Opening the Door to Algebra: The Role of Fraction Knowledge in Algebra Learning

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Abstract: We examine 6th, 7th, and 8th grade classes in order to determine how fraction knowledge improves naturally during business as usual instruction and whether naturally occurring improvements in fraction knowledge are related to improvements in algebra readiness/achievement outcomes.

Significance

Recent research has confirmed speculation of scholars and practitioners alike: Students' knowledge about fractions is predictive of their Algebra readiness (Booth & Newton, 2012), and performance and learning in algebra (Booth, Newton, & Twiss-Garrity, 2013). However, the mechanisms underlying the relation between fraction knowledge and improved algebra performance and learning are yet unclear, especially as the benefits of fraction knowledge appear even when students solve problems that do not involve fractions. In the present study, we examine naturally-occurring changes over the course of a school year in different types of fraction knowledge (magnitude knowledge, comparison, and arithmetic) and investigate whether and how those changes are predictive of changes to different types of algebra knowledge. This project fills a critical gap in the present knowledge on how fraction knowledge is related to Algebra performance and learning, including which types of fraction knowledge are critical for improving algebra learning and the mechanisms underlying these effects.

Theoretical background

Several mechanisms have been proposed to explain *why* fraction knowledge impacts algebra performance and learning. The most straightforward hypothesis is that lack of conceptual understanding of fractions "…limits students' ability to solve problems with fractions and to learn and apply computational procedures involving fractions (p. 6)" (Siegler et al., 2010). This would necessarily lead to difficulty solving problems in algebra, as they frequently require manipulation of fractions and other rational number representations. However, recent evidence demonstrated that the correlations between fraction knowledge and equation-solving skill were not limited to equations that contained fractions (Booth & Newton, 2012). This suggests that, while proficiency with fraction computation may certainly be one mechanism behind the connection between fractions and algebra learning, it may not be the only one.

Some scholars argue that, in addition to facility with computation, students need a deep understanding of the number system, including the place of fractions in that system, in order to succeed in algebra (Empson & Levi, 2011; Wu, 2001). Central to such a deep understanding of numbers is knowledge about magnitude (Siegler, Thompson, & Schneider, 2011). In particular, students' understanding that the magnitude of fractions is determined by a relationship between two numbers (a notion that is unfortunately difficult to grasp (Siegler & Pyke, 2012) is crucial for development of strong concepts of fraction equivalence and proportionality. Such proportional reasoning is thought to play an important role in algebra learning. For example, Empson & Levi (2011) suggest that understanding that "all fractional representations of 1/3 will fit into the equation, $x/y = 1/3$, which is equivalent to the equation $y = 3x$" (p.134) will help students to comprehend algebraic equations. Further, Wu (2001) suggests that rules and ideas learned when dealing with fractions are often applicable to algebraic topics, thus making that foundation knowledge about the rational number system crucial for algebra.

Here, we examine both student knowledge of fraction computation and knowledge of fraction magnitudes to determine whether changes in either or both are predictive of changes in algebra readiness or learning.

Method

Participants in this study are students in intact 6th, 7th, and 8th grade mathematics classrooms (N=45 classrooms) in diverse school districts. All students in participating classrooms are undergoing business as usual instruction at their grade level during the term. They completed fraction and algebra measures at the beginning of the year, and will complete those measures again at the end of the year, as well as a measure of grade-level mathematics competence. Demographic information is supplied by the school district.

Two fraction knowledge measures are used. *Fraction magnitude knowledge* is assessed using a number line estimation task (e.g., Siegler & Opfer, 2003). Students are presented with blank number lines with 0 marked at the left endpoint and 1 marked at the right endpoint. A fraction with a magnitude between 0 and 1 is printed on top of each line and students are asked to place a mark on the number line where they believe the indicated fraction belongs. *Fraction computation knowledge* is measured using items representative of the types of problems found in fraction arithmetic lessons (i.e., ½ + ¼; ⅔ x ⅝; 6÷ ⅓). Addition, subtraction, multiplication, and division with fractions are included.

Three algebra knowledge measures are used. *Equation Encoding* is assessed using a reconstruction task (e.g., McNeil & Alibali, 2004) in which students are presented with an equation or expression briefly and then asked to reconstruct the problem from memory immediately after it disappears from view; students complete this task for a series of equations with different structural formats and different placements of key problem features. *Feature knowledge* is assessed using previously established items designed to address students' understanding of concepts that have been identified in previous research as crucial for success in Algebra (e.g., the meaning of the equals sign, the significance of negatives in terms, identification of like terms, etc (Booth & Newton, 2012). Finally, *problem-solving* is assessed using items that measure students' ability to effectively carry out procedures to solve problems. These items are representative of the types of problems found in Algebra I textbooks and taught in Algebra I courses (i.e., $6x - 8 = 12$; $7x + 4 = 10x - 5$; $4 = 2/3x$).

General mathematics competence will be assessed using grade-specific mathematics ability tests comprised of items from that grade's mathematics textbooks aligned with that grade's CCSS-M content standards.

Anticipated results

We will assess how fraction knowledge related to improvements in various facets of algebra readiness and performance through a series of multiple regression analyses, regressing each end of year algebra score (# conceptual encoding errors, % correct on feature knowledge, and % correct on problem-solving) on students' start of year fraction and algebra measures, improvement on each of the fraction measures over the course of the school year, students' end of year math ability score, and relevant demographic variables (e.g., gender, ethnicity, SES). This will allow us to determine how initial fraction knowledge and improvements that result from traditional instruction are predictive of improvements in students' algebra readiness or performance.

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