Pairs Trading Model in R

A Statistical Arbitrage Strategy

Group members:
FOUKI Houssam, AMRAOUI Hassan, CONSOLI Lorenzo,
BADARU Oluwafisayomi, MAISURADZE Alexander

Professor Pierre JACOB
Forecasting & Predictive Analytics
ESSEC Business School - CentraleSupélec

Outline

1. **Basic Idea of Pairs Trading**
2. **Testing for Cointegration**
    - **The idea of Pairs Trading based on Cointegration**
    - **Pairs Selection**
    - **Least Squares (LS) regression**
    - **Augmented Dickey-Fuller (ADF) test**
    - **Phillips-Perron (PP) test**
3. **Trading Strategy Design**
    - **Trading the Spread**
    - **Pairs Trading Performance (Simulation)**
4. **Pros and Cons of Pairs Trading**
5. **Other Pairs Trading Implementations**

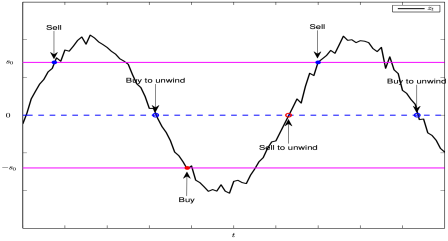
Basic Idea of Pairs Trading

Trading the spread

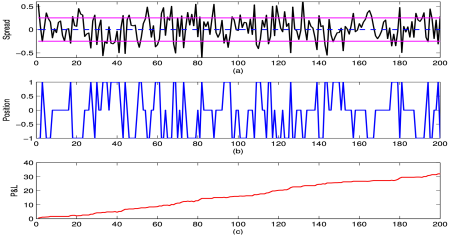
The idea behind pairs trading is to **short-sell the relatively overvalued stocks** and **buy the relatively undervalued stocks**, and also unwind the position when they are relatively fairly valued.

Illustration on how to trade the spread

$$z_t = x_t - \gamma y_t$$



Statistical arbitrage can be used in practice with profits



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The Idea of Pairs Trading based on Cointegration

Cointegration

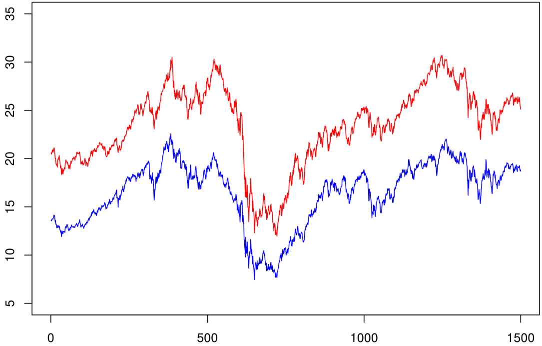
Consider a pair of time series, both of which are **non-stationary**. If we take a particular linear combination of these series, it can sometimes lead to a stationary series. Such a pair of series would then be termed **Cointegrated**.

Idea

While it may be difficult to predict individual stocks, it may be easier to predict **relative behavior** of stocks

Let x_t and y_t be two non-stationary $I(1)$ time series, with $\beta_0, \beta_1 \in \mathbb{R}$ constants. If the combined $y_t = \beta_0 + \beta_1 x_t + \epsilon_t$ series is stationary then we say that x_t and y_t are cointegrated.

Note: x and y here are clearly not stationary, but they seem to move together. In fact, they are cointegrated: $y_t - \beta_1 x_t - \beta_0$ should be stationary.



Illustrative Example

A drunk man is wandering the streets (random walk) with a dog. Both paths of man and dog are nonstationary and difficult to predict, but **the distance between them is mean-reverting and stationary.**

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Pairs selection: Normalized Price Distance

Design of Pairs Trading Strategy

In practice, pairs trading contains three main steps:

- **Pairs selection**: identify stock pairs that could potentially be cointegrated.
- **Cointegration test**: test whether the identified stock pairs are indeed cointegrated or not.
- **Trading strategy design**: study the spread dynamics and design proper trading rules.

- Normalized price distance (as a rough proxy to measure cointegration):

$$NPD = \sum_{t=1}^{T} (\tilde{p}_{1t} - \tilde{p}_{2t})^2$$

  where the normalized price $\tilde{p}_{1t}$ of stock 1 is given by $\tilde{p}_{1t} = p_{1t}/p_{10}$. The normalized prices of stock 2 defined similarly: $\tilde{p}_{2t} = p_{2t}/p_{20}$
- Now we can basically compute the NPD for all the possible combination of pairs and select some pairs with smallest NPD as the potentially cointegrated pairs.
- One can use a more refined measure of cointegration (more computationally demanding).

