Teaching about Confidence Intervals: How Instructors Connect Ideas Using Speech and Gesture

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Abstract: This work investigated how university-level statistics instructors use gestures along with speech to connect ideas when teaching about confidence intervals. We identified segments of classroom discourse in which instructors connected ideas, and coded the modalities they used to communicate those links. Most linked ideas were expressed multi-modally, typically in speech and gesture.

Introduction

Statistical reasoning is a critical skill for many career paths in STEM fields, and it is also essential in many other aspects of life for citizens in modern society. As in any mathematical domain, one hallmark of conceptual understanding in statistics is understanding *connections* among ideas. Course work in statistics at the high school and college levels seeks to foster students' building a rich, interconnected knowledge base to support their reasoning and problem solving.

But, what factors affect whether students grasp connections among ideas? Recent research has targeted teachers' instructional communication as one important factor. In mathematics and statistics instruction, teachers frequently highlight relationships among inscriptions, concepts, and procedures. In doing so, teachers typically use speech, and along with that speech they often produce spontaneous *gestures*, defined as movements of the hands and arms that are part of speaking. Teachers' gestures contribute to instructional communication in a variety of ways; they express emphasis, guide students' attention to features of the instructional context, and convey information (Alibali, Nathan & Fujimori, 2011). Gestures are ubiquitous in instructional communication, and research on gesture in instructional settings, including both naturalistic and experimental settings, is burgeoning (e.g., Flevares & Perry, 2001; Goldin-Meadow, Kim & Singer, 1999; Rasmussen, Stephan & Allen, 2004; Richland, Zur & Holyoak, 2007; Roth, 2001).

In this study, we investigate how university-level statistics instructors use gestures along with speech to connect ideas in introductory statistics courses. In particular, we focus on instruction about *confidence intervals* (CIs). There have been many calls for an increased focus on confidence intervals, both in statistics instruction and in reporting of research findings (e.g., APA, 2009). However, CIs are difficult to understand, and statistics students often hold misconceptions about CIs (e.g., Garfield & Ben Zvi, 2008). The research presented in this paper highlights potentially effective ways to make conceptual connections about CIs using speech and gesture.

Students' Difficulties with Confidence Intervals

Undergraduate students perform poorly on assessments of CI understanding (e.g., Castro Sotos, Vanhoof, Van den Noortgate & Onghena, 2007). Additionally, knowledge of CIs does not improve significantly over the course of a typical introductory statistics course (e.g., Delmas, Ooms, & Chance, 2007). Difficulties with CIs persist even beyond college; researchers in many domains have trouble making inferences from CIs (Cumming, Williams, & Fidler, 2004), and often incorrectly interpret CIs (e.g., Coulson, Healey, Fidler & Cumming, 2010). One reason for these difficulties may be inadequate or incorrect links between CIs and other statistical concepts and representations (e.g., Cumming et al., 2004).

Teachers Use Gestures to Connect Ideas in Instructional Communication

In this work, we explore how statistics instructors connect ideas in lessons about CIs. We focus not only on the connections that instructors make verbally, but also on the connections they make using spontaneous gestures. A large body of research demonstrates that gestures contribute to communication (for reviews, see Hostetter, 2011; Kendon, 1994). Moreover, experimental studies suggest that students benefit more from lessons with gestures than from lessons without gestures (e.g., Church, Ayman-Nolley & Mahootian, 2001; Cook, Duffy & Fenn, 2013; Singer & Goldin-Meadow, 2005; Valenzeno, Alibali & Klatzky, 2003). Students demonstrate better uptake of instructional information, more generation of new forms of reasoning, more generalization to new problem types, and greater retention of knowledge from lessons with gestures. Thus, teachers' gestures can have a substantial influence on students' learning. There is also evidence that teachers regularly express links between ideas multi-modally. In a recent analysis of 18 middle school mathematics lessons, drawn from 6 teachers, it was reported that when teachers linked ideas, they expressed both (or all) of the linked ideas multi-modally in 90% of cases (range 65%-100%) (Alibali, et al., 2014).

