Comparing Students' Solutions When Learning Collaboratively or Individually Within Productive Failure

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Abstract: In this article we focus on evaluating student solutions generated during the initial problem-solving phase of a Productive Failure study, where problem solving precedes instruction. In particular, we investigated whether collaboratively learning students came up with more solution ideas and generated qualitatively better solutions than individually learning students. Although descriptively the collaboratively learning students outperformed individually learning students with regard to both the quantity of solution ideas and the quality of solutions, we were unable to establish statistically significant differences. Moreover, we did not find correlations between students' solutions and their learning outcome. Possible reasons for the missing link are discussed. The analyses reported in this paper are based on yet unpublished data from a study by Mazziotti, Loibl & Rummel (2015).

Keywords: students' solutions, collaborative learning, Productive Failure

Introduction

Instructional approaches that comprise self-determined problem-solving prior to instruction often include small-group collaboration during problem solving. In the Productive Failure approach (PF, e.g., Kapur, 2012), for example, students first try to collaboratively solve a yet unknown problem (usually generating incomplete or erroneous solution approaches) and then receive explicit instruction building on their prior solution attempts. In order to shed more light on the role of small group collaboration for the effectiveness of PF Mazziotti, Loibl and Rummel (2015) conducted a 2x2 quasi-experimental study with the factors social form of learning (collaborative vs. individual) and timing of instruction (i.e., problem-solving prior to instruction, or PF vs. problem-solving after instruction, or Direct Instruction). We expected higher benefits from collaborative learning compared to individual learning (within PF) on students' conceptual knowledge acquisition because mutual collaborative explanations about students' problem-solving ideas trigger elaborative processes which in turn support students' conceptual knowledge acquisition (Teasley, 1995). However, in our previous study, individual students acquired descriptively more conceptual knowledge than their collaborative counterparts. Also when looking only at the students from the two PF-conditions, again, the individually learning students descriptively outperformed their collaborative counterparts.

In order to further unpack the role of collaboration within PF, we re-analyze the data from the Mazziotti et al. (2015) study. Our aim is to, go beyond investigating students' conceptual knowledge at the end of the intervention by analyzing students' solutions generated *during* the initial phase of PF. In line with previous research on PF, we investigate the quantity and quality of students' solutions and are thus able to make further comparisons between collaboratively and individually learning PF students. Investigating students' solutions is of particular interest as some previous PF-studies (e.g., Kapur, 2012) showed a positive link between the quantity and the quality of students' solutions and their learning outcome. For example, Wiedmann and colleagues (2012) showed that the number of different solutions as well as the number of high quality solutions were positively linked to students' learning outcomes.

Because in the collaborative problem-solving phase of PF, students are not only able to mutually explain their solutions but also to iteratively improve them by discussion, we hypothesize that within PF, collaboratively learning students generate qualitatively better solutions than their individual counterparts. Moreover, the collaboration may stimulate the solution generation, so we further hypothesize that collaboratively learning students come up with more solution ideas than their individual counterparts (as they can only rely on themselves). In line with previous research on PF, we further investigate whether there is a positive link between the quality and quantity of solutions on the one hand and their learning outcome on the other hand.

Methods

In order to investigate our hypothesis, we report unpublished data from the aforementioned study conducted by Mazziotti and colleagues (2015) by analyzing and comparing students' solutions.

Study design

In the aforementioned 2x2 quasi-experimental study (N= 228 fifth graders; Mazziotti et al., 2015), we varied the factors social form of learning (collaborative vs. individual) and timing of instruction (problem-solving prior to instruction, or PF vs. problem-solving after instruction, or DI), resulting in four experimental conditions (PF-Coll: N= 74 = 34 complete dyads, PF-Ind: N= 49, DI-Coll: N= 57 and DI-Ind: N=48). The experimental procedure comprised a problem-solving phase (either in dyads or individually) and an instruction phase (designed as a whole class lecture). While in both PF-conditions students first tried to collaboratively or individually solve a problem and then received instruction, in both DI-conditions students first received instruction and then tried to collaboratively or individually solve a problem. As the DI-conditions are beyond the scope of this paper, we here concentrate on a more detailed description of the PF conditions.

In the problem-solving phase (of the PF-conditions), students were engaged in solving a typical PF problem which is, in line with the PF design requirements (cf. Kapur & Bielacycz, 2012), a complex problem about equivalent fractions (see Figure 1). At the beginning of grade 5 students have not yet been formally introduced to equivalence of fractions and only have an initial understanding of fractions, so this problem was quite challenging for students and was not meant to be solvable by the students in the initial problem-solving phase. To further ensure complexity, our problem implied three (instead of one) problem-solving steps (which were not marked as such for the students; see Figure 1). By calculating a solution, applying logical reasoning, making use of a graphical representation (i.e. circle) or drawing various solution ideas, students were able to find multiple different solution paths (cf. design requirement Kapur & Bielaczyc, 2012, Mazziotti et al. 2015). During the instruction phase, the instructor built upon typical student solutions by comparing and contrasting them with the canonical solution (cf. Loibl & Rummel, 2014).

