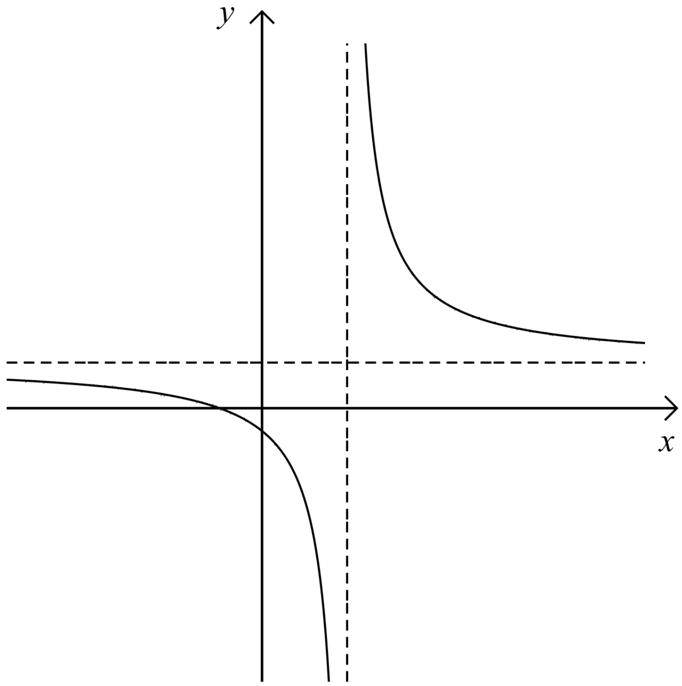
1. [Maximum points: 9]

Let $f(x) = \dfrac{x+1}{x-2}$. The diagram below shows the graph of $y = f(x)$.



(a) Find the equation of [3]

(i) the vertical asymptote

(ii) the horizontal asymptote

The graph of $y = f(x)$ is translated 3 units upwards to produce the graph of $y = g(x)$.

(b) Find the function $g(x)$. Write your answer in the form [3]

$$g(x) = \frac{ax+b}{x+c}$$

where $a, b, c \in \mathbb{Z}$.

The graph of $y = f(x)$ is reflected in the line $y = -3$ to produce the graph of $y = h(x)$.

(c) Find the equation of the resulting graph. Write your answer in the form [3]

$$y = \frac{dx+e}{x+f}$$

where $d, e, f \in \mathbb{Z}$.

2. [Maximum points: 6]

Let $f(x) = x^3 - 2x^2 + 1$. The graph of $y = f(x)$ is translated a units to the right to produce the graph of $y = g(x)$. If the coefficient of x in the function $g(x)$ is equal to 64 find the value of a.

3. [Maximum points: 6]

Let $f(x) = \log_4 x$ and $g(x) = 8^{x+2}$. Let $h(x) = (f \circ g)(x)$.

(a) Find the function $h(x)$. Write your answer in the form $h(x) = mx + c$ where $m, c \in \mathbb{R}$. [4]

(b) Hence describe a series of transformations that maps the graph of $y = x$ onto the graph of $y = h(x)$. [2]

4. [Maximum points: 6]

Let $f(x) = 3x + 1$. The graph of $y = f(x)$ is stretched horizontally by a factor of 9 and translated 4/3 units down to produce the graph of $y = g(x)$.

(a) Find the function $g(x)$ writing your answer in the form $g(x) = \dfrac{x+a}{b}$ where $a, b \in \mathbb{Z}$. [3]

(b) Find $f^{-1}(x)$. [2]

(c) Hence describe a single transformation that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$. [1]

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5. [Maximum points: 5]

Let $f(x) = \ln x$ and $g(x) = \mathrm{e}^{x+2}$. Find a single transformation that maps the graph of $y = x$ onto the graph of $y = (g \circ f)(x)$.

6. [Maximum points: 6]

Let $f(x) = 4^x$ and $g(x) = \frac{1}{x^2}$.

