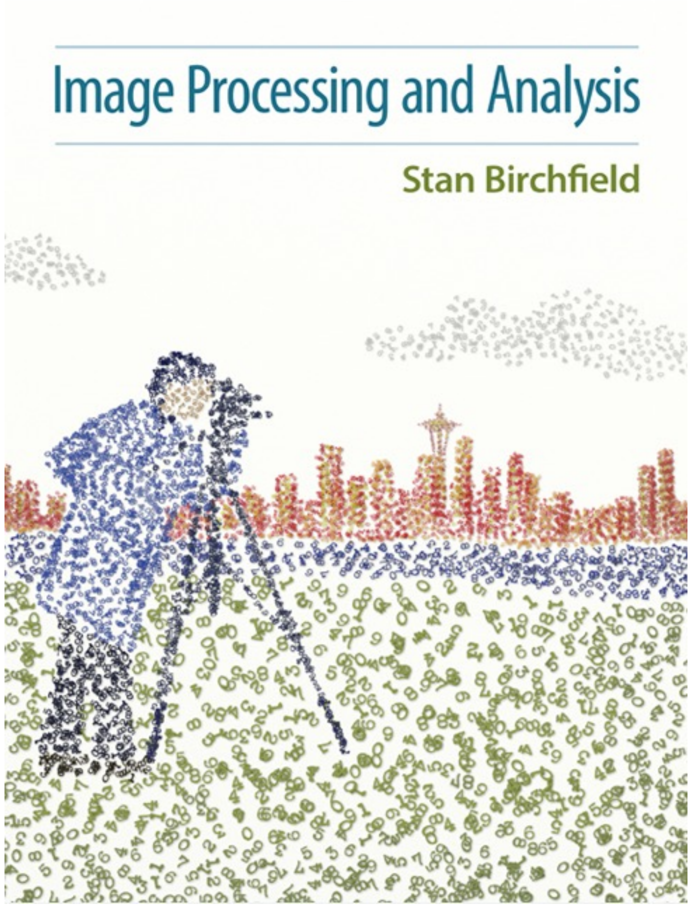
Prof. Kjersti Engan

ELE510 Image processing and computer vision

Frequency Domain processing, Frequency domain filtering (Chap6 Birchfield)
2020

University of Stavanger



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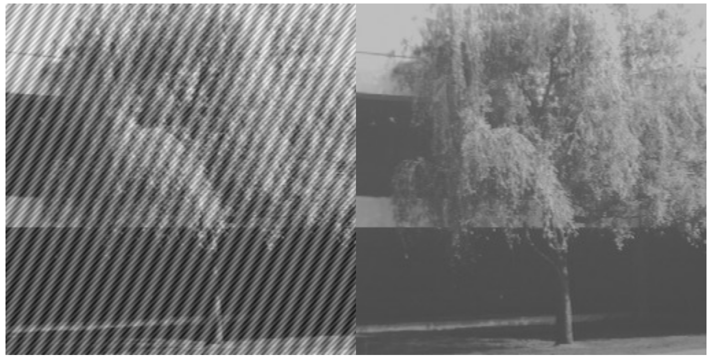
Filtering in the Frequency Domain

Three points from the topic:

1. How can we do the filtering in the frequency domain?

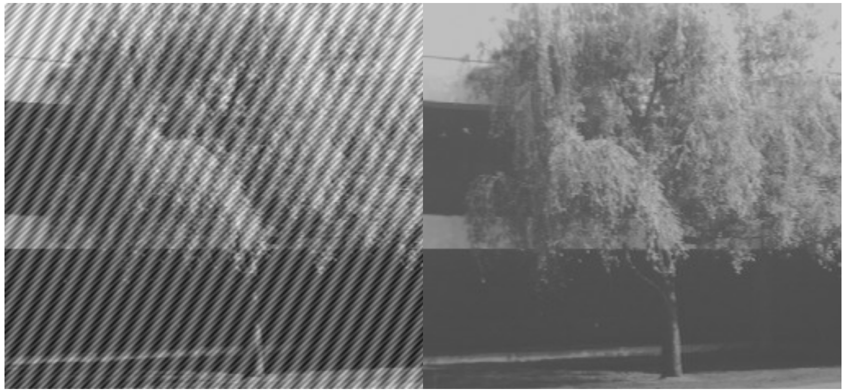
2. How can filtering infreq. domain help us understand what filtering in the spatial domain actually does to our data/images?

3. Which situations will filtering in freq. domain be an advantage over filtering in spatial domain?



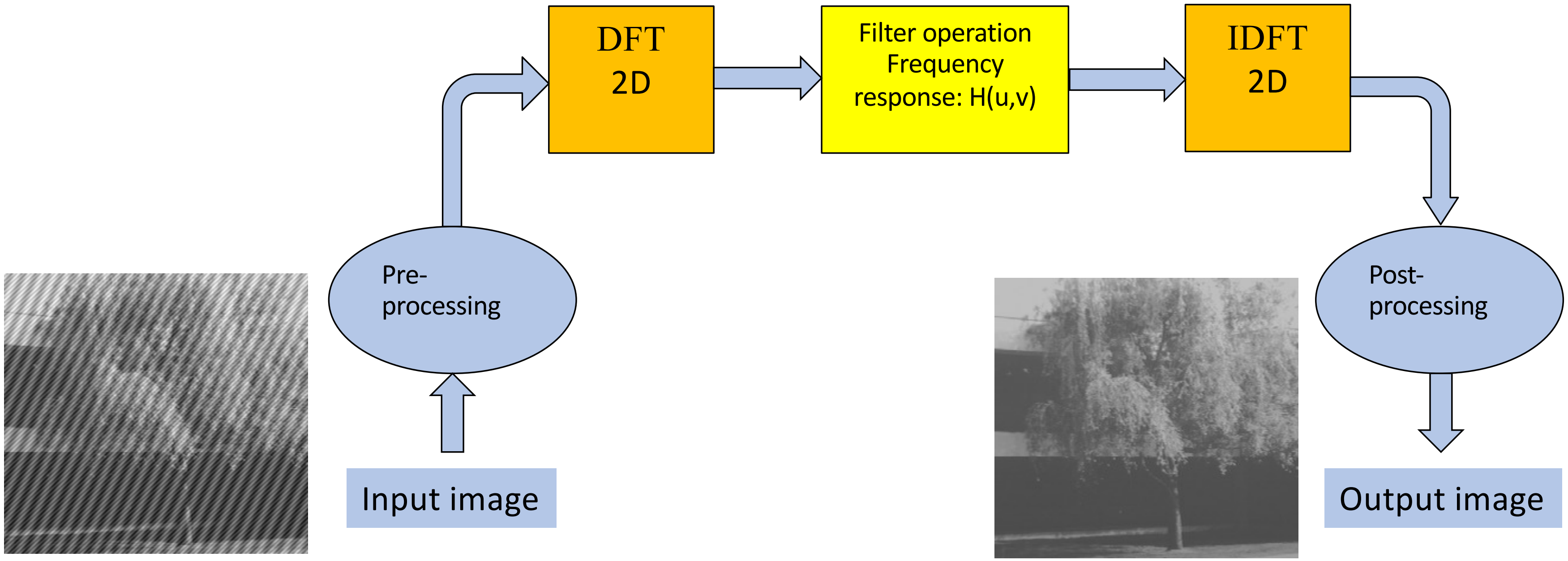
Filtering in the Frequency Domain

- Why do we filter?
  - Noise reduction (denoising)
  - Image enhancement
  - Effective representation
    - Compression
    - Feature extraction – classification
  - Etc..



