Exploring the Relationship Between Gesture and Student Reasoning Regarding Linear and Exponential Growth

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Abstract: Leading middle school students to effectively reason with magnitude, small and large-scale quantities, and linear and exponential growth is a challenge facing science teachers in the U.S. working to implement the NGSS and its associated focus on crosscutting concepts. This paper examines the relationship between middle school students' gestures and their reasoning about quantitative growth. For this study the authors developed interview protocols on several topics that involve numbers increasing linearly and exponentially. Students' verbal and gestural responses during the interviews were transcribed, coded, categorized and analyzed. Results showed that students display a breadth of context-dependent gestures, that their verbal reasoning is associated with the type of gesture they produce, and that gesture-speech mismatches are present in middle school students' reasoning about scale. Implications of this study include design guidelines for an interactive system that standardizes and structures scale-related gestures.

Keywords: quantitative growth, student gestures, gesture-speech mismatch

Introduction

There is strong support from multiple disciplines for the notion that gestures facilitate processes of thinking and learning (Roth, 2001), and there is building evidence that gestures can promote learning of specific ideas in STEM (e.g., Alibali & Nathan, 2012; Goldin-Meadow, Cook, & Mitchell, 2009). Some have argued that even abstract concepts in STEM come from our embodied interactions with the world (Lakoff & Nunez, 2000). In this paper we examine the notion of quantitative growth and how students make sense of increasing numbers and strategies for "scaling up" through verbal and gestural communication. In particular, we are interested in the ways in which gestures support or hinder student reasoning about large quantities in their efforts to solve scientific problems.

Several topic areas in science require students and practitioners to engage with increases in quantity and different types of scales (e.g., liner, exponential, logarithmic, etc.). Reflecting this multidisciplinary aspect, *scale, proportion, and quantity* are highlighted as crosscutting concepts in the Next Generation Science Standards in the U.S. (NGSS Lead States, 2013). And yet, research suggests that students across the K-12 curriculum often struggle with reasoning about and executing operations involving very large numbers (Tretter, Jones, & Minogue, 2006).

In this study we seek to gain greater insight on the challenges that students face when reasoning about increasing quantities, and whether or not the gestures students make during this type of reasoning appear to play a role in their thinking and learning. Specifically, the following questions are investigated: 1) What are the different ways by which students use gestures to convey their scientific reasoning of topics related to growth? 2) How does the context of the gesture (i.e., problem solving or communication) seem to affect the type of gestures students utilize?

Symbolic and concrete gestures

Figure 1. (left) A Symbolic gesture (right). A concrete gesture.

Studies of hand gestures have indicated that these embodied acts do more than simply echo verbal expressions; rather they are "symbols that exhibit meanings in their own right" (McNeill, 1992, p.4). The form of a gesture can vary depending upon the purpose of communication (Crowder, 1996). For the sake of examining gestures related to quantitative reasoning, we distinguish between symbolic and more "concrete" gestures. In this study, symbolic gestures are a representation of operations and/or numerical values, such as holding up fingers to represent a certain quantity (see Figure 1, left) or crossing two fingers to make an 'X' representing multiplication. Alternatively, concrete gestures represent objects or express ways that objects are manipulated (Figure 1, right). For example, placing one hand on top of the other to represent doubling as an operation, which adds a quantity to itself.

Methods

Participants

Fifteen (10 males and 5 females) middle-school students were interviewed for this study. Students came from different backgrounds from the surrounding area of a large Midwestern University in the United States. Student interviews were videotaped to aid in the analysis.

Instrument

This study employed semi-structured, task-based interviews involving problems about linear and exponential scales. This first question posed to students was a modified version of the 'grains of rice on a chessboard' problem, an exercise commonly used to teach exponential growth. In this version of the problem, students were asked to imagine that they had won a prize and were asked to choose whether they would prefer an option where they received \$1000 per day for 30 days, or a second option where they received \$1 on the first day and double the amount of the previous day, each day, for thirty days. Students were asked to choose one of the options and explain why they selected the option. They were then asked to explain how the total amount of money increased each day for both options. If students did not spontaneously use gestures in their explanations, they were prompted to gesture or act out how each option increased. Finally students were asked how they would convince a friend to choose the second option if they could not communicate verbally. In addition to facilitating a transition to having students think about different types of scales in various science contexts, this problem provided insight into how a participant makes sense of increasing quantities and the gestural resources they have available for thinking and communicating about them. After this initial question, three base problems contrasting linear and exponential growth were posed to the participants from various science domains, which employ different scales (e.g., The Richter scale, geologic time, population growth, etc.). Our analysis here focuses primarily on student reasoning about the initial question.

To analyze students' quantitative reasoning and gesture, we segmented the videos according to where a gesture related to describing a quantity that has increased or how it is increasing took place. Included in the segmentation of the video interviews was a detailed description of how the hands, fingers, etc. were employed in the gesture, as well as a transcription of the speech accompanying the gesture. Once coding of data was complete, students' gestures were categorized. Two raters categorized student reasoning and gestures with an initial 70.4% agreement. The raters then met to discuss problems on which there were disagreements in the ratings and to see if some common, agreed-upon rating could be reached. Following discussion 100% agreement was reached.

Analysis

Frequencies for each gesture category were computed for all participants and are shown in Table 1. The speech associated with each gesture act was coded for whether or not it showed correct reasoning, in the sense that it accurately described the mathematic relationships present in the problem. Additionally, each gesture was coded for whether it was aligned with the concurrent speech (i.e., did both the speech and the gesture appear to be describing the same quantity or relationship?). Table 2 shows frequencies for the different combinations of reasoning and alignment: (1) correct verbal reasoning on mathematical growth with aligned gesture, (2) correct verbal reasoning with an unaligned gesture, (3) incorrect verbal reasoning with an aligned gesture, and (4) incorrect verbal reasoning with an unaligned gesture. Note that alignment of the gesture was coded with respect to both alignment with correct reasoning and alignment with student verbal reasoning.

