











# A Decade of the Gdańsk School of Cellular Automata

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## Abstract

The Gdańsk School of Cellular Automata (GSCA) has, over the last decade, developed a coherent program that bridges theory with exploratory computation to study various aspects of Cellular Automata. In this paper, we review the key results of the group on (i) identifying cellular automata from partial, noisy observations, (ii) affine continuous cellular automata and their links to stochastic and deterministic rules, (iii) density classification, (iv) parity classification, and (v) number-conserving cellular automata across uniform, non-uniform, and non-square grids, including reversibility and split-and-perturb decompositions. This survey is written to honor the memory of prof. Danuta Makowiec who directly motivated the creation of GSCA and influenced the research methodology used by the group.

**Keywords:** cellular automata, affine continuous cellular automata, system identification, density classification problem, parity problem, number-conserving cellular automata, reversibility, non-uniform cellular automata

# 1 Introduction

The scientific contributions of prof. Danuta Makowiec to the theory and applications of Cellular Automata (CAs) are widely recognized through her numerous research projects, publications, and conference contributions. Beyond her outstanding academic achievements, prof. Makowiec was also an exceptional teacher and mentor, deeply respected by her students and collaborators alike. She delivered lectures and conducted computer laboratories within courses on “Cellular Automata” and “Complex Systems”, inspiring generations of students through her intellectual passion and personal dedication.

A distinguishing feature of her academic activity was a teaching philosophy rooted in the harmonious integration of rigorous mathematical reasoning with computer simulation and *experimentation*. While such a methodology is well established in the physical sciences, it represented an innovative and formative experience for young mathematicians and computer scientists attending her courses. For many of them, this approach not only shaped their understanding of complex systems, but also decisively influenced their scientific identity.

It was this unique educational atmosphere—and, above all, the research paradigm uniting theory with experimentation—that motivated one of prof. Makowiec’s former computer science students, Witold Bołt, and a theoretical mathematician, Barbara Wolnik, to pursue systematic investigations into the properties of CAs. This partnership gave rise to the Gdańsk School of Cellular Automata (GSCA). The research carried out within GSCA focuses primarily on theoretical aspects of CAs, with particular emphasis on number-conserving rules, global computational capabilities (notably density and parity classification), and the problem of rule identification. Over the past decade, members of GSCA have published more than 30 peer-reviewed articles and presented their results at more than 20 international conferences and workshops. Moreover, as a direct outcome of research conducted within the group, two members defended their master’s theses (Aleksander Bołt and Aleksander Wardyn), three members completed their PhD dissertations (Adam Dzedzej, Witold Bołt, and Marcin Dembowski), and finally, Barbara Wolnik obtained her habilitation.

This article surveys and synthesizes the results obtained by GSCA. Although prof. Makowiec was not directly involved in the work reported here, her profound and lasting influence on the group—as a mentor, colleague, and source of inspiration—permeates the research presented in this paper. In this sense, the work discussed in this article stands as a natural continuation of her scientific legacy.

The primary aim of this article is to honor the memory of prof. Danuta Makowiec and to express our deepest gratitude for her inspiration, mentorship, and enduring influence. We dedicate this work to her with respect, admiration, and heartfelt remembrance.

# 2 Identification of CAs

We built an identification pipeline that recovers CA rules from partial or noisy space–time traces. Evolutionary search handles bounded time gaps in incomplete

observations [1, 2], while frequency-based estimators address  $\alpha$ -asynchronous and diploid rules and can impute missing cells [3–5].

### 3 Affine Continuous CAs

Affine continuous CAs (ACCAs) can be expressed as convex combinations of classical deterministic rules, which links them conceptually to stochastic mixtures and clarifies how continuous dynamics relate to stochastic CA behavior [6]. Legal outer-totalistic ACCAs exhibit rich dynamics and multiple modes of sensitivity to parameters, grid size, and single-site perturbations [7]. These properties underpin later constructions for density and parity classification tasks.

### 4 Density Classification Problem

Our work began with the relaxed  $\alpha$ -DCT formulation for continuous CAs [8]. Finite-size ACCAs solve fixed-length instances [9], and most density-conserving ACCAs of radius one succeed on relaxed variants [10–12]. Together these results frame ACCAs as a viable alternative to classical binary rules for density classification.

### 5 Parity Problem

We closed long-standing gaps on the minimal neighborhood for parity classification: no one-dimensional CA with a six-cell neighborhood solves parity [13]. A proof-backed fix of the BFO construction reinstates a single-rule solution, confirming that parity is solvable by one CA rule [14].

