An Algebraic Process for Visualization Design

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Why theory?

Theory is the foundation.

Fundamental properties of what visualization is, and how it works.

Many Vis Theory papers are about taxonomies.

Our basic idea

Rethink theory of vis design. Not about:

"Dataset is X, so vis should be Y"

but rather:

"We can X the data; can we Y the image?"

Our design approach studies **changes**

in data to be visualized

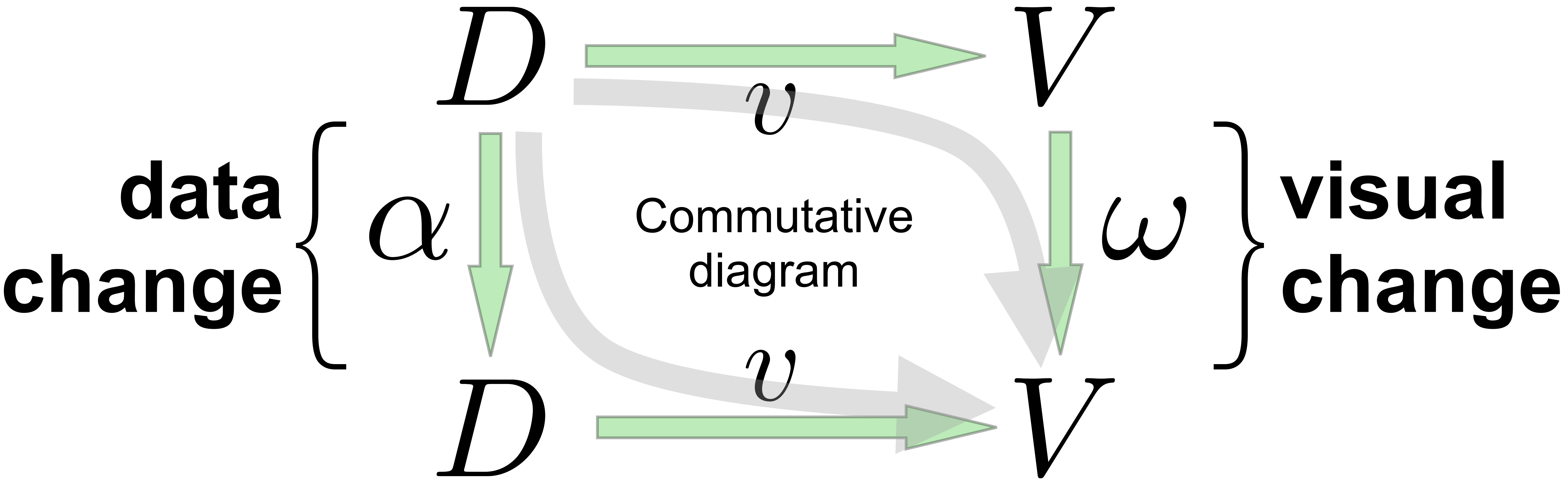
in images produced by visualization

The basic design question

"Are important data changes
well-matched
with obvious visual changes?

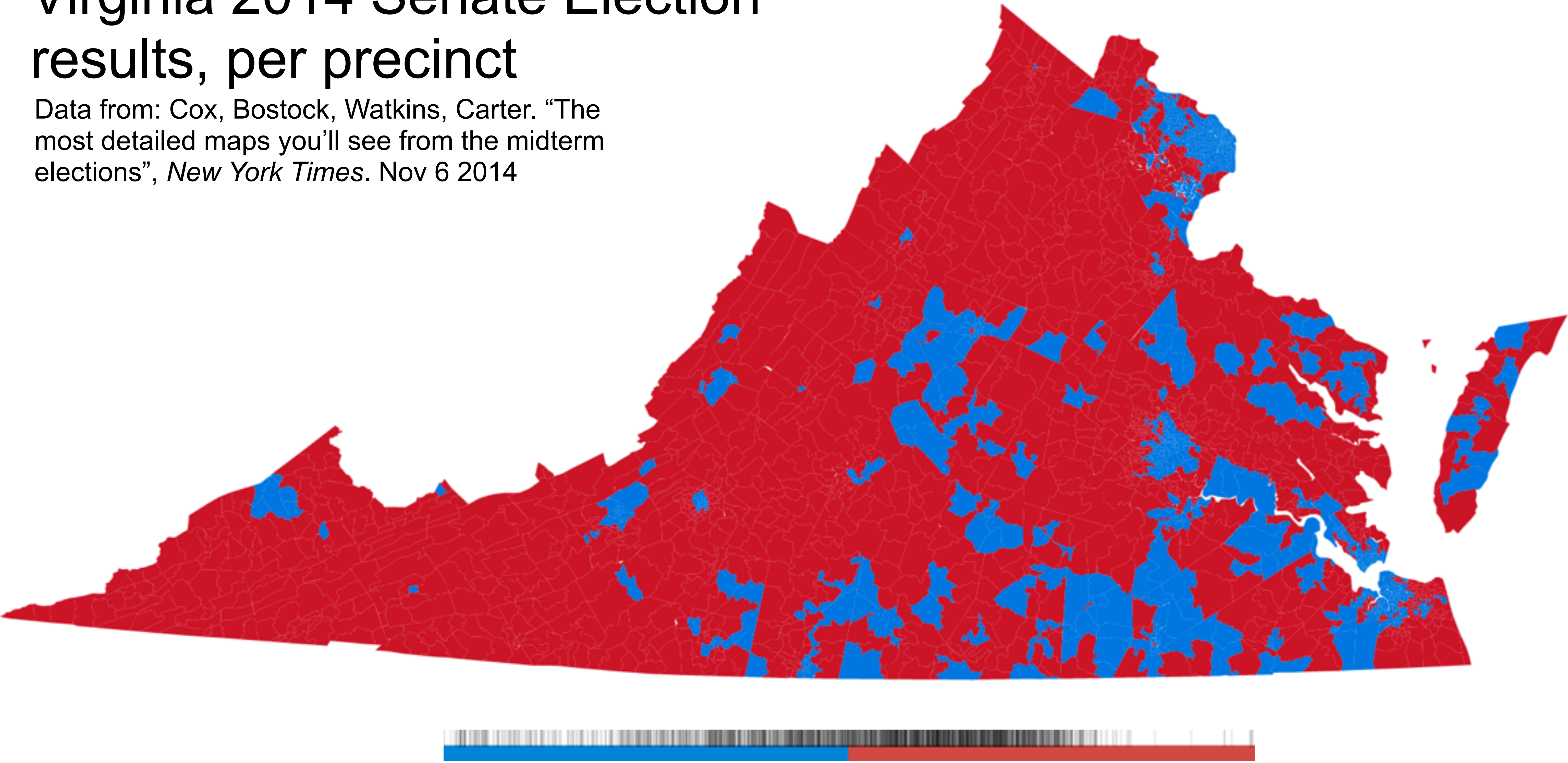
Algebraic visualization design

Contribution: a math vocabulary for targeted questions about a vis method, and for understanding why one method might be better, for a certain task



Virginia 2014 Senate Election results, per precinct

Data from: Cox, Bostock, Watkins, Carter. “The most detailed maps you’ll see from the midterm elections”, *New York Times*. Nov 6 2014



