

IFI 9000 Analytics Methods

Deep Learning in Computer Vision

by **Houping Xiao**

Spring 2021



Introduction

Computer vision is everywhere

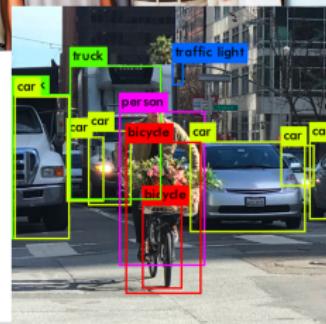
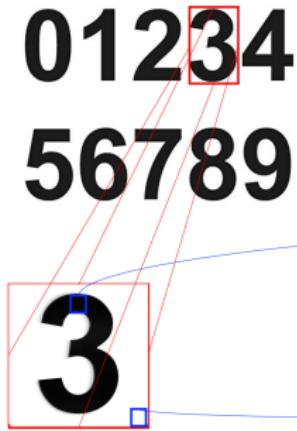


Image representation is the first step

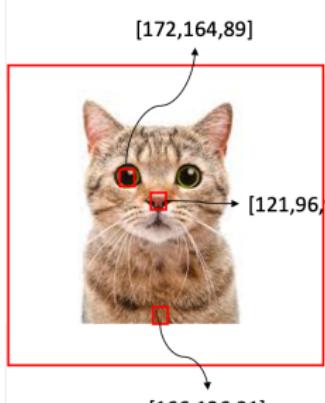


Consider an image as a function:
 $f(x, y): [a, b] \times [c, d] \rightarrow [0, 255]$

```
array([[255., 255., 255., 255., 252., 255., 255., 255., 255., 255.],  
       [255., 255., 253., 7., 4., 255., 14., 255., 255., 255.],  
       [255., 255., 123., 7., 250., 6., 7., 255., 255., 255.],  
       [255., 255., 249., 6., 255., 252., 13., 189., 255., 255.],  
       [255., 255., 255., 255., 32., 16., 255., 255., 255.],  
       [255., 255., 255., 14., 9., 253., 255., 255., 255.],  
       [255., 255., 255., 255., 255., 7., 6., 255., 255.],  
       [255., 255., 255., 255., 255., 7., 11., 255., 255.],  
       [255., 255., 8., 4., 255., 255., 7., 3., 255., 255.],  
       [255., 255., 10., 5., 8., 8., 6., 57., 255., 255.],  
       [255., 255., 253., 4., 11., 10., 13., 255., 255., 255.],  
       [255., 255., 255., 255., 255., 255., 255., 255., 255., 255.]],  
      dtype=float32)
```

Image representation: from gray-scale images to colorful images

- Extension of the grayscale function



```
array([[255., 255., 255., ..., 255., 255., 255.],  
       [255., 255., 255., ..., 255., 255., 255.],  
       [255., 255., 255., ..., 255., 255., 255.],  
       ...,  
       [255., 255., 255., ..., 255., 255., 255.],  
       [255., 255., 255., ..., 255., 255., 255.],  
       [255., 255., 255., ..., 255., 255., 255.],  
       ...,  
       [255., 255., 255., ..., 255., 255., 255.],  
       [255., 255., 255., ..., 255., 255., 255.],  
       [255., 255., 255., ..., 255., 255., 255.]], dtype=float32)
```

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

red channel
green channel
blue channel

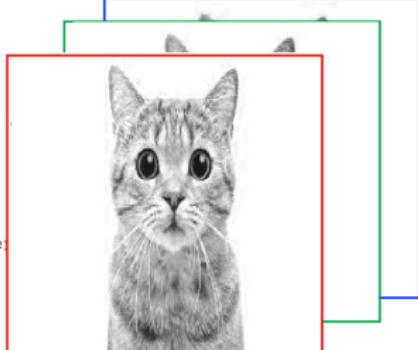


Image representation: a summary

- An image can be represented as a matrix or a tensor (a higher order matrix, usually 3-d in here) of pixel values



157, 153, 174, 168, 159, 152, 129, 151, 172, 161, 156, 156,
156, 182, 163, 74, 76, 62, 33, 17, 118, 218, 188, 154
188, 180, 50, 14, 34, 6, 18, 33, 48, 186, 159, 181
246, 109, 6, 124, 131, 111, 128, 284, 166, 15, 56, 188
194, 68, 137, 251, 237, 239, 239, 228, 227, 87, 71, 281
172, 106, 207, 233, 233, 214, 228, 239, 228, 58, 74, 266
188, 88, 179, 289, 186, 216, 211, 158, 139, 75, 28, 169
189, 97, 166, 84, 10, 168, 134, 11, 31, 62, 22, 148
199, 168, 191, 193, 158, 227, 178, 143, 182, 186, 36, 190
246, 174, 156, 252, 236, 231, 149, 178, 228, 43, 96, 234
198, 216, 116, 149, 236, 187, 86, 158, 79, 38, 218, 241
198, 224, 147, 188, 227, 218, 127, 182, 36, 101, 256, 224
198, 214, 173, 66, 183, 143, 96, 58, 2, 189, 249, 215
187, 196, 236, 75, 1, 81, 47, 8, 6, 217, 256, 211
183, 202, 237, 145, 8, 8, 12, 108, 108, 138, 243, 236
196, 206, 123, 207, 177, 121, 123, 208, 176, 13, 96, 218

157	153	174	168	150	162	129	157	172	161	156	156
155	182	163	74	75	62	35	17	110	210	180	154
180	180	60	14	34	6	30	48	106	189	181	181
206	158	3	134	131	111	120	204	166	15	54	180
194	68	137	261	237	239	239	238	227	87	71	201
172	104	207	233	230	214	220	239	228	48	74	206
188	88	179	209	186	216	211	188	179	75	29	169
189	97	168	84	10	168	134	31	31	62	22	148
199	168	191	193	158	227	178	143	182	104	36	190
206	174	168	252	236	231	149	178	228	43	91	234
190	216	116	140	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	103	36	101	358	224
190	214	173	66	103	143	96	90	2	109	249	215
187	196	236	75	1	81	47	8	6	217	256	211
183	202	237	145	8	8	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218

Image processing: image filtering and image warping

- **Image filtering:** change the range, i.e. the pixel values, of an image such that the colors of the image are changed without changing the pixel positions
- **Image warping:** change the domain, i.e. the pixel positions, of an image, where points are mapped to other positions without changing the colors

Image filtering: change the pixel values

- An example using “median filter”, replacing each entry with the median of neighboring entries
- used to remove noise from an image or signal
- preserves edges while removing noise

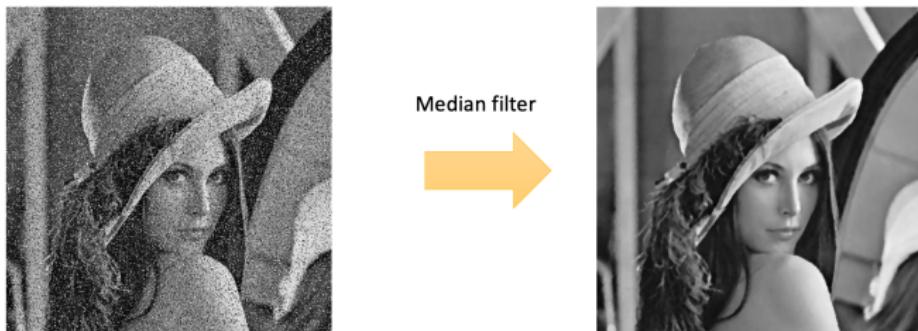


Image filtering: moving average

- A more smooth image with sharp features removed
- replace each pixel with the average pixel value of it and its neighborhood window of adjacent pixels

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	90	90	90	90	0	0	0
0	0	90	0	0	90	0	0	0
0	0	90	0	0	90	0	0	0
0	0	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0

Original



Averaging

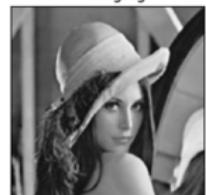
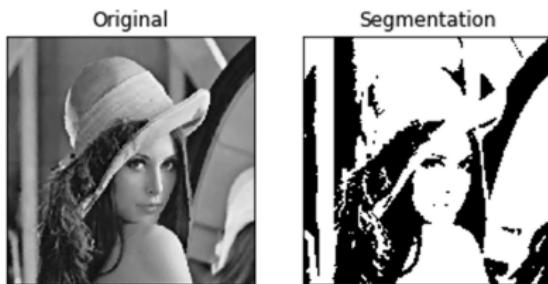


Image filtering: image segmentation filters

- Partition an image into regions where the pixels have similar attributes, so the image is represented in a more simplified way
- identify objects and boundaries more easily

$$f(x, y) = \begin{cases} 255, & \text{if } f(x, y) > 100 \\ 0, & \text{otherwise} \end{cases}$$



2d convolution filter: works on a input and a kernel image

- Filters can be expressed in a principal manner using 2d convolution, such as smoothing and sharpening images, and detecting edges



Sharpening filter

- Original image - smoothed image = details
- Original image + details = sharpened image



An application: edge detection



Image warping (scaling)

- Digitally manipulating an image, such as resizing the image (subsampling)
 - any shapes portrayed in the image have been significantly distorted



Subsampling/downsampling

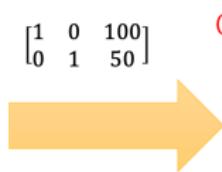


256x256x3

Translation

- Shifting of an object location

- transformation matrix: $M = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix}$