(a) Find the function $(g \circ f)(x)$ writing your answer in the form 4^{ax} where $a \in \mathbb{Z}$. [3]

(b) Describe a series of transformations that maps the graph of $y = f(x)$ onto the graph of $y = (g \circ f)(x)$. [3]

7. [Maximum points: 6]

Let $f(x) = \sqrt{x}$, $g(x) = x + 3$ and $h(x) = 2^x$.

Find a single transformation that maps the graph of $y = h(x)$ onto the graph of

(a) $y = (h \circ g)(x)$ [2]

(b) $y = (f \circ h)(x)$ [4]

8. [Maximum points: 7]

Let $f(x) = x + 3$, $g(x) = \ln x$ and $h(x) = x^4$.

Find a single transformation that maps the graph of $y = g(x)$ onto the graph of

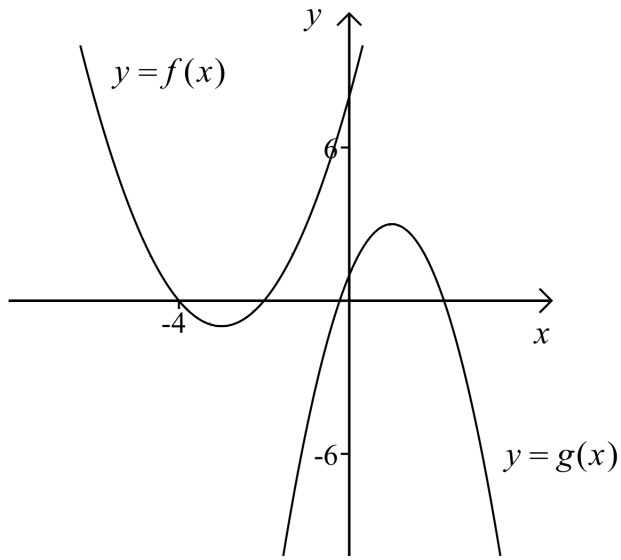
(a) $y = (f \circ g)(x)$ [2]

(b) $y = (g \circ f)(x)$ [2]

(c) $y = (g \circ h)(x)$ [3]

9. [Maximum points: 17]

Let $f(x) = x^2 + 6x + 8$ and $g(x) = -2x^2 + 4x + 1$. The following diagram shows the graphs of $y = f(x)$ and $y = g(x)$.



(a) Write the following functions in the form $y = a(x-p)^2 + q$ where $a, p, q \in \mathbb{Z}$ [4]

(i) $f(x)$

(ii) $g(x)$

(b) Describe a series of transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$. [4]

The graph of $y = g(x)$ is translated k units upwards so that it intersects with the graph of $y = f(x)$ at least once.

(c) Show that $k \geq \dfrac{20}{3}$. [5]

(d) If the translated graph intersects with the graph of $y = f(x)$ exactly once find the x-coordinate of the point of intersection. [4]

10. [Maximum points: 8]

Let $f(x) = x^3 - 9x^2 + 27x - 27$ and $g(x) = \ln x$.

(a) Given that $f(3) = 0$ factorise $f(x)$. [2]

(b) Find the domain of [3]

(i) $g(x)$

(ii) $(g \circ f)(x)$

(c) Describe a series of transformations that maps the graph of $y = g(x)$ onto the graph of $y = (g \circ f)(x)$. [3]

11. [Maximum points: 8]

(a) Expand $(x+2)^3$. [3]

Let $f(x) = \ln\left((x+2)^2\right)$, $g(x) = \ln(x+2)$ and $h(x) = f(x) + g(x)$.

(b) Find $h(x)$ in the form $h(x) = \ln\left(ax^3 + bx^2 + cx + d\right)$ where $a, b, c, d \in \mathbb{Z}$. [2]

(c) Find a series of transformations that maps the graph of $y = \ln x$ onto the graph of $y = h(x)$. [3]

12. [Maximum points: 8]

(a) Solve the equation $4^x = 8$. [2]

The graph of $y = 4^x$ is stretched vertically by a factor of 8.

