Rituals, Explorations, and Cultural Resources in the Mathematics Classroom: When Arguing Does Not Help Learning

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Abstract: This study examines "ritual" and "explorative" participation in math classroom. We use the case of adult ultra-Orthodox studying algebra for the first time, to present a rare example of ritual teaching which is responded to by explorative participation. We explain it by addressing students' "intellective identities" as they stem from students' experience in Talmud studies, a main practice of the ultra-Orthodox culture. We conclude by discussing the potential relevancy of the case to mathematics education.

Objective and theoretical framework

"Rituals" and "explorations" are described by the commognitive framework (Sfard & Lavie, 2005, Sfard, 2008) as mathematical routines, distinguished from each other by (A) learners' goals, (B) the objects that are being talked about, (C) learners' flexibility in use of the routines, (D) authority, and (E) whether the focus is on steps and procedures, or on producing new mathematical narratives. Explaning the necessity of rituals as a stage towards explorative routines, Sfard & Lavie (2005) suggest that rituals are "probably the only possible departure points toward a meaningful discourse" (p. 283). In a recent work, Heyd-Metzuyanim and Graven (2016) have broadened this lens to link between ritual participation, South African post-apartheid context and learners' identity. The current paper continues this line of inquiry, aiming to examine connections between the ultra-Orthodox culture, which has some known explorative characteristics, and participation in mathematics learning. This relation is examined on adult Ultra-Orthodox males studying secondary mathematics for the first time in their lives at a preparatory college program.

Research context and method

For religious and political reasons, most ultra-Orthodox males in Israel do not study mathematics after the age of 11-12. Instead, they devote all their time to the study of the Talmud, the central holy text of Judaism. The Talmud is a Jewish anthology of texts – mostly conversational - from 200-500 AD, with later critical interpretations of these discussions. The text usually present conflicting viewpoints and describes the process by which participants in the discussion have arrived at their conclusions. Doubts, questions, challenges, disagreements, clarifications, and even humor are all legitimate parts of conversations (Schwarz, 2015).

Pre-academic programs are the means for young adults that have completed their Yeshiva studies to access post-secondary education and employment. The episode presented here is taken from a study of a classroom of 10 men in such a program. The study followed the last 4 months in a 12 months course instructed by the first author. Collected data included six video-recorded lessons and four interviews with 3 students regarding their experience as Talmud and math learners. All data is originally in Hebrew and has been translated by the authors.

Previously (Ehrenfeld & Heyd-Metzuyanim, in press) we have analyzed how cultural resources from both Talmudic and the mathematics studies, were used by students to construct a new hybrid discourse, and the relation of this process to students' intellective identities (Greeno, 2002). In this paper we use a complementary theoretical framework: explorative vs. ritual partcipation (Sfard, 2008). The commognitive lens (Sfard, 2008) conceptualizes mathematics learning as participation in a discourse characterized by certain routines. Two types of routines are relevant for our study: ritual and explorative. Ritual routines aregeared towards connecting with others or being identified as a competent learner while explorative routines are geared towards constructing mathematical narratives for their own sake. Building on Greeno (2002), and Sfard & Prusak (2005), we link students forms of participation (ritual vs. explorative) to their *intellective identities*, which are those narratives that participants use (or refer to tacitly) to describe themselves (and others) as competent or incompetent. We then suggest how the relation between participation and intellective identities can inform our thinking about cultural resources in the classroom.

Results and scholarly significance

Our data shows that some students in the rather traditionally-set mathematics preparatory classroom were bringing with them Talmudic discourse practices to the teacher-directed lessons. In particular, the authority structure of Talmudic studies, where the students argue with each other over the text without the interference of a teacher,

could be seen occurring spontaneously, even while the teacher did not plan (or implicitly opposed) any discussion to occur.