### 6 Number-Conserving CAs

This line spans characterization, decomposition, reversibility, and non-uniform settings. Survey chapters summarize the landscape and open problems [15, 16]. Geometric conditions characterize von Neumann rules and show binary number-conserving CAs are intrinsically one-dimensional [17, 18]. Split-and-perturb methods yield complete lists and reversible families, including four-state 2D and higher-state cases [19–24]. Rotation-symmetric exploration uncovers a seven-state example beyond septenary isomorphism and establishes dimensional thresholds [25, 26]. Extensions to triangular grids provide decompositions and enumerations [27], while non-uniform elementary rules are fully characterized on finite and infinite grids [28, 29]. Reversibility on triangular and hexagonal grids is settled via constructive counterexamples and complete tests [30–32].

## References

- [1] Bolt, W., Baetens, J.M., De Baets, B.: An evolutionary approach to the identification of cellular automata based on partial observations. In: 2015 IEEE Congress on Evolutionary Computation (CEC), pp. 2966–2972 (2015). <https://doi.org/10.1109/CEC.2015.7257258>

- [2] Bolt, W., Baetens, J.M., De Baets, B.: Identification of cellular automata based on incomplete observations with bounded time gaps. *IEEE Transactions on Cybernetics* **50**(3), 971–984 (2020) <https://doi.org/10.1109/TCYB.2018.2875266>
- [3] Bolt, W., Wolnik, B., Baetens, J.M., De Baets, B.: On the identification of  $\alpha$ -asynchronous cellular automata in the case of partial observations with spatially separated gaps. In: *Studies in Computational Intelligence*, pp. 23–36 (2016). [https://doi.org/10.1007/978-3-319-30165-5\\_2](https://doi.org/10.1007/978-3-319-30165-5_2)
- [4] Bolt, W., Bolt, A., Wolnik, B., Baetens, J.M., De Baets, B.: A statistical approach to the identification of diploid cellular automata. In: *Lecture Notes in Computer Science*, pp. 37–48 (2017). [https://doi.org/10.1007/978-3-319-71069-3\\_3](https://doi.org/10.1007/978-3-319-71069-3_3)
- [5] Bolt, W., Bolt, A., Wolnik, B., Baetens, J.M., De Baets, B.: A statistical approach to the identification of diploid cellular automata based on incomplete observations. *Biosystems* **186**, 103976 (2019) <https://doi.org/10.1016/j.biosystems.2019.103976>
- [6] Bolt, W., Baetens, J.M., De Baets, B.: On the decomposition of stochastic cellular automata. *Journal of Computational Science* **11**, 245–257 (2015) <https://doi.org/10.1016/j.jocs.2015.09.004>
- [7] Wolnik, B., Dembowski, M., Augustynowicz, A., De Baets, B.: A complete description of the dynamics of legal outer-totalistic affine continuous cellular automata. *Nonlinear Dynamics* **110**(1), 589–610 (2022) <https://doi.org/10.1007/s11071-022-07642-w>
- [8] Bolt, W.: In: Myśliński, A. (ed.) *Solving the density classification task with the use of the continuous cellular automata. Techniki informacyjne: teoria i zastosowania – wybrane problemy*, vol. 4. IBS PAN, Warsaw, Poland (2014)
- [9] Dembowski, M., Wolnik, B., Bolt, W., Baetens, J.M., De Baets, B.: Affine continuous cellular automata solving the fixed-length density classification problem. *Natural Computing* **17**(3), 467–477 (2017) <https://doi.org/10.1007/s11047-017-9631-4>
- [10] Wolnik, B., Dembowski, M., Bolt, W., Baetens, J.M., De Baets, B.: Density-conserving affine continuous cellular automata solving the relaxed density classification problem. *Journal of Physics A: Mathematical and Theoretical* **50**(34), 345103 (2017) <https://doi.org/10.1088/1751-8121/aa7d86>
- [11] Wolnik, B., Dembowski, M., Bolt, W., Baetens, J.M., De Baets, B.: The density classification problem in the context of continuous cellular automata. In: *Lecture Notes in Computer Science*, pp. 79–87 (2016). [https://doi.org/10.1007/978-3-319-44365-2\\_8](https://doi.org/10.1007/978-3-319-44365-2_8)

