# A Tale of Two Worlds: Using Bifocal Modeling to Find and Resolve "Discrepant Events" Between Physical Experiments and Virtual Models in Biology

Tamar Fuhrmann, Shima Salehi and Paulo Blikstein
Transformative Learning Technologies Lab, Stanford University, 520 Galvez Mall, CERAS 232, CA, USA.
tamarrf@gmail.com, salehi@stanford.edu, paulob@stanford.edu

Abstract: In this paper we demonstrate an approach to supporting students' engagement in combined physical experimentation and virtual modeling. We present a study that utilizes a scientific inquiry framework which links students' physical experimentation with their use of computer modeling in real time, which we call "Bifocal Modeling." In the case of the Bifocal Modeling activities discussed here, a group of high-school students designed computer models of bacterial growth through reference to a physical experiment they were conducting, and they were able to validate the effectiveness of their model against the results from their experiment. Our findings suggests that as students compare their virtual models with physical experiments, they encounter "discrepant events" that contradict their existing conceptions and elicit a disequilibrium. This experience of conflict encourages students to further examine their ideas and hypothesis, seek more accurate explanations of the observed natural phenomena, improving the design of their computer models.

#### Introduction

In recent years virtual experimentation has received increasing emphasis as an alternative to conducting experiments in a physical environment. Much of the research in this area has focused on the question of the advantages of virtual experiments over physical experimentation (Jaakkola & Nurmi, 2008; Klahr et al., 2007), but more recently researchers have started to examine the effects of combining the virtual and physical modalities (Zacharia et al., 2008; Jaakkola et al., 2011; Liu, 2006). Additionally, studies report that alternation between these distinct experimental modalities in the course of individual experiments can often improve learning outcomes (Zacharia, 2012; Smith, 2010; Gire et al., 2010). Zacharia and colleagues (2008, 2012) suggested that the best way to develop a framework portraying the optimal combinations of physical and virtual manipulation is to employ the learning objectives of each experiment as the criteria for blending them. Nevertheless, there are two under researched areas in this literature. The first is that most of the virtual experiments were simulated versions of a physical experiment, often mimicking the appearance of the lab equipment, with the goal of trying to make the physical and virtual experiments as similar as possible (Blikstein, 2014). The second is that the research has mostly focused on predesigned physical and virtual experimentation. Simulation tools have been among the preferred means for providing environments for virtual experiments, but the rules and models behind these simulations often remain hidden from the students. Recent advances in inquiry learning research have sought to implement activities in which virtual experimentation is supplemented by opportunities to design computer models (Mulder et al., 2011), and the aim of our work is to examine the learning outcomes of designing these computer models that are explicitly meant to be different than the physical ones, in order to promote students' critical stance towards their own models and hypothesis. The creation and critical evaluation of models are important components of scientific practice which have been increasingly recognized as a valued educational goal (Levy & Wilensky, 2008; Blikstein, 2010). Predesigned models can scaffold and direct students to attend to relevant aspects of a phenomenon, but they do not offer students opportunities to externalize and debug their models, or to evaluate their assumptions and their limitations.

Bifocal Modeling (Blikstein et. al., 2012; Blikstein, 2010, 2014) is an approach to inquiry-driven science learning that challenges students to design and compare in real time physical and virtual models in order to identify their respective differences and limitations. In these activities, students explore scientific phenomena such as heat diffusion, the properties of gases, and wave propagation by conducting physical experiments, designing virtual models, and connecting the experiments with the models in real time through iterative comparisons with empirical data. During the physical phase of the process, students design and develop their physical experiment, and they run the experiment while collecting data with embedded sensors or a time lapse camera. Concurrently, they design and develop a virtual model for the same phenomenon, and compare the behavior of the virtual model with their observations from their physical experiment (figure 1). When they identify a discrepancy, students have the opportunity to redesign their models and re-iterate the process.