(100,50)



Rotation

- Rotation θ can be achieved by the transformation of the form
- $M = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ or $M = \begin{bmatrix} \alpha & \beta & (1-\alpha)x - \beta y \\ -\beta & \alpha & \beta x + (1-\alpha)y \end{bmatrix}$, where $\alpha = \text{scale} * \cos \theta$, $\beta = \text{scale} * \sin \theta$ and (x, y) is the center

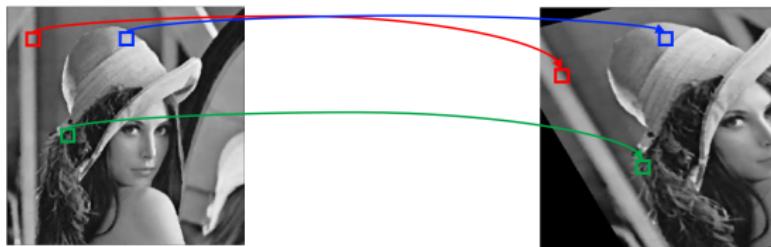


$\theta=180$
Scale = 1
(x,y) is center of image



Affine and perspective transformation

- Affine: similar to rotation, all parallel lines in the original image will still be parallel



- Perspective: zoom out for a specific range defined by four points

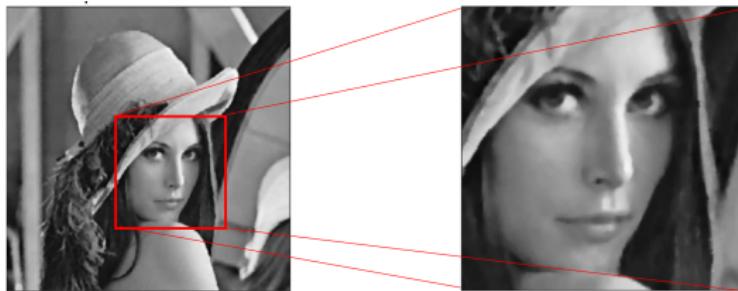
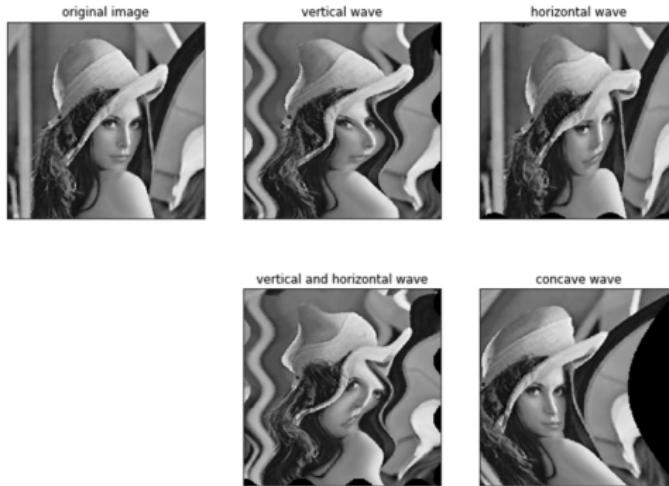
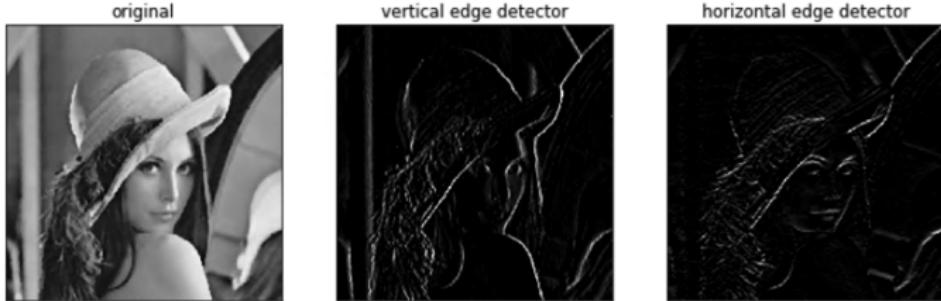


Image warping



Demo on image processing; check the python code!

Filters: a motivating example of edge detection



- vertical edge detection

$$\begin{array}{c} \text{input } 6 \times 6 \\ \begin{array}{|c|c|c|c|c|c|} \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline \end{array} \end{array} * \begin{array}{c} \text{filter } 3 \times 3 \\ \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 1 & 0 & -1 \\ \hline 1 & 0 & -1 \\ \hline \end{array} \\ \begin{array}{|c|c|c|} \hline \text{white} & \text{gray} & \text{black} \\ \hline \end{array} \end{array} = \begin{array}{c} \text{output } 4 \times 4 \\ \begin{array}{|c|c|c|c|} \hline 0 & 30 & 30 & 0 \\ \hline 0 & 30 & 30 & 0 \\ \hline 0 & 30 & 30 & 0 \\ \hline 0 & 30 & 30 & 0 \\ \hline \end{array} \\ \begin{array}{|c|c|c|} \hline \text{white} & \text{dark gray} & \text{white} \\ \hline \end{array} \end{array}$$

More edge detection filters

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

*



1	0	-1
1	0	-1
1	0	-1

=

0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0



0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10

*



1	0	-1
1	0	-1
1	0	-1

=

0	-30	-30	0
0	-30	-30	0
0	-30	-30	0
0	-30	-30	0



More edge detection filters

1	0	-1
1	0	-1
1	0	-1

Vertical

1	1	1
0	0	0
-1	-1	-1

Horizontal

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10

*

1	1	1
0	0	0
-1	-1	-1

=

0	0	0	0
30	10	-10	-30
30	10	-10	-30
0	0	0	0

1	0	-1
1	0	-1
1	0	-1

1	0	-1
2	0	-2
1	0	-1

Sobel filter

3	0	-3
10	0	-10
3	0	-3

Scharr filter

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

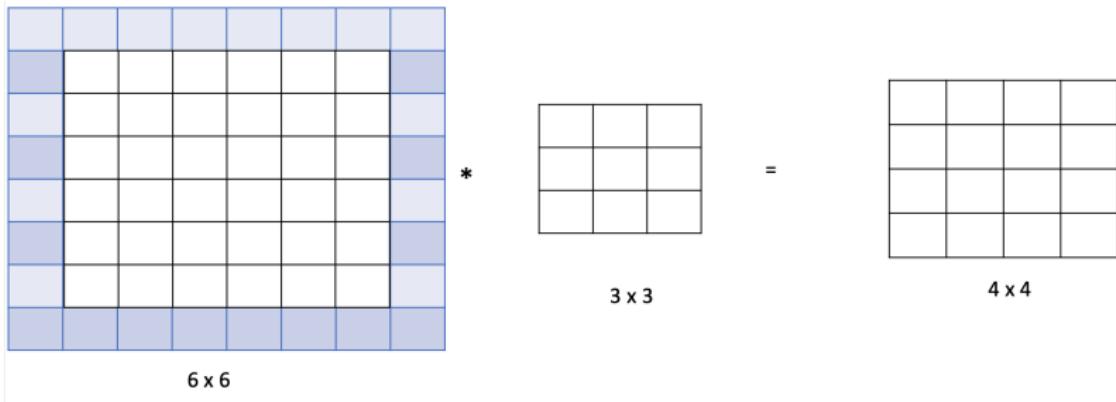
*

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

=

Padding

- Shrinkage output
- through away information from edge



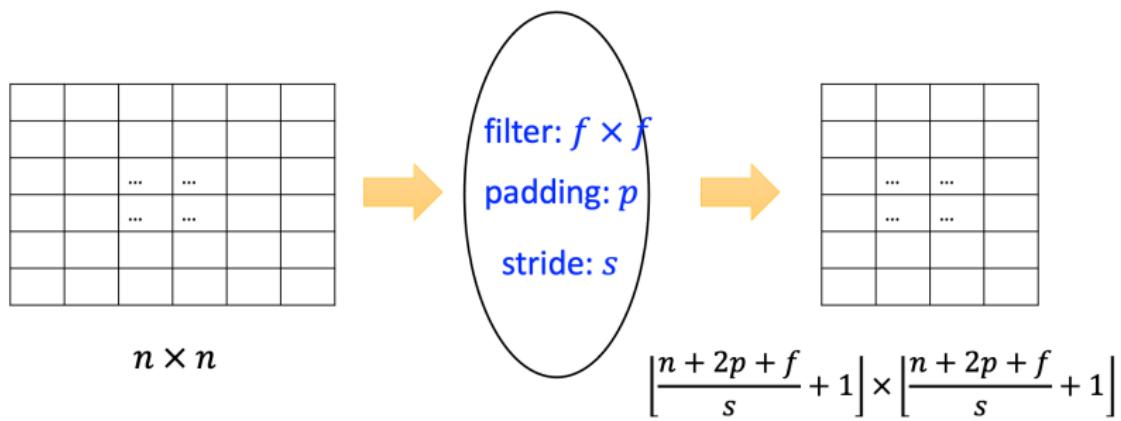
- Options: Valid and Same padding
- Valid: no padding, and output $(n - f + 1) \times (n - f + 1)$
- Same: output feature map stays the same size as the input image (feature map), and output $(n + 2p - f + 1) \times (n + 2p - f + 1)$
 - f usually odd

