# Becoming More Mathematical: New Directions for Describing and Designing for Positive Dispositions Toward Mathematics

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Abstract: This symposium brings together researchers interested in studying mathematical proficiency through a focus on students' dispositions toward mathematics - their ideas and affect about mathematics and their patterns of engagement with it. While dispositions are useful for connecting important aspects of students' proficiency, they are also broad and challenging to operationalize in empirical contexts. This symposium aims to promote dialogue around the meaning of dispositions as an analytic construct and to explore their relationship to other constructs relevant to helping learners become mathematical. We also aim to demonstrate that different methodological orientations allow us to see different aspects of what we all agree to call "dispositions". We present empirical work which address two foci within research on dispositions —operationalizing in ways that enable documentation in a classroom context, and designing learning environments which foster positive dispositions to mathematics - in addition to a piece that bridges work between the two.

### Introduction

Proficiency in mathematics is a longstanding but elusive goal for education. Many students fail to reach proficiency in math, and indeed do not move past introductory algebra. This lack of mathematical proficiency has consequences for school and career choices. Researchers now agree that becoming proficient in math depends at least partially on how students' come to see themselves as math learners, what they think it means to do math, and their patterns of engagement with mathematics. In recent years, the construct of *dispositions* has emerged to refer to this collection of ideas and attitudes about mathematics and math learning. The idea of dispositions places social, affective and motivational factors as central to what students learn and who they come to be as mathematical thinkers (Gresalfi, 2009). Research on dispositions explores the interconnections among students' patterns of thought, affect and action and the classroom environment through a focus on moment-to-moment interactions and longer-term patterns over time (Gresalfi, 2009; Gresalfi & Cobb, 2006). While the construct of a disposition is useful to connect students' ideas and affect about mathematics with their patterns of engagement, it is also broad and challenging to operationalize in empirical contexts. Also, several related lines of research address similar issues with different terms, such as identity (Langer-Osuna, accepted; Nasir 2002; Sfard & Prusak 2005), goals (Nasir, 2002), affect (Sengupta-Irving & Enyedy, under review), and social and intellectual authority (Langer-Osuna & Iuhasz, 2013; Yackel & Cobb, 1996).

This symposium brings together researchers studying mathematical dispositions, with the goal of promoting a dialogue about the meaning of dispositions as an analytic construct, and to explore its relationship to other related constructs. There are two related foci regarding students' dispositions, each requiring different types of research. The first focus is: How do we operationalize dispositions in ways that enable their documentation in a classroom context? The second focus is: How can we design learning environments to foster positive dispositions to mathematics?" Our symposium addresses both of these issues through presentations that address each foci, accompanied by commentary from two discussants who are well-versed in issues of promoting productive engagement with content.

### **Background**

Mathematical proficiency, as defined in the new Common Core State Standards for Mathematics (CCSSM), includes being able to innovate, solve problems and engage in mathematical discussions (CCSS, 2011). Yet international and national assessments indicate that many students are not reaching advanced levels of problem solving (Gonzales et al., 2008; U.S. Department of Education, 2011). In addition, there is growing concern that many students are developing a passive, disengaged relationship with mathematics (Boaler & Greeno, 2000) which stands in stark contrast to the confident, flexible use of mathematics required for advanced mathematical work (Boaler, 2008). It is becoming clear that focusing instruction on students' learning of skills and knowledge alone greatly underdetermines their subsequent success in mathematics (Franke, Kazemi, & Battey, 2007). There are important aspects of the mathematical thinkers that students are becoming, including their attitudes toward mathematics, their ideas about what it means to practice, and their patterns of engagement with mathematics, which are increasingly recognized as important aspects of their development and an emerging focus for research.

Building on a socio-cultural perspective to learning, which focuses on the role of social interaction and cultural history in individual learning (e.g. Vygostky, 1978), we argue that becoming a strong mathematical thinker is as much a process of acquiring "habits and dispositions of interpretation and sense-making" (Resnick, 1988, p. 58) as about learning particular skills and knowledge. From a sociocultural perspective, students develop understanding of what it means to do mathematics through their interactions with others, and these interactions take place within and are shaped by the practices of a specific community over time. As students are exposed to and take up specific ideas about the value and purpose of math and what it means to practice mathematics, they develop an orientation to mathematics themselves. The development of this orientation, situated within the context of the classroom, is the primary focus of this symposium.

