# Assignment 2

**LI XINYA (G2004358J)**

Q1.


$z_1 = w_1 \cdot x + b_1, h_1 = \partial(w_1 \cdot x + b_1)$
$z_2 = w \cdot h_1 + b, h_2 = \partial(w \cdot h_1 + b)$
$z_3 = w \cdot h_2 + b, h_3 = \partial(w \cdot h_2 + b)$
$z_4 = w \cdot h_3 + b, h_4 = \partial(w \cdot h_3 + b)$
$z_5 = w \cdot h_4 + b, y = \partial(w \cdot h_4 + b)$

$$\frac{\partial y}{\partial z_5} = \partial'(z_5)$$

$$\frac{\partial y}{\partial z_4} = \frac{\partial y}{\partial z_5} \cdot \frac{\partial z_5}{\partial h_4} \cdot \frac{\partial h_4}{\partial z_4} = w \cdot \partial'(z_5) \cdot \partial'(z_4)$$

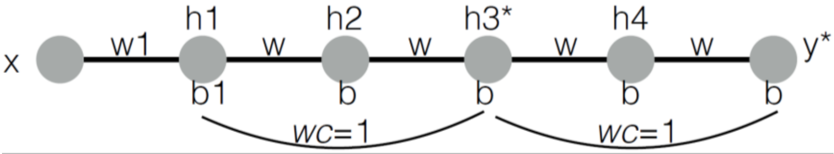
$$\frac{\partial y}{\partial z_3} = \frac{\partial y}{\partial z_4} \cdot \frac{\partial z_4}{\partial h_3} \cdot \frac{\partial h_3}{\partial z_3} = w^2 \cdot \partial'(z_5) \cdot \partial'(z_4) \cdot \partial'(z_3)$$

$$\frac{\partial y}{\partial z_2} = \frac{\partial y}{\partial z_3} \cdot \frac{\partial z_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial z_2} = w^3 \cdot \partial'(z_5) \cdot \partial'(z_4) \cdot \partial'(z_3) \cdot \partial'(z_2)$$

$$\frac{\partial y}{\partial z_1} = \frac{\partial y}{\partial z_2} \cdot \frac{\partial z_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_1} = w^4 \cdot \partial'(z_5) \cdot \partial'(z_4) \cdot \partial'(z_3) \cdot \partial'(z_2) \cdot \partial'(z_1)$$

$$\frac{\partial y}{\partial w_1} = \frac{\partial y}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1} = x \cdot w^4 \cdot \partial'(z_5) \cdot \partial'(z_4) \cdot \partial'(z_3) \cdot \partial'(z_2) \cdot \partial'(z_1)$$

$$\frac{\partial y}{\partial b_1} = \frac{\partial y}{\partial z_1} \cdot \frac{\partial z_1}{\partial b_1} = w^4 \cdot \partial'(z_5) \cdot \partial'(z_4) \cdot \partial'(z_3) \cdot \partial'(z_2) \cdot \partial'(z_1)$$

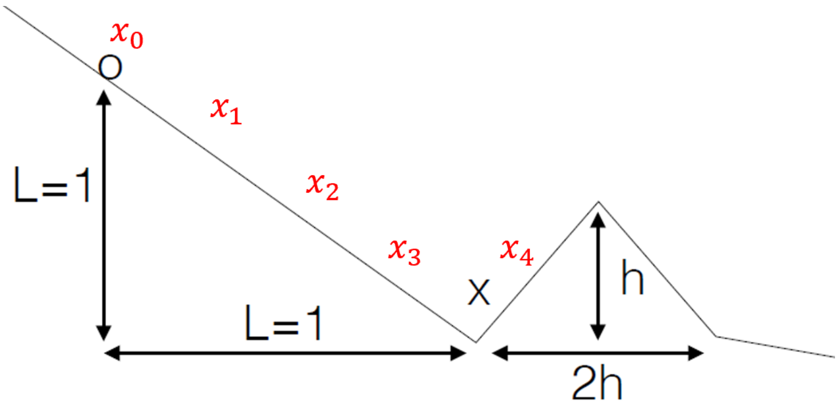



$z_1 = w_1 \cdot x + b_1, h_1 = \partial(w_1 \cdot x + b_1)$
$z_2 = w \cdot h_1 + b, h_2 = \partial(w \cdot h_1 + b)$
$z_3^* = w \cdot h_2 + h_1 + b, h_3^* = \partial(w \cdot h_2 + b)$
$z_4 = w \cdot h_3^* + b, h_4 = \partial(w \cdot h_3^* + b)$
$z_5 = w \cdot h_4 + h_3^* + b, y^* = \partial(w \cdot h_4 + h_3^* + b)$