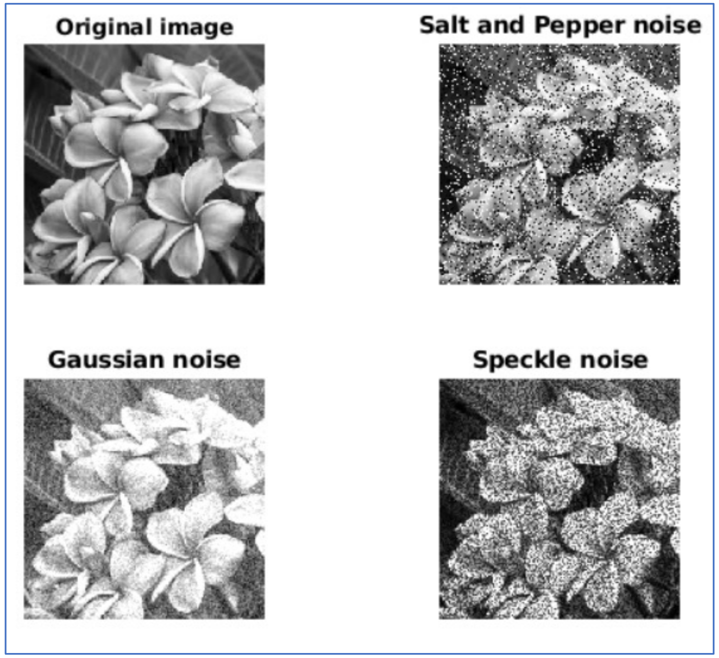
(6.4) Filtering in the Frequency Domain

By using the convolution theorem we can do linear filtering in the frequency domain. This is more computational efficient than spatial domain filtering when the filter kernel is larger than approximately (8x8) pixels.



Noise

- Additive $g(x, y) = f(x, y) + n(x, y)$
- Multiplicative $g(x, y) = f(x, y) \cdot n(x, y)$



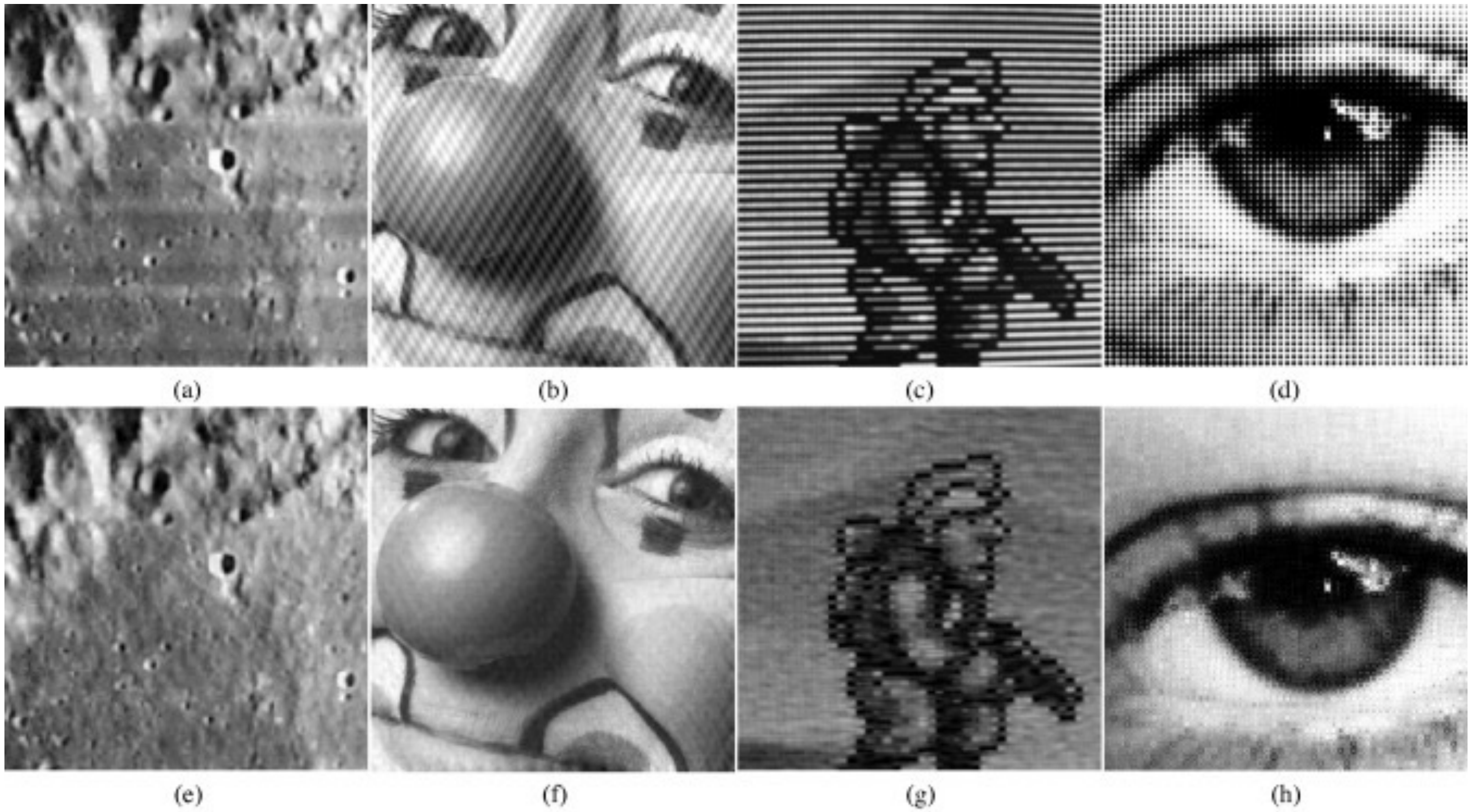
- **Gaussian**
  - Each pixel an additive noise component drawn from a Gaussian dist.
  - If homogeneous: $n(i, j) \in N(\mu(i, j), \sigma(i, j))$
    $\mu(i, j) = \mu \text{ and } \sigma(x, y) = \sigma \text{ for all } i, j$
- **Salt and pepper / impulse**
  - Some pixels have noise. These are white or black.
  - Probability for a pixel of being influenced by noise is often modelled as a Poisson distribution
- **Speckle noise** – multiplicative noise. Granular noise texture degrading the quality as a consequence of interference among wavefronts in coherent imaging systems, such as radar, synthetic aperture radar (SAR), medical ultrasound and optical coherence tomography.
- **Blur / motion blur –** more complicated to model

More on noise

- Homogeneous – noise model/paramteres are the same for all pixels.
- Zero-mean = unbiased noise
- White noise = spectrum = all frequencies has equal noise power. flat power
- Uncorrelated and zero-mean = white noise
  - Can be dependent even if it is uncorrelated
- iid – independent identically distributed (If the noise is iid and zero-mean it is white noise)
  - Independent noise: if the value of the noise at one pixel position is independent from the noise value at other pixel positions, the noise is independent.

Bandlimited noise

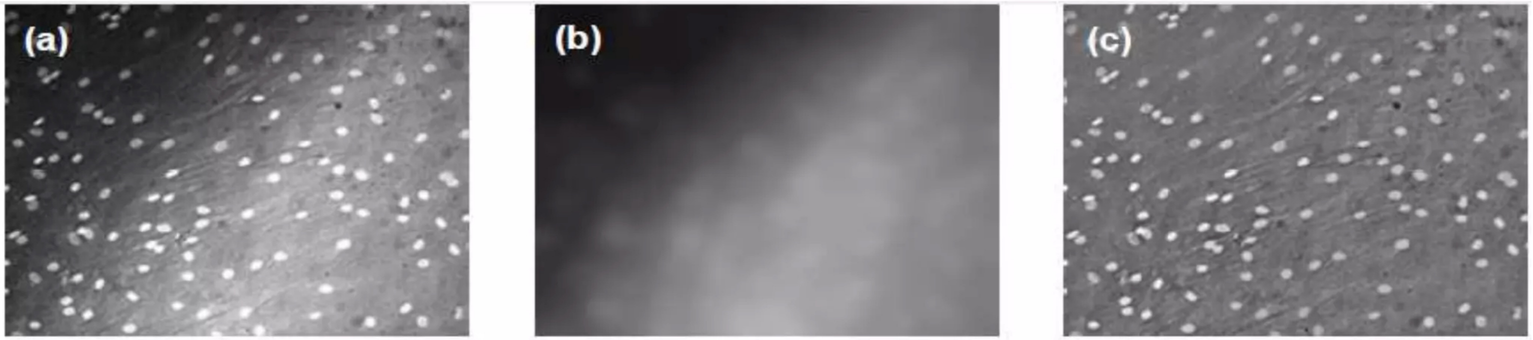
- Noise that apears localized in the frequency domain