This symposium aims to bring together several lines of related research that address aspects of dispositions. One of the goals of the symposium is to provide an opportunity for cross-pollination of ideas both between presenters and attendees. A second goal of the symposium is to demonstrate that different methodological orientations allow us to see different aspects of what we all agree to call "dispositions". The construct of dispositions is useful in focusing on important factors of students' developing mathematical thinking. At the same time, understanding, describing and designing for the development of productive dispositions will require a joint coordinated effort, which requires dialogue and understanding across related lines of work.

#### **Presentations**

We begin with work that addresses the first focus of operationalizing dispositions in the classroom. Two presentations focus on empirical descriptions of students' relationships with mathematics and relate this to their engagement in a classroom environment. Tesha Sengupta-Irving and Noel Enyedy will present findings from a teaching experiment with 5<sup>th</sup> grade students in which they examine contrasts in affect between groups who had the guided vs. self-directed pedagogical approach, and relate contrasts in affect to students' patterns of disciplinary engagement. Melissa Kumar will present findings from a year-long study with 3rd grade students in which she relates students' reported adoption of four positive goals for learning with their patterns of engagement in problem solving as a window on their dispositions, looking specifically at how students with different goal profiles varied in patterns of engagement. We then turn to the second foci, designing to support development of positive dispositions. Melissa Gresalfi will describe a design experiment with 8<sup>th</sup> grade students. She examines how the different design of tasks, one which asked students to practice using a formula and one which asked them to invent a formula to describe density, supported different forms of engagement, specifically decision making during problem solving. The final presentation bridges the two research foci, beginning with descriptions of students' dispositions and moving to more recent work describing a design experiment based on findings about students' dispositions. Jennifer Langer-Osuna will present results of an analysis of how social and intellectual forms of authority related to different types of engagement in problem solving. She then reports on how a design experiment in which each student was offered opportunities to take on the group leader role supported more equitable opportunities for students to construct forms of authority.

#### **Symposium Structure**

The session will take the form of a traditional 90-minute paper session, with brief remarks by the chair followed by twelve minutes per paper presentation. We have included two discussants with different perspectives in our proposal because one of our goals is specifically to facilitate dialogue around the overlap and contrast of different approaches to study what we agree to call dispositions, and we believe that this decision will facilitate a richer session to this end. Each discussant will have 15 minutes to respond to the papers, after which we will open the space to a discussion with the audience. Our first discussant, Anna Sfard, is a leading authority on mathematics discourse in thinking and learning. She has examined the relationship between students' identities and their engagement with mathematics, illuminating the way that group narratives shape the dispositions of

individual students (Sfard & Prusak, 2005). Our second discussant, Kris Gutierrez, is a leading authority on literacy and learning. Her description of the 3rd space, where the narratives and identities of the teacher and students can coexist, has facilitated the examination of how students' actively take up or repurpose available ideas, values and discourses within the classroom (Gutierrez, Rymes & Larson, 1995). Each discussant will speak to common themes in the papers, offer commentary on areas for additional research and bring up questions to be pursued in the general discussion.

## Smiles Don't count: A Case Study Unifying Disciplinary Engagement with Dispositions in the Study of Mathematical Learning

Tesha Sengupta-Irving, University of California, Irvine and Noel Enyedy, University of California, Los Angeles

Whether seen as a triumph of Western traditions separating and valuing cognition over emotion, or the logical consequence of increasingly technocratic systems of accountability in public schools, the national discourse of achievement has grossly obscured the importance of advancing student thinking *and* cultivating positive feelings toward a discipline (Vadeboncoeur & Collie, 2013). In mathematics, we risk a generation of learners who score among the highest averages in achievement but who are also among the most negative in their attitudes toward the discipline, as seen of other countries in international and comparative studies (e.g., Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996; Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrostowski, & Smith, 2000). This inverted relationship – achievement without enjoyment – is both learned and problematic: innovation is cultivated through rewarding curiosity and effort, not disinterest and detachment. This risk is underappreciated in the national discourse on mathematics education – whether looking to the National Research Council's five strands of mathematical proficiency (Kilpatrick, Swafford & Findell, 2001), or math practice standards of the Common Core Standards (CCSI, 2011), evaluating if students *like* learning is rarely discussed. Recent studies on the notion of dispositions in mathematics education (e.g., Boaler & Greeno, 2000; Gresalfi, 2009), however, demonstrate the potential for research in the learning sciences to anticipate this risk, and identify means of mitigating it.