Figure 1. Two examples of Bifocal Models made by students for gas laws (left) and Newton's cradle (right)

Depending on the nature of the phenomenon, for a bifocal activity, students can use different programming languages to implement their virtual models. A common implementation software is NetLogo (Wilensky, 1999), a free, open-source environment for scientific modeling. NetLogo models typically consist of a set of autonomous agents (such as gas particles or bacteria entities) moving through a virtual world and interacting to produce emergent outcomes. Students define the variables for both the agents and their worlds, and specify sets of rules for agent-level behavior, such as, "if two gas particles collide, they exchange energy and bounce off each other." The students' goal is to build a model with a behavior that matches or imitates the physical data they collect. Through the comparison of the design of the virtual model and their experimental data, the students engage in the discovery of discrepancies between the results of each modality. Piaget (1985) argued that to foster conceptual change students must be confronted with "discrepant events" that contradict their conceptions and invoke a "disequilibration or cognitive conflict". Following the forms of equilibration in Piaget's theory, researchers (Hewson, 1988) identified two distinct types of cognitive conflicts: the conflict between the internal and external worlds of a student's conception and experiences, and the purely conceptual conflict between two different cognitive structures related to the same phenomenon. In our study, we found that when students were instructed to design a virtual model that imitates the bacterial growth curve, they used their previously acquired knowledge about the curve and the physical appearance of the bacteria in the Petri dish as reference patterns to indicate what their model should generate. When the virtual model data did not match the observed data, they were confronted by the discrepancy between the physical and conceptual worlds which led to the conceptual mismatch between two cognitive structures related to the same phenomenon.

#### **Bifocal Modeling Framework**

Given that Bifocal Modeling comprises many different tools and techniques, there are multiple possibilities for classroom implementation of each modality. To structure our studies, we divided the physical and the virtual assignments into a sequence of shorter activities (Figure 2):

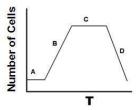


Figure 2. General structure of a Bifocal Modeling activity (Blikstein et al., 2012)

- a. *Design*: Students select research question, plan their observation, generate hypotheses, and design experiments and virtual models that will potentially confirm them. In designing the virtual model, students define variables, and conceptualize micro-rules or equations to describe the phenomenon.
- b. Construct: Students structure both their physical experiment and virtual model.
- c. *Interact*: Students interact with their experiments through direct observation or embedded sensors/cameras, and interact with their computer models by changing parameters, and recording data.

# **Subject Matter of the Activity: Bacterial Growth**

We chose bacterial growth for their simple cellular structure, rapid reproduction, and complex ecological dynamics. The goal was for students to recognize the four distinctive patterns of the bacterial growth curve (Figure 3), understand the variables underlying them, and explain the underlying mechanisms of each stage.



<u>Figure 3</u>. Bacterial growth curve over time (the number of cells is in the log scale)

- A. *Lag phase*: Population remains temporarily unchanged; in this phase the bacteria are adjusting to their new environment, repressing or inducing enzyme synthesis, and initiating chromosome replication.
- B. *Log phase*: Bacteria growth proceeds by the division of bacterium into pairs in a process known as "binary fission." Exponential growth cannot continue indefinitely because the medium is soon depleted of nutrients, which are replaced by waste products.
- C. Stationary phase: The population remains constant because the bacterial growth rate is equivalent to the death rate.
- D. Death phase: In this final stage, the bacteria have exhausted their nutrients, lose their ability to divide, and die off. As in the rapid growth phase, the decay pattern characterizing the death phase is exponential (1)