- [12] Dembowski, M., Wolnik, B., Bolt, W.T., Baetens, J., De Baets, B.: Two-dimensional affine continuous cellular automata solving the relaxed density classification problem. *Journal of Cellular Automata* **14**(3-4), 191–212 (2019)
- [13] Nenca, A., Wolnik, B., De Baets, B.: No six-cell neighborhood cellular automaton solves the parity problem. *Theoretical Computer Science* **1021**, 114923 (2024) <https://doi.org/10.1016/j.tcs.2024.114923>
- [14] Wolnik, B., Nenca, A., Balbi, P.P., Baets, B.D.: Cellular automata can really solve the parity problem (2025). <https://arxiv.org/abs/2501.08684>
- [15] Wolnik, B., Bolt, W., De Baets, B.: In search of cellular automata that preserve numbers: Advances, limitations, and open problems. In: *Lecture Notes in Computer Science*, pp. 43–58 (2025). [https://doi.org/10.1007/978-3-032-01570-9\\_4](https://doi.org/10.1007/978-3-032-01570-9_4)
- [16] Wolnik, B., Bolt, W., De Baets, B.: Recent insights into number-conserving cellular automata. In: *Emergence, Complexity and Computation*, pp. 399–422 (2025). [https://doi.org/10.1007/978-3-031-78757-7\\_14](https://doi.org/10.1007/978-3-031-78757-7_14)
- [17] Wolnik, B., Dzedzej, A., Baetens, J.M., De Baets, B.: Number-conserving cellular automata with a von neumann neighborhood of range one. *Journal of Physics A: Mathematical and Theoretical* **50**(43), 435101 (2017) <https://doi.org/10.1088/1751-8121/aa89cf>
- [18] Wolnik, B., De Baets, B.: All binary number-conserving cellular automata based on adjacent cells are intrinsically one-dimensional. *Physical Review E* **100**(2) (2019) <https://doi.org/10.1103/PhysRevE.100.022126>
- [19] Dzedzej, A., Wolnik, B., Dziemiańczuk, M., Nenca, A., Baetens, J.M., De Baets, B.: A two-layer representation of four-state reversible number-conserving 2d cellular automata. *Journal of Statistical Mechanics: Theory and Experiment* **2019**(7), 073202 (2019) <https://doi.org/10.1088/1742-5468/ab25df>
- [20] Wolnik, B., Nenca, A., Baetens, J.M., De Baets, B.: A split-and-perturb decomposition of number-conserving cellular automata. *Physica D: Nonlinear Phenomena* **413**, 132645 (2020) <https://doi.org/10.1016/j.physd.2020.132645>
- [21] Dzedzej, A., Wolnik, B., Nenca, A., Baetens, J.M., De Baets, B.: Efficient enumeration of three-state two-dimensional number-conserving cellular automata. *Information and Computation* **274**, 104534 (2020) <https://doi.org/10.1016/j.ic.2020.104534>
- [22] Wolnik, B., De Baets, B.: Ternary reversible number-conserving cellular automata are trivial. *Information Sciences* **513**, 180–189 (2020) <https://doi.org/10.1016/j.ins.2019.10.068>

- [23] Dzedzej, A., Wolnik, B., Nenca, A., Baetens, J.M., De Baets, B.: Two-dimensional rotation-symmetric number-conserving cellular automata. *Information Sciences* **577**, 599–621 (2021) <https://doi.org/10.1016/j.ins.2021.06.041>
- [24] Wolnik, B., Dziemiańczuk, M., Dzedzej, A., De Baets, B.: Reversibility of number-conserving 1d cellular automata: Unlocking insights into the dynamics for larger state sets. *Physica D: Nonlinear Phenomena* **429**, 133075 (2022) <https://doi.org/10.1016/j.physd.2021.133075>
- [25] Wolnik, B., Nenca, A., Dzedzej, A., De Baets, B.: Seven-state rotation-symmetric number-conserving cellular automaton that is not isomorphic to any septenary one. *Physical Review E* **107**(2) (2023) <https://doi.org/10.1103/PhysRevE.107.024211>
- [26] Wolnik, B., Mrozek, N., Dzedzej, A., De Baets, B.: Three-dimensional rotation-symmetric number-conserving cellular automata. *Journal of Cellular Automata* **15**(4), 243–259 (2020)
- [27] Wolnik, B., Nenca, A., De Baets, B.: A decomposition theorem for number-conserving multi-state cellular automata on triangular grids. *Theoretical Computer Science* **953**, 113795 (2023) <https://doi.org/10.1016/j.tcs.2023.113795>
- [28] Wolnik, B., Dziemiańczuk, M., De Baets, B.: Non-uniform number-conserving elementary cellular automata. *Information Sciences* **626**, 851–866 (2023) <https://doi.org/10.1016/j.ins.2023.01.033>
- [29] Wolnik, B., Dziemiańczuk, M., De Baets, B.: Non-uniform number-conserving elementary cellular automata on the infinite grid: A tale of the unexpected. *Information Sciences* **649**, 119680 (2023) <https://doi.org/10.1016/j.ins.2023.119680>
- [30] Wolnik, B., Augustynowicz, A., Dziemiańczuk, M., De Baets, B.: Reversibility of non-saturated linear cellular automata on finite triangular grids. *Chaos: An Interdisciplinary Journal of Nonlinear Science* **31**(1) (2021) <https://doi.org/10.1063/5.0031535>
- [31] Wolnik, B., Dziemiańczuk, M., De Baets, B.: Recurrent misconceptions in the study of ca reversibility on triangular grids. *International Journal of Bifurcation and Chaos* **31**(01), 2150014 (2021) <https://doi.org/10.1142/s0218127421500140>
- [32] Augustynowicz, A., Baetens, J., De Baets, B., Dzedzej, A., Nenca, A., Wolnik, B.: A note on the reversibility of 2d cellular automata on hexagonal grids. *Journal of Cellular Automata* **13**(5-6), 521–525 (2018)