(b) Find the equation of the resulting graph. [2]

(c) Find a different transformation that can be applied to the graph of $y = 4^x$ that results in the same graph. [4]

13. [Maximum points: 6]

The graph of $y = 2^x$ is translated 3 units to the right.

(a) Find the equation of the resulting graph. [2]

(b) Find a different transformation that can be applied to $y = 2^x$ will result in the same graph. [4]

14. [Maximum points: 5]

Let $f(x) = 2x^3 - 4x^2 - 3x + 1$.

(a) Find the sum of the zeros of $f(x)$. [2]

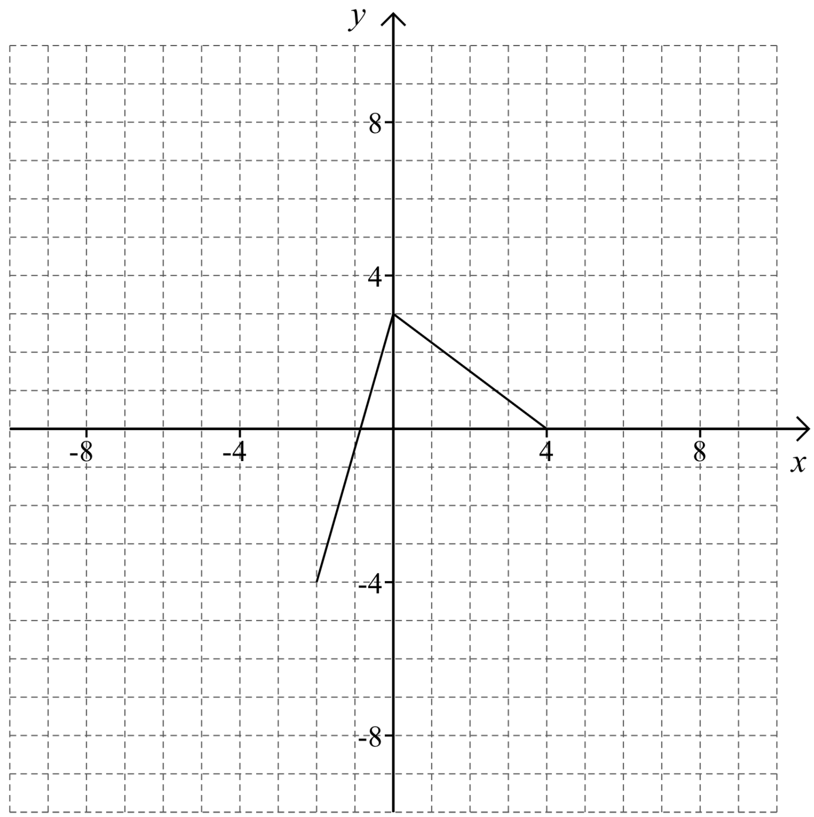
(b) Find the sum of the zeroes of $f(x + 4)$. [3]

15. [Maximum points: 6]

Find a single transformation that maps the graph of $y = 4^x$ onto the graph of $y = 7^x$.

16. [Maximum points: 4]

The graph below shows the function $y = f(x)$.



On the same set of axes draw the graph of $y = -f\left(\dfrac{x}{2} + 3\right)$.

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17. [Maximum points: 7]

Let $f(x) = x^3 - 5x^2 - 2x + 10$.

(a) Find the remainder when $f(x)$ is divided by $x + 2$. [2]

(b) Hence find a single transformation that can be applied to the graph of $y = f(x)$ so that the resulting graph has an x-intercept at $(-2,0)$. [2]

(c) Find the other two x-intercepts of the transformed graph. [3]

18. [Maximum points: 7]

Let $f(x) = x^3 - 3x^2 + 4x + 1$.

(a) Find the remainder when $f(x)$ is divided by $x + 1$. [2]

(b) Hence find a single transformation that can be applied to the graph of $y = f(x)$ so that the resulting graph has an x-intercept at $(-1,0)$. [2]

(c) Show that the transformed graph has no more x-intercepts. [3]

19. [Maximum points: 6]

(a) Solve the equation $4 = 3^c$ for c. [2]

(b) Hence find a transformation that maps the graph of $y = 3^x$ onto the graph of $y = 4^x$. [4]

20. [Maximum points: 20]

Let $f(x) = x^2 + 2x - 3$ and $g(x) = -x^2 + 8x - 12$.