Pairs selection: Normalized Price Distance

Results were generated and we picked the pair with the lowest NPD: **('JPM', 'GS')**. This comes as no surprise since both stocks are finance related and these companies can be said to be competitors. $(Log-)$prices of both stocks are shown below:

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Least Squares (LS) regression

- If the spread z_t is stationary, it can be written as:

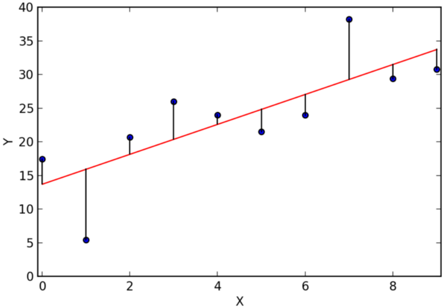
$$z_t = x_t - \gamma y_t = \mu + \epsilon_t$$

  where μ represents the equilibrium value and ϵ_t is a zero-mean residual.

- Equivalently, it can be written as:

$$x_t = \mu + \gamma y_t + \epsilon_t$$

  which now has the typical form of linear regression.



- Least Squares (LS) regression over T observations:

$$\min_{\mu,\gamma} \sum_{t=1}^{T} (x_t - (\mu + \gamma y_t))^2$$

- LS regression is used to estimate the parameters μ and γ, obtaining the estimates $\hat{\mu}$ and $\hat{\gamma}$.

Least Squares (LS) regression

LS Regression Results					
Coefficients:					
	Estimate	Std. Err	t-value	Pr. (>| t |)	
(Intercept)	2.4507	0.015	160.779	0.0000	***
JPM	0.6565	0.004	162.954	0.0000	***

Significant Codes: 0 '***'

From this we can establish the spread as:

$$z_t = log(GS)_t - (0.6565)log(JPM)_t$$

Now that we have the spread equation ready, we can run the Augmented Dickey-Fuller test and other unit roots statistical tests to test for Cointegration.

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Augmented Dickey-Fuller (ADF) test

Mathematically, the ADF is based on the idea of testing for the presence of a unit root in an autoregressive time series sample. It makes use of the fact that if a price series possesses mean reversion, then the next price level will be proportional to the current price level. A linear lag model of order is used for the time series:

$$\Delta y_t = \alpha + \beta_t + \gamma y_{t-1} + \delta_1 \Delta_{t-1} + ... + \delta_{p-1} \Delta y_{t-p+1} + \epsilon_t$$

Where α is a constant, β represents the coefficient of a temporal trend and $\Delta y_t = y(t) - y(t-1)$. The role of the ADF hypothesis test is to consider the null hypothesis that $\gamma = 0$, which would indicate with $\alpha = \beta = 0$ that **the process is a random walk and thus non mean reverting.** The first task is to calculate the test statistic (ADF_τ), which is given by the sample proportionality constant $\hat{\gamma}$ divided by the standard error of the sample proportionality constant:

$$ADF_\tau = \frac{\hat{\gamma}}{SE(\hat{\gamma})}$$

The test statistic is a negative number and thus in order to be significant beyond the critical values, the number must be **less than the critical values.**

Augmented Dickey-Fuller (ADF) test

ADF Test Result (Residuals)		
p-value	0.01494	
Critical Value	-3.2985	At 1%: -3.4316
		At 5%: -2.8621
		At 10%: -2.5671

The p-value is small and hence we have evidence to **reject the null hypothesis that the series possesses a unit root**. This means that the spread is stationary and hence **the J.P Morgan and Goldman Sachs stocks are indeed cointegrated**. Now we try the Phillips-Perron Unit root test.

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Phillips-Perron (PP) test

The ADF test is based on the assumption that the error terms have constant variance and are statistically independent. The PP test, however, which was developed as a generalization of the ADF test, has a milder assumption regarding the error terms. The regression test takes the following $AR(1)$ form:

$$y_t = a_0 + a_1 y_{t1} + \epsilon_t$$

where the null is that $a_1 = 1$ and the alternative that $a_1 < 1$. Rejecting the null will indicate that y_t does not have a unit root and is therefore stationary.

Whereas the ADF test adds lagged differentiated terms to handle higher-order correlations, the PP test modifies the coefficient a_1 from the $AR(1)$ regression for the serial correlation in ϵ_t. The derivation of the PP-test is beyond the scope of this thesis.