#### **Methods**

## **Audience and Resources**

The study was conducted with 13 students (4 females and 9 males) and lasted for a total of three days in a university laboratory setting. Students came from a 70% minority high school located in a predominantly Latino urban setting and volunteered for a 4-week, 30 hour/week digital fabrication workshop at a local university. This workshop took place during the school's intersession, during which all students were required to enroll in a month-long extracurricular course or internship outside of school. The selection of students was governed by a complex allocation system developed by the school; consequently, since not all students ended up being able to enroll in their preferred choices, the final group was rather diverse in terms of school achievement. All students were videotaped and recorded during all activities, their computer usage was logged with screen-capture software, the researchers conducted informal interviews and kept field notes, and all student notes and sketches were saved. The entire data (15 hours of video recordings) was analyzed by the researchers following transcriptions. Episodes explicitly showing moments of comparison between the virtual model and the real experimentation were the focus of this study. In these episodes, in order to reveal the discrepancies, we analyzed the content and the context of the situations in the videos to identify iterative moments of comparison.

## **Instructional Sequence**

The total time designated for the activity was 15 hours, divided across four smaller activities (Figure 4):

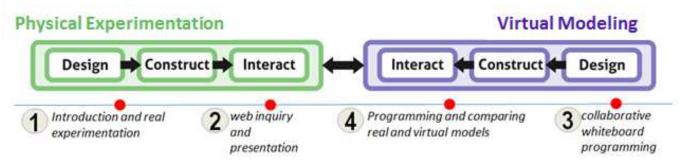


Figure 4. Bacteria Growth-Bifocal Modeling – the four activities

- 1. Introduction and physical experimentation. After the class was given an introduction to bacterial growth, the students' had to grow real bacteria. They prepared a Petri dish with agar and collected a bacteria sample from an object likely to be contaminated (e.g., a door knob, keyboard, toilet.). They were provided a time-lapse camera to capture images at 30-minute intervals over seven days. The images were automatically compiled into a video. In response to time restrictions, we also showed the students a video of a bacterial growth experiment conducted previously by the research team in the same lab with the same toolkit.
- 2. Web inquiry and presentation. Students were grouped into pairs and asked to make a list of questions about bacteria and bacterial growth. They were asked to conduct web research to answer their questions, and presented their information to the entire class in short slideshows.
- 3. Collaborative "whiteboard programming." Students were divided into three groups, each group with a dedicated facilitator from the research team. The task for the groups was to come up with rules that govern bacteria growth. First, students listed all variables that they thought would affect bacterial growth. Next, the facilitator proposed the iterative building of a block-based "computer" program on the whiteboard (figure 5), in which the students should generate the main stages in bacterial growth, as well as account for how each stage would develop, and how the variables would interact. After three hours of "whiteboard modeling," the groups split, and shared their ideas with two members from the

- other groups in a 45-minute discussion panel. After receiving feedback on their initial ideas from members of other groups, the groups reconvened and began programming their virtual experiment in NetLogo.
- 4. Programming and comparing experiments and virtual models. In this last phase, the facilitator would sit before the each group in front of a large television used for displaying code; the facilitator's role here was to "translate" the ideas of the students into NetLogo code. These last three hours of the study were dedicated to coding the students' virtual model, and comparing the coding results with the data collected from the experiment in the Petri dish. Students discussed the results, developed hypotheses for approaches to the validation of both models, and made changes to the virtual model in order to make it similar to the real bacterial growth observed in the Petri dish.

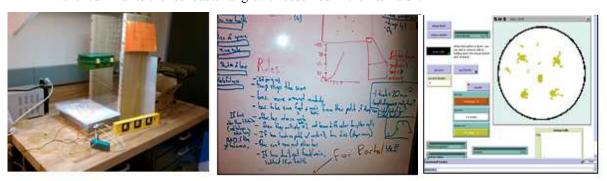


Figure 5. Physical experiment with time-lapse camera, "whiteboard modeling," and virtual model

#### **Data and Discussion**

During the unit, students created a whiteboard model; translated the whiteboard rules into a model's specifications, "ran" the model to envision how bacteria would multiply according to the model; and compared the modeled results with both the observed real Petri dish growth patterns and with the growth curve the students generated from the physical data. Finally, students refined the virtual model by adding rules and variables to address the perceived differences between the model and the experiment. In figure 6, we present a chronological list of the additions made by the students in one of the groups (4 students). This group repeated this processes a total of four times during the 1.5 hours of the final session (activity four) and, in the process; these students increased the accuracy of the model.

