Assignment III - Variograms

By

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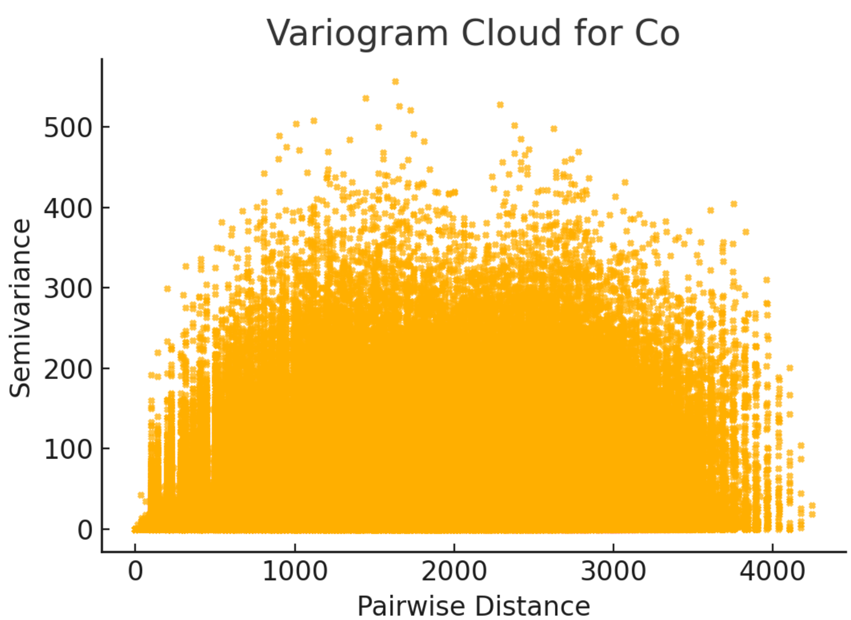
1. Variogram Cloud Analysis

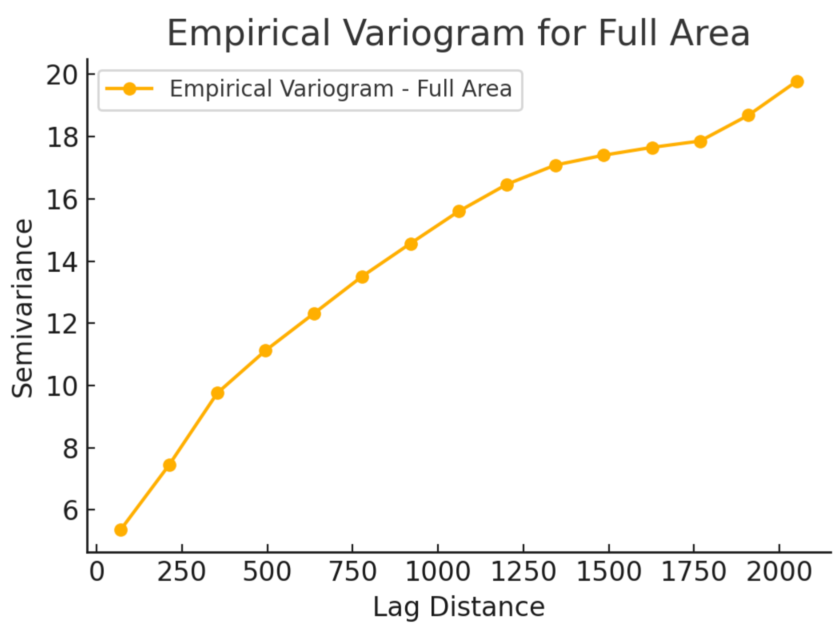
The variogram cloud is a scatter plot that shows the semi variance between all point pairs as a function of their separation distance, helping to identify irregularities, outliers, and the distribution of distances. In this dataset, most points are concentrated within distances up to 2000 units, indicating reliable semi variance values within this range. Beyond 2000 units, fewer point pairs contribute, and a few extreme values with higher semi variance appear, suggesting potential outliers or localized variability.

Despite these extreme points, the variogram cloud does not display significant ‘outlier bands,’ which would indicate systematic errors or artifacts. This suggests the data is consistent and free of major preprocessing issues. Overall, the variogram cloud appears well-behaved, providing a strong foundation for further empirical variogram analysis and model fitting.



