136 5 5 0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106	0 1.414214 3 2 2.236068 3.162278 3.605551 4.123106 4.472136 6 0 2.236068 1.414214 3 2.828427 3.605551	0 2.236067977 1.414213562 3 2.828427125 3.605551275 4.242640687 7 0 1 2.828427125	0 1 2.82842 1 2 1.41421 2.23606 8	0 7 2.2360 1.4142 2.2360 4 2.2360 8 2.8284 9 9 8 0 4 2.2360 8 2 8 3.1622	068 008 008 008 008 008 008 008 008 008	0068	0 1 1 1.41421	4	0 414214 1	0 1	

Third Step	7,8									
	1,2	3,4	5	6	7,8	9	10	11	12	1
1,2	0									i i
3,4	1	0								
5	1	1	0							
6	2	1	1.414214	0						[
7,8	3	2.236067977	2	1.414214	0					
9	2.828427125	3.16227766	2.236068	3	2.236067977	0				
10	4.123105626	3.605551275	3.162278	2.828427	1	2.236068	0			
11	4.472135955	4.242640687	3.605551	3.605551	2	2	1	0		i i
12	5.099019514	4.472135955	4.123106	3.605551	1.414213562	3.162278	1	1.414214	0	
13	5.385164807	5	4.472136	4.242641	2.236067977	3	1.414214	1	1	
										i i
Fourth Step	10,11									
	1,2	3,4	5	6	7,8	9	10,11	12	13	
1,2	0									
3,4	1	0								
5	1	1	0							
6	2	1	1.414214	0						
7,8	3	2.236067977	2	1.414214	0					
9	2.828427125	3.16227766	2.236068	3	2.236067977	0				
10,11	4.123105626	3.605551275	3.162278	2.828427	1	2	0			
12	5.099019514	4.472135955	4.123106	3.605551	1.414213562	3.162278	1	0	0	
13	5.385164807	5	4.472136	4.242641	2.236067977	3	1	1	0	
Fifth Step	12,13									
	1,2	3,4	5	6	7,8	9	10,11	12,13		
1,2	0									
3,4	1	0								
5	1	1	0							
6	2	1	1.414214	0						
7,8	3	2.236067977	2	1.414214	0					
9	2.828427125	3.16227766	2.236068	3	2.236067977	0				
10,11	4.123105626	3.605551275	3.162278	2.828427	1	2	0			
12,13	5.099019514	4.472135955	4.123106	3.605551	1.414213562	3	1	0		

Sixth Step	1,2,3,4	5	6	7,8	9	10,11	12,13
1 2 2 4	1,2,3,4))	0	7,8	y	10,11	12,13
1,2,3,4	0	<u>-</u>					
5	1	0	_				
6	1	1.414213562	0				
7,8	2.236067977	2	1.414214	0			
9	2.828427125	2.236067977	3	2.236068	0		
10,11	3.605551275	3.16227766	2.828427	1	2	0	
12,13	4.472135955	4.123105626	3.605551	1.414214	3	1	0
Seventh Step	1,2,3,4,5						
	1,2,3,4,5	6	7,8	9	10,11	12,13	
1,2,3,4,5	0						
6	1	0					
7,8	2	1.414213562	0				
9	2.236067977	3	2.236068	0			
10,11	3.16227766	2.828427125	1	2	0		
12,13	4.123105626	3.605551275	1.414214	3	1	0	
·							
Eighth Step	1,2,3,4,5,6						
	1,2,3,4,5,6	7,8	9	10,11	12,13		
1,2,3,4,5,6	0	.,,,					
7,8	1.414213562	0					
9	2.236067977	2.236067977	0				
10,11	2.828427125	1	2	0			
12,13	3.605551275	1.414213562	3	1	0		
12,13	3.003331273	1.414213302		-			
Ninth Step	10,11,12,13						
Militi Step	1,2,3,4,5,6	7,8	9	10,11,12,13			
1,2,3,4,5,6	1,2,3,4,3,0	7,8		10,11,12,13			
7,8	1.414213562	0					
		2.236067977	0				
	2.236067977		0				
10,11,12,13	2.828427125	1	2	0			

Tenth Step	10,11,12,13,7,8		
0	1,2,3,4,5,6	7,8,10,11,12,13	9
1,2,3,4,5,6	0		
7,8,10,11,12,13	1.414213562	0	
9	2.236067977	2	0
11th Step	4,5,6,7,8,10,11,	12,13	
1,2,3,	4,5,6,7,8,10,11,	9	
1,2,3,4,5,6,7,8,10,11,12,13	0		
9	2	0	
1			
12nd Step	All		
1,2,3,4			
1,2,3,4,5,6,7,8,9,10,11,12,13	0		