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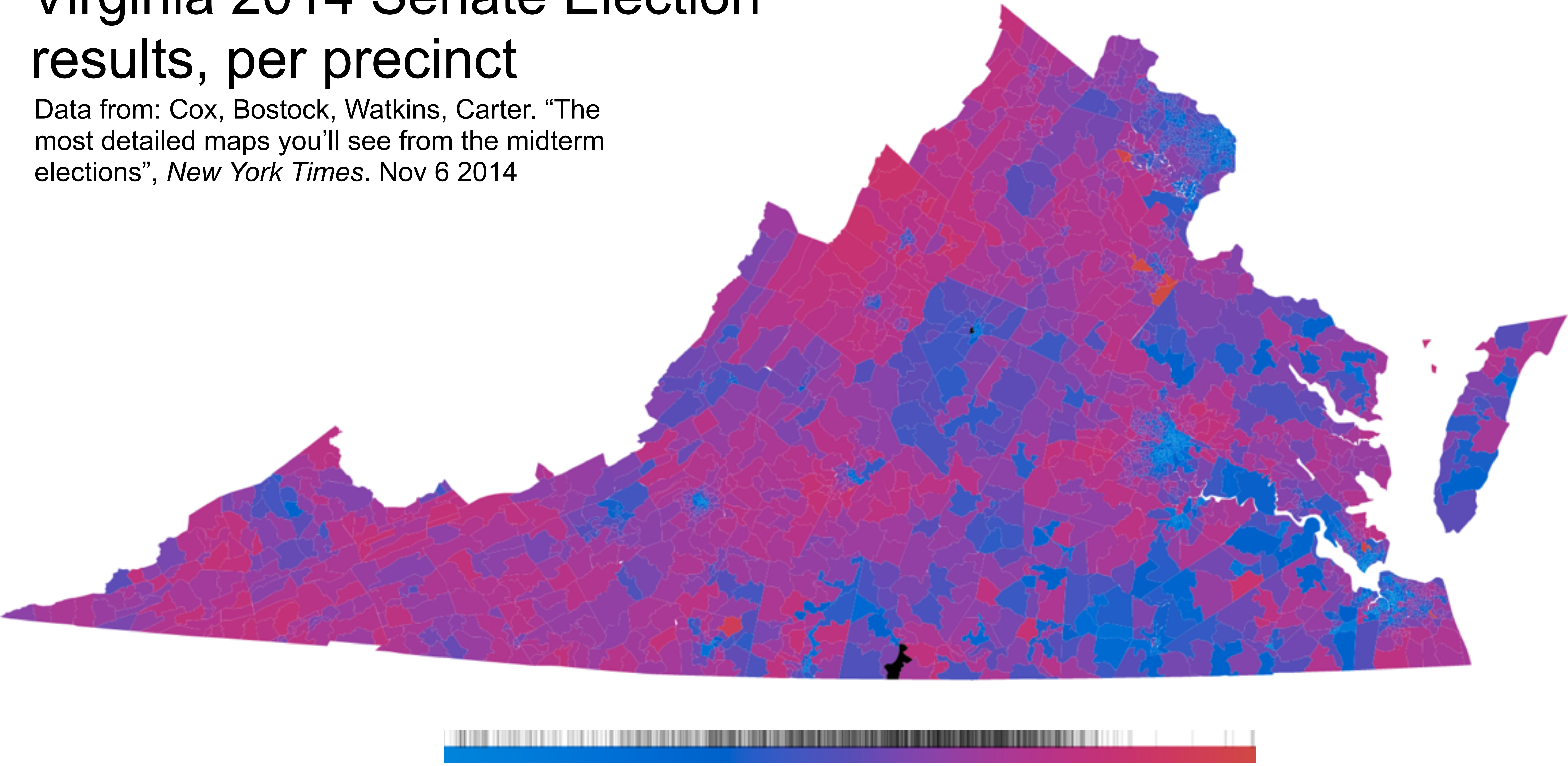

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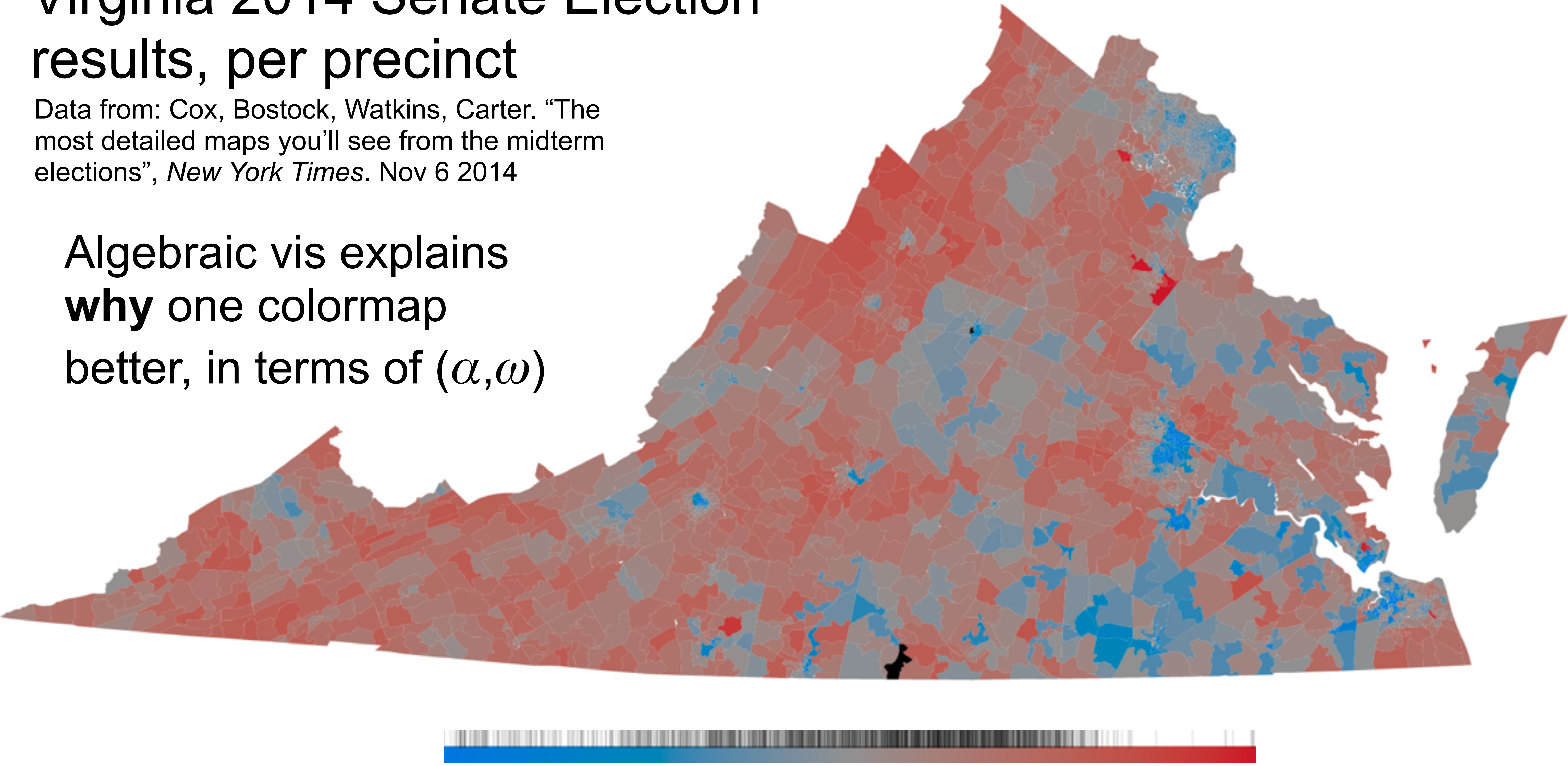
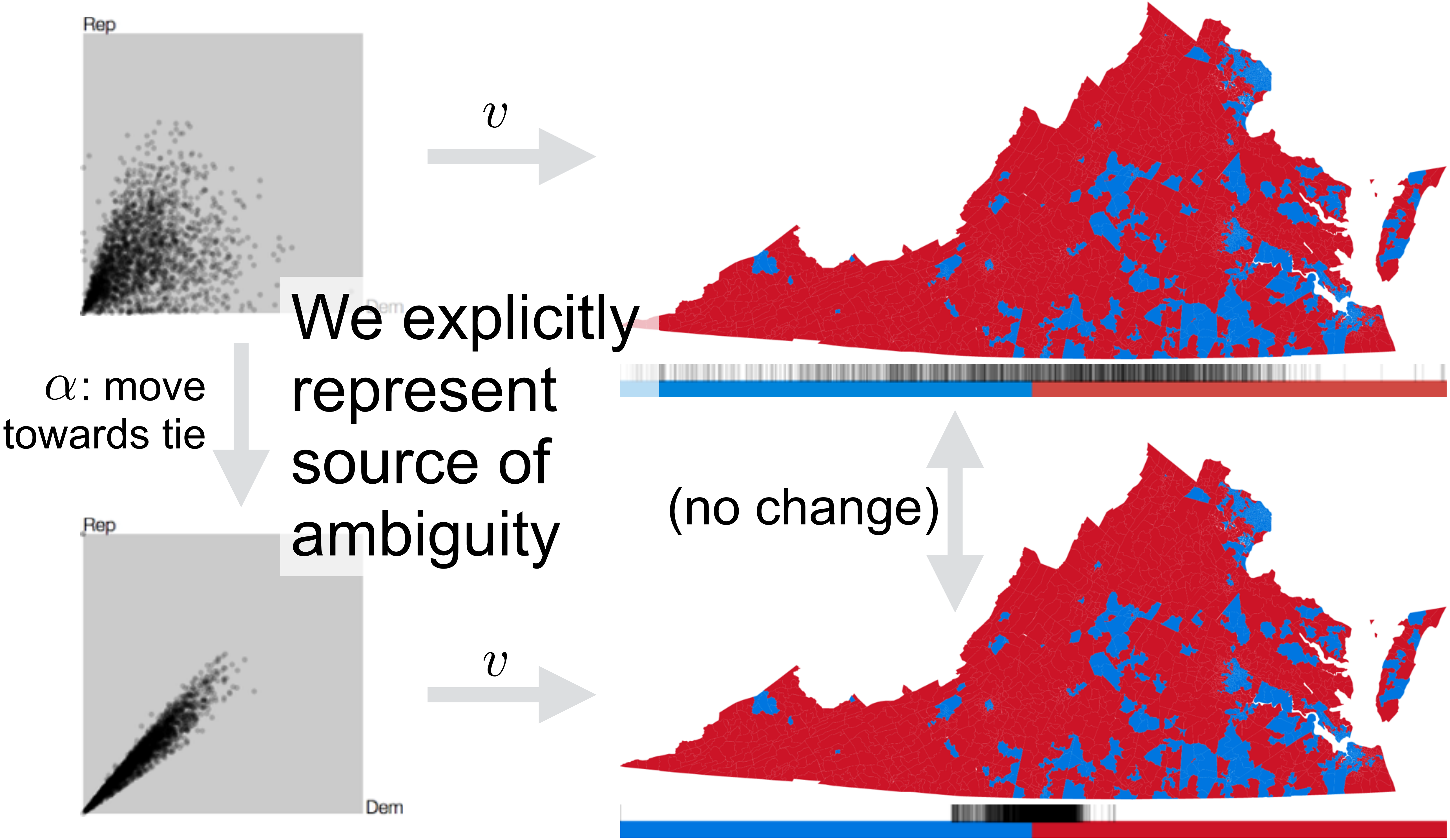
Algebraic vis explains **why** one colormap better, in terms of (α,ω)