J. Varghese, "Adaptive threshold based frequency domain filter for periodic noise reduction", 2016

Bandlimited noise

- Low frequency interference
  - arises for example due to uneven illumination



Frequency domain filtering – 1D

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

$$y(n) = \sum_k h(k)x(n-k)$$

$$F(y(n)) = Y(w) = H(w)X(w)$$

Linear filters : convolution in time (space) domain is mulitplicatioin in freq.domain. (1D and 2D)

Frequency domain filtering

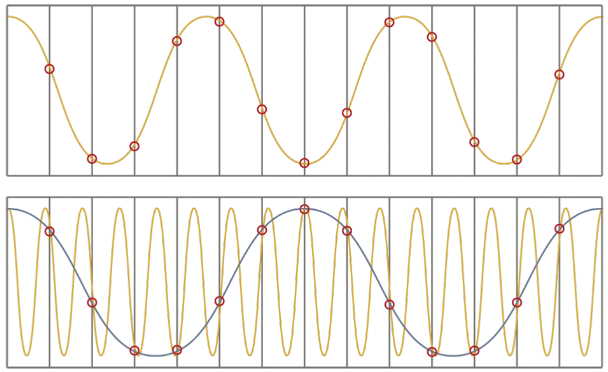
Digital images/signals –> digital frequency

Remember the aliasing discussion.

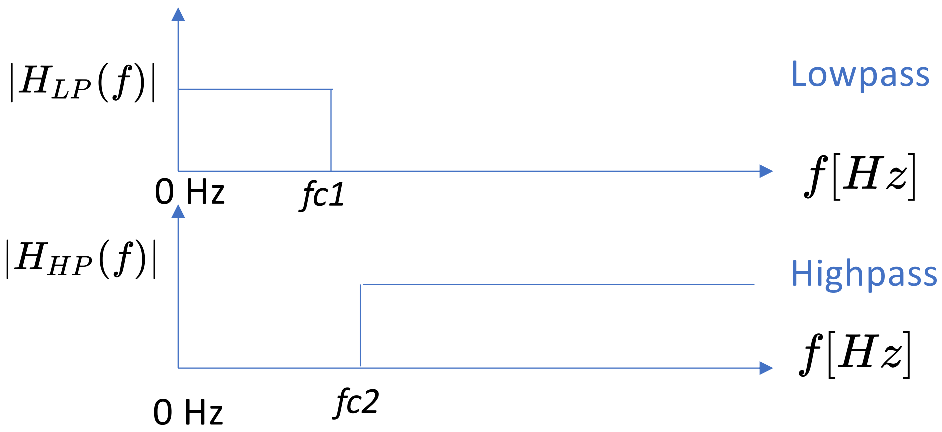
Can only differentiate between [0, Fs] where Fs is sampling freq.

Equivalent to: [-Fs/2, Fs/2] (periodic)

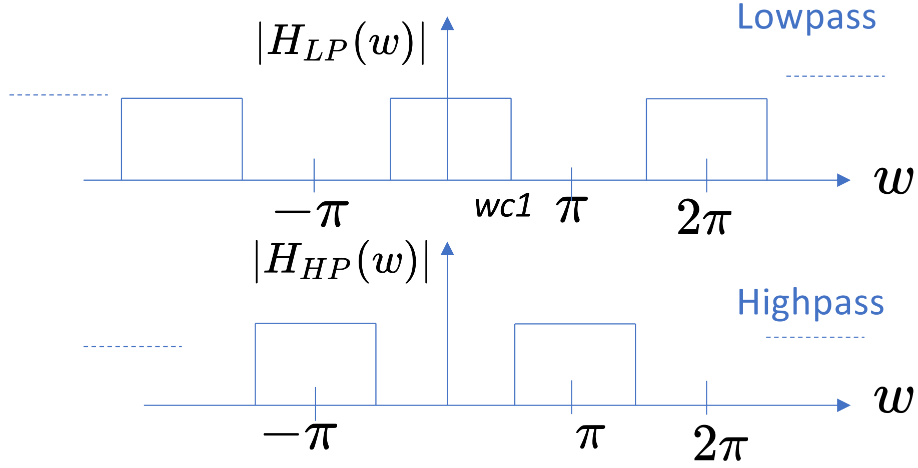
We often describe this as the digital angular frequency, ω : [0, 2π] or [-π, π].



Analog filters:



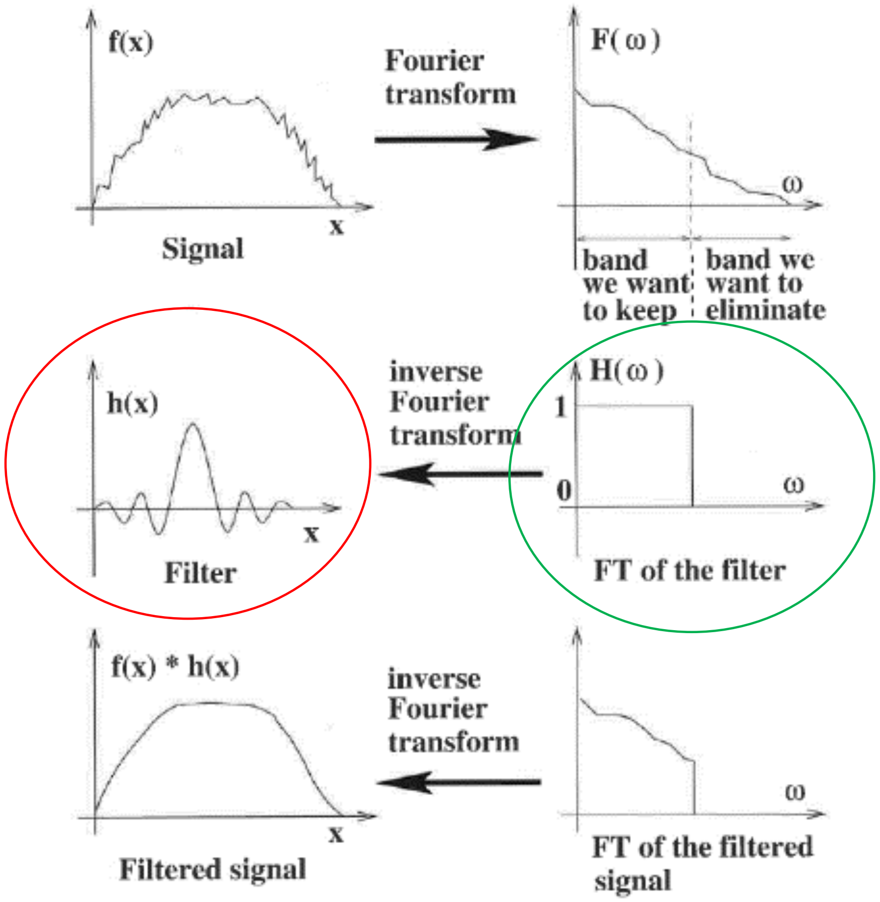
Digital filters:



Ideal filters

What does it mean in the space/time domain if we demand ideal filters in the frequency domain?

