Exploring Relevant Problem-Solving Processes in Learning From Productive Failure

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Abstract: Productive Failure (PF) is an instructional design which uses a problem-solving phase to support the acquisition of conceptual knowledge from a following instruction phase. By activating prior knowledge and raising awareness of knowledge gaps, the PF problem-solving phase is assumed to help students acquire conceptual knowledge. However, it is still unclear which specific problem-solving processes prepare learning from the subsequent instruction. To explore the role of problem-solving behaviours during the initial phase of PF, the process data of 24 participants of a quasi-experimental study were analysed. We hypothesised that evaluating their own problem-solving during the initial phase of PF facilitates students' awareness of their knowledge gaps and, thus, is associated with learning from the subsequent instruction. However, our data analyses did not support this hypothesis. Further explorations showed the importance of students achieving some understanding of the goal state of the problem they solve in the initial phase of PF.

Theoretical background

Previous research has already provided a large amount of evidence that the instructional design of productive failure (PF) facilitates conceptual knowledge acquisition in the mathematical domain (cf. Kapur 2012, Kapur & Bielaczyc, 2012). In order to support learning of previously unknown content, PF uses a problem-solving phase to prepare the students for a subsequent instruction phase. This paper focuses on the preparatory mechanisms involved in the initial problem-solving phase of PF. During the problem-solving phase, students are instructed to generate as many solutions to a problem as possible (Kapur & Bielaczyc, 2012). It is assumed that by this, students activate relevant prior knowledge (Kapur, 2012; Loibl, Roll & Rummel, 2017). However, as the students do not have sufficient prior knowledge for solving the given problem, their solutions are mostly flawed or incomplete (Kapur & Bielaczyc, 2012). Due to the erroneousness of the students' solutions and their lack of prior knowledge, students might realise that their solutions were insufficient and become aware of their knowledge gaps (Loibl et al., 2017). Yet, it is unclear how exactly students become aware of their lack of knowledge. Loibl et al. (2017) presume that these preparatory mechanisms, the activation of prior knowledge and an awareness of knowledge gaps, prepare students for learning from the subsequent instruction during the problem-solving phase. In line with this assumption, a study by Loibl and Rummel (2014) reveals that the subsequent instruction becomes more effective if the students' knowledge gaps are addressed by discussing typical student solutions. Nevertheless, the premises by Loibl et al. (2017) still lack empirical evidence. However, to gain empirical evidence, firstly, it is necessary to acquire further details about specific problem-solving behaviours that are relevant for preparing learning from the subsequent instruction during the PF problem-solving phase.

The work presented in this paper therefore explores the PF problem-solving process to address the question what kinds of problem-solving behaviours prepare students for learning from the following instruction phase. For a theoretical foundation, research on problem solving is reconsidered and placed into the context of PF research. Our aim was to explore the theoretical assumptions by Loibl et al. (2017) in further detail to generate more insights into the relevance of the problem-solving process in PF.

Problem solving is defined as a process in which a problem is solved by finding an operator that can reduce the previously examined gap between the current state of a problem and its expected goal state (Sweller, 1998). In PF, the goal state represents the canonical solution of the given mathematical problem. In this study, the mathematical problem deals with the concept of variance, i.e. the reliability of the performance of football players. Therefore, the canonical solution and goal state of this problem is the formula for standard deviation. However, as the students face a completely unknown content (Loibl & Rummel, 2014), they cannot be certain which kind of goal state is to be expected, nor can they be sure that the operators they might be able to apply to the problem are appropriate. Even though students likely come up with a set of intuitive ideas that might guide the problem-solving process (cf. Jonasson, 2000), this lack of knowledge makes it necessary to rely on further strategies in order to monitor one's own problem-solving process. Firstly, to reduce the range of possible operators, students' intuitive ideas need to be structured and formed into criteria for the desired goal state, that is, what it looks like and how it might be achievable. Based on these criteria, the problem solvers are able to select the first problem-solving operator and generate the first solution of their set of solutions to the given problem during the PF problem-solving phase. Nevertheless, as the students' prior knowledge is limited, an assessment of the quality of the generated solution is required. For this, the students need to evaluate the current state of the problem produced by their selected operator(s). Schoenfeld (1992) emphasised the significance of evaluating one's own problem-solving behaviour for successful problem solving. However, in PF, students' problem-solving processes are generally not successful, but rather erroneous and incomplete due to the lack of prior knowledge and information at hand (Kapur, 2012). For this reason, it is likely that already while facing their solutions' results, students become aware of conflicts in their solutions. This awareness of conflicts might trigger a thorough analysis and evaluation of the own problem-solving process and the selected operator(s).