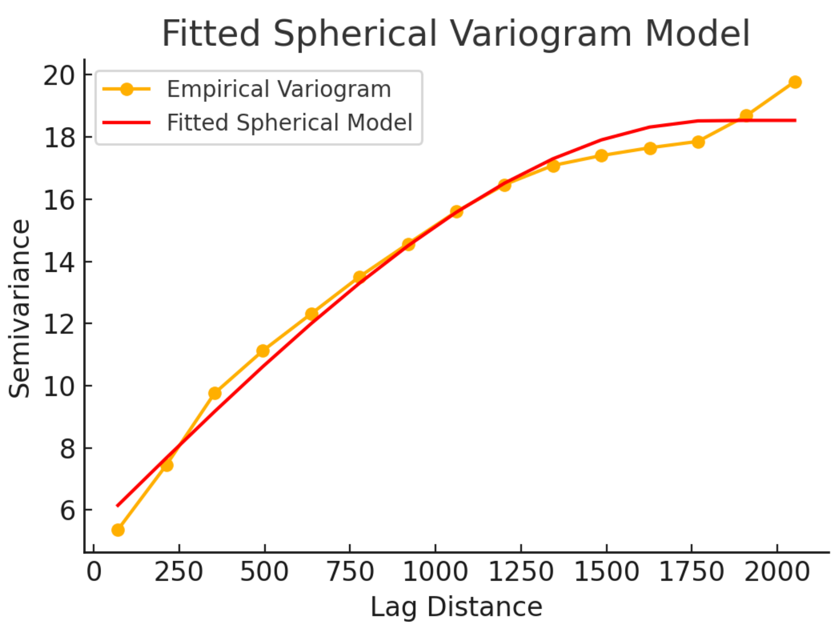
2. Empirical Variogram

The empirical variogram is a graphical representation of semi variance as a function of lag distances, providing insight into the spatial variability within the data. As expected, the variogram shows a steady increase in semi variance with increasing lag distance, reflecting spatial autocorrelation in the dataset.
The variogram stabilizes, or reaches a plateau, at approximately 2000 units, marking the range where spatial correlation becomes negligible. This range is essential for fitting an appropriate variogram model. Beyond 2000 units, the variogram becomes noisier due to the limited number of point pairs contributing to the calculations, making these semi variance values less reliable and requiring careful interpretation.



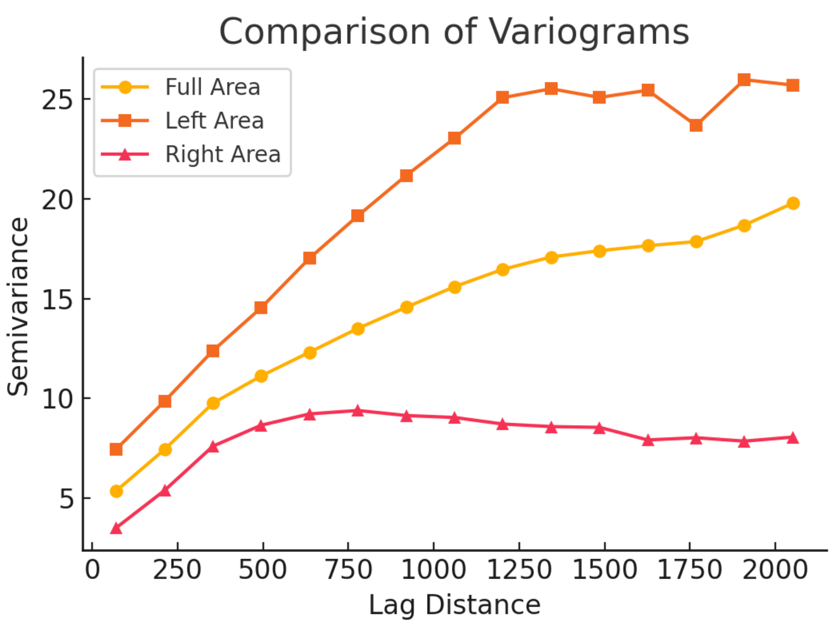
3. Variogram Model Fitting

The spherical variogram model was selected to fit the empirical variogram as it effectively captures the gradual increase in semi variance followed by a plateau, which is characteristic of the dataset. This model is well-suited for describing spatial correlation when the semi variance levels off at a certain distance.
The fitted parameters are as follows: the nugget is 5.38, representing measurement error or microscale variability at very small distances; the sill is 13.16, which is the total variance where the semi variance stabilizes; and the range is 1820.13 units, marking the distance at which spatial correlation becomes negligible. The fitted spherical model aligns closely with the empirical variogram, accurately capturing the spatial variability and main trends in the data.