Research Questions

The primary aim of this research was to characterize how statistics instructors connect ideas during instruction about CIs. We investigated whether instructors typically used multiple modalities (e.g., speech, gesture or writing/drawing) to express the linked ideas, or whether they sometimes expressed linked ideas in a single modality. In light of past research showing an abundance of multi-modal linking in mathematics lessons (Alibali, et al., 2014), we hypothesized that instructors would tend to express linked ideas multi-modally.

Note that we construed the notion of "linked ideas" broadly, to include inscriptions that are traditionally used in statistics (e.g., equations, graphs of distributions), and also to include verbal expressions and pictorial and gestural depictions of mathematical or statistical entities (e.g., a diagram drawn on the board or in the air). We focused not only on links across different external representations (e.g., between an equation and graph), but also on links within a single representation (e.g., links between two elements within the same graph).

A secondary aim of this research was to investigate the range of ways instructors used gestures to connect ideas. We focused on the *types* of gestures (in particular, pointing vs. depictive gestures). People often use gestures to indicate objects, inscriptions, or locations in the physical world via pointing. They also commonly use gestures to depict or represent actions, objects or events. We predicted that instructors would use both of these types of gestures in communicating key ideas in statistics instruction; for example, they might point to elements of a graph when referring to those elements, or they might represent the boundaries of a CI using depictive gestures, to evoke a mental image of an interval in their students' minds.

In focusing on teacher's communicative behaviors in statistics instruction, this work addresses authentic practices in a key domain of student learning. Past research suggests that teachers use gestures to help scaffold students' understanding, particularly for material that is challenging for students (e.g., Alibali, et al., 2013). We suggest that better understanding of instructor practices in connecting ideas will yield insights into how best to organize such practices for optimal student learning.

Methods

We videotaped four university-level instructors as they taught about confidence intervals in introductory statistics courses. Three of the instructors were employed at a large public university in a mid-size Midwestern US city, and one instructor was employed at a mid-sized public university in a large Midwestern US city. The videotapes were transcribed and analyzed using Transana video analysis software. Although other parts of the lessons were viewed for context, for the purposes of this study, only segments of the lessons explicitly addressing CIs were analyzed. Since different instructors organized the lessons in different ways, the overall duration of the CI portions varied. We recorded the word count and length of each of these sections.

Identifying Linking Episodes

To analyze the data, one member of the research team first viewed the video segments and wrote narrative descriptions of what occurred. Next, this researcher and a second member of the research team went through the transcripts and video (using Transana) and identified *linking episodes*. Linking episodes were defined as segments of discourse in which instructors explicitly made connections between two or more different ideas or representations. These ideas or representations included representations of key statistical concepts, such as *confidence interval*, *sample mean*, and *population distribution*. A linking episode was a segment of discourse in which links between such statistical representations were set up or described. Within each linking episode, the researchers identified the *target link(s)*. A *target link* was the portion of the discourse in which the link between the ideas or representations was directly expressed. Some linking episodes contained more than one target link, either because the instructor explicitly stated the link multiple times, or because the instructor stated links among multiple ideas or representations in varying subsets (e.g., for a linking episode that connected ideas A, B, and C, the instructor might first state the target link between A and B, and then that between B and C, for a total of two target links).

The two researchers worked together to come to consensus on the portions of the lesson that were considered linking episodes and which segments within those episodes were considered target links. Three potential linking episodes were excluded from analysis because the researchers could not come to consensus, either that a particular episode contained a link or that a segment of a broader linking episode was a target link.

Identifying the Modalities in which Linked Ideas Were Expressed

We recorded the modalities instructors used to express each of the linked ideas in each target link. Each linked idea could be expressed in speech, gesture, writing or drawing, or in multiple modalities. Gestures were defined as movements of the hands or arms that co-occurred with speech (e.g., spreading both hands apart while saying “the confidence interval is a range”). For target links that contained at least one idea expressed in gesture, we coded the gestures using an adaptation of McNeill’s (1992) gesture coding scheme. Gestures were classified as points, depictive gestures, or writing gestures. Points were defined as hand movements that indicated a referent, usually with the index finger. Depictive gestures were defined as gestures that represented semantic features of the referents via handshape or motion trajectory (e.g., holding both hands apart with palms facing one another to show the range of a CI). Writing gestures occurred when the instructor made markings on the board *while speaking* that either directed attention to particular piece of information (e.g., using chalk to mark a dot repeatedly on the symbol μ) or emphasized the relationship between two or more things that were on the board (e.g., drawing a line from the mean (μ) to the upper or lower limit of the CI). We included writing gestures as a separate category because these actions seemed similar to hand gestures that occurred with speech and qualitatively different from writing to put content (e.g., equations or figures) on the board.