Placing these theoretical assumptions into the context of the above discussed preparatory mechanisms that are assumed to be relevant to PF, leads to hypotheses concerning specific required PF problem-solving processes: Firstly, upon receiving the problem, students might begin with a planning phase. For this, they would generate intuitive ideas for their solutions by activating relevant prior knowledge. Next, they would need to form criteria for the desired goal state of the problem in order to be able to choose from available problem-solving operators. In a following implementation phase, they would then select an operator and apply it to the problem. Subsequently, students would evaluate the outcome of their action. At this point, students might detect conflicts in their erroneous solution and possibly engage in an analysis of these. An analysis of conflicts could raise the students' awareness of their knowledge gaps and thus play an essential part in the preparation for learning from the following instruction phase (Roll, Holmes, Day & Bonn, 2012; Loibl et al., 2017). Based on this theoretical conceptualisation of specific required PF problem-solving processes we formulated three hypotheses. First, we assumed that the more solutions include conclusions drawn from the results, the more awareness of conflicts is raised and the more analyses of conflicts are performed (hypothesis 1). Secondly, we hypothesized that the higher the amount of solutions with conflict awareness and analyses, the higher the students' awareness of knowledge gaps (hypothesis 2). Lastly, we assumed that the amount of solutions including conflict analyses is associated with the students' conceptual knowledge at the end of the instruction phase (hypothesis 3).

Method

To investigate our hypotheses, we drew from the data of a quasi-experimental study (Hartmann, Rummel & Van Gog, 2017) with 75 10^{th}-graders of two secondary schools in Germany. However, this paper focusses on the data of 24 students who worked in the PF condition as described above. The study was conducted in a school setting on two different days. On the first day, the participants received a short prior-knowledge test in order to assess their knowledge on different concepts of variance. Subsequently, the students engaged in the initial phase of the study, the problem-solving phase. In this phase, the students received a task on the concept of standard deviation, which was so far unknown to them. The task demanded to find the most reliable football player amongst three players. The students received a list of each of the player's amount of goals per year over a period of ten years. They were instructed to work on this problem individually and to find as many solutions to it as possible. In order to capture the problem-solving process, the students were asked to write their solutions on tablets and to express all of their thoughts out loud. Both students' thoughts and solutions were recorded. After 45 minutes, the participants received a short test in order to measure, among other variables, their awareness of knowledge gaps by self-report with a 6-point Likert scale. On the second day of the study, the participants commenced the subsequent phase, in which they received an instruction about the concept of standard deviation. The instruction was built on typical student solutions, picked up common mistakes and misconceptions and contrasted them with the conceptual features of the canonical solution (see Loibl et al., 2017). After this, a short practice phase was implemented before the students engaged in a 40-minute post-test, which measured, besides other types of knowledge, the students' conceptual knowledge. The validity of the used tests, as well as the internal consistency was revised in Loibl and Rummel (2014) and found satisfactory. However, in this study, the internal consistency slightly decreased for the awareness of knowledge gaps (Cronbach's alpha = .71) and plummeted for conceptual knowledge (Cronbach's alpha = .41).

As a means to make central cognitive processes visible, we used think-aloud data (Ericsson & Simon, 1980) for the following process analysis. The analysis was conducted with the help of a self-constructed coding scheme. The coding scheme divides the problem-solving process into three phases: a planning phase, an implementation and an evaluation phase. Each phase describes different problem-solving behaviours based on the theoretical outline discussed earlier e.g. criteria for desired goal state (phase 1), drawing conclusions (phase 2) and conflict awareness or analysis (phase 3). Students' problem-solving behaviour was coded dichotomously

(1 = present; 0 = non-present). The coding scheme was applied to each solution of each student. As the students each generated two to seven solutions, the coding scheme was applied multiple times per person.

Findings

The coded data was analysed in three ways. First, we analysed the data descriptively. Then, for hypotheses testing we ran a series of correlations. Finally, we explored the data in order to gain more insights into the problem-solving behaviour of PF students. The descriptive analyses of the problem-solving variables revealed that all students drew conclusions in at least one of their solutions to the problem. Furthermore, for 91.7% of the participants, conflict awareness or conflict analyses could be identified in one or more solutions. However, conflict analyses were only implemented by 54.2% of the students.

For hypothesis testing, we used the rank correlation coefficient Spearman's rho, as none of the variables were normally distributed. The relationship between the variables was calculated by using the sum of solutions per person in which a specific problem-solving behaviour could be found. However, we did not find significant correlations for our four hypotheses (see Table 1).