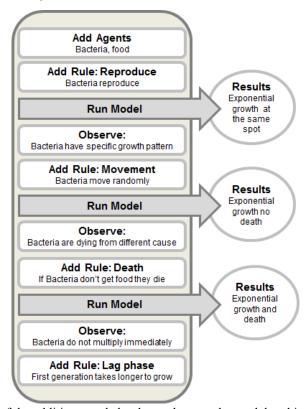


Figure 6. Chronological list of the additions made by the students to the model and instances in which they ran it

In what follows, we present three sample episodes that demonstrate this iterative process by one of the groups. This collection is presented as a representative example of the comparison moments and the discrepancy events the students encountered during their participation in the activities.

## Episode # 1: How Do Bacteria Move?

#### Context

In the first episode, students were in the initial stages of designing their virtual model on the whiteboard. They decided to add agents such as bacteria and food, as well as a rule regarding reproduction. However, they remained uncertain about the mechanism for bacterial motion: do bacteria move at all? Do they move toward food? Do they move randomly? How do they actually move? Do they recognize one another?

Following is students' design process which we broke down into three phases for the purposes of this analysis: design, observe, and revise.

## **Design Process**

- Design: as a first step, students made their virtual bacteria reproduce. 'Running' this model resulted in exponential growth. Each tick (time step in NetLogo) resulted in an increased bacteria population confined to its original location on the virtual Petri dish.
- Observe: while corroborating the virtual model with the experiment in the Petri dish, students observed a mismatch; they saw that in the Petri dish, real bacteria colonies did not resemble the virtual colonies of their model. In the experiment bacteria appear to have a specific and unique growth pattern. They do not grow on top of each other; rather they spread throughout the dish. At this point students asked questions and sought to explain the phenomenon. How do bacteria's unique patterns develop?
- Revise: students added a new rule to their virtual model, which helped them simulated bacteria moving
  randomly over the virtual Petri dish, resulting in colonies that spread across the virtual dish in a pattern
  resembling that of the real dish.

#### Excerpts from the Episode

Student 1:	Do we know if they	move around randomly	though, or —

Student 2: How else would they move around?

Student 4: Maybe in specific ways that we could understand ...

Student 1: I guess, like, where they scooted, go toward the food, but it could just do that...

Student 2: What makes you think this is fine or not? How do you know?

Student 2: I think it doesn't really matter how they move.

Instructor: Doesn't really matter, for what?

Student 2: What do you mean?

Student 1: Like, how do you know it doesn't really matter, you know? Student 2: Well, I mean, they'll eventually find food by moving randomly

#### Researcher's Observation

In this episode students face a specific conflict while comparing the virtual design results to the real colonies in the Petri dish; they discovered that bacteria (in the physical Petri dish) do not reproduce in the same location; rather, they migrate. This comparison and observation of the experimental results to the modeling suggested the idea that bacteria do not grow on top of each other. While observing the real Petri dish and confronting the mismatch with their virtual modeling results, students discussed possible mechanisms for bacterial motion in order to debug their model, which, in turn, made them seek explanations for the natural process. The discussion then progressed to physical mechanisms that might assist bacterial motion (e.g. the bacteria might be flagellates with whip like cilia at their anterior ends.)

## Episode # 2: Do Bacteria Have an Infinite Life Cycle?

#### Context

While examining and running their virtual model, the students discovered that the bacteria would never die. Students then sought to understand how to make bacteria die by manipulating their food resources. After facing this discrepancy between what was observed in their virtual modeling as opposed to the physical experimentation, they began questioning the issues of bacterial death and life cycle.

Following is students' design process which we broke down into the same three phases.