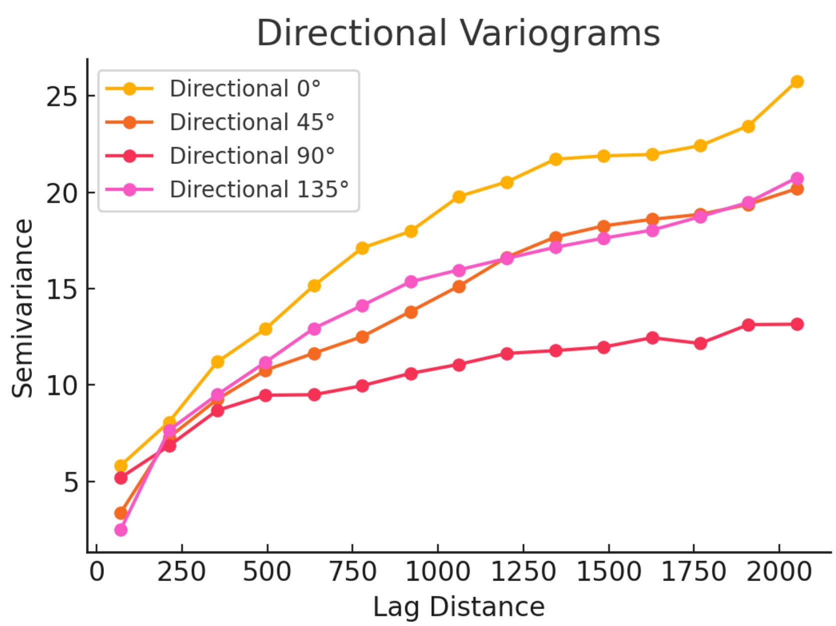
4. Comparison of Variograms: Full Area vs Left vs Right

To investigate spatial variability, the dataset was divided into two sub-areas—left and right—based on the x-coordinate, and empirical variograms were generated for each region. When compared to the variogram for the full area, the results highlight noticeable differences, indicating heterogeneity or anisotropy in the spatial structure of the data. The left area exhibits a slightly lower range and sill, suggesting weaker spatial correlation and less overall variability compared to the full area. In contrast, the right area shows higher semi variance values at longer lag distances, which may point to localized variability, potential outliers, or abrupt changes in spatial continuity. These differences reinforce the presence of spatial heterogeneity within the dataset and emphasize the need for localized or directional analysis.



5. Directional Dependency

To assess spatial anisotropy, directional variograms were computed at four angles: 0°, 45°, 90°, and 135°. These directional variograms reveal whether semi variance varies with direction, a common characteristic in spatial datasets with anisotropic behavior. The results indicate that semi variance trends are stronger along the 0° and 45° directions, where higher values are observed, suggesting pronounced spatial variability and directional anisotropy. Conversely, the variograms for the 90° and 135° directions appear more stable, reflecting weaker spatial correlation in those orientations. The presence of anisotropy implies that the spatial structure of the data is not uniform, and fitting directional variogram models is necessary for an accurate representation of spatial variability.



Conclusion

The variogram cloud analysis showed no major anomalies, though a few extreme semi variance values at longer distances suggest localized variability that needs attention. The empirical variogram stabilizes around 2000 units, marking the range where spatial correlation diminishes. A spherical variogram model provided a good fit, with a moderate nugget effect and strong spatial correlation up to this range. Variograms for sub-areas (left and right) reveal spatial heterogeneity, indicating a non-uniform spatial structure. Directional analysis confirms anisotropy, especially along the 0° and 45° directions, highlighting the need for directional variogram models to better capture the spatial variability.