This paper draws on a 10 day teaching experiment comparing two pedagogical approaches (one guided and one self-directed) used to teach data and statistics to 5<sup>th</sup> grade students (n=52). This study took place in a socioeconomically diverse elementary school in a large urban center in California. The progressive school encouraged teachers to adopt research-based teaching practices. The teacher, an African-American woman, had fifteen years of teaching experience in mathematics.

Elsewhere, we have described learning results for the teacher (Sengupta-Irving, Redman, & Enyedy, 2013) and students (Sengupta-Irving & Enyedy, under review). Of relevance, we found that students in both treatments (n=27, n=25) improved in their proficiency in data and statistics, but reported significantly different orientations in how much they liked learning the lessons. In this paper we take up the methodological question that emerged as a consequence of this result: How can we use video-based interactional analyses to characterize why students were "liking" what (or how) they were learning? We see this methodological question as part of the larger project of promoting research that uniformly attends to disciplinary engagement and cultivating positive affect among mathematics learners.

In the study, we used video-based interactional analyses to explore why students had different affective responses to treatments in the experiment (with no corresponding difference in performance). In the process, we made several important methodological and practical decisions in order to operationalize "liking" that could then be characterized and quantified comparatively. What began with a tongue-in-cheek discussion of 'Can't we just count the smiles?' evolved into a complex constellation of cultural, disciplinary, and pragmatic decisions about how we characterize students "liking" (or not) what they are learning. Our paper discusses the evolution and results of these discussions, which directly relate to a guiding focus of the symposium: How do we operationalize dispositions for individuals in a classroom context? In our work, we narrow the theoretical scope of dispositions to focus specifically on how we characterized, contextualized, and quantified students' engagement in the mathematical tasks for each treatment. Moreover, we speak to patterns of engagement within the context of a 10-day pedagogical experiment, and not patterns over time and across learning contexts. We drew significantly from Engle and Conant's (2002) principles of productive disciplinary engagement, and developed our own measures of student autonomy (i.e., continuous activity without adult intervention) and ontask behaviors, which were grounded in the design of the experiment. What results is a depiction of learning that inextricably links affect and patterns in disciplinary engagement to the classroom context and organization of learning therein.

This case study offers a model of how research can promote a perspective on learning that draws together students' disciplinary engagement and affect toward the discipline. We discuss the design of learning environments and tasks that support this perspective, which speak both to research and teaching. Finally, we raise several questions and potential directions for future studies in the learning sciences that can help mitigate

the widening gap between promoting disciplinary proficiency and developing disciplinary dispositions in our national discourse.

### Elementary Students' Dispositions, Goals and Engagement in Problem Solving in Mathematics

Melissa Kumar, University of California, Los Angeles

Mathematical proficiency by today's standards includes being able to innovate, problem solve, and collaborate around challenging problems. One important question is, What drives students' engagement in these activities? In this paper I describe work from my dissertation in which I build on the construct of dispositions to explore the goals that motivate student's engagement in math class, such as wanting to understand concepts or solve challenging problems. Studying students' goals facilities more specific insight into the motives that students have for participating in mathematics, and how their relationships with mathematics emerge from a specific cultural practice. I focus on 3<sup>rd</sup> grade students because research points to an important shift (often negative) in perceptions of self-efficacy and value of mathematics at this age (Eccles, Wigfield, Harold, & Blumenfeld, 1993). I examine students' adoption of four positive goals for mathematics and relate this to engagement in problem solving as a window on their dispositions to mathematics. The study was guided by the questions, (1) What goals are appropriated by the students? And to what degree do these goals align with the values of the reform-oriented mathematics education community? And (2) How do different profiles of goals relate to students' engagement in problem solving?