Results

How Many Links Did Instructors Produce?

There was substantial variability across instructors, both in the amount of time they spent on CIs and in the number of links they produced during instruction on CIs. Time spent on CIs ranged from 16:54 (min:sec) to 37:59, with an average of 25:03. The number of links expressed ranged from 5 to 27 links, with an average of 15.5. Because different instructors spent different amounts of time on CIs, we also consider these findings in terms of the rate of links per 10 minutes of instruction. The instructors produced from 2.96 to 11.70 links per 10 minutes of instruction, with an average of 6.57. By definition, each link connected two or more ideas; some links incorporated as many as 5 distinct ideas. Averaged across instructors, the mean number of ideas expressed per link was 2.68 (range 2.5 – 3.0).

Did Instructors Express Most Links Multi-Modally?

We next examined whether instructors tended to use multiple modalities to express the linked ideas, or whether they sometimes expressed linked ideas in a single modality. Given past findings on multi-modal linking in middle-school teachers (Alibali, et al., 2014), we hypothesized that statistics instructors would tend to express linked ideas multi-modally. Table 1 presents the distribution of links for each instructor, classified according to the modalities in which each of the linked ideas was expressed. The upper portion of the table includes links in which at least one of the linked ideas was expressed in a single modality, and the lower portion includes links in which both (or all) of the linked ideas were expressed multi-modally. Overall, in 55% of links, *all* of the linked ideas were expressed multi-modally. Note that this value is much lower than the comparable value of 90% for the middle-school lessons (Alibali, et al., 2014). There was substantial variation across instructors in the proportion of links they expressed multi-modally, from a low of 40% to a high of 100% (mean = 62%).

Table 1: Target links produced by Instructors A through D, classified in terms of the modalities in which the linked ideas were expressed.

Modalities in which linked ideas expressed	*A*	*B*	*C*	*D*
Both/all in speech alone	0	2	0	0
All in either speech alone or gesture alone	1	0	0	0
Speech alone, gesture alone *and* speech+gesture	0	1	0	0
Speech alone *and* speech+gesture	11	6	0	2
Speech alone *and* speech+gesture *and* speech+writing *or* speech+gesture+writing	0	2	0	0
Gesture alone *and* speech+gesture *or* speech+gesture+writing	2	1	0	0
Total uni-modal links	**14**	**12**	**0**	**2**
Both/all in speech+writing	1	2	1	0
Both/all in speech+gesture	10	4	5	2
Speech+gesture *and* speech+writing	2	2	0	1
Speech+gesture *and* speech+gesture+writing	0	0	4	0
Total multi-modal links	**13**	**8**	**10**	**3**

What Types of Gestures Did Instructors Use to Express Linked Ideas?

Instructors used different types of gestures to express linked ideas. Depictive gestures were most common overall (58% of the 253 coded gestures), followed by points (32%) and writing gestures (10%). Instructors often mixed gesture types within the same link; 45% of target links included gestures of multiple types, while 23% included pointing gestures only, 19% included depictive gestures only, and 3% included writing gestures only. The four instructors varied in their gesture patterns; one produced more than four times as many pointing gestures than depictive gestures, while the other instructors produced more depictive gestures than pointing gestures. These differences in gestural styles could be related to individual differences in the instructors' use of other visual representations in instruction.