Stride

$$\begin{array}{|c|c|c|c|c|c|c|} \hline 2 & 3 & 7 & 4 & 6 & 2 & 9 \\ \hline 6 & 6 & 9 & 8 & 7 & 4 & 3 \\ \hline 3 & 4 & 8 & 3 & 8 & 9 & 7 \\ \hline 7 & 8 & 3 & 6 & 6 & 3 & 4 \\ \hline 4 & 2 & 1 & 8 & 3 & 4 & 6 \\ \hline 3 & 2 & 4 & 1 & 9 & 8 & 3 \\ \hline 0 & 1 & 3 & 9 & 2 & 1 & 4 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 3 & 4 & 5 \\ \hline 1 & 0 & 2 \\ \hline -1 & 0 & 3 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 91 & 100 & 83 \\ \hline 69 & 91 & 127 \\ \hline 44 & 72 & 74 \\ \hline \end{array}$$

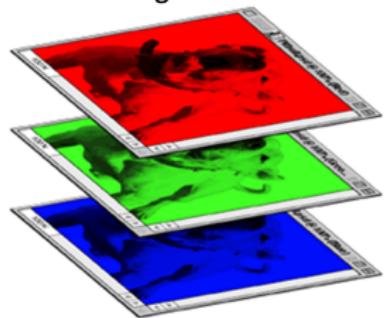
- output: $(\frac{n+2p-f}{s} + 1) \times (\frac{n+2p-f}{s} + 1)$

Output dimension after a convolutional layer



Convolutions on RGB (colorful) images

Colorful image



$$\begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{matrix}$$

*

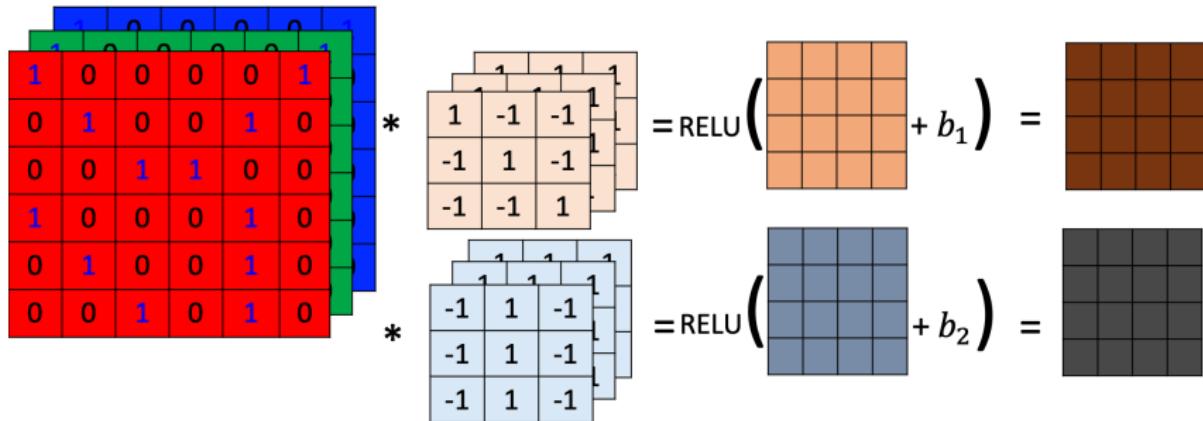
$$\begin{matrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & -1 \end{matrix}$$

=

$$\begin{matrix} & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{matrix}$$



Convolutions on RGB (colorful) images

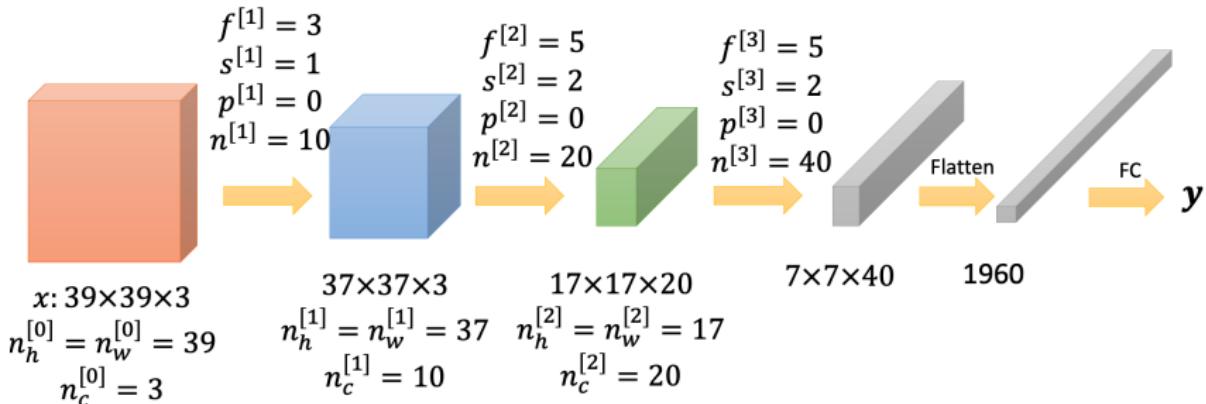


- Number of parameters in one layer
 - consider one convolutional layer with 10 filters that are 3x3, how many parameters we need to train?

Notations

- If layer l is a conv layer:
- $f^{[l]}$ = filter size
- $p^{[l]}$ = padding
- $s^{[l]}$ = stride
- $n_c^{[l]}$ = number of filters
- Each filter is: $f^{[l]} \times f^{[l]} \times n_c^{[l-1]}$
- Activations: $a^{[l]} \rightarrow n_h^{[l]} \times n_w^{[l]} \times n_c^{[l]}$
- Weights: $f^{[l]} \times f^{[l]} \times n_c^{[l-1]} \times n_c^{[l]}$
- Bias: $n_c^{[l]}$
- Input (RGB images): $n_h^{[l-1]} \times n_w^{[l-1]}$
- Output: $n_h^{[l]} \times n_w^{[l]} \times n_c^{[l]}$
- $n_{h \setminus w}^{[l]} = \left\lfloor \frac{n_{h \setminus w}^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$

An example of ConvNet



- Convolution (ConV), complicated
- Pooling (Pool), easy
- Fully Connected (FC), easy

Pooling layers (Pool)

- reduce the size of image representation, speed up the computation, and robust feature detection

1	3	2	1
2	9	1	2
1	3	3	2
6	5	2	1

Max pool

Hyperparameters:

$$f = 2 \\ s = 2$$



9	2
6	3

1	3	2	1
2	9	1	2
1	3	3	2
6	5	2	1

Average pool

Hyperparameters:

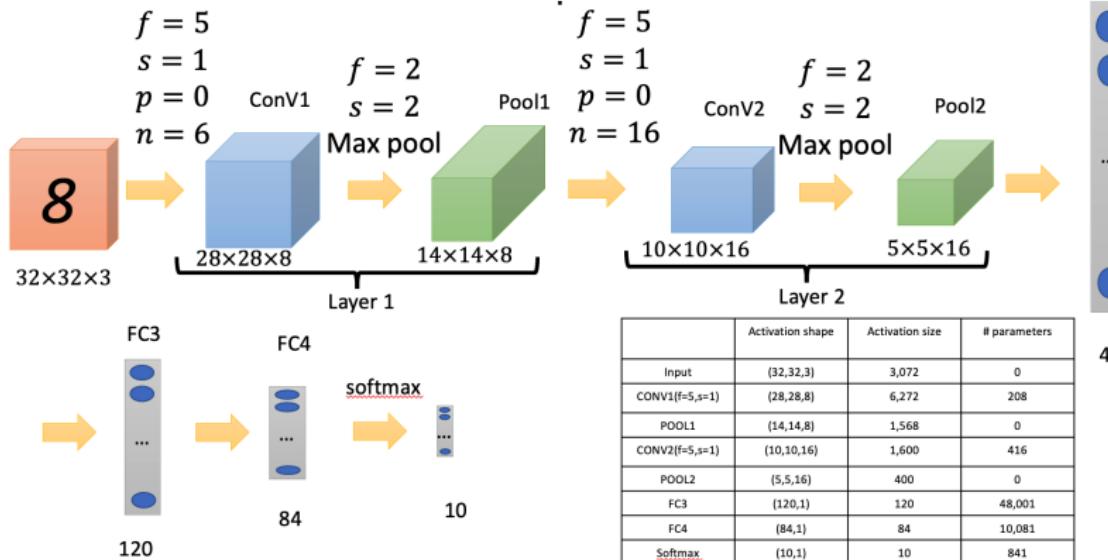
$$f = 2 \\ s = 2$$



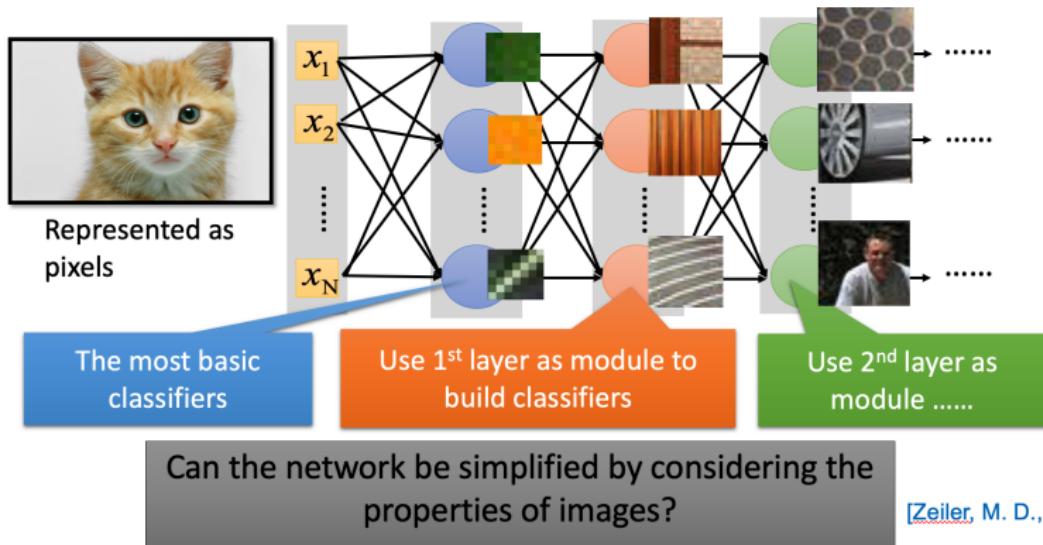
3.75	1.25
4	2

- No parameters to learn, output $\lfloor \frac{n+2p-f}{s} + 1 \rfloor \times \lfloor \frac{n+2p-f}{s} + 1 \rfloor$