Goals have been used as a way to understand the link between cultural activity and individual cognition beginning with early Soviet psychology (Wertsch, 1981). More recent work has examined goals as a mediating link between society, culture and thinking (Saxe, 1999, as cited in Nasir, 2002) and to understand the concurrent development of students' identities, mathematical goals, and learning in informal mathematics practices (Nasir, 2002). Nasir (2002) argues that goals are critical to understanding learning because humans act specifically to accomplish goals in social activity, and these goals "help structure the nature of the thought and problem solving of individuals" (p. 216). The current study builds on the view that goals emerge through participation in cultural practice, and links goals both to the larger construct of disposition as well as the moment-to-moment problem solving activity of individuals. I contend that studying students' goals can shed light on aspects of their dispositions because the more "enduring" aspects of dispositions—the "ideas about" and "perspectives on" mathematics—must be transformed into engagement in the moment. Goals are a likely construct to mediate this transformation from ideas and perspectives to action and thus may illuminate what students think is valued in mathematics in their classroom, how they appropriate these available goals, and how aspects of the context and dispositions combine to direct student effort in problem solving. Looking at students' goals is therefore a useful lens in which to examine students' dispositions in a mathematics classroom.

While there are multiple goals that drive students' engagement in math class, I focus on four goals that I derived from the math education and educational psychology goal-orientation literature, the CCSSPM, and from descriptions of the practices and values of mathematicians. The goals are understanding the mathematics at a conceptual level (CCSI, 2011; Boaler, 2008), solving challenging problems (CCSI, 2011; Lampert, 1990), exercising agency and creativity in solving math problems (CCSI, 2010; Boaler, 2002), and being viewed as a valuable contributor in classroom discussions (CCSI, 2010; Boaler, 2002; Boaler & Greeno, 2000). These goals are meant to represent the motivation that students would have as a part of a positive disposition to mathematics in alignment with the reform-oriented mathematics education community. The position of this paper is that all of these goals are important to be successful in mathematics practice. These goals form the framework of my exploration of students' reported goals and observation of their patterns of engagement in problem solving.

I will report on data that was collected in a 3<sup>rd</sup> grade classroom (n=27) over the 2013-2014 school year. I selected the site because of its progressive, constructivist teaching philosophy and because it follows the principles of Cognitively Guided Instruction (Carpenter, Fennema, & Franke, 1996; Carpenter, Fennema, Franke, Levi, & Empson, 1999) a curriculum that attends both to fostering problem solving and the dispositions of students to be problem solvers. The teacher embraces these philosophies and draws on her CGI training in her instruction. Data sources include a video record and field notes of class activities, with a focus on six case study students during problem solving, student surveys and interviews, and student work. Video was collected throughout the year, and surveys and interviews were administered three times during the year. I focused on collaborative problem solving because it is central to the practice of mathematics and because it affords multiple opportunities to see students pursue goals in specific conditions. The survey focused on students' adoption of the four positive goals and two goals of relative performance using a Likert-type scale from "not at all true" to "very true". The interview focused on clarifying and expanding survey responses and asking students about their problem solving using video clips from class.

To characterize students' engagement in problem solving, I identified problem solving sessions that were challenging, open-ended, and had multiple possible solutions from the first and second half of the year for

three case study students. I identified decision points and created a "roadmap" for each problem, with particular attention to students' entry into the problem. Decision points included: new information from other groups, peers or the teacher, roadblocks in problem solving that prompted the student to change or modify the strategy, revisions to the group work plan, and moments of insight. I then characterized students' engagement at each decision points, including the degree to which they showed perseverance, creativity and flexibility in their thinking.

Preliminary analysis of survey results suggests that students in the class did indeed have different goal profiles. To create students' goal profiles, I first created composite scores for each goal, and then I categorized students by how many of the goals had a high, medium or low score. Nine students were categorized as "high" because they had high scores for 3 or 4 goals. Three students were categorized as "medium high" because 2 goals had high scores and the other two were medium or low. Seven students were "medium" because they either had medium scores for all goals, or a mix of high and low scores, and seven students were "low" because they had mostly low scores for each goal. These preliminary results will be compared with students' survey results in the middle and end of the year. In addition, I will present preliminary analysis of the connection between students' goals and pattern of engagement in problem solving in the classroom based on the video analysis described above.