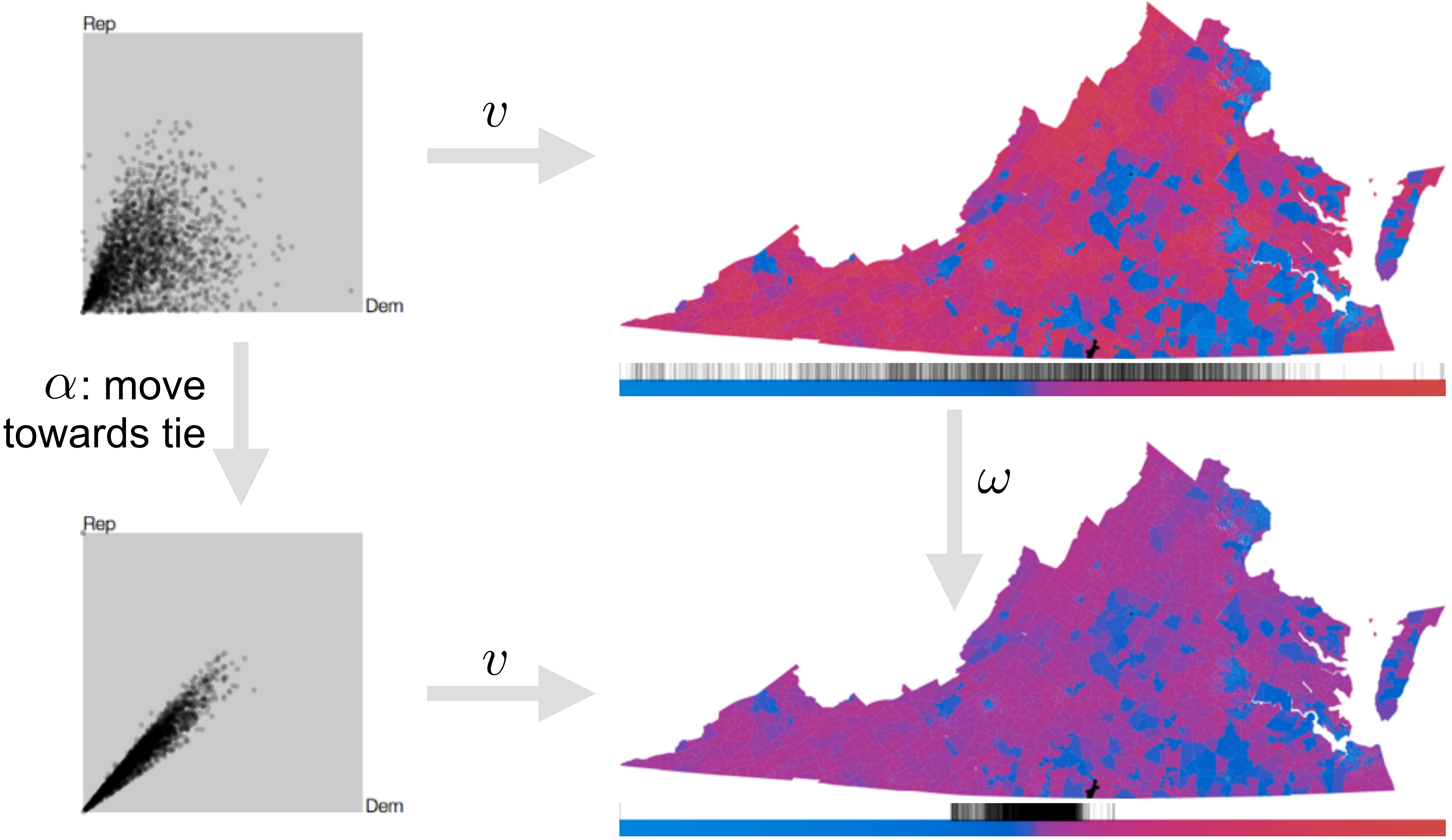


Let’s change the data

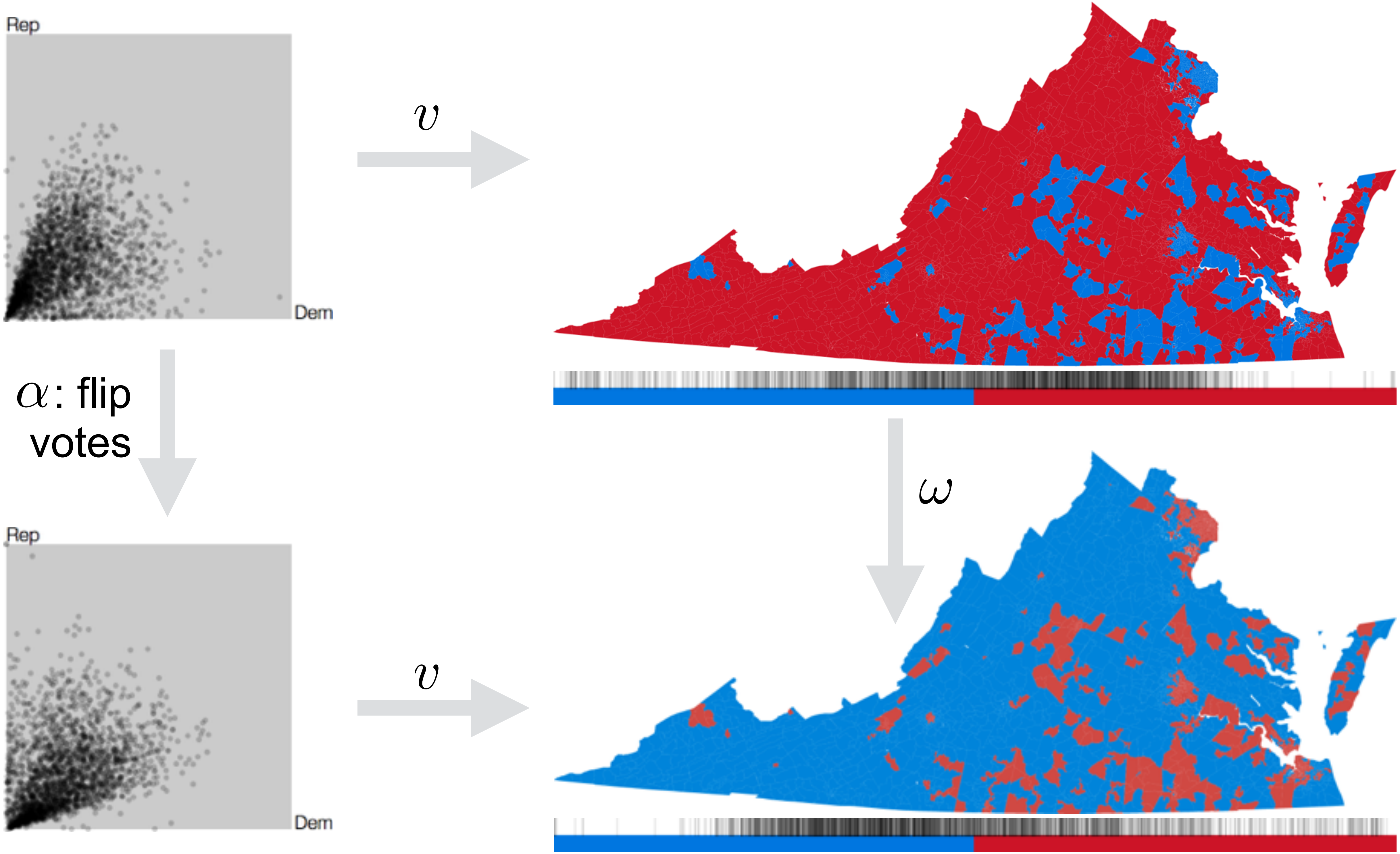