(a) Write the following functions in the form $a(x-p)^2 + q$ where a, p and q are integers to be determined. [4]

(i) $f(x)$

(ii) $g(x)$

(b) Determine the x-intercepts of the graph of [4]

(i) $y = f(x)$

(ii) $y = g(x)$

(c) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same set of axes. The scale on both the x-axis and the y-axis should be from -10 to 10. [6]

(d) A vertical translation is applied to the graph of $y = f(x)$ resulting in it touching the graph of $y = g(x)$ at least once. Comment on this translation. [6]

21. [Maximum points: 5]

The parabola $y = x^2 - 3x + 4$ is translated 2 units to the left, reflected in the y-axis and stretched horizontally by a factor of 1/2. Determine the equation of the resulting parabola in the form $y = ax^2 + bx + c$ where a, b and c are integers to be determined.

22. [Maximum points: 6]

Describe a set of transformations that maps the parabola $y = x^2 + 4x - 1$ onto the parabola $y = 3x^2 + 9x + 2$.

23. [Maximum points: 7]

Consider the parabola $y = 2x^2 + 4x + 5$.

(a) Show that the parabola does not intersect the x-axis. [3]

(b) The parabola is translated so that it intersects the x-axis twice. Comment on this translation. [4]

1. (a)

(i) $x = 2$ A1

(ii) Since

$$f(x) = \frac{1 + \frac{1}{x}}{1 - \frac{2}{x}}$$ M1

The equation is

$$y = 1$$ A1

(b) We have

$$g(x) = \frac{x+1}{x-2} + 3 = \frac{x+1+3(x-2)}{x-2} = \frac{4x-5}{x-2}$$ M1A1A1

(c) Reflect the graph of $y = g(x)$ in the x-axis and translate 3 units downwards. M1

So we have

$$y = \frac{5-4x}{x-2} - 3 = \frac{11-7x}{x-2}$$ A1A1

2. We have

$$g(x) = (x-a)^3 - 2(x-a)^2 + 1 \qquad \text{A1}$$

Use binomial expansion. The coefficient of x is therefore M1

$$^3\mathrm{C}_2 a^2 + 4a \qquad \text{A1}$$

So we have

$$3a^2 + 4a = 64 \qquad \text{A1}$$

Therefore

$$(3a+16)(a-4) = 0 \qquad \text{M1}$$

So

$$a = 4 \qquad \text{A1}$$

3. (a) We have

$$h(x) = \log_4\left(8^{x+2}\right) = (x+2)\log_4 8 \qquad \text{M1A1}$$

Since $\log_4 8 = 3/2$ this simplifies to M1

$$h(x) = \frac{3x}{2} + 3 \qquad \text{A1}$$

(b) A horizontal stretch by a factor of 2/3 followed by a translation of 3 units upwards. A1A1

OR

A vertical stretch by a factor of 3/2 followed by a translation of 3 units upwards.

4. (a) We have

$$g(x) = 3\left(\frac{x}{9}\right) + 1 - \frac{4}{3} = \frac{3x-3}{9} = \frac{x-1}{3}$$ M1A1A1

(b) We have

$$x = 3y + 1$$ M1

So

$$f^{-1}(x) = \frac{x-1}{3}$$ A1

(c) A reflection in the line $y = x$. A1

5. We have

$$(g \circ f)(x) = \mathrm{e}^{\ln x + 2} = \mathrm{e}^2 \mathrm{e}^{\ln x} = \mathrm{e}^2 x \qquad \text{M1A1A1}$$

The transformation is a vertical stretch by a factor of e^2. A1A1

6. (a) $\frac{1}{(4^x)^2} = \frac{1}{4^{2x}} = 4^{-2x}$ M1A1A1

(b) A reflection in the y-axis and a horizontal stretch by a factor of 1/2. A1A1A1