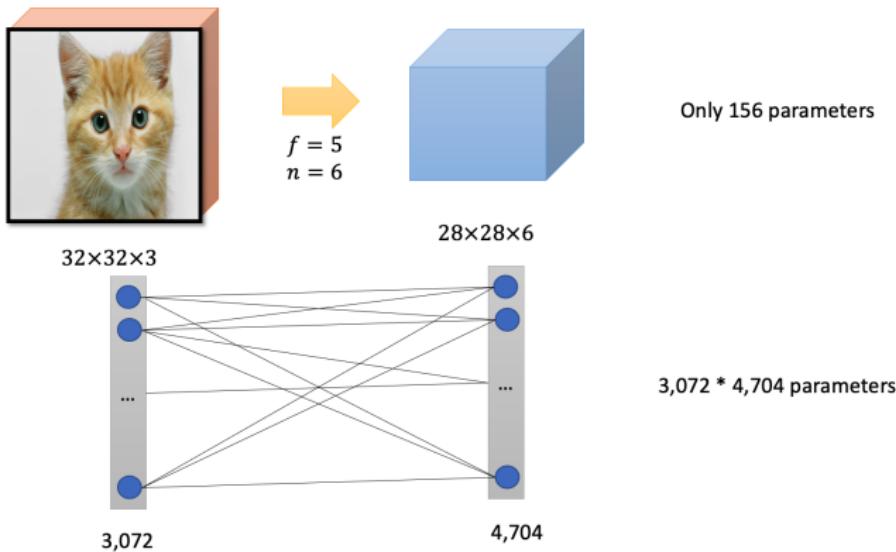
An example



Why convolutions



Why convolutions

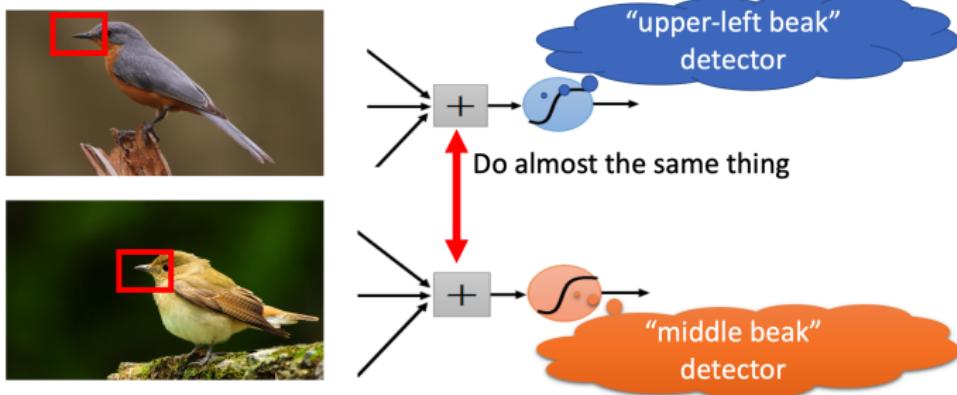


Convolutions enable parameter sharing

- A feature detector (such as the vertical edge detector) that's useful in one part of the image is probably useful in another part of the image

$$\begin{array}{|c|c|c|c|c|c|} \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 1 & 0 & -1 \\ \hline 1 & 0 & -1 \\ \hline \end{array} = \begin{array}{|c|c|c|c|} \hline 0 & 30 & 30 & 0 \\ \hline 0 & 30 & 30 & 0 \\ \hline 0 & 30 & 30 & 0 \\ \hline 0 & 30 & 30 & 0 \\ \hline \end{array}$$

- The same patterns appear in different regions

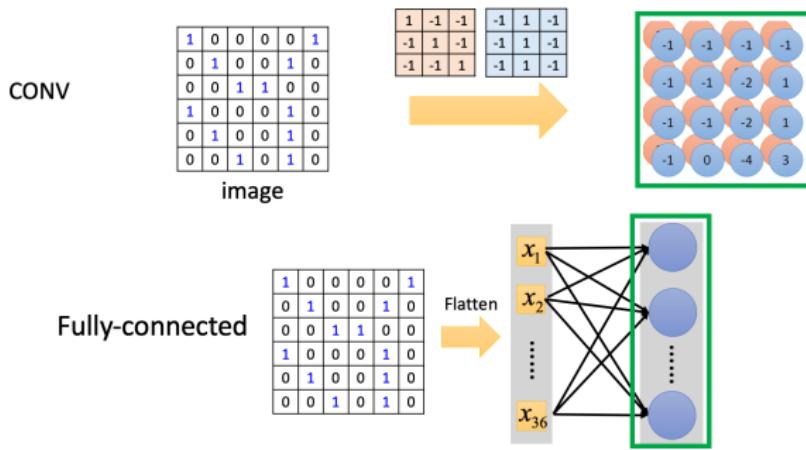


Convolutions enable sparsity of connections

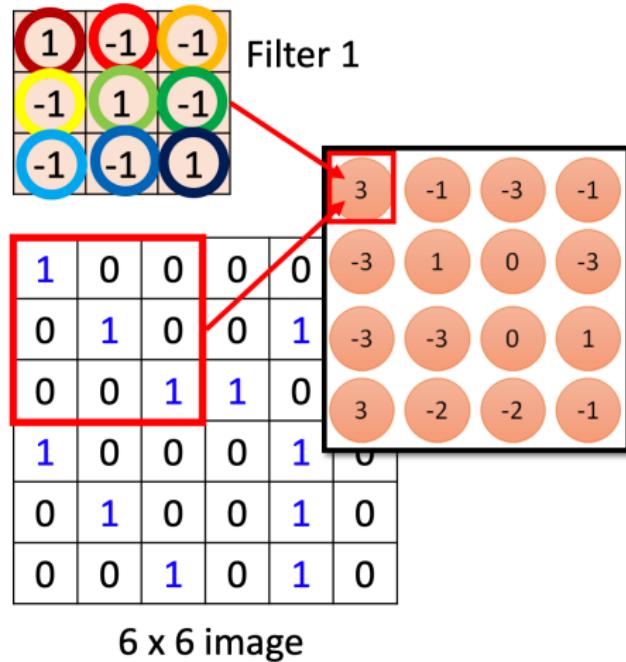
- in each layer, each output value depends only on a small number of inputs

$$\begin{array}{|c|c|c|c|c|c|} \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline 10 & 10 & 10 & 0 & 0 & 0 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 1 & 0 & -1 \\ \hline 1 & 0 & -1 \\ \hline \end{array} = \begin{array}{|c|c|c|c|} \hline 0 & 30 & 30 & 0 \\ \hline 0 & 30 & 30 & 0 \\ \hline 0 & 30 & 30 & 0 \\ \hline 0 & 30 & 30 & 0 \\ \hline \end{array}$$

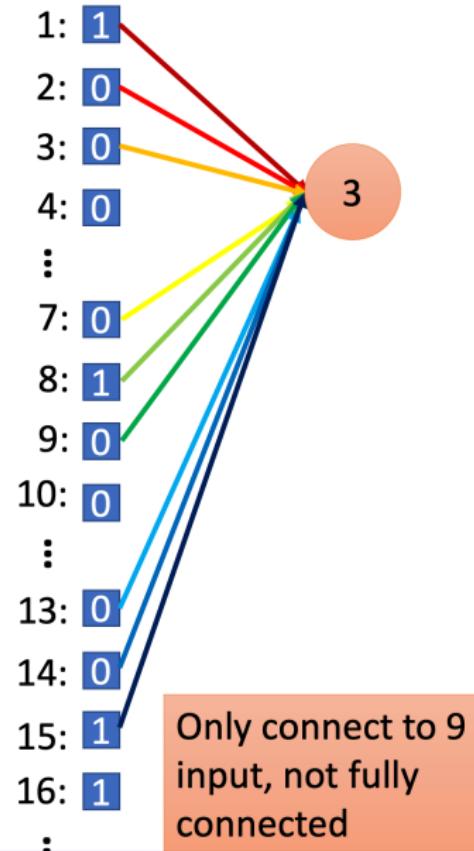
- Sparsity of connections: ConV v.s. FC



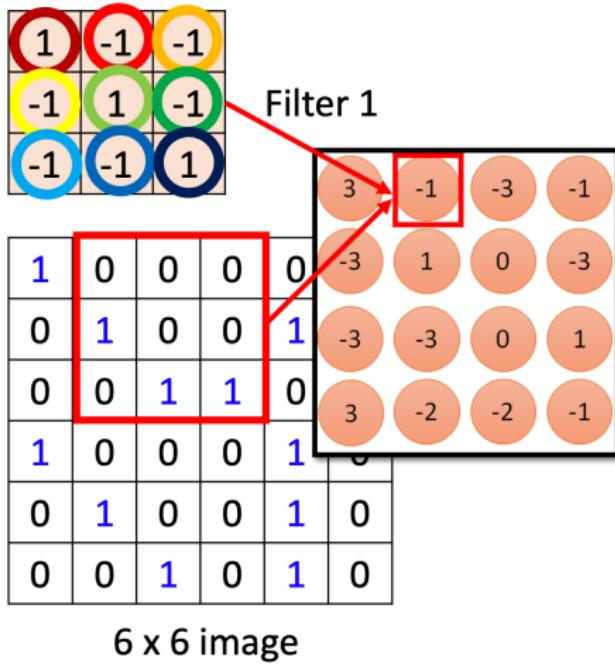
Benefits of using ConV



Less parameters!