We explicitly represent source of ambiguity

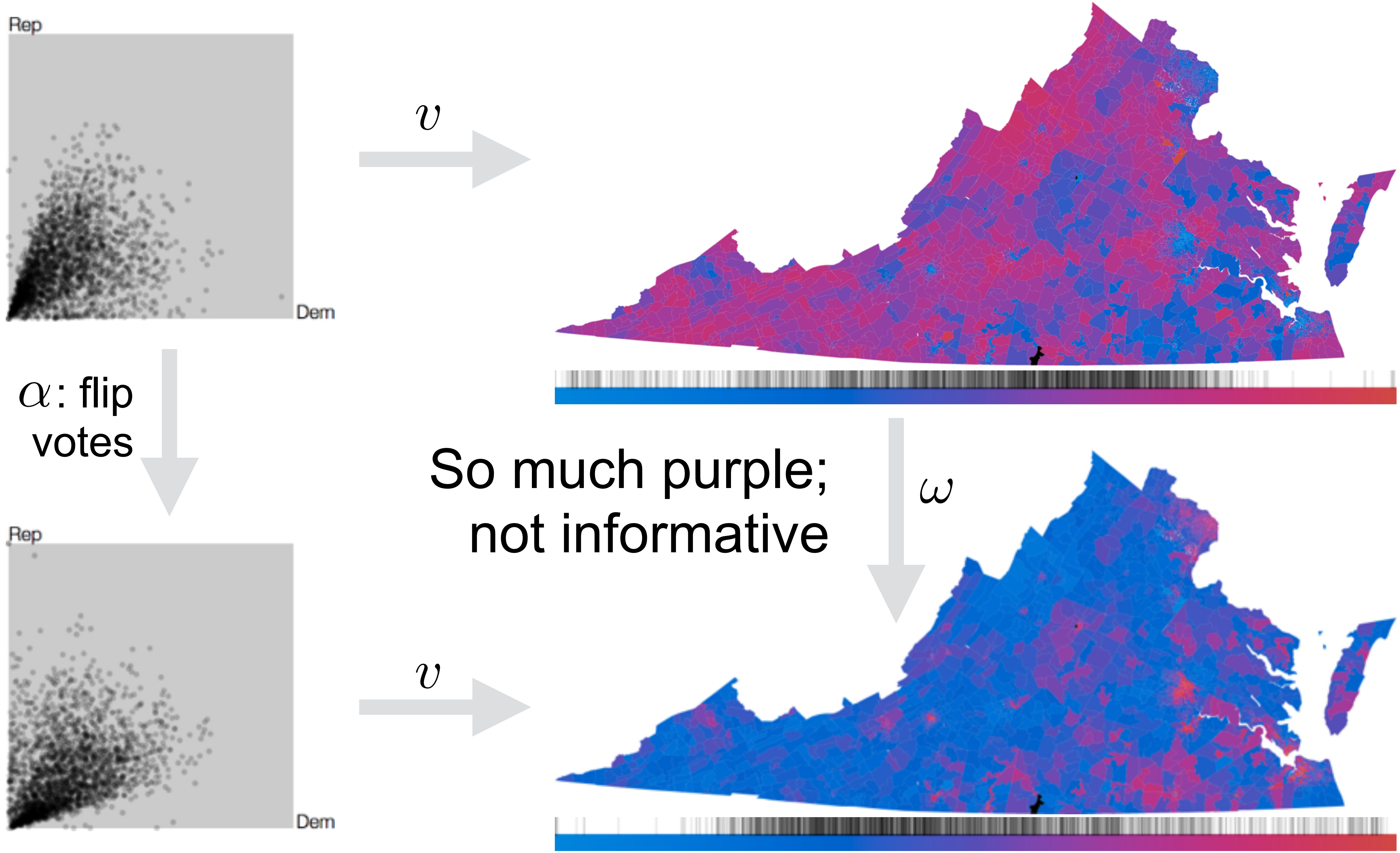
… now with a different visualization