This presentation describes empirical connections between the goals that students have for mathematics, specifically the four goals considered positive for mathematical proficiency, and their engagement in problem solving as a window on their dispositions to mathematics. In doing so, I address the first goal of this symposium, which is to address methodological issues of operationalizing dispositions in a classroom environment. In addition, this paper presents a unique approach to focusing on students' motivation in math class by proposing four goals synthesized from the reform-oriented mathematics education community, and connecting student's adoption of the positive goals to their engagement in problem solving. This is significant for those who wish to understand what motivates student engagement in math class in order to design instruction that promotes understanding and supports positive dispositions to mathematics. Clarifying what positive goals for mathematics look like, how they translate into problem solving actions, and how they relate to other aspects of the classroom ecosystem is valuable to educators, researchers and parents, as well as students in their later years.

### Framing Engagement Through The Design of Tasks

Melissa Gresalfi, Vanderbilt University

Student mathematics learning and achievement is a topic of significant study and concern in both the United States and around the world. Although a range of issues is often discussed, one that yields some of the greatest debate regards what and how students should know and understand mathematics. The past twenty years have brought technological innovations that have changed the world as we know it. Continuous access to online information has supported a transformation of the relationship between individuals and knowledge; with information so readily accessible, people have been repositioned to move beyond *acquisition* of facts to consider when to access those facts, interrogate them, and integrate them into activity. For these reasons, knowledgeable participation in mathematics must involve more than proficiency using key procedures. Instead, knowledgeable participation must involve engaging in acts of decision-making, determining which procedures enable the resolution of defensible solutions, how, and why. Lampert (1990) has defined such mathematical activity as being "courageous and modest in making and evaluating [one's] own assertions and those of others, and in arguing about what is mathematically true" (p. 33).

It is clear that supporting students to engage with mathematics in this way is not simply a matter of teaching them more mathematics. Instead, it is crucial that students have opportunities to learn new content in ways that are consistent with how we actually want them to use that information (Boaler, 2000; Bransford & Schwartz, 1999; Greeno, 1991; Lave, 1997). When creating new curricula, it is therefore crucial that designers attend not just to the content goals of the unit (the mathematical ideas you want students to learn and understand), but also dispositional goals of the unit (how you hope students will engage). This does not mean that becoming a successful engineer, for example, requires learning all mathematics at the elbow of a practicing engineer. Instead, this suggests that the kinds of *practices* that a practitioner is expected to leverage (such as experimenting, reviewing, collaborating, justifying, testing, and inventing) are the practices engaged in during the learning experiences.

In my work I have found it productive to consider how students are engaging information as an explicit framework to both support design and to analyze student learning (Gresalfi & Barab, 2011; Gresalfi & Barnes, 2012; Gresalfi, Barnes, & Cross, 2013). Specifically, we have focused on four forms of engagement with mathematics: Procedural, Conceptual, Consequential, and Critical. Although procedural and conceptual engagement are familiar ideas, consequential and critical engagement are not. *Consequential engagement* involves recognizing the usefulness and impact of disciplinary tools; being able to connect particular solutions

to particular outcomes. Finally, *Critical engagement* involves choosing particular tools and interrogating their impact in attaining desired ends. Related to consequential engagement, critical engagement captures the decision making involved in problem-solving, and at some level involves critiquing the method itself in relation to the particular context in which it's being used.

In this presentation, I draw on this framework to consider how the design of particular tasks frame students' engagement with problems related to understanding ratio and density, drawing on data from a study reported by Schwartz, Chase, Oppezzo, & Chin (2011). This study included students in four different 8<sup>th</sup> grade classes taught by the same teacher. Students in two of the classes completed a series of tasks that allowed them to practice using the formula for density across three different kinds of worksheets (the "tell" condition). Students in the other two classes used the same worksheets, but were instead asked to "invent an index" that could describe the density of different items (the word density in this case was not used (the "invent" condition). In analyzing these data, Schwartz et al. (2011) found that while students in both groups demonstrated proficiency at using the given formulas on word problems (a "near transfer" task); students in the "invent" condition were able to transfer the underlying idea of ratio to new content (a "far transfer" task). The analysis reported here focuses on videotapes of groups of students in both conditions solving these tasks over three days.