$$\frac{\partial y^*}{\partial w_1} = \frac{\partial y^*}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1} = x \cdot w^4 \cdot \partial'(z_5) \cdot \partial'(z_4) \cdot \partial'(z_3^*) \cdot \partial'(z_2) \cdot \partial'(z_1) + x \cdot w^2 \cdot \partial'(z_5) \cdot \partial'(z_4) \cdot \partial'(z_3^*) \cdot \partial'(z_1) + x \cdot w^2 \cdot \partial'(z_5) \cdot \partial'(z_3^*) \cdot \partial'(z_2) \cdot \partial'(z_1) + x \cdot \partial'(z_5) \cdot \partial'(z_3^*) \cdot \partial'(z_1)$$

$$\frac{\partial y^*}{\partial b_1} = \frac{\partial y^*}{\partial z_1} \cdot \frac{\partial z_1}{\partial b_1} = w^4 \cdot \partial'(z_5) \cdot \partial'(z_4) \cdot \partial'(z_3^*) \cdot \partial'(z_2) \cdot \partial'(z_1) + w^2 \cdot \partial'(z_5) \cdot \partial'(z_4) \cdot \partial'(z_3^*) \cdot \partial'(z_1) + w^2 \cdot \partial'(z_5) \cdot \partial'(z_3^*) \cdot \partial'(z_2) \cdot \partial'(z_1) + \partial'(z_5) \cdot \partial'(z_3^*) \cdot \partial'(z_1)$$

Q2. (1) It will be stuck at point “x”.




$$\frac{\partial Loss(x)}{\partial x} = \begin{cases} -1\ (x < 1) \\ 1\ (1 < x < 1+h) \\ -1\ (1+h < x < 2h) \\ -0.3\ (x > 1+2h) \end{cases}$$

$$x_0 = 0$$
$$x_1 = 0 - 0.3 \times (-1) = 0.3$$
$$x_2 = 0.3 - 0.3 \times (-1) = 0.6$$
$$x_3 = 0.6 - 0.3 \times (-1) = 0.9$$
$$x_4 = 0.9 - 0.3 \times (-1) = 1.2$$
$$x_5 = 1.2 - 0.3 \times (1) = 0.9$$

“Stuck”

(2) The max height ‘h’ is approximately “0.41019”.

```
def gradient(w,h):
    if w<1:
        return -1
    elif 1<=w<(1+h):
        return 1
    else:
        return -1
num_steps=100000
for h in np.arange(0.31,1,0.0001):
    w=0
    moment1=0
    moment2=0
    beta1=0.9
    beta2=0.999
    lr=0.3
    for t in range(1,num_steps):
        beta1=0.9
        beta2=0.999
        lr=0.3
        dw=gradient(w,h)
        moment1=beta1*moment1+(1-beta1)*dw
        moment2=beta2*moment2+(1-beta2)*dw*dw
        moment1_unbias=moment1/(1-beta1**t)
        moment2_unbias=moment2/(1-beta2**t)
        w-= lr*moment1_unbias/(np.sqrt(moment2_unbias))
        if w>(1+h):
            break
    if w<(1+h):
        print(h)
        break
```

```
0.41019999999998896
```

Q3. According to the “comp_tree.py” file, the relationship between $x_1$, $x_2$, $x_3$ and $x_4$ is shown as below, the meaning of all the nodes is marked in the plot:

$$x_1 = bx_0 + c$$

$$x_2 = x0 + x_1f$$

$$x_3 = (x_0\text{-}ex_1)^a + \sin(dx^2)$$

$$x_4 = x_3x_2 + dx_1$$