#### **Design Process**

• Design: students made their virtual bacteria move randomly and reproduce after encountering specific environmental conditions (food, water, etc.)

- Observe: It appears that the bacteria in the physical experiment are dying from different causes. Bacteria do not live forever. Students observed that in the Petri dish, the bacteria colony remained the same size for several days. Additionally, they noticed the bacterial growth curve, which includes a death phase.
- Revise: Students added the "food" variable. The corresponding rule is that when food is exhausted, and no new food resources become available, the bacteria die. The new design included bacteria that do not have unlimited lifespans.

## Excerpts from the Episode:

Student 2: Look at the death.

Student 1: Death?

Student 4: What should happen is that they run out of food.

Student 3: Okay. How should we – how should we do that? Can we make some – write some

imaginary code for that?

Student 2: Made some of the patches disappear [patches in the NetLogo code represent food]

Instructor: So can you give me more? Imagine that I'm, like, really like a dumb computer. You

need to tell me the steps I need to take. Is it, like, when all that is gone, then they all

die?

Student 1: The bacteria.

Student 4: They slowly die. They still reproduce, but they slowly die.

Student 1: Okay. And when is it, like, every tick or...?

Student 2: Every ten.

Student 3: If all patches – all 100 patches are gone, then bacteria die.

Instructor: If all food is gone, then all bacteria die. Okay. Let's run the model in our heads and

think about how we're going to do it. So all the food is gone...eventually when they run out of food, boom, they die. They all die. That's the code we have right now.

#### Researcher's Observation

In this episode the students explore the significant effect of food resources on the bacteria population. In this specific excerpt of the dialog, the students were asked to think of a way to "translate" the role of resources into code in Netlogo. They use "patches" (2) as "bacteria's food", and explained that when all food is gone, the bacteria die.

## Episode # 3: Is There a Lag Phase in Bacteria Growth Pattern?

## Context

At one point after "running" the virtual model, a student observed a mismatch: the growth curve was increasing exponentially from the start. She noted that this finding was incorrect because the real growth curve had an initial flat "lag phase" before beginning to grow. After a long discussion between group members, the students attempted to explain the lag phase of the bacterial growth, which commenced with the inoculation of the Petri dish.

Following is students' design process:

#### **Design Process**

- *Design:* students made their virtual bacteria begin reproducing as soon as they are introduced into the Petri dish.
- Observe: In comparing with the physical experiment, students become aware of the lag time that occurs before the bacterial reproduction becomes apparent. The students discovered that it takes about five days before they are able to detect a colony on their Petri dish. The growth of the microscopic bacteria remains invisible until the population grows into the millions, at which point the colony has become sufficiently large to be visible. This discovery led the students to realize that specific conditions must be met for bacteria to reproduce.
- Revise: Students add variables and rank them so their modeled bacteria will reproduce only under favorable conditions. For example, if the model's food value is greater than 10, the bacteria will reproduce. If this value is less than 10, the bacteria will first enter a lag phase.

#### Excerpts from the Episode

Instructor: What about the, I am asking again because I'm really trying to make a point here,

remember they didn't start like this in the graph? They didn't just reproduce? ... and we did it like that and we had this phase which they don't change, ... yeah. What

happen there?

Student 4: The lag?

Student 3: What is happening? Yeah, what is happening to them, the bacteria in real bacteria

dish?

Student 2: Because it takes a while for it to form and like reproduce, as soon as they get the hang

of it, they're like, yeah, to make more.

Student 3: So they get used to their, like they get used to their environment.

Student 4: Their place.

Instructor: So how can we do it in program? What do we need to add there?

Student 3: Maybe like a spurt where they're having a bunch of babies and they kind of stop

having babies, then they start having babies again. Have it slowly...

Student 4: Slowly so they won't start at the beginning?

Student 2: Yeah then they start and then they don't and then they start.

Student 1: Are you trying to make it like this?

Instructor: How can we turn this idea of the lag phase into a code?