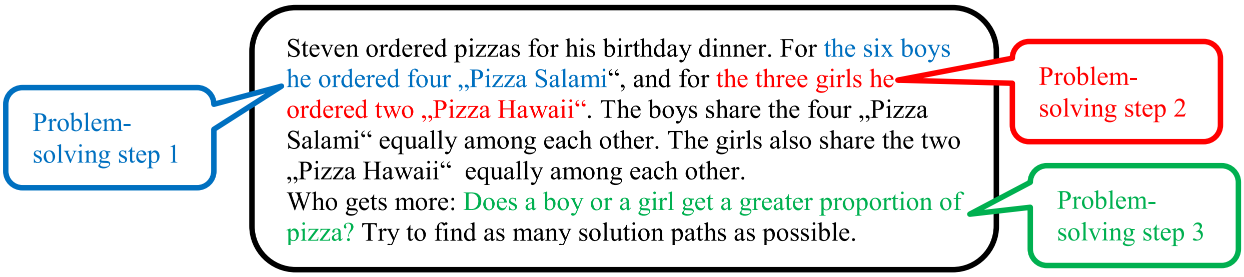

Figure 1: The equivalent fraction problem for the problem-solving phase of PF.

Overall, the study took place in three mathematics lessons (3x 45 minutes) and followed this procedure: In the mathematics lesson preceding the study, students were asked to fill out a pretest (10-15 minutes) that assessed students' mathematical prerequisites regarding fractions equivalence. The problem-solving phase took 30 minutes including 15 minutes introduction. The instruction phase took 45 minutes. All students had 45 minutes time to answer the posttest that assessed student's knowledge of equivalent fractions with open text-based items (6 items, some with multiple subtasks; a maximum of 17 points could be reached).

Quality and quantity of solutions

In order to investigate whether collaboratively learning students generated more and qualitatively better solutions compared to individually learning students, we analyzed the quality and quantity of students' solution solutions (cf. Kapur, 2012, Wiedmann et al., 2012). Naturally, quality and quantity are closely linked to the to-be-solved problem (see Figure 1).

For *solution quality*, we rated how well students divided the pizzas for boys and girls (cf. problem-solving steps 1 and 2), and how they compared the two proportions (cf. problem-solving step 3). Table 1 illustrates how many credits students could receive for each problem-solving step. In sum, assessing the quality of all problem-solving steps (dividing by number of boys, dividing by number of girls, comparing) leads to a maximum possible score of 9. As unit of analysis we used a single solution, that is, students' attempt to engage in all three problem-solving steps before restarting a new solution by engaging again in problem-solving step 1. Because 76% of the students across both conditions generated only one solution (i.e. attempt to solve *all three* problem-solving steps), we only rated the quality of this first solution.

To determine the *quantity of solution* we did not focus on the entire solution as unit of analysis (i.e., students attempt to engage in all three problem-solving steps), but used a more fine-grained approach in determining the quantity of *solution ideas* per problem-solving step. This was more appropriate for our target group because coming up with a new idea for a *single* problem-solving step rather than coming up with three completely new ideas for all three problem-solving steps (i.e., a complete solution) is already challenging and therefore not highly likely (cf. 76% of all students generated only a first solution). In particular, Ruwisch highlights that young students have difficulties to desist from an idea and to re-orientate towards a new one (Ruwisch, 1999). As a coding example, drawing the four pizza Salami and dividing them for the six boys into halves was coded as one solution idea for problem-solving step 1, and drawing and dividing the four pizza Salami for the six boys into quarters as an alternative way was coded as another solution idea for problem-solving step 1. We summarized the number of solution ideas across all problem-solving steps into one sum score.

Table 1. Rating of the quality of students' solutions per each problem-solving step

Problem-solving step	**Rating**
Problem-solving steps 1& 2: Dividing the pizza for boys and girls	Dividing equally, fairly and without a remainder (e.g., into sixths, thirds, twelfths). (1 point for trying to divide + 1 point for dividing equally + 1 point for dividing without leaving a remainder = 3 points)
	Diving equally but unfairly (e.g., halves, quarters, fifths). (1 point for trying to divide + 1 point for dividing equally = 2 points)
	Avoidance strategies: Stating that some children are not hungry, or ordering more pizza so that children get whole pizzas. (1 point for trying to divide whole pizzas amongst the children = 1 point)
	Not trying to divide (0 points).
Problem-solving step 3: Comparing two proportions of pizza	Both pizzas are divided into the same amount of pieces (e.g., both pizza Salami and pizza Hawaii are divided into twelfths). Students can thus "prove" that each boy and girl receives the same proportion of pizza. (1 point for trying to compare + 1 point for comparing the equal amounts of pieces + 1 point for reaching the correct conclusion that each child receives the same number of pieces = 3 points)
	The two pizzas are divided into different numbers of pieces (e.g., pizza Salami into thirds and pizza Hawaii into twelfths). Due to a superficial comparison, students conclude correctly that each boy and girl receives the same proportion of pizza. (1 point for trying to compare + 1 point for reaching the correct conclusion that each child receives the same number of pieces = 2 points)
	The two pizzas are divided into different numbers of pieces (e.g., pizza Salami into thirds and pizza Hawaii into eights). Due to a superficial comparison, students conclude incorrectly that either a boy or a girl receives a smaller or greater proportion. (1 point for trying to compare = 1 point)
	Not trying to compare (0 points).