Ideal filter, frequency domain

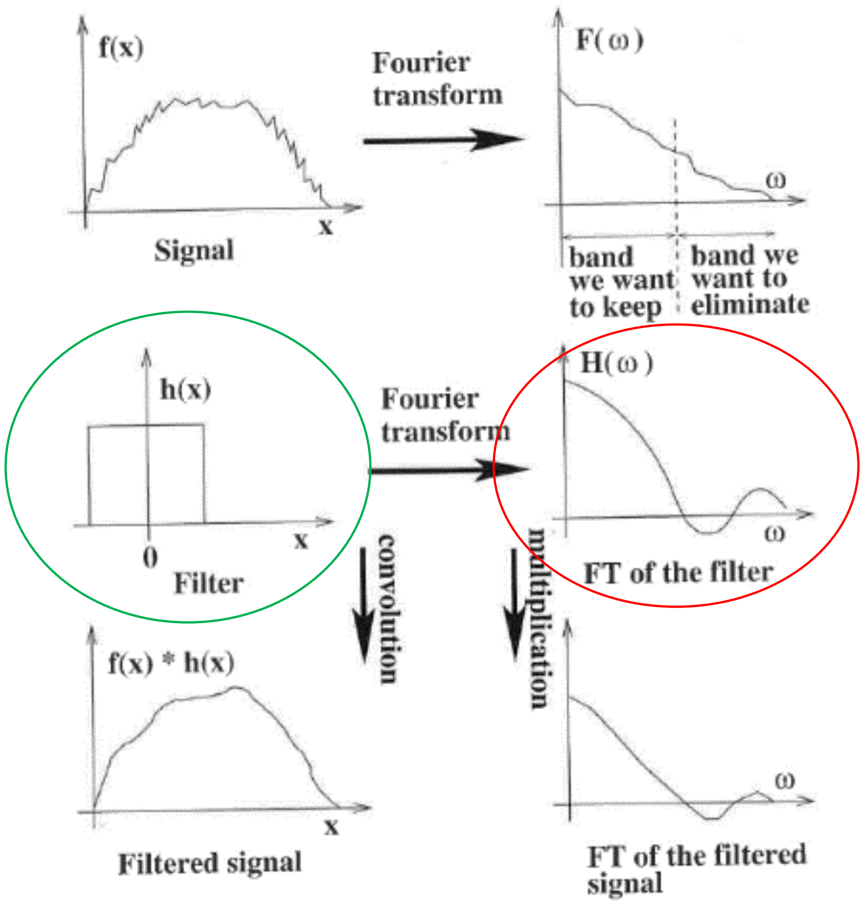


An ideal filter in the frequency domain corresponds to an infinite impuls respons (IIR).
for 2D: infinite point spread function

Simple filter in the space domain

What does a box-filter in space domain mean for the frequency domain?

Simple filter in time/space domain



An simple mean filter in the time/space domain corresponds to high sidelobes in the frequency domain

No filter can have finite extension in both spatial and frequency domain.

Any filter:

- infinite in space domain
- infinite in frequency domain
- or both.

What is a good trade off?

Fourier transform of a Gaussian i still a Gaussian.

For image processing Gaussian filters turns out to be a good balance, but not very steep cut-off.

Low pass filtering for noise reduction

White noise affects all frequencies.

Information of the image is often in low freq.

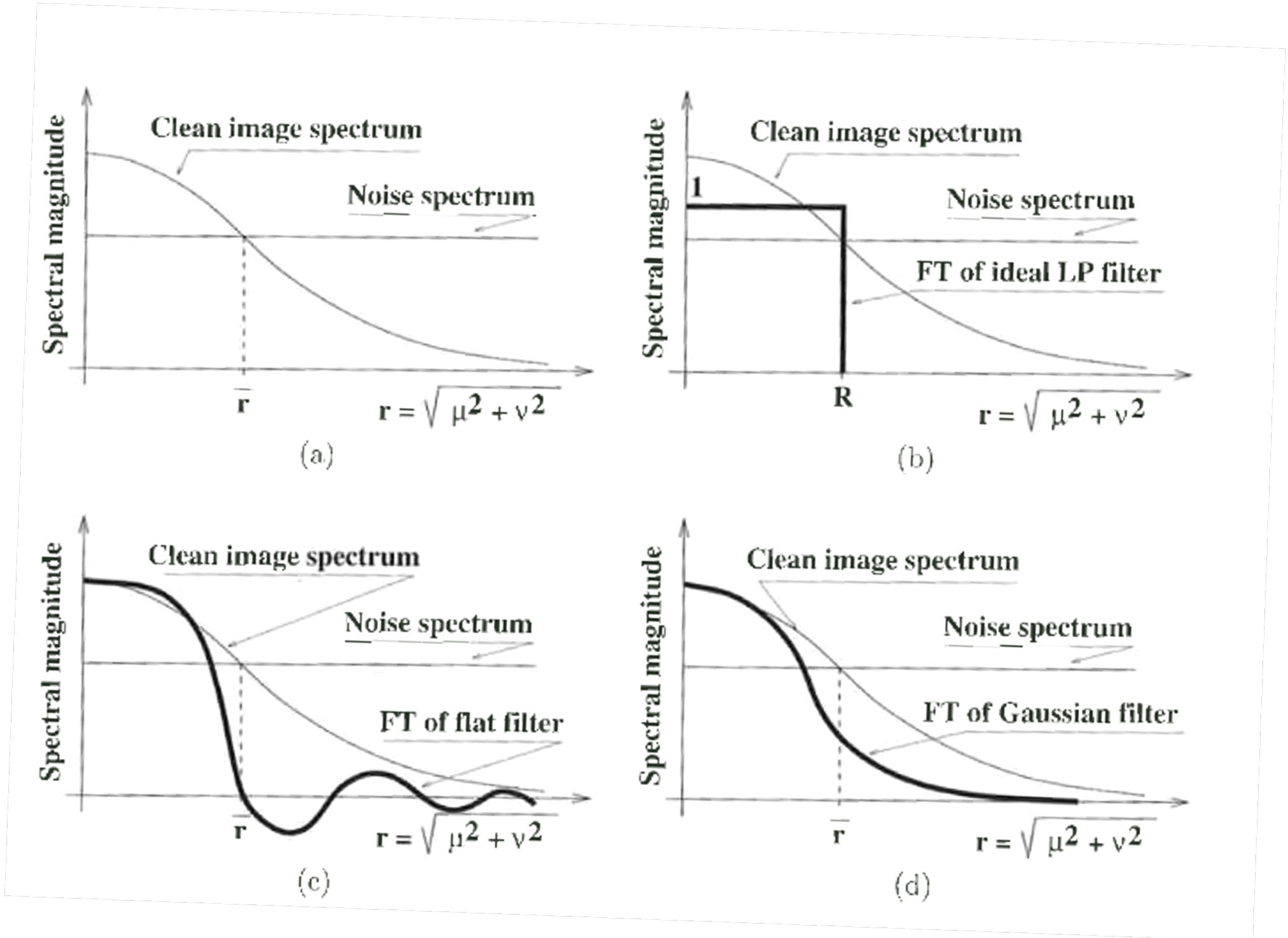
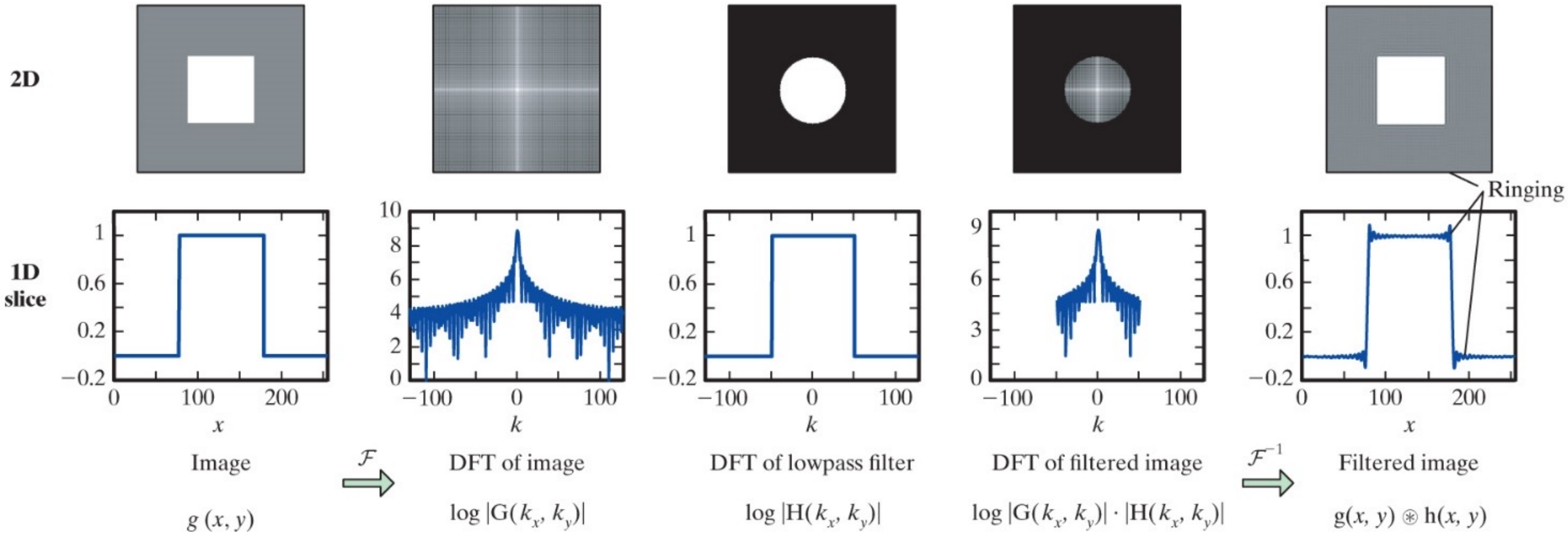
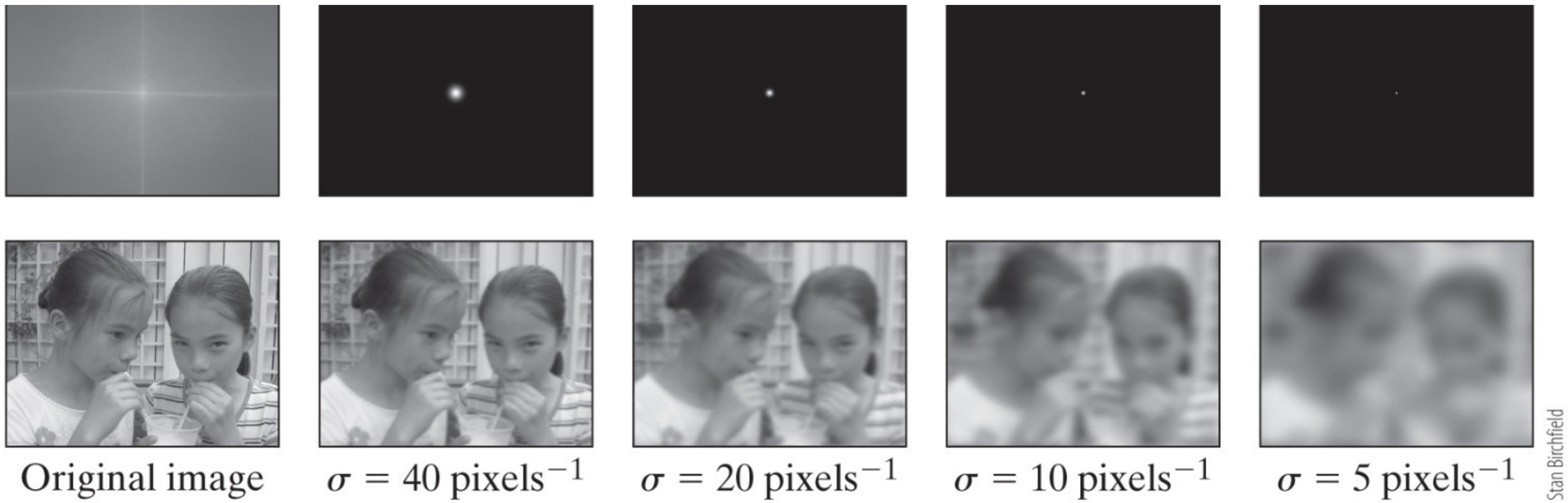