Table 1: Hypotheses Testing (Spearman's rho; N=24)

Variable 1	**Variable 2**	r_s	**p**
Drawing conclusions	Conflict awareness & analysis	.105	ns
Conflict awareness	Awareness of knowledge gaps	-.120	ns
Conflict analysis	Conceptual knowledge	.023	ns

In the last step of the data analysis, the data was explored beyond the hypotheses of this paper. Our aim was to examine further problem-solving behaviours that could be relevant for learning in PF. For this, a series of correlation matrices was generated. As in hypothesis testing, we used the sum of solutions per person with a specific problem-solving behaviour. However, of all variables, none could be associated with awareness of knowledge gaps and only one with conceptual knowledge. This variable measured the amount of solutions that included criteria for the desired goal state of the problem. Between the variables criteria for the goal state and conceptual knowledge, we found a moderate positive correlation (r_S= .462) as well as a moderate negative relationship between criteria for the desired goal state and the number of solutions (r_S = -.446). This means that a higher number of solutions, which include criteria for the desired goal state, goes along with a fewer number of solutions. On a descriptive level, nine out of 24 participants formulated criteria in at least one solution, whereas 15 out of 24 participants did not name criteria for the desired goal state at all. Thus, this nearly presents a naturally occurring median split with almost half of the participants showing this behaviour. When comparing these two groups, it becomes evident that those students who formulated criteria for the desired goal state performed almost one point better in the conceptual knowledge test than those who did not use criteria (see Table 2 for means and standard deviations).

Table 2: Descriptive Analysis – Conceptual Knowledge of Students With/Without Criteria for the Desired Goal State

Manifestation of the variable 'criteria for desired goal state'	**n**	**M**	**SD**
No criteria for desired goal state	15	2.87	1.37
Criteria for desired goal state (in at least one solution)	9	3.83	1.30
Total	24	3.23	1.40

Discussion

Prior to the discussion of results, some limitations of the study need to be considered. Firstly, as this paper only focused on 24 participants, the small sample size has to be noted. This ultimately reduces the statistical power of the study, which may be an explanation why none of the stated hypotheses could be confirmed. Another reason for this could be the debatable internal consistency of the scale measuring conceptual knowledge. Furthermore, the gathered think-aloud data needs to be regarded with caution. As Nisbett & Wilson (1997) indicated, it is unclear to what extent students are aware of their cognitive processes and are able to verbalise them sufficiently. Thus, central cognitive processes could remain concealed in the analyses. As it is generally unclear how beneficial problem-solving processes might manifest, this ultimately also impedes the validation of the constructed coding scheme. As this study was a first attempt at taking a closer look at students problem-solving process in PF, the newly constructed coding scheme needs yet to be tested for validity and (interrater) reliability in further uses.

Even though we did not find significant results in support of our hypotheses, further data exploration presented valuable insights into the PF problem-solving process that need closer examination. The correlation between criteria for the desired goal state and conceptual knowledge as well as the clear descriptive performance differences between the participants who did and did not use identified criteria in their solutions, highlight this type of planning behaviour in PF. This finding is in line with the findings of Roll et al. (2012) and Schoenfeld (1992), who stressed the importance of planning in math problem solving. It could be assumed that the construction of criteria for the desired goal state has a central function in the problem-solving process as a means to structure it. A deeper understanding of the desired goal state might help to guide the problem-solving process – and particularly the search of an adequate problem-solving operator – despite the limited amount of prior knowledge for and information about the given problem. Furthermore, the moderate negative correlation between criteria for the desired the goal state and the number of solutions suggest the possible efficiency of posing criteria. As a high number of solutions with criteria for the desired goal state is associated with a low number of solutions but also a high conceptual knowledge score, this gives rise to the assumption that, in a structured problem-solving process, a small number of efficient solutions suffices to prepare learning from the subsequent instruction phase successfully during the problem-solving phase. This might result from a deeper processing and higher awareness of one's own solutions and their shortcomings when generating only a small amount of structured solutions, supporting the integration of new knowledge during the instruction. Though this idea stands out against current research, which presumes that a higher diversity of solutions fosters learning (Kapur & Bielacyzc, 2012), it might allow to deduce fruitful new hypotheses on PF problem solving.

Current PF research still largely neglects a thorough analysis of tangible problem-solving behaviours during the problem-solving phase. This study served as a first exploration into this area. Future research will need to further investigate the potential relevance of students' planning behaviour that relates to the problem's goal state for the PF problem-solving phase. Even though finding criteria for the desired goal state appeared in our explorative analyses to promote the acquisition of conceptual knowledge, it was not linked to an awareness of knowledge gaps. As this awareness is, however, assumed to be essential for the preparation for learning from the subsequent instruction phase (Loibl et al., 2017), this opens up further questions about the validity of the used instruments and if there might be more suitable ways to operationalise awareness of knowledge gaps. The results of this study show that, despite a theoretically founded examination of the PF problem-solving phase, it is difficult to reliably and validly assess specific problem-solving behaviours, and to identify behaviours that prepare for subsequent learning. This highlights the necessity of further research for a deeper understanding of essential prerequisites in the problem-solving phase for a successful PF effect.

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