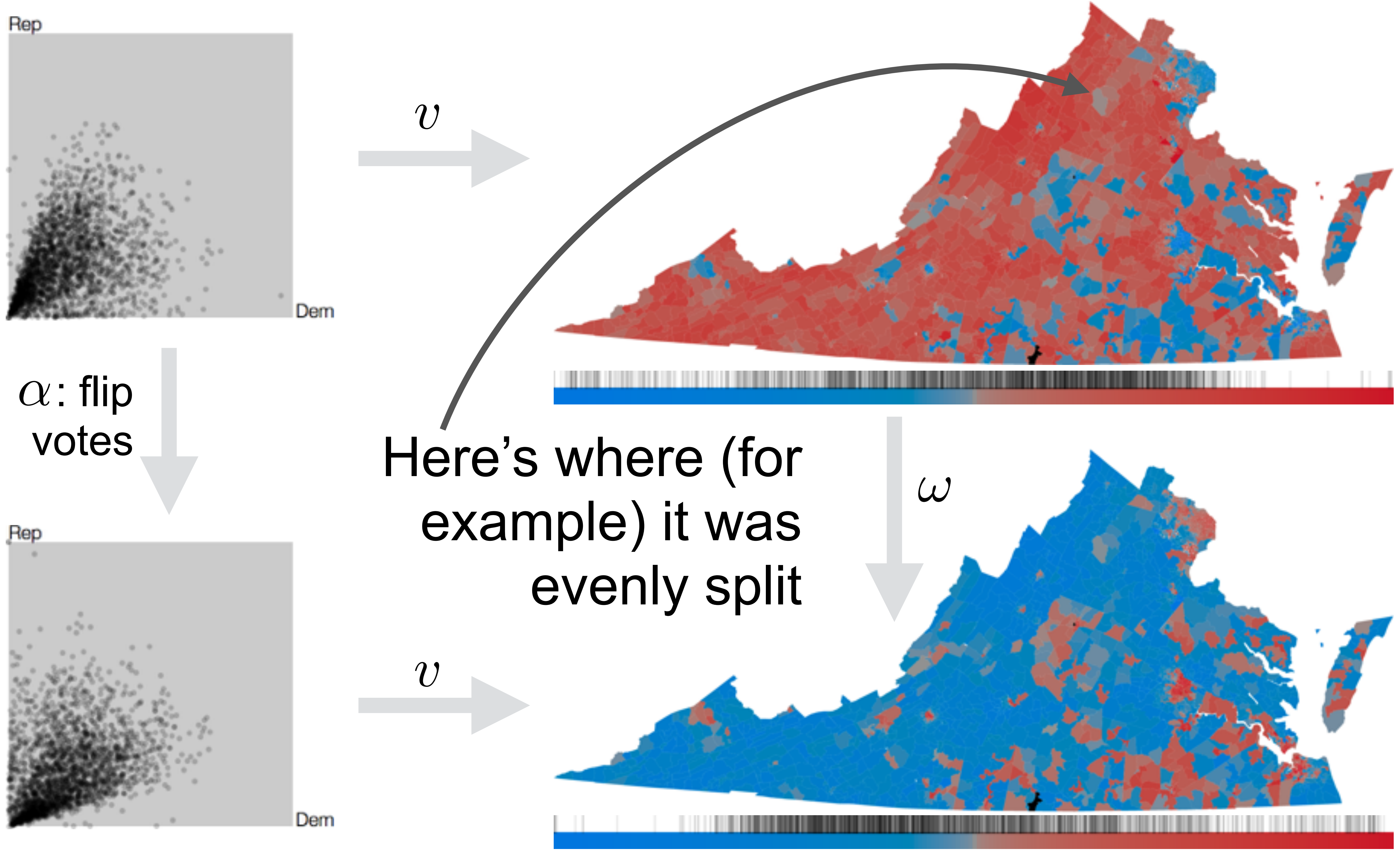
Where was vote evenly split?



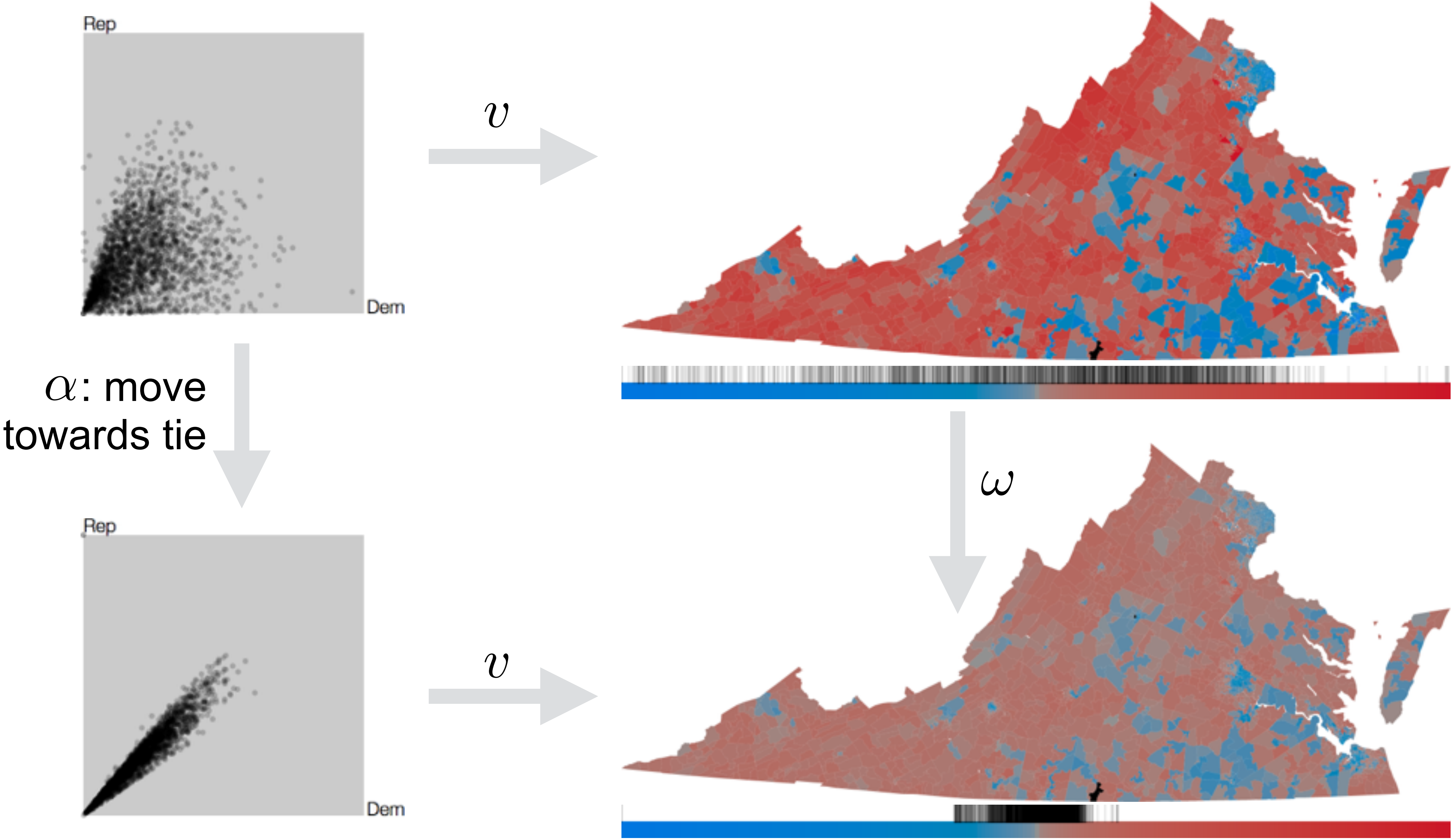
Where was vote evenly split??