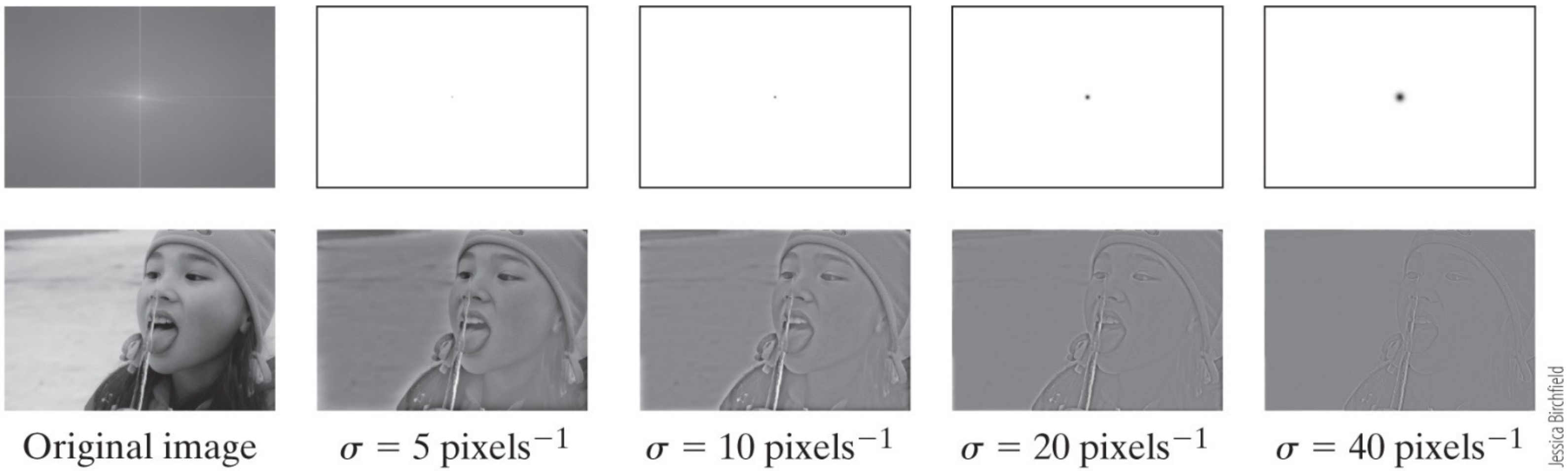
Figure 6.14 The process of frequency-domain filtering. From left to right: The DFT of the image is computed and multiplied by the frequency-domain filter, followed by the inverse DFT to yield the filtered image. Notice in this example that the ideal lowpass filter causes significant ringing in the output.





Above: Gaussian LP filtering in frequency domain, below: Gaussian HP filtering in frequency domain.

Top rows: DFT of image and filters, where dark values corresponds to attenuated frequencies.



$$|H_{HP}(w)| = 1 - |H_{LP}(w)|$$

Reducing low frequency interference and high pass filtering

- Convenient high pass filters with good properties in the frequency domain are various derivatives of the Gaussian function, truncated and discretized (as the Gaussian filter)

- Low frequency interference arises for example due to uneven illumination
- Uneven distance to the illumination source
  - A common problem indoor. Outdoor in natural light that is normally not a problem

- Shadows and ambient light
  - Problem both indoor and outdoor
  - Can also be mistaken for boundaries and edges