Two coders were trained to assess the quality of students' first solution and the quantity of students' solution ideas. To determine interrater reliability for the quantity of solution ideas, the raters both coded 43% of the paper-based data (i.e. PF-Coll and PF-Ind condition) and reached satisfactory interrater reliability (quantity of solution ideas $\kappa = .65$). To determine inter-rater agreement for the quality of the first solution, they both rated a subset of 20% of students' first solution. Agreement was high ($ICC_{absolute} = .79$; 95%-CI [.53; .92]). Disagreements were resolved by discussion.

Results and discussion

To test whether collaboratively learning students or individually learning students generated a higher quantity of solution ideas and a higher quality of the first solution, we calculated two univariate ANOVAs with the factor condition (PF-Coll vs. PF-Ind) and either quantity of solution ideas or quality of the first solution as the dependent variable. Table 2 shows means and standard deviations of both PF-conditions. Descriptively, the collaboratively learning students outperformed individually learning students with regard to both quality of their solution ($F[1,81]=2.178$, $p=.144$) and the quantity of students' solution ideas ($F[1,81]=0.949$, $p=.333$), but the differences were not statistically significant.

In line with previous PF-studies, we calculated correlations between students' learning outcomes and their solutions. There was no significant correlation between students' learning outcome and the quality of students' solution ($r(117)=-.011$, $p=.902$), or the quantity of students' solution ideas ($r(117)=.081$, $p=.385$). Looking at each PF-condition separately, we did not find significant correlations between students' learning outcome and the quality of their solution (PF-Coll: $r(68)=.084$, $p=.495$; PF-Ind: $r(49)=-.086$, $p=.555$), or the quantity of solution ideas (PF-Coll: $r(68)$ =.32 p=.793; PF-Ind: $r(49)$=.173, p=.233). Overall, there was no positive link between students' solutions and their learning outcome.

Table 2. Quantity and quality of solutions from individually and collaboratively learning students

	PF-Coll			**PF-Ind**		
	M	SD	N (dyads)	M	SD	N
Quality of solutions	4.97	2.1	34	4.18	2.57	49
Quantity of solution ideas	4.53	1.8	34	4.12	1.92	49

In conclusion, when comparing whether collaboratively or individually learning students generated more and qualitatively better solutions, we found a small and statistically non-significant advantage of collaborative over individual learning for both the quantity of solution ideas and quality of the first solution. Still, during the collaborative problem-solving process, students might have iteratively improved their solution by discussing strengths and weaknesses of their solution in ways that our analysis could not yet capture. In order to support this conclusion, our next step is to investigate how collaboratively learning students developed their solutions in the course of the discussion, by analyzing students' verbal process data.

In line with the aforementioned research on PF, we further investigated whether there was a positive link between students' solutions and the learning outcome. We were not able to replicate this link, as other PF-researchers before us (e.g., Loibl & Rummel, 2014). One possible explanation for the missing correlation in our and other previous studies might be the short learning time and in our case a domain specific issue that is the persistence of students' misconceptions about fractions which were reflected in their solutions. In other words: Students' misconceptions were too persistent to be resolved prior to the test phase. An indication in this direction is that students' average scores on the post-test still fell short of the maximum score (PF-Coll: M= 10.31, SD = 2,45; PF-Ind: M = 10.8, SD= 2.41 out of 17 points). In a next step of analysis we thus aim to investigate how students' misconceptions which will be extracted from the verbal process data are linked to students' posttest scores.

Another possible explanation may be that although students were asked about the same knowledge components (KC; Koedinger, Corbett & Perfetti, 2012) as for example comparing two fractions with unlike denominators and numerators during the problem-solving and the test phase, the differences of superficial characteristic between the PF problem (i.e., concrete situation) and some test items (i.e., abstract situation) was too large. Therefore, students may not have been able to recognize the KCs in the learning and test phases as identical. For future research, we thus propose to either keep also superficial characteristics of the to-be-solved problem and test items as identical as possible or to ask students' to generate the KCs by themselves while solving test items. In sum, the question of how students' solutions are linked to students' learning outcome cannot yet be answered.

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