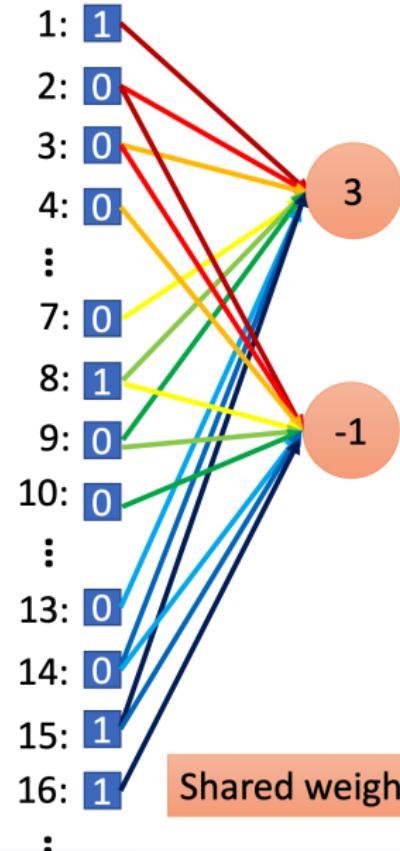


Benefits of using ConV



Less parameters!

Even less parameters!



Why pooling?

- Subsampling the pixels will not change the object

bird



subsampling

bird



We can subsample the pixels to make image smaller



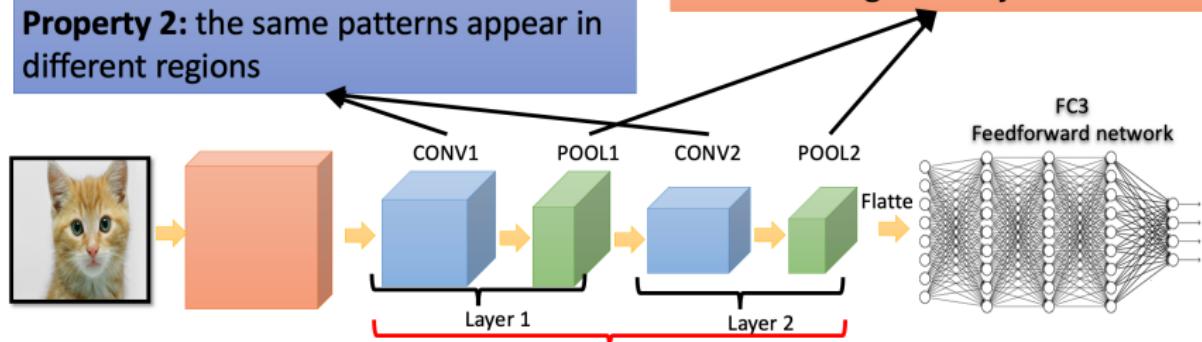
Less parameters for the network to process the image

The whole CNN architecture

Property 1: some patterns are much smaller than the whole image

Property 2: the same patterns appear in different regions

Property 3: subsampling the pixels will not change the object

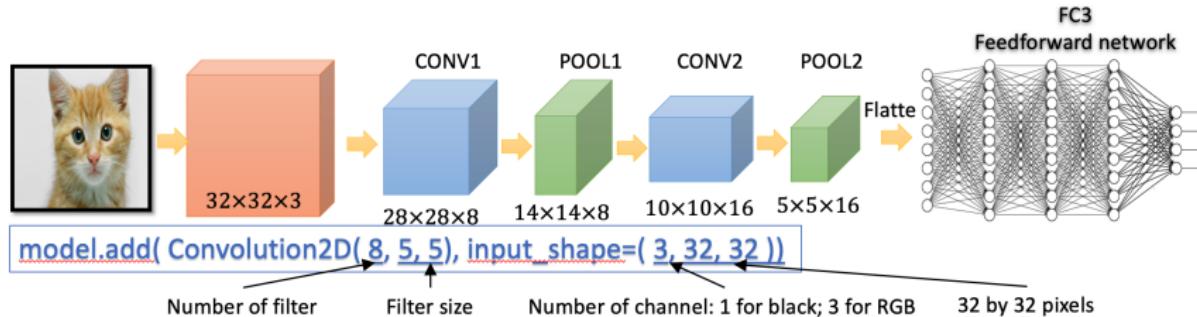


$$\text{Cost } J = \frac{1}{m} \sum_{i=1}^m \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

Can repeat many times

Use gradient descent to optimize parameters to reduce J

The whole CNN architecture use Keras



model.add(MaxPooling2D((2,2))

model.add(Convolution2D(16, 5, 5))

model.add(MaxPooling2D((2,2))

model.add(Dense(output_dim = 100))

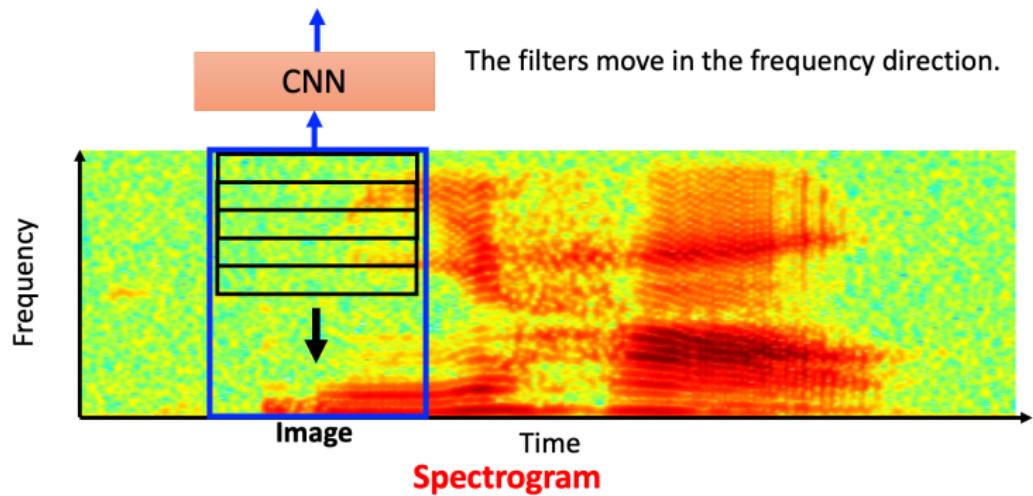
model.add(Activation('relu'))

model.add(Dense(output_dim=10))

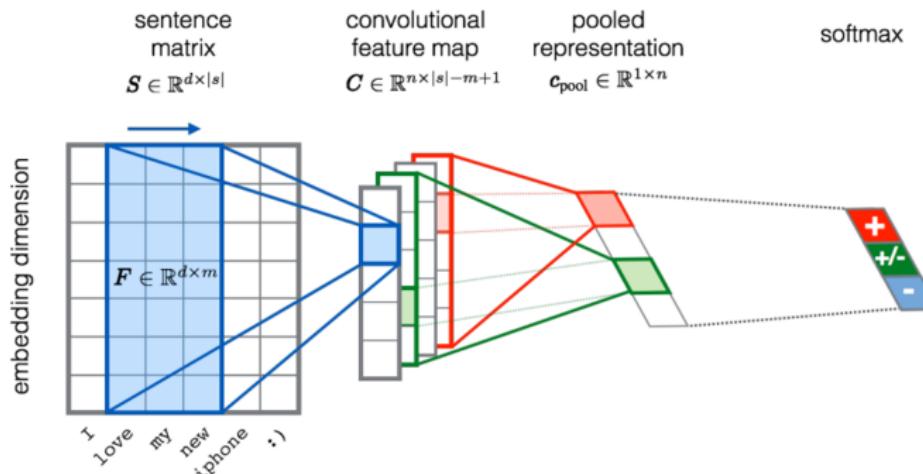
model.add(Activation('softmax'))

model.add(Flatten())

CNN for speech



CNN for text



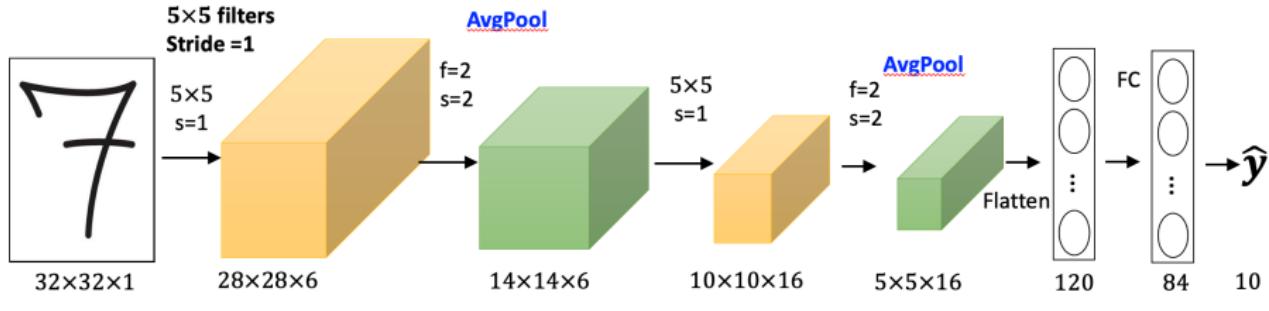
Source of image:

<http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.703.6858&rep=rep1&type=pdf>

Classic Networks

LeNet-5, AlexNet, VGG, GoogleNet, ResNet

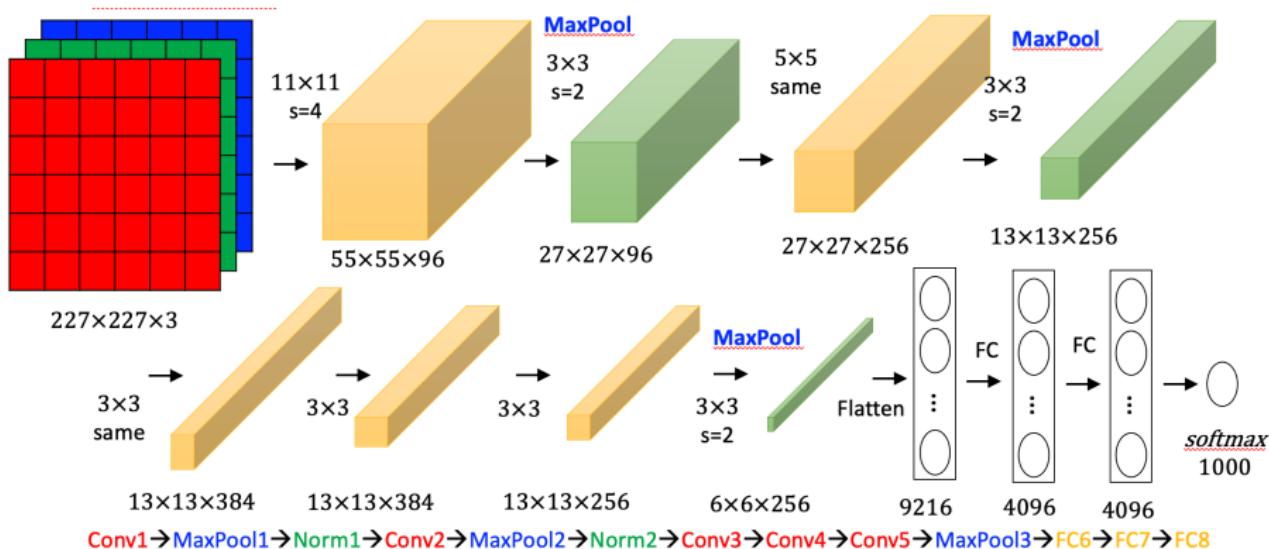
LeNet-5



digital recognition

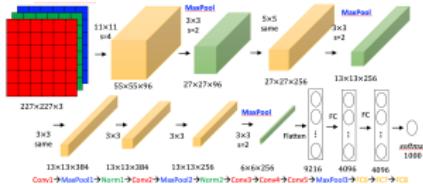
Conv1 → AvgPool1 → Conv2 → AvgPool2 → FC3 → FC4 → Softmax

AlexNet



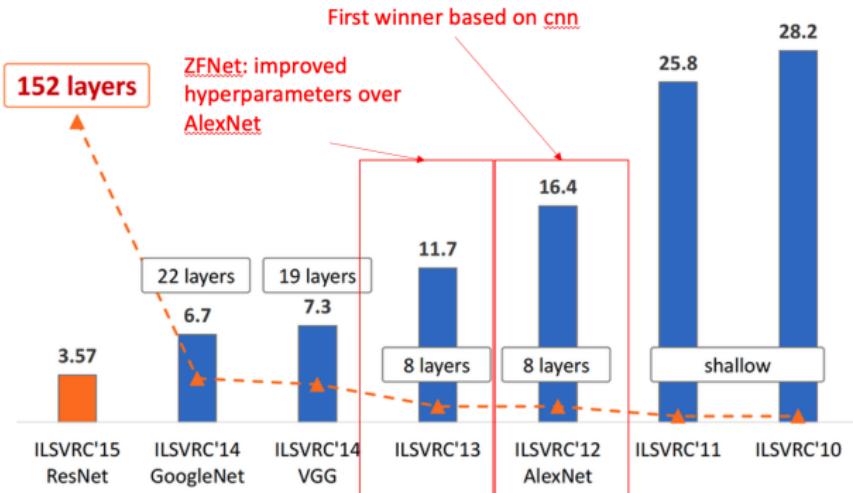
- First large scale CNN to do well in image classification!

AlexNet



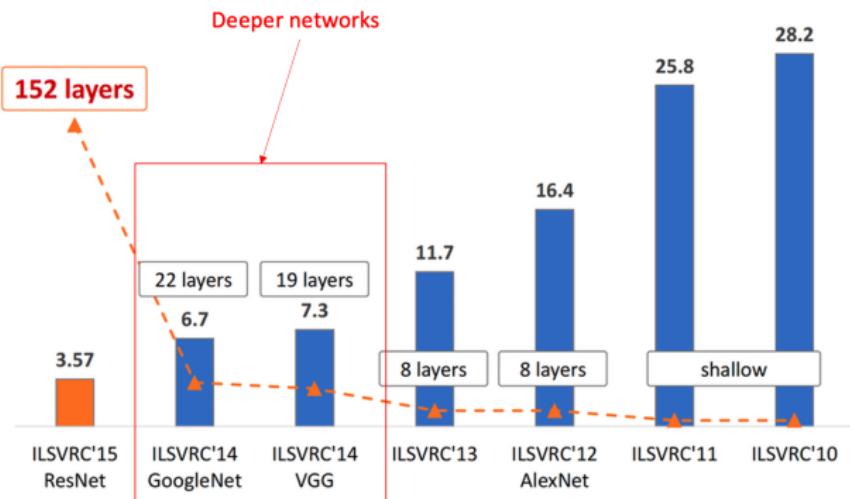
- Input: $227 \times 227 \times 3$ images; First layer (Conv1): 96:11x11 filters with stride = 4
 - Q: what is the output volume size? $55 \times 55 \times 96$
 - Q: how many parameters? $(11 \times 11 \times 3) \times 96$
- Second layer (MaxPool1): 3x3 filters with stride = 2
 - Q: what is the output volume size? $27 \times 27 \times 96$
 - Q: how many parameters? 0
- Details:
 - First use of ReLU; Used norm layers (not common anymore); Heavy data augmentation; Dropout=0.5; Batch size = 128; Sgd momentum = 0.9; Learning rate 0.01, reduced by 10 manually when val accuracy plateaus; Le weight decay 0.0005; 7 cnn ensemble, accuracy improved by around 3

ImageNet large scale visual recognition challenge winners



- ZFNet has the same structure with AlexNet, but
 - Conv1: 11×11 filers with stride 4 $\rightarrow 7 \times 7$ filers with stride 2
 - Conv3, 4, 5: number of filers 384, 384, 256 \rightarrow 512, 1024, 512
- accuracy improved by 4.7%

ImageNet large scale visual recognition challenge winners



VGG-16

VGG-16

Small filters, but deeper networks

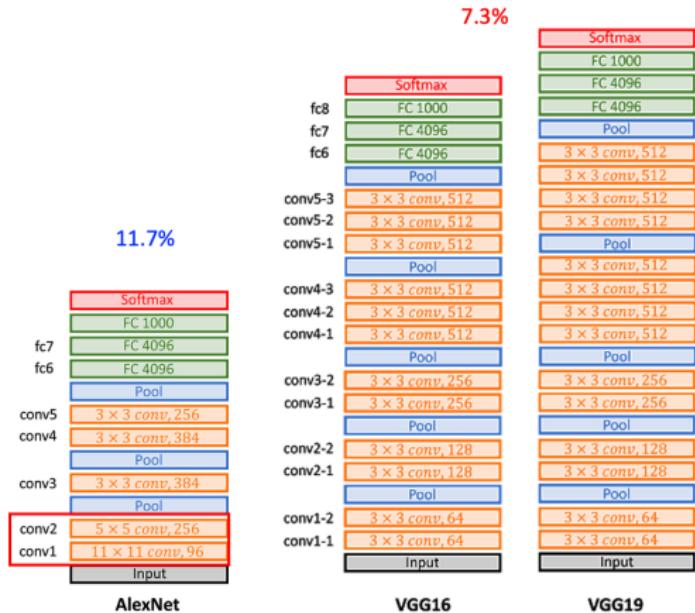
Q: why use smaller filters?