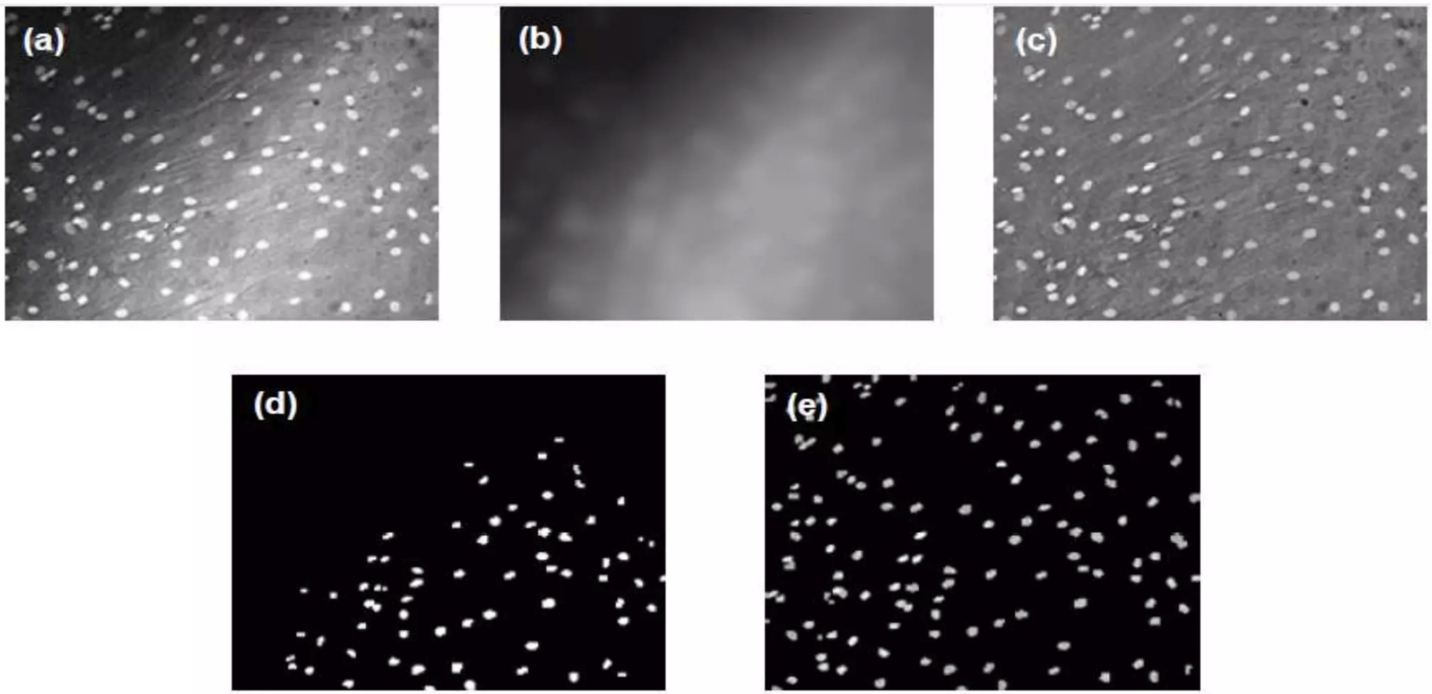
Uneven illumination

- Lowpass-filtering

- Unsharp masking
- Retinex
- Homomorphic filtering

- Flatfielding: Correct an image so that it behaves as it was captured under uniform illumination. If possible a reference image can be captured on a uniform colored pice of paper and used as a model of the illumination.

Illumination correction by lowpass filtering



Low-pass filter correction of an image of nuclei with uneven illumination. (a) A field of nuclei with uneven illumination; (b) Estimated background image generated from a low-pass filter on (a); (c) Correction of uneven illumination by subtracting (b) from (a); (d) Segmentation image by thresholding image (a), note that, there are a lot of nuclei missed from the dark background on the left-top corner of the image; (e) Segmentation image by thresholding image (c).

https://www.slideshare.net/tawosetimothy/image-segmentation-34430371

Unsharp masking / high boost filtering

Unsharp masking

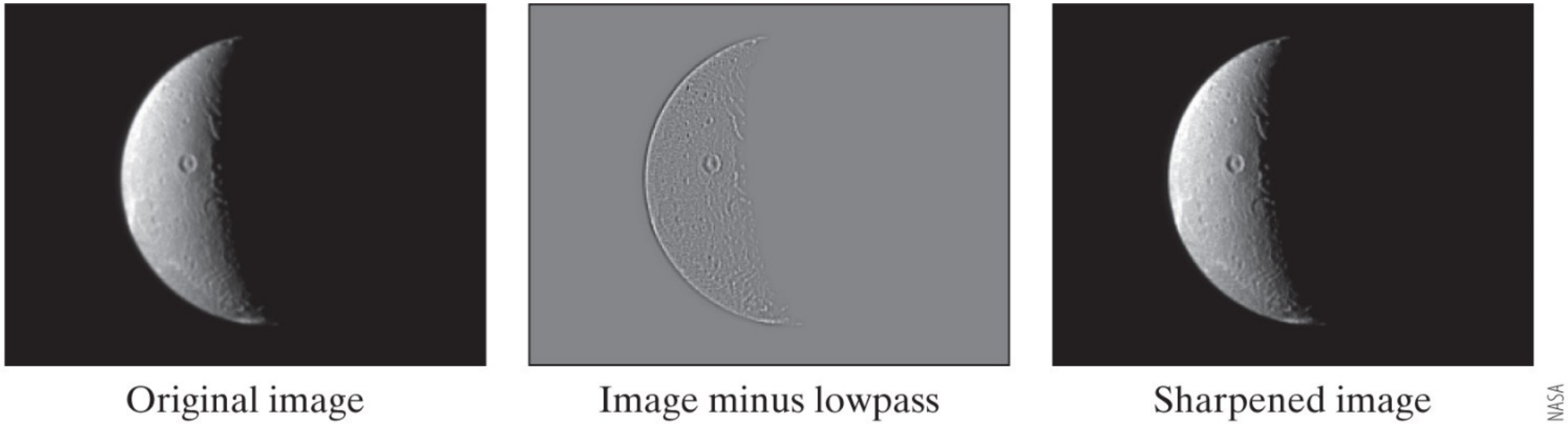
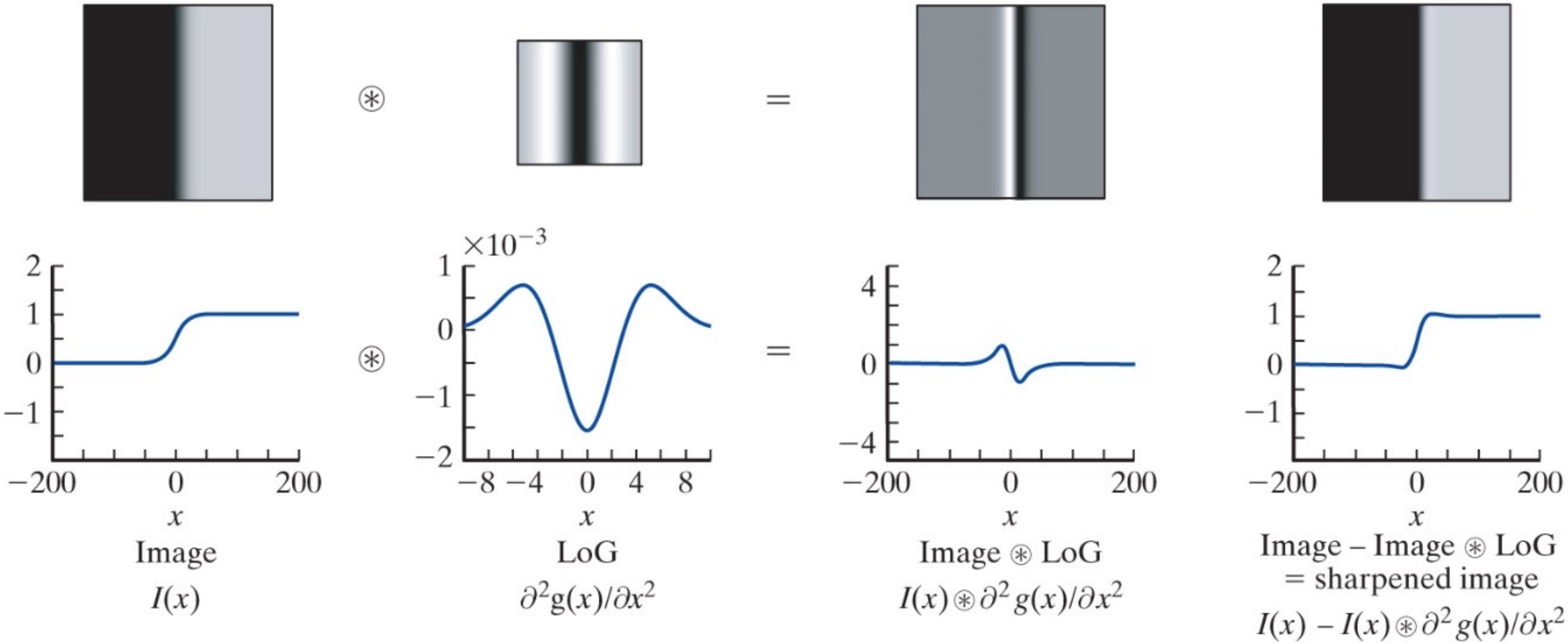


Figure 6.21 From left to right: An image of Saturn's moon Dione, the result of subtracting the low-pass filtered version of the image from itself, and the sharpened image resulting from adding this subtraction back to the original image.

Sharpening can be done in spatial or frequency domain. Spatial: subtract a blurred version of the image from the image, add back to image. Mathematically as seen on notes.

Figure 6.22 The process of image sharpening: The image is convolved with the LoG, and the result is subtracted from the original image. The edge in the right column appears sharper than that in the left column.