7. (a) We have

$$(h \circ g)(x) = 2^{x+3} \qquad \text{M1}$$

So, the transformation is a translation of 3 units to the left. A1

(b) We have

$$(f \circ h)(x) = (2^{x})^{1/2} = 2^{x/2} \qquad \text{M1A1}$$

So, the transformation is a horizontal stretch by a factor of 2. A1A1

8. (a) We have

$$(f \circ g)(x) = \ln x + 3 \qquad \text{A1}$$

So, a translation of 3 units up. A1

(b) We have

$$(g \circ f)(x) = \ln(x+3) \qquad \text{A1}$$

So, a translation of 3 units left. A1

(c) We have

$$(g \circ h)(x) = \ln(x^4) = 4\ln x \qquad \text{A1A1}$$

So, a vertical stretch by a factor of 4. A1

9. (a)

(i) $y = (x+3)^2 - 1$ A1A1

(ii) $y = -2(x-1)^2 + 3$ A1A1

(b) A translation of 4 units to the right. A1

A vertical stretch by a factor of 2. A1

A reflection in the x-axis. A1

A translation of 1 unit up. A1

(Note that the vertical translation can happen before the stretch if the size of the translation is 1/2, and before the reflection if the direction of the translation is down).

(c) We have

$$x^2 + 6x + 8 = -2x^2 + 4x + 1 + k$$ M1

Rearrange

$$3x^2 + 2x + 7 - k = 0$$ A1

There is at least one solution so the discriminant cannot be negative. R1

$$4 - 12(7-k) \geq 0$$ M1

So

$$k \geq \frac{20}{3}$$ A1

(d) We have

$$x^2 + 6x + 8 = -2x^2 + 4x + 1 + \frac{20}{3}$$ M1

Rearrange

$$9x^2 + 6x + 1 = 0$$ A1

Factorise

$$(3x+1)^2 = 0$$ M1

So

$$x = -\frac{1}{3}$$ A1

10. (a) We have

$$x^3 - 9x^2 + 27x - 27 = (x-3)(x^2 - 6x + 9) = (x-3)^3$$ M1A1

(b)

(i) $x > 0$ A1

(ii) Since $(\mathrm{g} \circ f)(x) = \ln\left((x-3)^3\right)$ the domain is $x > 3$. A1A1

(c) We have

$$(g \circ f)(x) = \ln\left((x-3)^3\right) = 3\ln(x-3)$$ M1

The transformations are therefore a vertical stretch by a factor of 3 and a translation to the right by 3 units. A1A1

11. (a) Use binomial expansion M1

$$x^3 + 3 \cdot 2 \cdot x^2 + 3 \cdot 2^2 \cdot x + 2^3 = x^3 + 6x^2 + 12x + 8$$ A1A1

(b) $h(x) = \ln\left((x+2)^2\right) + \ln(x+2) = \ln\left((x+2)^3\right) = x^3 + 6x^2 + 12x + 8$ M1A1

(c) We have

$$h(x) = 3\ln(x+2)$$ A1

The transformations are a vertical stretch by a factor of 3 and a translation 2 units left. A1A1