… now with a different visualization



How about with the first α?



Design goal: Task $\rightarrow \alpha$, $\omega \rightarrow$ affordance

Low-level abstract tasks

[Munzner 2009]
[Meyer et al. 2012]

Perception, Affordances

[Cleveland & McGill 1984]
[Gibson 1986] [Ware 2012]

$$
\begin{tikzcd}
D \arrow[r, "v"'] \arrow[d, "\alpha"'] & V \arrow[d, "\omega"] \\
D \arrow[r, "v"] & V
\end{tikzcd}
$$

data change

visual change

Three Algebraic Design Principles

Derived from: "Are important α well-matched with obvious ω ?"

$$\begin{tikzcd} D \arrow[r, "v"'] \arrow[d, "\alpha"'] & V \arrow[d, "\omega"] \\ D \arrow[r, "v"] & V \end{tikzcd}$$

Does ω **make sense**, given α ?

$\rightarrow$ 1. Principle of Visual-Data Correspondence

For all important α, is ω obvious?

$\rightarrow$ 2. Principle of Unambiguous Data Depiction

Can obvious ω arise without data change (α=1)?

$\rightarrow$ 3. Principle of Representation Invariance

1. Principle of Visual-Data Correspondence

Important α produce obvious and meaningful ω

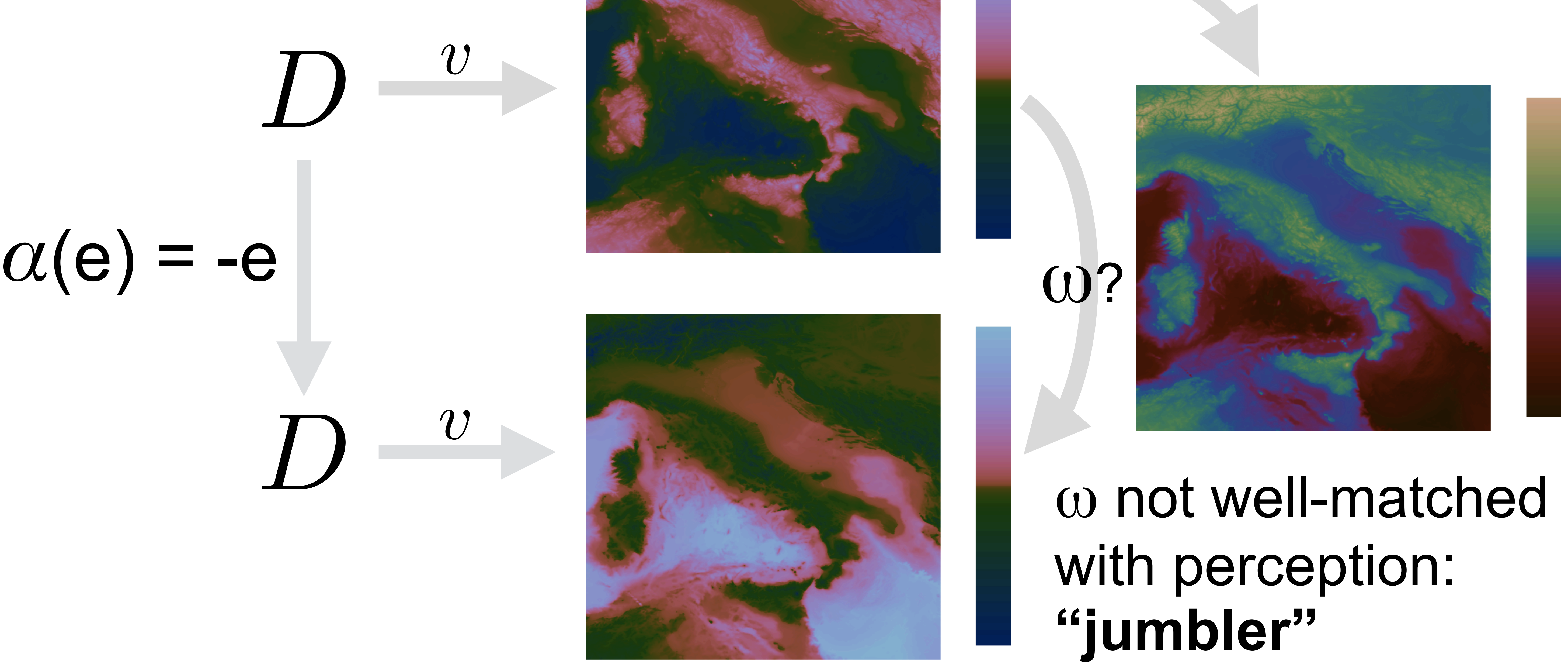
$$\begin{tikzcd} D \arrow[r, "v"'] \arrow[d, "\alpha"'] & V \arrow[d, "\omega"] \\ D \arrow[r, "v"] & V \end{tikzcd}$$

- α and ω well-matched, "$\alpha \cong \omega$"
- ω makes sense, given α

- **Congruence**: visual (external) structure $\cong$ viewer's mental (internal) structure [Tversky et al. 2002]
- **Effectiveness**: important data attributes mapped to readily perceived visual attributes [Mackinlay 1986]
- **Visual embedding**: visualization preserves distance (in spaces of data, perception) [Demiralp et al. 2014]

Correspondence example: elevation colormap

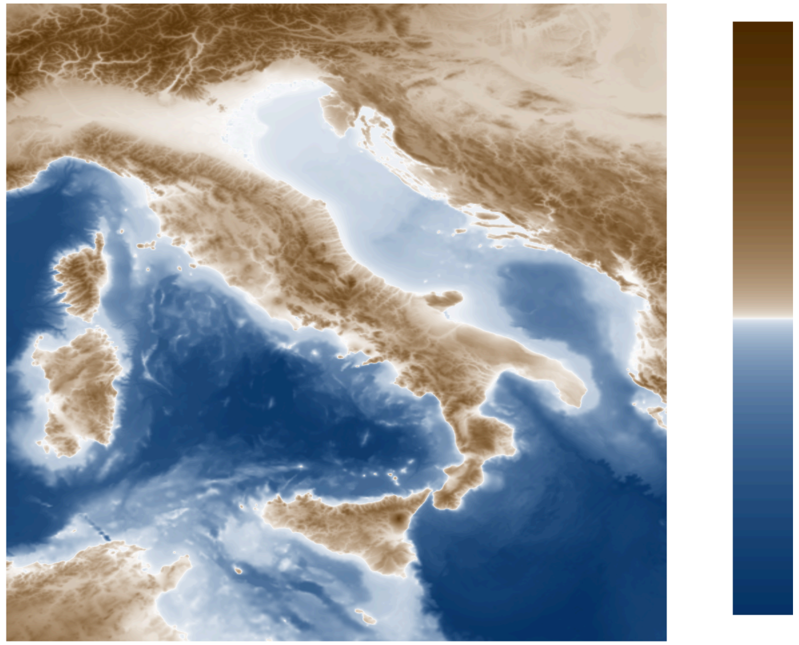
Data: signed elevation relative to sea level