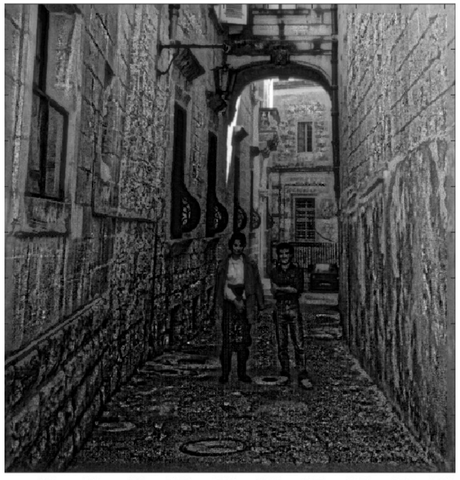
Retinex algorithm

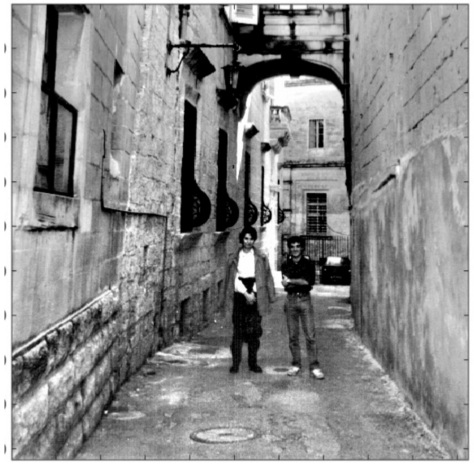
For enhancement and for removing uneven background illumination

Unsharp masking and Retinex alg.

Original

Unsharp – global





Unsharp – local

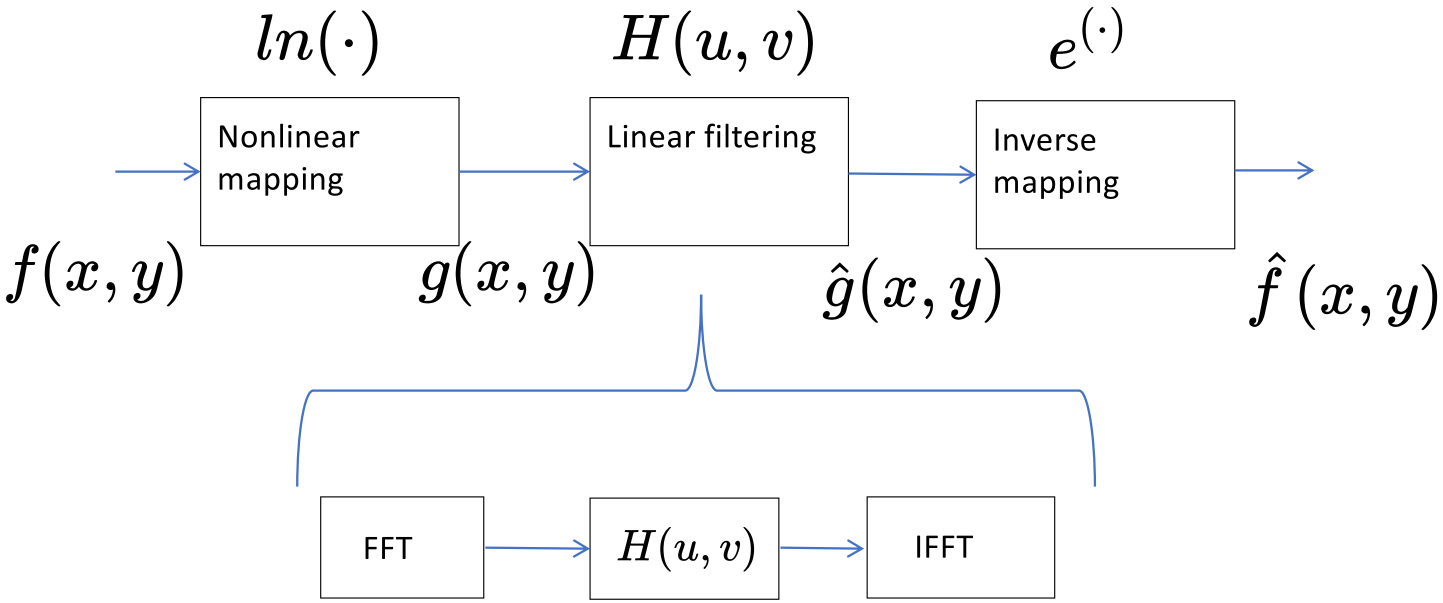
Retinex

Homomorphic filtering

- Homomorphic filtering is a technique used in signal and image processing:
  1. Use a non-linear maping to a different domain
  2. Thereafter use a linear filter
  3. Map back to the original domain

- Can be useful when signals are mixed by multiplication, multiplicative noise (speckle noise), or with uneven illumination.
- Logaritmic function makes multiplication become addition !

Homomorphic filtering



Homomorphic filtering – uneven illumination

$$f(x,y) = i(x,y) \cdot r(x,y)$$

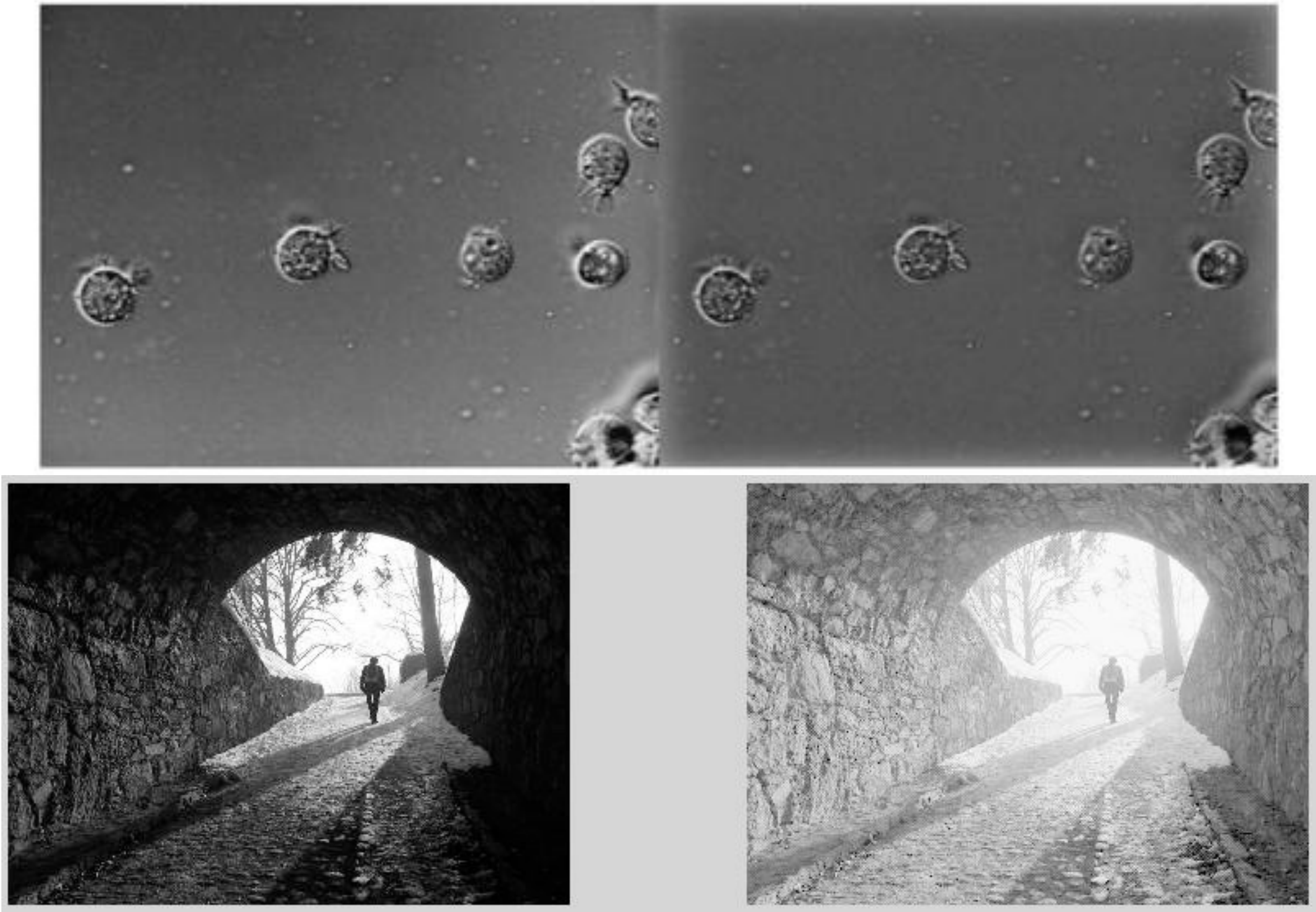
Illumination i(x,y) is generally of low frequency content.
Reflectance r(x,y) have more high frequency content due to edges between objects etc.