Student 2: I guess we can use a wait about like twenty ticks oh that's a lot, a lot of wait alike ten

ticks to get used to the environment so they can just say wait... ten days before

starting to reproduce?.

Instructor: Good idea.

#### Researcher's Observation

It took time for students to realize that there is "lag phase" at the outset of the bacterial growth process. In the real Petri dish, it took five days before the students observed visible alterations and growth. However, in students' initial model bacteria grew and reproduced immediately. After the comparison between their computer model and the results of both the experiment and a bacterial growth curve, students realized that the initial stage of the physical experiment evidenced no apparent change in bacteria population. This conflict engaged the students in rethinking the phenomena they were attempting to model, and it also led them to revise their model according to their observations of the physical experiment. In order to succeed in this task, they had to find an explanation for the stable phase for inclusion in their virtual model in order to achieve a better correspondence with their observed results. In the process of generating a virtual model that better emulated the phenomena, the students added behavior parameters and behavior sequences in ways that related explicitly to real-world behaviors or included real-world constraints. In addition, they conducted similar processes to add the other relevant variables to their model.

#### Conclusion

This study demonstrates one of the main elements of the Bifocal Modeling framework: how discrepancies between a virtual model and physical experimentation can be generative. One of the main features of this framework is the explicit comparison of virtual models designed by students with physical experiment in real time. Our results suggest that the use of physical experimentation as a reference pattern in the creation and refinement of the virtual model is effective. In designing a virtual model that recreated the bacterial growth curve, students used their previously acquired knowledge about the curve and the physical cues of the bacteria colonies as an initial reference pattern, which indicated what their model should generate. When the model behaviors did not match the observed ones, students faced a discrepant event that required resolution (Piaget, 1985; Hewson, 1988). This mismatch led to debugging (Papert, 1980) and encouraged students to question their assumptions, to rethink the results, to consider alternative conceptions. In this process, students were actively engaged in hypothesis generation and testing. Throughout the entire activity, students acquired specific and detailed evidence regarding the behavior of bacteria, but the value of this evidence almost exclusively became apparent to them during their attempt to make their virtual model match their empirical observations. During their web research and physical experiment, the students took note of the fact that bacteria grow in specific patterns, do not remain in their original spot, and do not grow indefinitely. All these detailed bits of evidence would have remained inert knowledge had the students not engaged in "model matching"—and as a result, the students acknowledged this evidence opportunely when such knowledge was needed to design a more accurate

In addition, without the real-time comparison with the physical model, their virtual model would have contained a number of misconceptions. Studies report that students are capable of designing correct models (Mulder, 2011), but that they often fail to relate their knowledge of natural events to their models (e.g., Sins, Savelsbergh, & van Joolingen, 2005). This incomplete understanding of the modeling process could present serious obstacles to learning through modeling. Traditionally, researchers have tried to argue that by making virtual models very similar to physical phenomena, and thus backgrounding their differences, learners could achieve equally in both virtual and physical experimentation. Conversely, we argue that these differences should be foregrounded and made apparent to students, and that combining the virtual and physical modalities and

encouraging students to look for mismatches offers a promising way to make learning with models more effective.

What is more, the process of model comparison encouraged the students to become engaged in the discovery of "discrepant events" in a manner that is congruent with scientific professional practice. Students' desire to "fix" their models developed spontaneously throughout the activity. Even though we acknowledge that further research is needed to fully validate our framework, the data seems to suggest that the main feature of Bifocal Modeling—real-time model comparison—was an effective in the generation of model debugging moments that engaged students in rich, agentive, and generative intellectual work.

#### **Endnotes**

- (1) All phases can be slowed down by lowering the temperature leaving the culture at an optimum temperature for growth over long periods will simply accelerate the death of the culture. Most dead bacteria cells closely resemble living cells, so a normal appearance of a colony on the Petri dish is no indication that its cells are actually alive.
- (2) The NetLogo world is a two dimensional world that is made up of "turtles" (moveable agents) and patches (stationary agents). The patches are the "ground" over which the turtles move.