Correspondence example: elevation colormap

Data: signed elevation relative to sea level

$D \xrightarrow{v}$

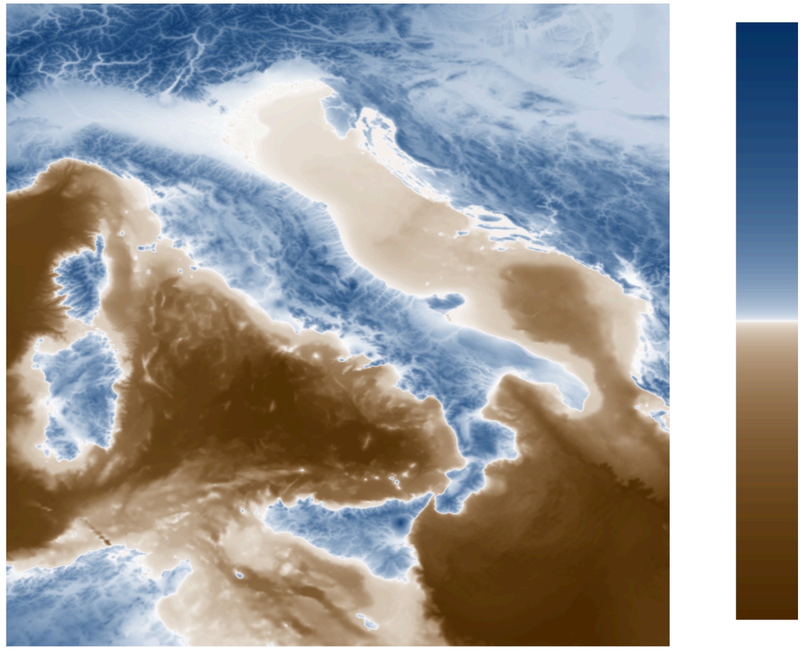


diverging colormap

α(e) = -e

ω: negate hue

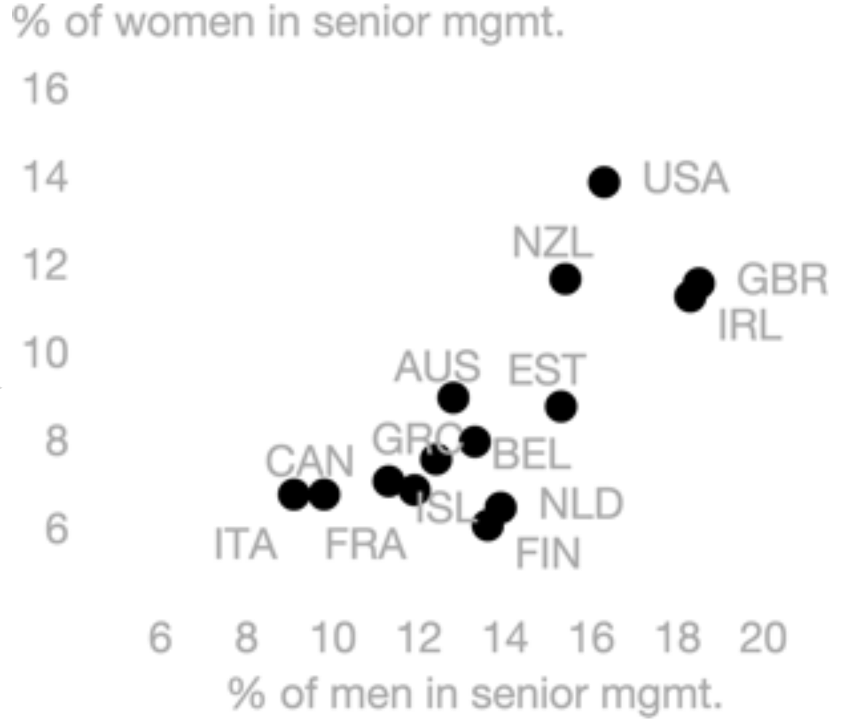
$D \xrightarrow{v}$



Correspondence example: scatterplots

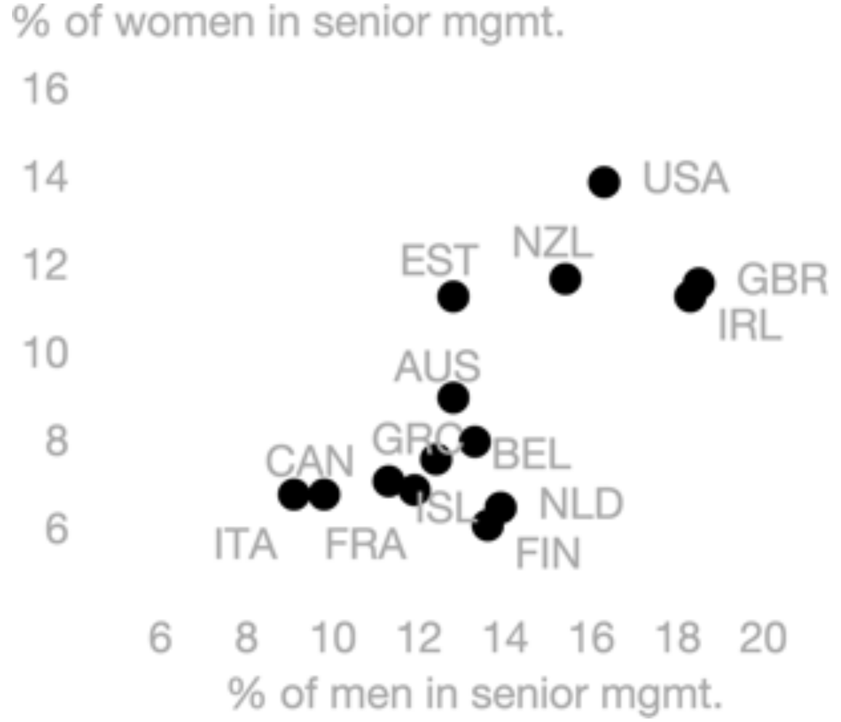
Data: % men vs women employed as senior managers in various countries

$D \xrightarrow{v}$



α: decrease gender gap for one country: EST

$D \xrightarrow{v}$



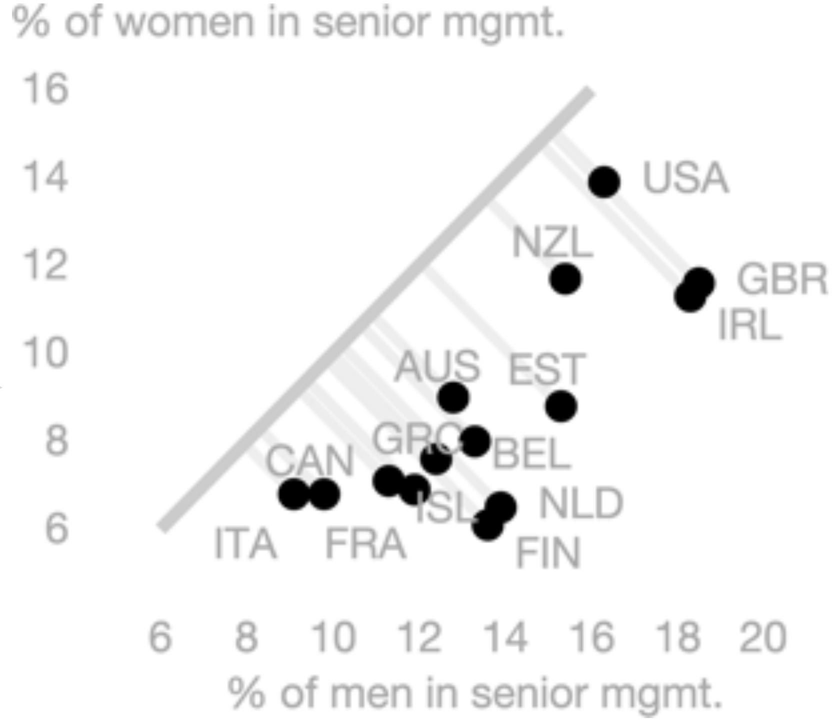
ω? Not clear how big that change was

http://economix.blogs.nytimes.com//2013/04/02/comparing-the-worlds-glass-ceilings/?_r=0