12. (a) We have

$$2^{2x} = 2^3$$ M1

So

$$x = 3/2$$ A1

(b) $y = 8 \cdot 4^x$ A1A1

(c) We have

$$8 \cdot 4^x = 4^{3/2} \cdot 4^x = 4^{x+3/2}$$ M1A1

The translation is a translation of 3/2 units to the left. A1A1

13. (a) $y = 2^{x-3}$ A1A1

(b) We have

$$y = 2^{x-3} = 2^{-3} \times 2^{x} = \frac{2^{x}}{8}$$ M1A1

The transformation is a vertical stretch by a factor of 1/8. A1A1

14. (a) $-\frac{-4}{2}=2$ M1A1

(b) The zeros will all reduce by 4. R1

So the sum will be $2-3\times 4=-10$. M1A1

15. Write one base in terms of the other M1

$$7 = 4^{\log_4 7}$$ A1

So

$$7^x = (4^{\log_4 7})^x = 4^{x \log_4 7}$$ M1A1

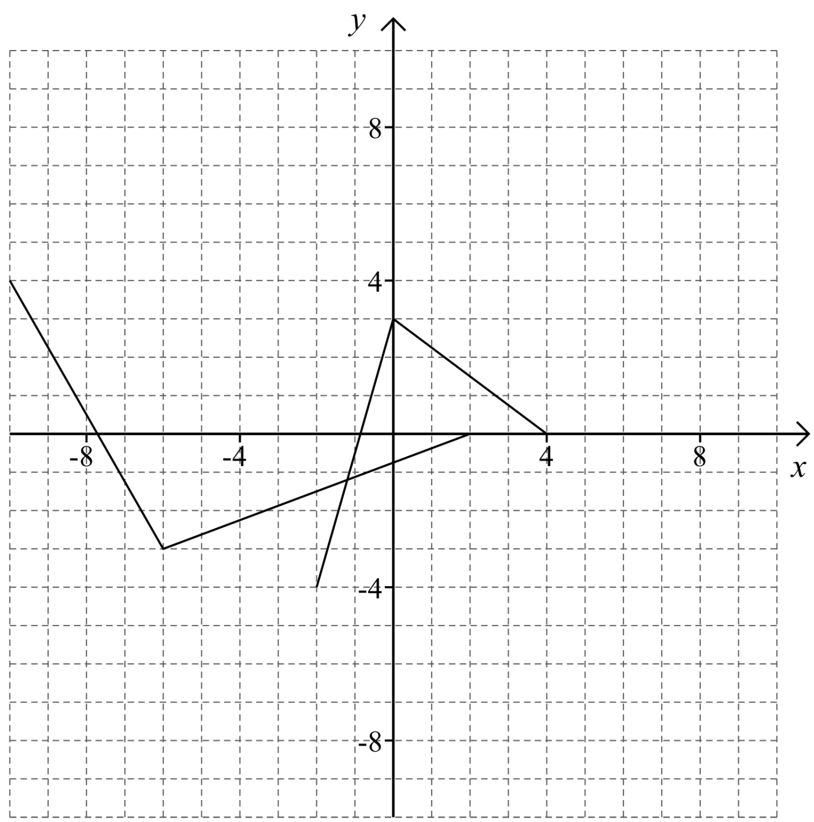
The transformation is a horizontal stretch by a factor of $\frac{1}{\log_4 7}$. A1A1

16. The point $(-2,-4)$ changes to $(-10,4)$. A1

The point $(0,3)$ changes to $(-6,-3)$. A1

The point $(4,0)$ changes to $(2,0)$. A1

The points are connected by straight lines. A1



17. (a) $(-2)^3 - 5(-2)^2 - 2(-2) + 10 = -14$ M1A1

(b) A translation of 14 units upwards. A1A1

(c) We have

$$x^3 - 5x^2 - 2x + 24 = (x+2)(x^2 - 7x + 12) = (x+2)(x-3)(x-4) \qquad \text{M1A1}$$

The other two x-intercepts are therefore $(3,0)$ and $(4,0)$. A1

18. (a) $(-1)^3 - 3(-1)^2 + 4(-1) + 1 = -7$ M1A1

(b) A translation of 7 units upwards. A1A1

(c) We have

$$x^3 - 3x^2 + 4x + 8 = (x + 1)(x^2 - 4x + 8)$$ M1

Since the discriminant of $x^2 - 4x + 8$ is negative there are no more x-intercepts. R1 A1

19. (a) $a = \log_3 4$ A1A1

(b) We have

$$4^x = (3^{\log_3 4})^x = 3^{x \log_3 4}$$ M1A1

So the transformation is a horizontal stretch by a factor of $\dfrac{1}{\log_3 4}$. A1A1

20. (a)

(i) $f(x) = (x+1)^2 - 4$ A1A1

(ii) $g(x) = -(x-4)^2 + 4$ A1A1

(b)