Phillips-Perron (PP) test

Phillips-Perron (PP) Test Result		
p-value	0.040	
Critical Value	-2.944	At 1%: -3.43
		At 5%: -2.8621
		At 10%: -2.5671

At a 95% confidence level, Phillips-Perron test is below the critical value, therefore it is safe to **reject the null hypothesis that the spread contains a unit root** and hence assume that the spread was generated by a **stationary process**.

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Trading the spread

From our spread results, we standardize the spread series and set upper and lower boundaries to ± 1. Thus, our trading signals occur at $+1$ and -1. The strategy is the following:

- When the spread is above $+1$, then we sell the spread which means that we sell GS and buy JPM with units corresponding to cointegrating coefficient as explained above.
- When the spread is below -1, then we buy the spread which means that we buy GS and sell JPM with units corresponding to cointegrating coefficient as explained above.
- Whenever the spread converges again to 0, then we close the position.

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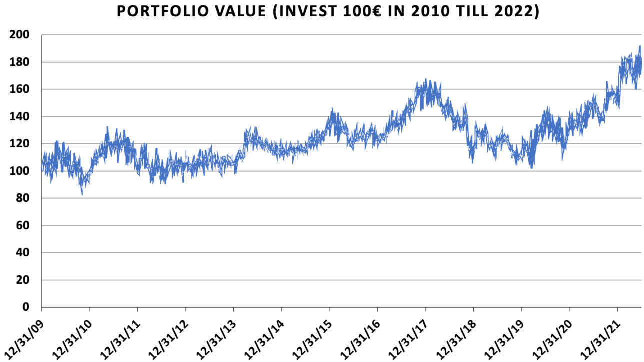
Pairs Trading Performance (Simulation)



Strategy return	**5%**
Beta	**-0.011875**

There are still fluctuations which shows that although the strategy is market-neutral, an investor is still exposed to **idiosyncratic risk** and the strategy is only market neutral as it provides a **return of 5% at an almost zero risk exposure** which is comparable to the market returns.

Pros and Cons of Pairs Trading

Pros

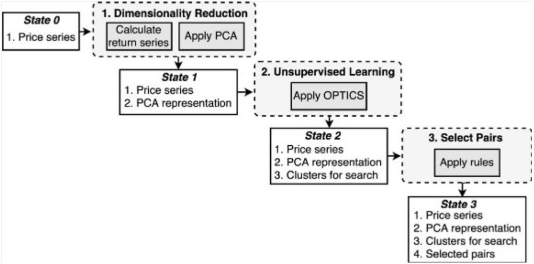
- Able To Mitigate Potential loses and Risks (one underperforms other possibly taking loses
- Profit Earning : regardless of the market
- Trader is completely hedged

Cons

- High commission charges (double of the fee of standard trade)
- Price Filling : Low margins-¿ large quantities . Risk of filling the stock orders at the desired price
- Hard to find rewarding pairs, because of the growing data and that it relies on high statistical correlation

Another example of Pairs Trading Implementation ML

- Limit space search PCA (High Dim –irrelevant features and curse
- Density based clustering algorithm
- Cointegration : Engle and Granger
- Hurst exponent 0-0.5 TS - mean-reverting
- Coherence Between mean-reversion time and trading period
- Threshold by Quantiles deciles or profit mix optimization
- Artificial Neural Networks: LSTM (Sequential dependency, History)



$$\{\alpha_S, \alpha_L\} = \underset{q}{\operatorname{argmax}} R^{\text{val}}\left(q\right),$$

$$q \in \left[\left\{Q_{f^-(x)}\left(0.20\right), Q_{f^+(x)}\left(0.80\right)\right\}\left\{Q_{f^-(x)}\left(0.10\right), Q_{f^+(x)}\left(0.90\right)\right\}\right]$$

where R^{val} is the return obtained in the validation set.

Another example of Pairs Trading Deep Reinforcement Learning

- DQN
- Produces Q function which maximizes profit
- Pairs-dickey fuller is in [0,0.05],std/mean¿0.5
- Returns next state and profit
- Updates input weights (ADAM optimizer) and repeats
- OPENAI gym environment

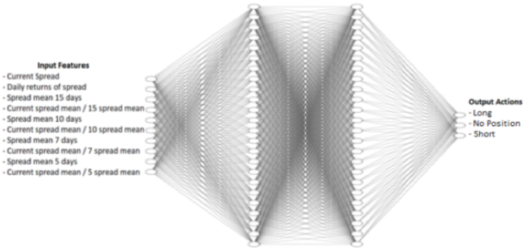


Figure 10: DQN NN structure

$$training\ rewards = action \times spread\ returns \times negative\ returns\ multiplier$$

$$testing\ rewards = action \times spread\ returns$$