Stack of three 3×3 Conv (stride 1) layers has same effective receptive field as one 7×7 Conv layer

Q: what is the effective receptive field of three 3x3 Conv (stride 1) layers? 7x7

Pros:

- more non-linearities
 - fewer parameters: $3 \times (3^2 \times C)$ vs $7^2 \times C$ where C is the number of channels

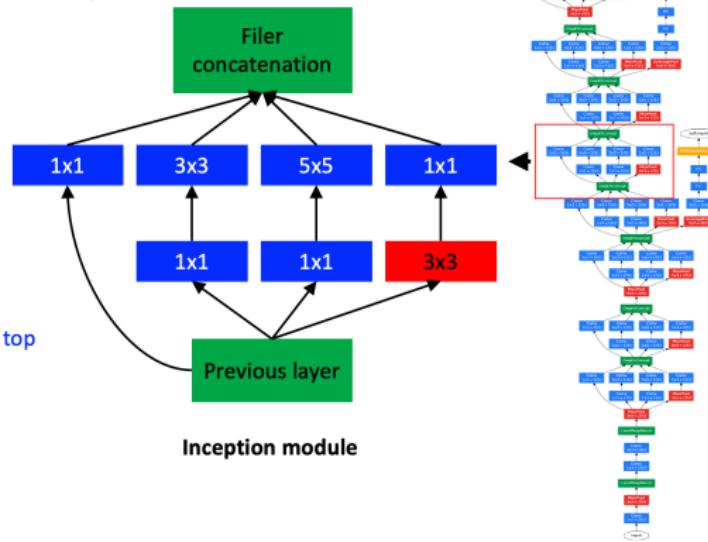


GoogleNet

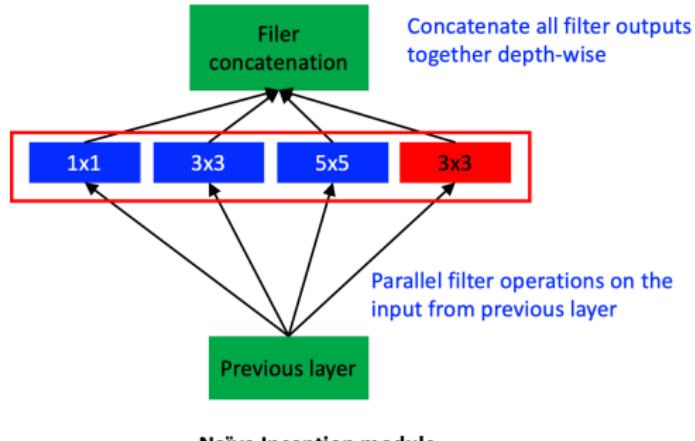
Deeper networks with computational efficiency

- 22 layers
- Efficient inception module
- No fully connect layers
- Only 5m parameters (much less than alexnet)
- ILSVRC'14 classification winner (6.7%)

Inception module: a good local network topology and then stack these modules on top of each other



Novelty of GoogleNet: Inception Module



[Szegedy et al., 2014]

Q: what is the problem with this naïve inception module?
Computational complexity

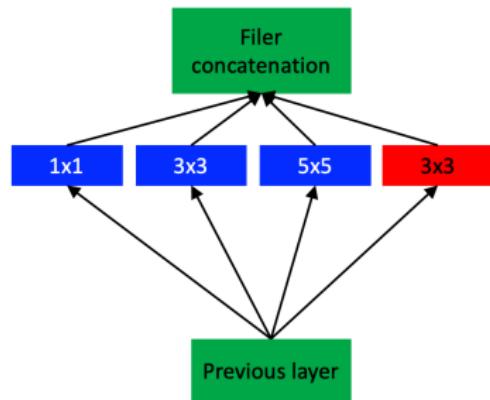
Input: $28 \times 28 \times 256$
126 1x1 Conv same padding
192 3x3 Conv same padding
96 5x5 Conv same padding
3x3 Pool

What's the output size?
 $28 \times 28 \times 672$

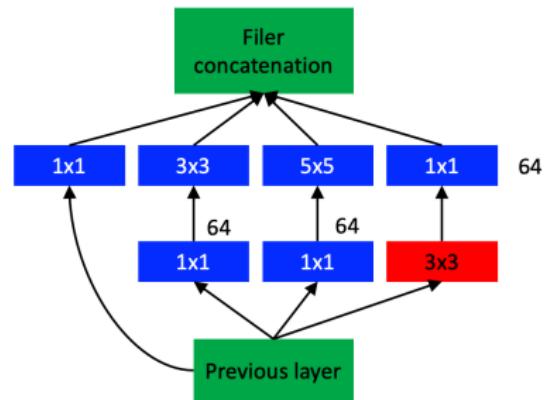
Pooling layer also an issue. Why?

Google solution: use bottleneck layers that use 1x1 Conv to reduce feature depth

Novelty of GoogleNet: Inception Module



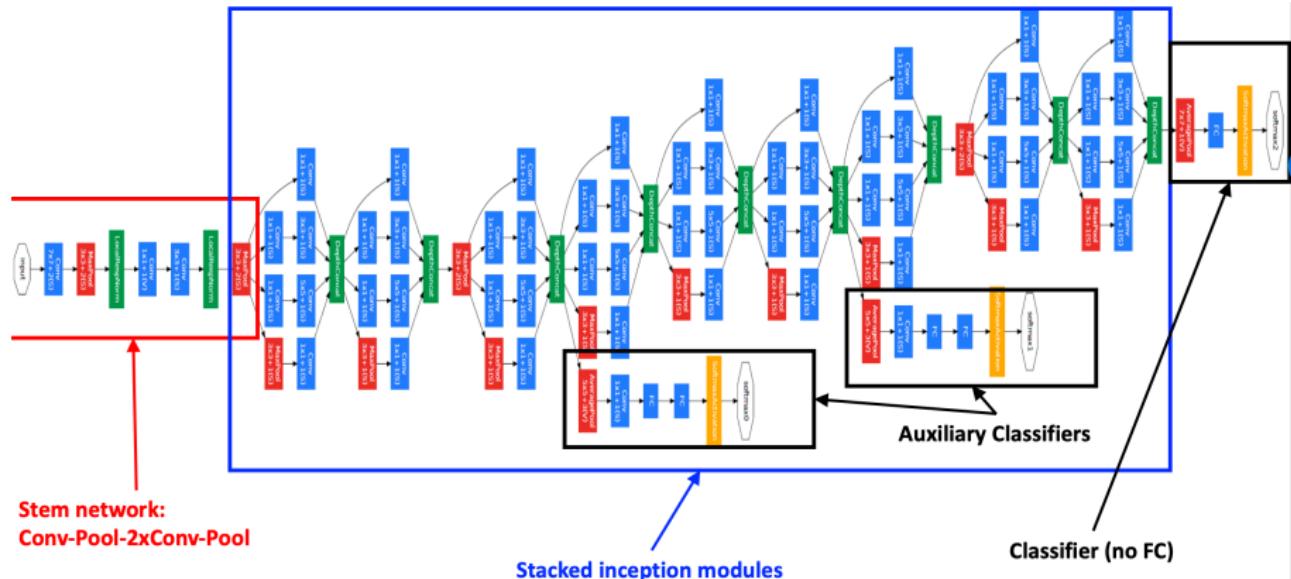
Naïve Inception module



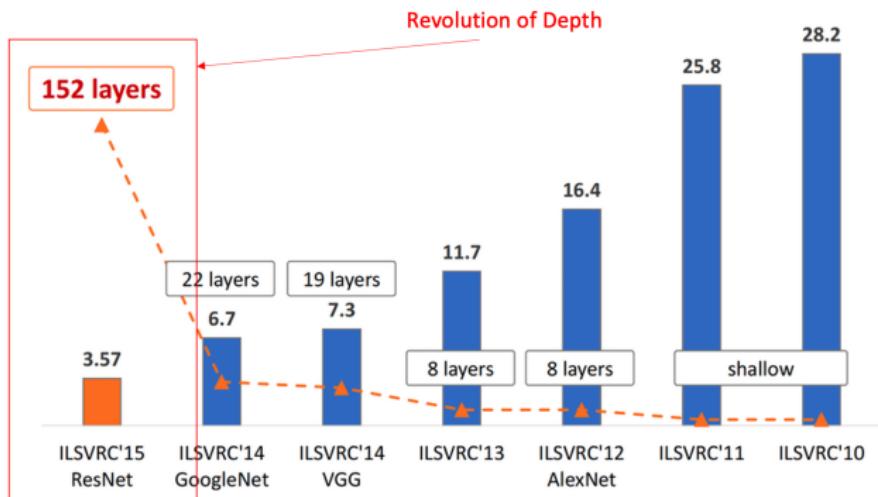
Inception module: dimension reduction

[Szegedy et al., 2014]

GoogleNet



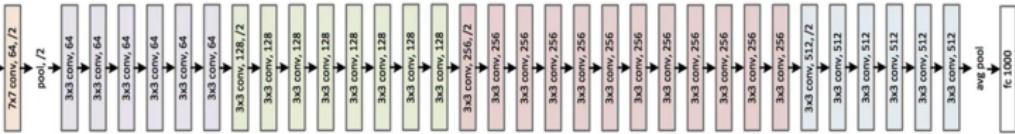
ImageNet large scale visual recognition challenge winners