Student interaction was coded using both apriori and emergent schemes; drawing on coding from previous work (Gresalfi, Barnes, & Cross, 2012; Gresalfi & Barnes, 2012), we classified students' engagement with the task as either procedural, conceptual, consequential, or critical. We then examined instances of student talk in relation to these codes in order to better understand what about the task was contributing to their engagement. Our analysis specifically considers the affordances of the tasks for supporting different forms of engagement, and considers whether and how those different affordances are realized through the group's interactions. In so doing, we consider both how students' engagement changes over the course of three days, and how that engagement is framed by the tasks they are working on.

### **Supporting the Construction of Student Authority During Collaborative Mathematics Problem Solving**

Jennifer Langer-Osuna, University of Florida

The proposed presentation will focus on how students construct relations of authority in interaction, and the ways in which differential authority relations serve to marginalize or privilege particular students' engagement in collaborative mathematical problem solving. In particular, I will present preliminary results of a design experiment meant to structure how students become positioned with forms of authority in an effort to support equitable and productive engagement in collaborative mathematics activity.

Students positioned with intellectual authority participate more frequently in small groups, are more able to gain access to and hold the conversational floor and decide what is correct, tend to be seen as contributing more meritorious ideas, and become more influential than students perceived as having less intellectual authority (Engle, Langer-Osuna, McKinney de Royston, in press; Inglis & Mejia-Ramos, 2009). And though forms of student authority other than intellectual authority are rarely studied in mathematics education research, emerging research is beginning to show that social forms of authority—in particular being positioned with the right to issue directives to peers—is fundamentally related to both student engagement levels and the development of intellectual authority itself (Langer-Osuna, 2011; Langer-Osuna & Iuhasz, 2013).

### Designing for Equitable Distribution of Intellectual and Directive Authority

The proposed presentation builds on the results of a previous analysis focused on how social and intellectual forms of authority relate to the construction of particular mathematical solution paths (Langer-Osuna & Iuhasz, 2013). Results highlighted how particular interactions supported the construction of forms of authority – both the social authority to issue directives to peers and the intellectual authority of being positioned as a credible source of information – and its role in determining whose ideas were taken up as part of the solution path. In this presentation, I briefly present and then build on these results to frame a new design experiment meant to support more equitable opportunities for students to construct forms of authority, such that multiple student ideas are attended to, debated and considered as potential contributions to a problem's solution path.

The study context is a weekly afterschool math program for third through fifth graders at two public elementary schools with diverse student populations. Students worked individually and in groups of three. Below I detail how we framed the learning context for interactions around both intellectual and directive authority.

Preliminary video-based interaction analysis will be presented, focused on how students positioned themselves and one another with forms of intellectual and directive authority, how those positionings were taken up by peers, and their effect on patterns of engagement and whose ideas were taken up as part of the solution path. Results will be illustrated through vignettes of how students constructed forms of authority and

the ways in which these positions affected both student engagement and the construction of particular mathematics solution paths based on what (and whose) ideas became influential.

### Fostering and Examining Positions of Intellectual Authority

Group participation structures such as distributed expertise (Brown et al. 1993) can support the development of student intellectual authority. In classrooms that utilize distributed expertise, students are treated as researchers who gather, analyze, and share information about a particular topic, gradually developing expertise in that specialty area, increasing the likelihood that all students become positioned with intellectual authority.

Here, a key element of the mathematical activity is an explicit focus on representations as vehicles for mathematical thinking and communication. The design experiment created opportunities for each member of each student group to develop expertise in a particular representation to then bring back to the collaborative problem-solving process.

### Fostering and Examining Positions of Social Authority

Despite the varied forms of authority associated with particular kinds of participation structures (such as group roles), there is no research focused on examining the conditions under which leader roles, which enable the leader to issue directives to peers, support or discourage equitable group work. The role of group leader has often been included in research on group work in ways that assume its benefit. Though such participation structures may afford students positions of authority, the role of leader is not only up to the student assigned that role but also depends on how the other group members take up the leader's directives rather than reject or modify such positions (Langer-Osuna, 2011). Here, each student was offered opportunities to take on leader-related roles that contributed to the management of the problem-solving process of their group.

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