As an example of a linking episode in which the linked ideas were expressed in depictive gestures, consider Figure 1, which presents a target link in which the instructor connected point estimates to interval estimates. The specific words that accompanied each individual gesture are marked in square brackets in the utterances quoted below. Just prior to this segment, the class had worked through an example in which the mean value was 21.73. As the instructor began the discussion of CIs, he explained that the point estimate was this mean, saying, "And so our *point estimate* in this case is going to be the mean of the sample. So that's where you're going to start with the confidence interval, giving that [point estimate]. That's your best guess." With this utterance, he put his fingers (of both hands) together in front of him, representing a point (Figure 1, first panel). He went on to note, however, that the mean is an overly precise estimate, and that another sample would not necessarily yield the same value (21.73) again. Then he introduced the idea of a CI, saying, "And so what we do with the confidence interval [is just to widen that], [make] a little bracket. And that becomes our confidence interval." With this utterance, he simultaneously extended both hands downward at 45-degree angles, ending by representing an interval "bracket" (Figure 1, second and third panels). Thus, in this example, the instructor linked point estimates and interval estimates using depictive gestures that expressed each concept, and that also expressed the notion of *widening*. This series of gestures could help students understand the quantitative relations between point and interval estimates, by presenting a visuospatial representation of the key ideas being linked. The visual representations the instructor expressed in gestures could clarify the verbal labels and support students' understanding of how point estimates and interval estimates are related.

This instructor returned repeatedly to the idea captured by these gestures throughout the lesson, and in fact, he repeated this basic set of gestures three additional times, producing what McNeill and Duncan (2000) termed a gestural *catchment* – a set of gestures in which some physical features of the gestures, or the entire gestures, are repeated. Catchments function to increase cohesion of the discourse—in, this case, perhaps, to foster a cohesive understanding of the relations between point estimates and interval estimates.

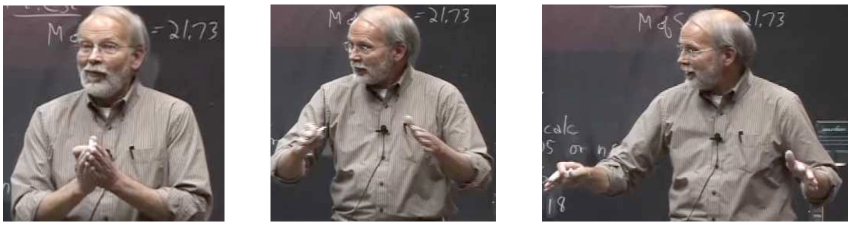

Figure 1. Target link in which both linked ideas (point estimate and interval estimate) were expressed in depictive gestures.

Discussion

Statistical knowledge is indispensable in many ways in modern society, from helping researchers in the learning sciences to conduct sound studies, to making culturally and politically responsible citizens. Among statistical topics, CIs are of central importance, but students often have misconceptions about them. This research bridges statistics education, embodied cognition, and gesture studies, and in so doing contributes to a broader perspective on teaching practices, and a specific focus on aspects of such practices that might affect student learning. Specifically, this work documents that instructors use gesture, as well as speech and writing, to connect ideas multi-modally in instruction on CIs.

Past research in other mathematical domains has suggested that teachers' gestures serve to *scaffold* students' understanding of central concepts in the lessons (e.g., Alibali et al., 2013). One reason gestures may support students' comprehension and learning of instructional material is because they offer an alternative, visuo-spatial format for expressing ideas. Statistics involves many visuo-spatial representations (such as number lines and graphs of distributions) so gesture is a potentially valuable communicative modality in this domain.

Although the statistics instructors in this study expressed a majority of links multi-modally, they used many fewer multi-modal links than the middle school teachers studied in previous work (55% of links for statistics instructors vs. 90% for middle school teachers) (Alibali, et al., 2014). Many factors might account for this difference. Younger students may benefit more from teachers' gestures than older students (Hostetter, 2011), and teachers may recognize this implicitly and adjust their use of gestures to match their students' needs. Alternatively, differences in content may account for some of the difference.

In sum, we have shown that gesture is pervasive in college-level instruction on CIs, and that it is used to convey important information about connections among ideas. This work paves the way for future studies that will test the implications of variations in instructors' linking for students' learning about CIs. It is our hope that ultimately, this line of inquiry will lead to empirically based recommendations regarding effective modes of communicating links among ideas in this important STEM domain.

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