In order to examine further the first research question, a Fisher's exact test between students' gesture type (concrete vs. symbolic) and the alignment of their gestures with their reasoning on mathematical growth was conducted. The frequency table can be found in Table 3. To answer the second research question, a Chi-Square test of independence was used to investigate the association between the type of gesture used by the student and the context of the question (whether a student spontaneously gestures or whether s/he is prompted to gesture).

Results

In the context of the quantitative growth problem, students generated a total of 161 gestures. Table 1 shows the most used gesture ($N = 44$) was the symbolic gesture of using fingers to indicate numbers or mathematical symbols (Figure 1 left). Conversely, there were only 5 instances of gestures indicating exponential growth by using a hand to show the curve of an exponential graph. Other commonly used gestures included different ways by which students represented stacking or used a referent, which they manipulated to indicate growth.

Table 1: Student gesture categories and frequency

Gesture Category	Total	Gesture Category	Total
1. Fling hands and/or fingers to show a random increase	14	5. Adding from one hand to another to indicate an increase in the form of stacking	29
2. Using fingers to indicate numbers, numerical or mathematical symbols	44	6. Vertical/horizontal parallel hands to show a certain amount	25
3. Vertical/horizontal parallel hands to show a linear increase	20	7. Wide arms spread gesture	6
4. Using hand as a curve for an exponential growth	5	8. Placing a referent in different places to represent time and growth	13
		9. An iterative process to indicate increase	5

Table 2 below shows that the vast majority of students' gestures (129 out of 161) revealed correct reasoning about mathematical growth and produced gestures aligned with their reasoning. An interesting finding was that 15 out of 161 gestures revealed cases where although students incorrectly reasoned about mathematical growth, their gestures matched the correct reasoning. For example, Kathy (pseudonym) incorrectly articulates her reasoning about doubling. According to her verbal statements, doubling is adding 2 to the initial amount. As she makes this statement, however, she produces a series of gestures where she uses her hands to set a fixed amount, and then she systematically extends the distance between her hands to twice the initial amount. This gesture-speech mismatch was evident in 15 places in which students' verbal reasoning (speech) is incorrect but their gestures are indicative of (aligned with) the correct reasoning. This finding is in line with previous research showing gesture-speech mismatches as an index of transitional knowledge (Perry, Church, & Goldin-Meadow, 1992).

Table 2: Students' levels of sophistication

	Correct Reasoning	Incorrect Reasoning	Reasoning Not Applicable	Total
Gesture and Reasoning Aligned	129	15	3	147
Gesture and Reasoning Unaligned	4	5	2	11
Alignment Not Applicable	0	0	3	3
Total	133	20	8	161

Fisher's exact test between students' gesture type (concrete vs. symbolic) and the alignment of their gestures with their reasoning on mathematical growth was conducted. The test revealed significant relationship between gesture type and alignment (χ^2 (4) = 14.58, $p < .05$). Specifically we found that concrete gestures are more likely to be aligned with correct reasoning than symbolic gestures in this problem-solving context. In addition, a pair-wise test of correlation shows that there is a significant positive association ($p < .05$) between gesture type and alignment. This suggests that the use of concrete gestures may be indicative of more correct reasoning when describing quantitative increase. We make the observation that even though symbolic gestures were more common, concrete gestures had a stronger association with correct reasoning in topics related to growth.

Table 3: Relationship between students' type of gesture and the gesture alignment with reasoning

Alignment	Gesture Type			
	Concrete	Symbolic	Concrete and Symbolic	Total
Aligned	96	27	24	147
Unaligned	9	2	0	11
N/A	0	3	0	3
Total	105	32	24	161

To answer the second question, an examination of the relationship between gesture type and the question context was examined. The questions either asked students to explain their reasoning or to try and communicate a specific quantity or quantitative operation to another person (e.g., how would you show someone 'doubling' non-verbally). Table 4 shows the result of the Chi-Square test of independence revealed an association between the type of gestures used by students and the context of the question (χ^2 (4) = 35.36, $p < .01$). Specifically, students were more likely to produce concrete gestures when they were engaged in reasoning, and more likely to produce symbolic gestures when engaged in communication about specific quantities or operations.

Table 4: Relationship between students' type of gesture and the gesture alignment with reasoning

Context	Gesture Type			
	Concrete	Symbolic	Concrete and Symbolic	Total
Reasoning	59	3	2	64
Communication	45	32	20	97
Total	104	35	22	161

Conclusions and implications

We make the following inferences from the findings of this study: (1) Concrete gestures were more common in reasoning (as opposed to communication) acts. In addition, when using these concrete gestures for reasoning, students' reasoning was accurate in almost all cases (2) The variation of concrete gestures seems to provide students with multiple potential pathways for doing their reasoning, whereas symbolic gestures only give a constrained set of numbers and operations (3) Gesture-speech mismatch exists in the context of middle school students reasoning about increasing quantities. Further research is needed on whether these mismatches can be leveraged as transitional points in the learning trajectory for students' understanding about scale and magnitude.

Students used a breadth of gestures to reason and communicate about growth. A meaningful pattern in communications about increasing quantities was evident in this study. This implies that in topics such as mathematical growth, we can benefit from a system that standardizes and structures this pattern. Finally, we think that students may benefit from opportunities to use more concrete gestures, but in a more structured and consistent way. This study gave us many ideas for the kinds of gestures that might be incorporated into a gesture recognition interface. In fact, our research team is currently in the process of building a prototype interface that allows students to define and use concrete gestures for reasoning about problems across science topics.

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