Our analysis of such episodes shows a mixture of ritual and explorative routines, with a surprisingly prominent aspect of explorative ones. However, the prominence of explorative interactional routines, where students question each other and the teacher's mathematical claims, did not always lead to productive learning. At times, it actually raised obstacles for the lesson. These are exemplified in a case where students were introduced to the use of 'f(x)' instead of 'y'. The teacher's rational for introducing 'f(x)' was that the new sign might visually support students' in learning derivatives. The new convention was described by the teacher as mere "convenience". Yet, the ultra-Orthodox students resisted dealing with symbols unrelated to the mathematical meanings. For example, Joshua (pseudonym) marked, half humorously: "and why? because the sky is blue" (Hebrew: "ve'lama? Kova af la'gova."), an idiom that signs accepting something without reason. The fact that Joshua mentions he accepts a change without an explanation is evidence that this process is a conscious rather than automatic one. While the teacher attempted to avoid substantiating his usage of mathematical signifiers, students' discourse continued to be focused on the rationale for using the f(x) sign instead of y. For example, Abraham asked "what did you gain?" , and Micky asked "why didn't you do f of y?".

We acknowledge that for some readers, this might seem as ideal students' participation since students actively seek reasons for certain mathematical actions. However, we suggest a different point of view. Given the teacher's plan to teach derivatives, these explorative tendencies come at a particular time of the lesson where ritual rule following might actually have been more conducive for the students. This episode, as well as others observed in this study, may support Sfard's (2008) claim that certain types of learning need to progress from ritual to explorative participation since to engage with talking about certain mathematical objects, learners first need to familiarize themselves with these objects through thoughtful imitation of more experienced others. Our data thus points to the occasional necessity of balancing students' explorations, in particular due to what we hypothesize as the importing of Talmudic authority structure and "cultural preference for disagreement" (Blum-Kulka, Blondheim, & Hacohen, 2002).

We gained insight into some of the students' insistence on questioning and arguing with the teacher through interviews that explored their intellective identities. Through these interviews, we found that the ultra-Orthodox students' idea of learner competency was imported by them from the Talmudic world to mathematics. For example, one of the students referred to Talmud learning as endless exploration, telling about the Talmudic habit of constantly asking 'why?': "Anything you learn, you ask yourself 'why'? And then you try to obtain a book, and whatever it says about it, you also ask 'why'? And that's something that is endless. It's very enjoyable and very nice." We found a link between students' reported insistence on questioning and arguing in Talmudic studies, and their repeating of these interactional routines in the mathematics classroom. This link points to intellective identities and cultural resources as important domains for inquiry in the search for mechanisms by which learners become critical and explorative thinkers. Furthermore, the uniqueness of ultra-Orthodox students' cultural preference for disagreement might enhance our thinking of both opportunities and constraints that such discursive moves can place on mathematics learning.

References

Blum-Kulka, S., Blondheim, M., & Hacohen, G. (2002). Traditions of dispute: From negotiations of talmudic texts to the arena of political discourse in the media. *Journal of Pragmatics*, *34*(10–11), 1569–1594.

Ehrenfeld N. & Heyd-Metzuyanim E. (in press). Intellective Identities in the Construction of a Hybrid Discourse: the Case of an Ultra-Orthodox Jewish Mathematics Classroom. *International Journal of Science and Mathematics Education.*

Greeno, J. G. (2002). Students with competence, authority, and accountability: Affording intellective identities in classrooms. *The College Board*, 1–29.

Heyd-Metzuyanim, E., & Graven, M. (2016). Between people-pleasing and mathematizing - South African learners' struggle for numeracy. *Educational Studies in Mathematics*, *91*(3), 349–373.

Sfard, A. (2008). *Thinking as communicating*. New York: Cambridge University Press.

Sfard, A., & Lavie, I. (2005). Why cannot children see as the same what grown-ups cannot see as different? — Early numerical thinking revisited. *Cognition and Instruction*, *23*(2), 237–309.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational researcher*, 34(4), 14-22.

Schwarz, B. B. (2015). Discussing argumentative texts as a traditional Jewish learning practice. In L. B. Resnick, C. Asterhan, & S. Clarke (Eds.), *Socializing Intelligence Through Academic Talk and Dialogue*. American Educational Research Association.