Make illumination and reflectance additive by logarithmic funciton, therafter high pass filter to remove uneven illumination, and transform back.

$$g(x,y) = \ln(f(x,y)) = \ln(i(x,y)) + \ln(r(x,y))$$

Original

After homomorphic filtering for removing uneven illumination



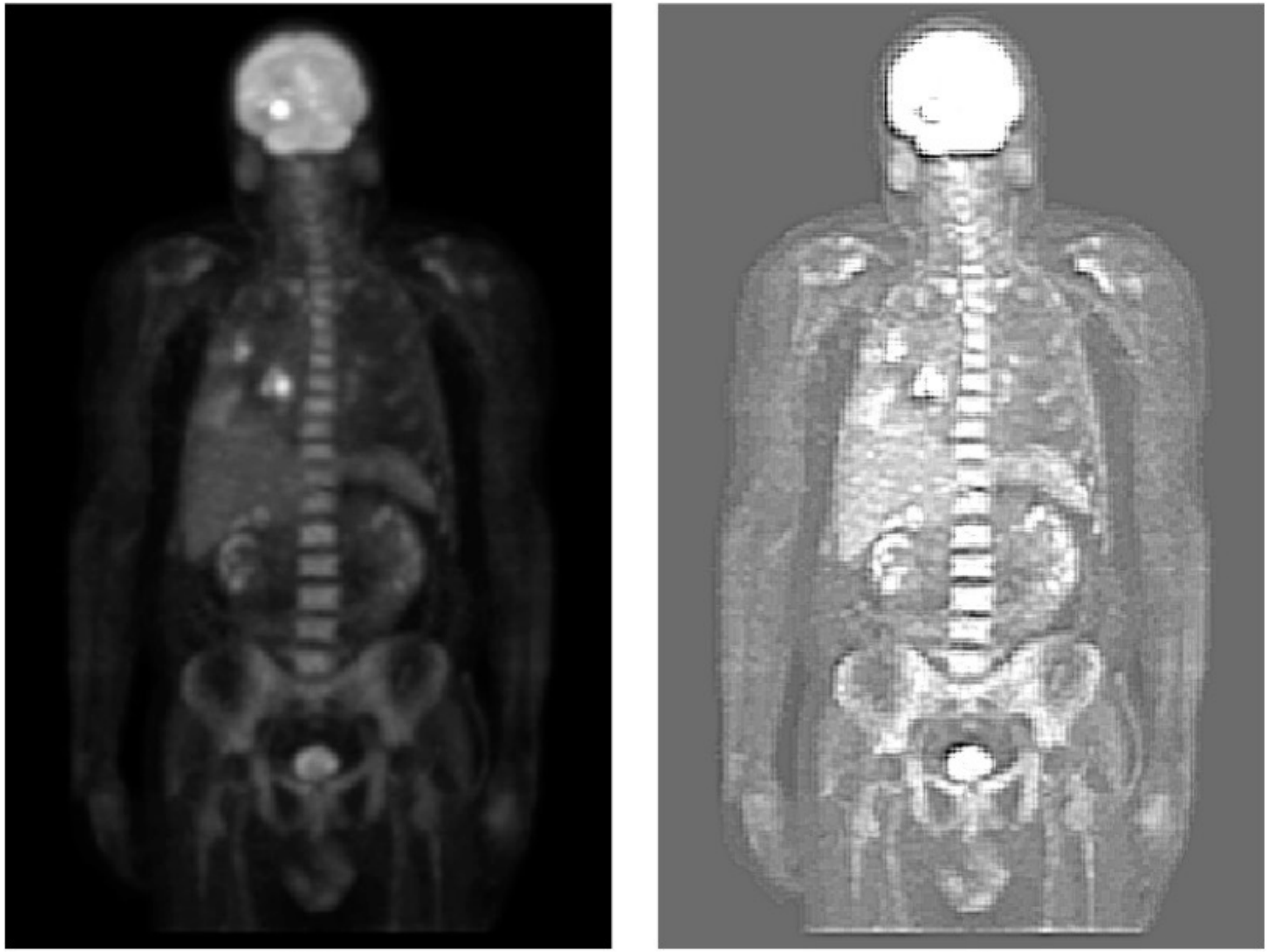
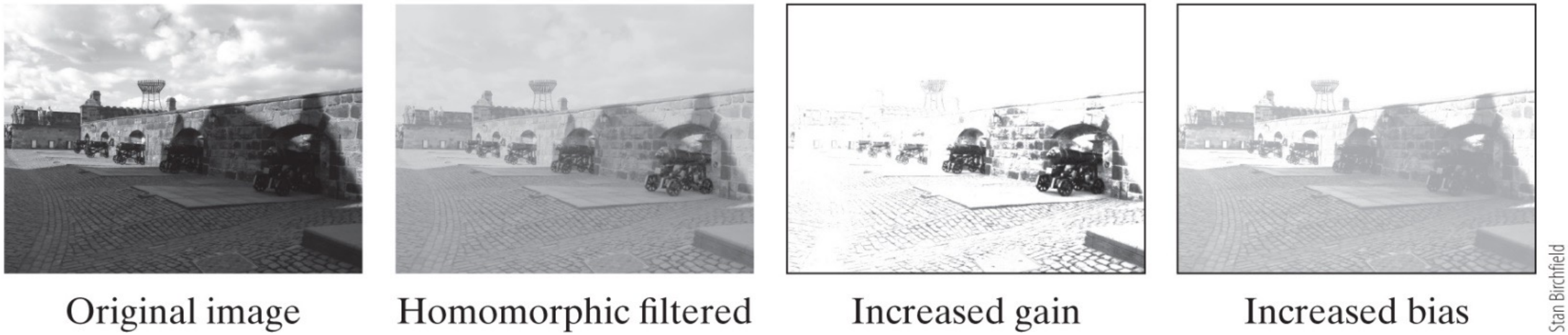


Figure 6.23 Left: An image with severe shadows, and the result of homomorphic filtering using a high-frequency filter to reduce the influence of lighting. Right: the result of multiplying the image by a constant and adding a constant to the image, for comparison. Note the ability of homomorphic filtering to reveal details in the shadow of the canon that are not visible in any of the other images.

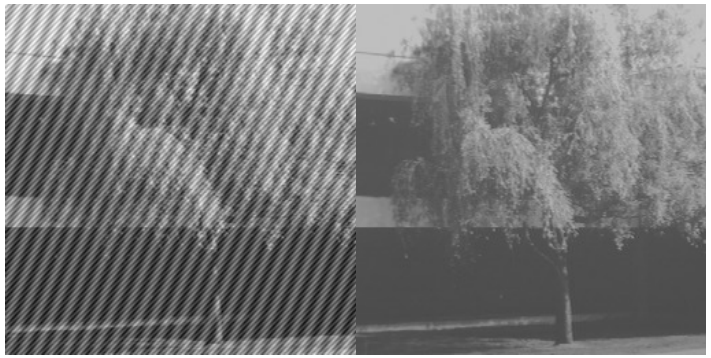


Filtering in the Frequency Domain

Three points from the topic:

1. How can we do the filtering in the frequency domain?
   - ✓ Convolution in spatial domain is multiplication in freq. domain. Multiply filter with image.

2. How can filtering infreq. domain help us understand what filtering in the spatial domain actually does to our data/images?
   - ✓ By studying both domains we see why box filters introduce ringing and artifact in the other domain



3. Which situations will filtering in freq. domain be an advantage over filtering in spatial domain?
   - ✓ Some examples: Large filter kernels (computationally better). Noise that are localised in frequency domain, hard/impossible in spatial domain but easy with freq.domain filters.