(i) Solve the equation $f(x) = 0$ using any method e.g.

$$(x-1)(x+3) = 0 \qquad \text{M1}$$

So $x = -3$ or 1. A1

(ii) Solve the equation $g(x) = 0$ using any method e.g.

$$-(x-4)^2 + 4 = 0 \qquad \text{M1}$$

So 2 or 6. A1

(c) The vertex of $y = f(x)$ is $(-1,-4)$. A1

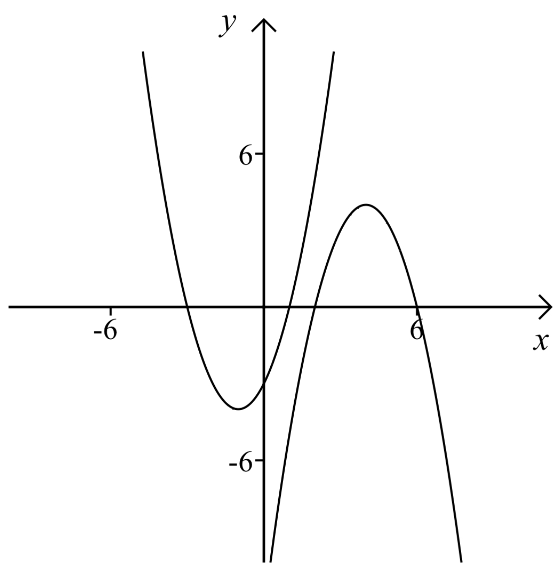
The x-intercepts are $(-3,0)$ and $(1,0)$. A1

The graph of $y = f(x)$ is concave upwards. A1

The vertex of $y = g(x)$ is $(4,4)$. A1

The x-intercepts are $(2,0)$ and $(6,0)$. A1

The graph of $y = g(x)$ is concave downwards. A1



(d) Let the graph of $y = f(x)$ be translated a units downwards. We therefore have

$$x^2 + 2x - 3 - a = -x^2 + 8x - 12$$ M1

Rearrange

$$2x^2 - 6x + 9 - a = 0$$ A1

There has to be at least one solution to this equation so

$$(-6)^2 - 4 \cdot 2 \cdot (9 - a) \geq 0$$ M1

The solution to this is $a \geq 4.5$. A1

So the translation is at least 4.5 units downwards. A1A1

21. After a translation of 2 units left we have

$$y = (x+2)^2 - 3(x+2) + 4 \qquad \text{A1}$$

After a reflection in the y-axis we have

$$y = (-x+2)^2 - 3(-x+2) + 4 \qquad \text{A1}$$

After a horizontal stretch by a factor of 1/2 we have

$$y = (-2x+2)^2 - 3(-2x+2) + 4 \qquad \text{A1}$$

Expand

$$y = 4x^2 - 8x + 4 + 6x - 6 + 4 \qquad \text{M1}$$

Simplify

$$y = 4x^2 - 2x + 2 \qquad \text{A1}$$

22. Write each parabola in the form $y = a(x-p)^2 + q$.

So we have

$$y = (x+2)^2 - 5 \qquad \text{A1}$$

and

$$y = 3\left(x + \frac{3}{2}\right)^2 - \frac{19}{4} \qquad \text{A1}$$

So we need a vertical stretch by a factor of 3. A1

A translation to the right by 1/2. A1

(The equation will then be $y = 3\left(x + \frac{3}{2}\right)^2 - 15$). R1

So we need a translation up by $\frac{41}{4}$. A1

23. (a) If we wish to determine any x-intercepts then we need to solve the equation

$$0 = 2x^2 + 4x + 5$$ A1

Calculate the value of the discriminant. M1

$$4^2 - 4 \times 2 \times 5 = -24$$

Since the value is negative there are no solutions to the equation so the parabola does not intersect with the x-axis. A1

(b) Rewrite the equation of the parabola. M1

$$y = 2(x+1)^2 + 3$$

So the vertex is at $(-1,3)$. A1

The translation must be more than 3 units downwards. A1A1