## References

- Blikstein, P. (2010). Connecting the science classroom and tangible interfaces: the Bifocal Modeling framework. In K. Gomez, L. Lyons, & J. Radinsky (Eds.), *Learning in the Disciplines: Proceedings of the 9th International Conference of the Learning Sciences* (ICLS 2010) (pp. 128-130). University of Illinois at Chicago: International Society of the Learning Sciences: Chicago, IL.
- Blikstein, P., Fuhrmann, T., Greene, D., & Salehi, S. (2012). Bifocal modeling: mixing real and virtual labs for advanced science learning. In *Proceedings of the 11th International Conference on Interaction Design and Children* (pp. 296-299). ACM.
- Blikstein, P. (2014). Bifocal Modeling: Promoting Authentic Scientific Inquiry Through Exploring and Comparing Real and Ideal Systems Linked in Real-Time. In A. Nijholt (Ed.), *Playful User Interfaces* (pp. 317-352): Springer Singapore.
- Gire, E., Carmichael, A., Chini, J. J., Rouinfar, A., Rebello, S., Smith, G., & Puntambekar, S. (2010). The Effects of Physical and Virtual Manipulatives on Students' Conceptual Learning about Pulleys. In K. Gomez, L. Lyons, & J. Radinsky (Eds.), *Learning in the Disciplines: Proceedings of the 9th International Conference of the Learning Sciences* (ICLS 2010) (pp. 937-943). University of Illinois at Chicago: International Society of the Learning Sciences: Chicago, IL.
- Hewson, P., & Hewson, M. (1984). The role of conceptual conflict in conceptual change and the design of science instruction Instructional Science 13 (1), *American Journal of Physics*, Vol. 71, 618-29.
- Jaakkola, T., Nurmi, S., & Veermans, K. (2011). A comparison of students' conceptual understanding of electric circuits in simulation only and simulation-laboratory contexts. *Journal of research in science teaching*, 48(1), 71-93.
- Klahr, D., Triona, L. M., & Williams, C. (2007). Hands on what? The relative effectiveness of physical versus virtual materials in an engineering design project by middle school children. *Journal of Research in Science Teaching*, 44(1), 183-203.
- Liu, X. (2006). Effects of combined hands-on laboratory and computer modeling on student learning of gas laws: A quasi-experimental study. *Journal of Science Education and Technology*, vol. 15, 89-100.
- Levy, S. T., & Wilensky, U. (2008). Inventing a "mid-level" to make ends meet: Reasoning through the levels of complexity. *Cognition and Instruction*, 26(1), 1-47.
- Mulder, Y. G., Lazonder, A. W., & De Jong, T. (2011). Comparing two types of model progression in an inquiry learning environment with modelling facilities. *Learning and Instruction*, 21, 614-624.
- Olympiou, G., & Zacharia, Z. C. (2012). Blending physical and virtual manipulatives: An effort to improve students' conceptual understanding through science laboratory experimentation. *Science Education*, 96(1), 21-47.
- Piaget, J., (1985). *The equilibration of cognitive structures* (B. Terrance & K. J. Thampy, Trans.) Chicago, IL:University of Chicago Press.
- Sins, P., Savelsbergh, E., & Van Joolingen, W. (2005). The Difficult Process of Scientific Modelling: An analysis of novices' reasoning during computer based modeling. *International Journal of Science Education*, 27(14), 1695-1721.
- Wilensky, U. (1999). *NetLogo* [computer software]. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
- Zacharia, Z. C. & Constantinou, C. P. (2008). Comparing the influence of physical and virtual manipulatives in the context of the Physics by Inquiry curriculum: The case of undergraduate students' conceptual understanding of heat and temperature. *American Journal of Physics*, 76(4&5), 425-430.