ResNets

34-layer plain

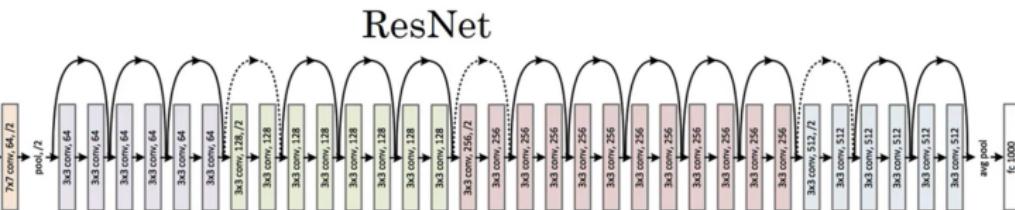
Image



Plain

34-layer residual

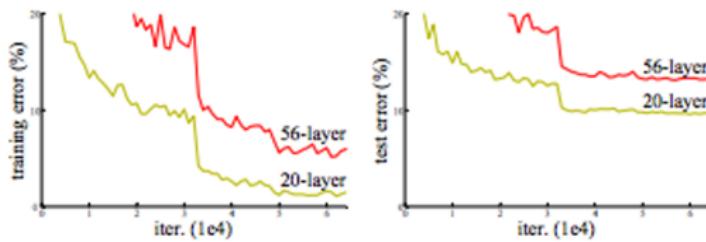
Image



ResNet

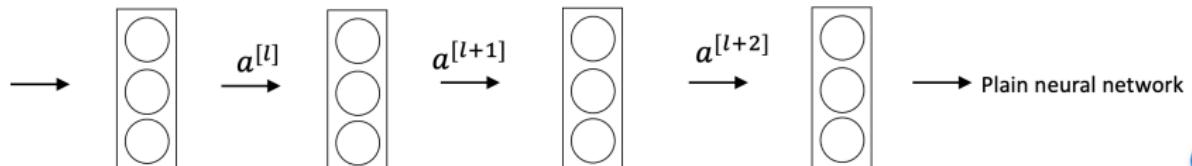
Residual Network

- Very deep neural network is difficult to train because of vanishing and exploding gradients
- ResNet is able to train very deep neural network



- 56-layer model performs worse on both training and test error
- The deeper model performs worse, but it's not caused by overfitting, more because of optimization issue

Residual block

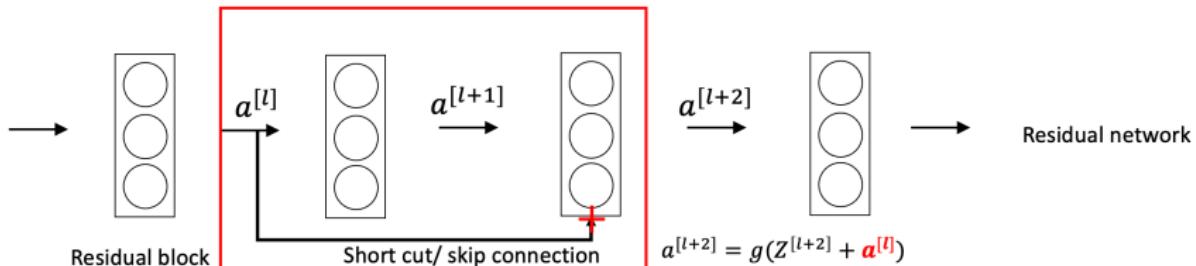


$$Z^{[l+1]} = W^{[l]}a^{[l]} + b^{[l+1]}$$

$$a^{[l+1]} = g(Z^{[l+1]})$$

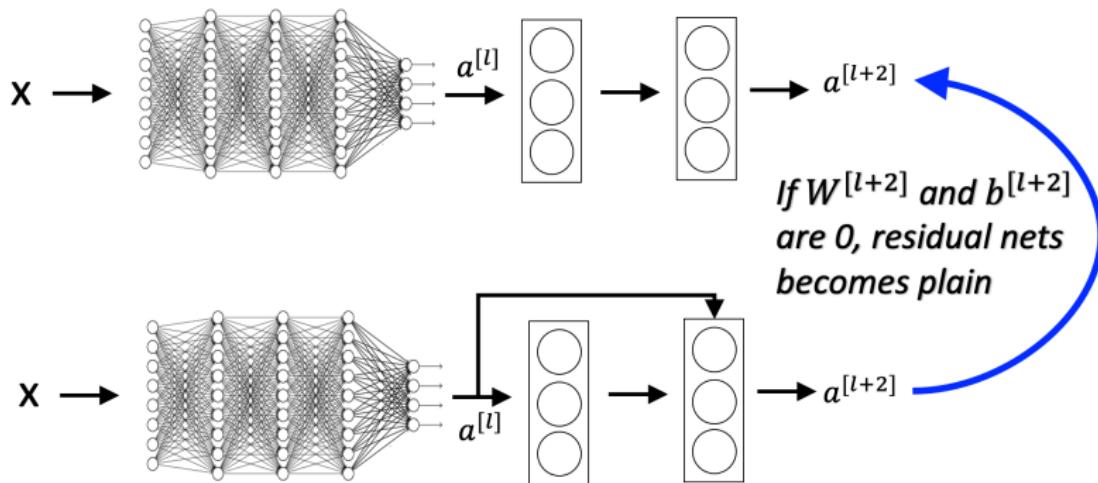
$$Z^{[l+2]} = W^{[l+1]}a^{[l+1]} + b^{[l+2]}$$

$$a^{[l+2]} = g(Z^{[l+2]})$$

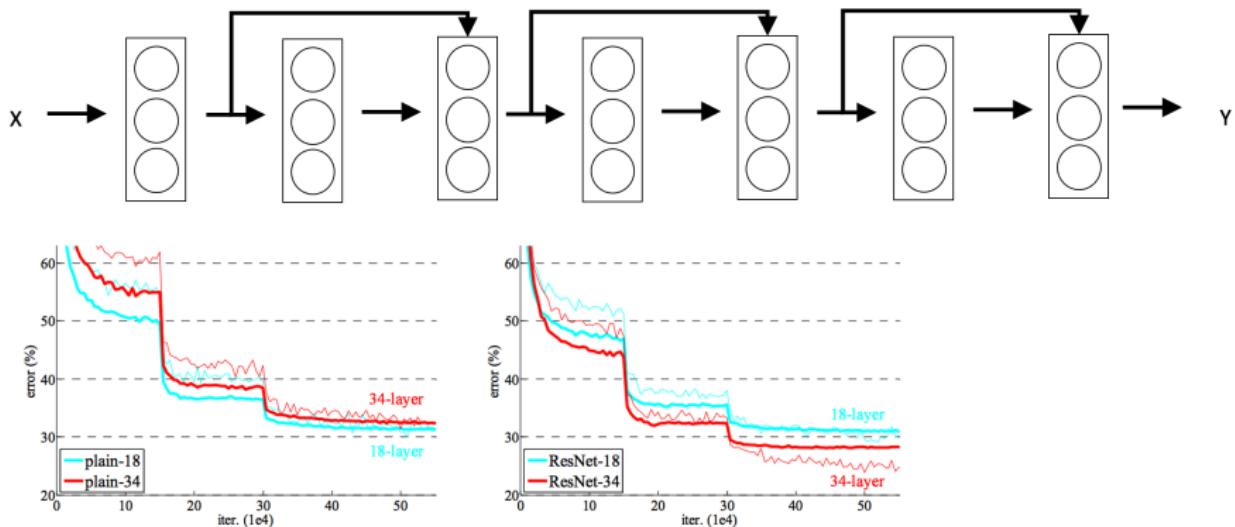


[He et al., 2015, deep residual networks for image recognition]

Why ResNets work?

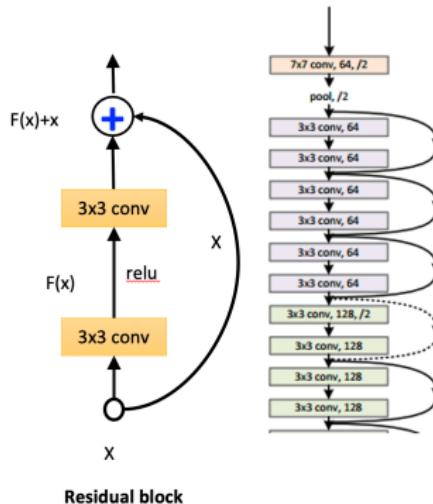


Why ResNets work?



[He et al., 2015, deep residual networks for image recognition]

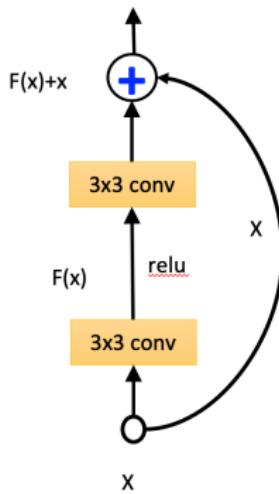
ResNets



- stack residual blocks; every residual block has 2: 3×3 ConV layers
- periodically, double number of filters and downsample spatially using stride 2
- additional ConV layer at the beginning and no FC layers at the end

ResNets

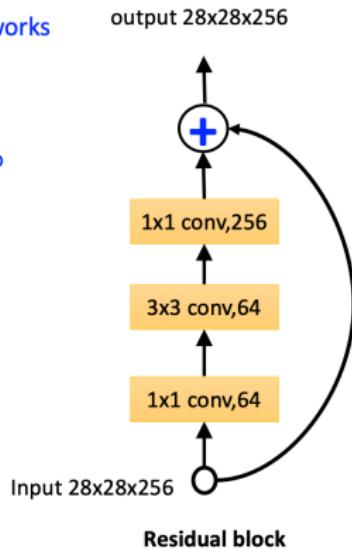
Similar to GoogLeNet, for deeper networks (more than 50 layers), ResNet use bottleneck layer to improve efficiency



1×1 conv, 256 filters to project back to $28 \times 28 \times 256$

3×3 conv (only 64 feature maps)

1×1 conv, 64 filters to project to $28 \times 28 \times 64$



Residual block

The End