Correspondence example: scatterplots

Data: % men vs women employed as senior managers in various countries

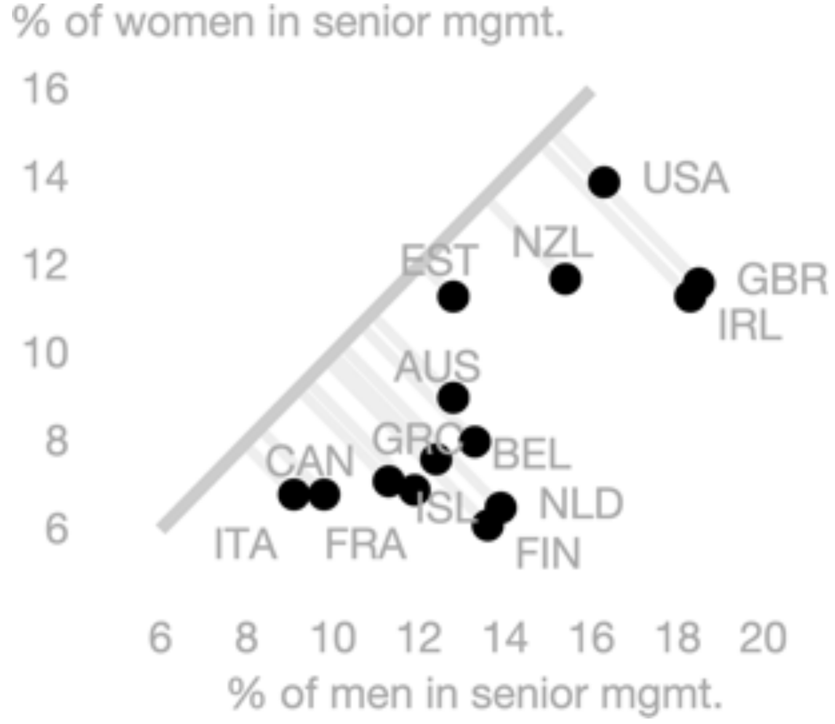
$D \xrightarrow{v}$



add diagonal line (%men = %women) and support lines

α: decrease gender gap for one country: EST

$D \xrightarrow{v}$



ω: change in position along a common scale [Cleveland & McGill 1984]

http://economix.blogs.nytimes.com//2013/04/02/comparing-the-worlds-glass-ceilings/?_r=0

2. Principle of Unambiguous Data Depiction

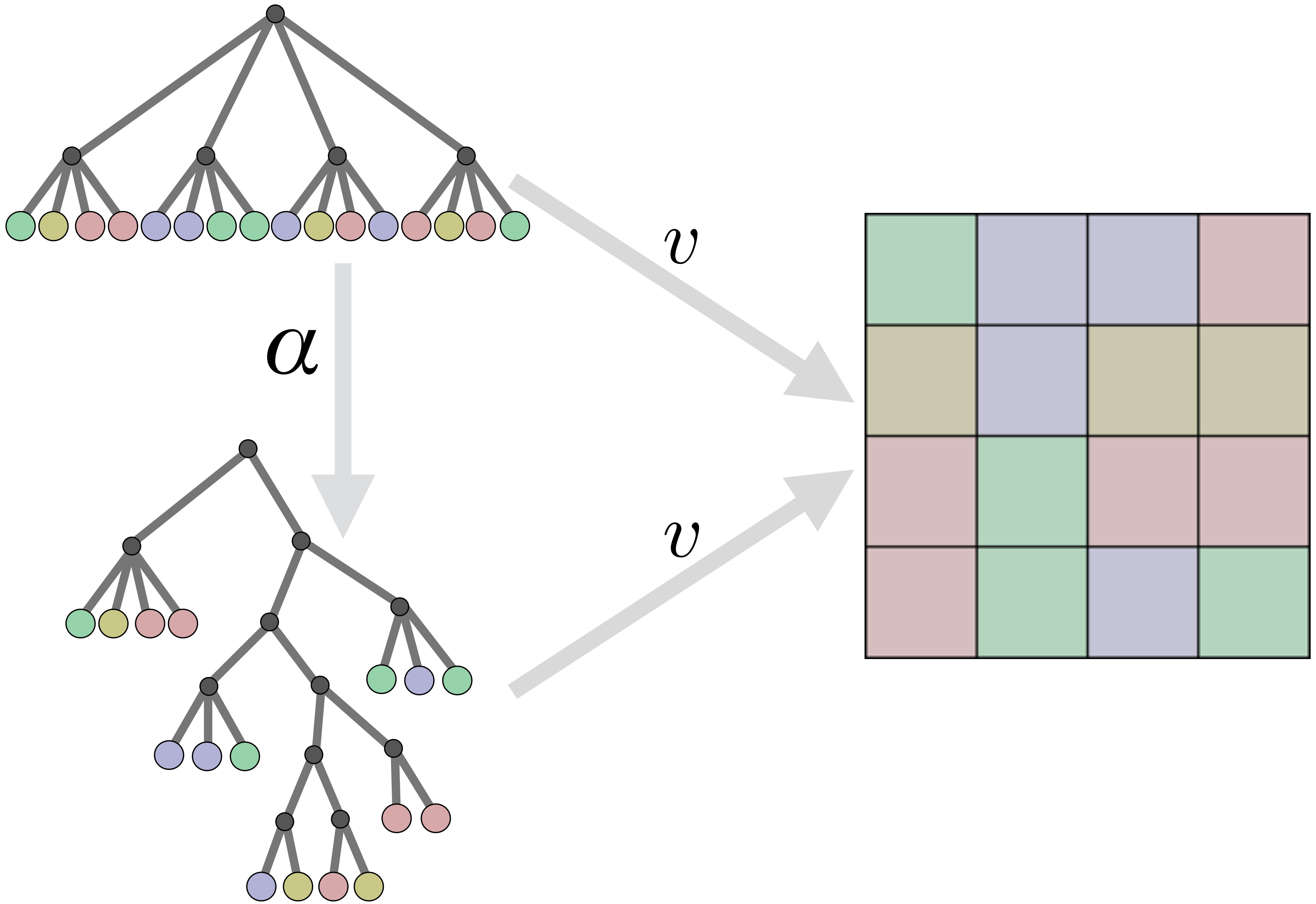
$$\begin{tikzcd}
D \arrow[dr, "v"] \arrow[dd, "\alpha"'] & \\
& V \\
D \arrow[ur, "v"'] & (\omega = 1)
\end{tikzcd}$$

Important α map to obvious ω.

If ω=1, then α=1.

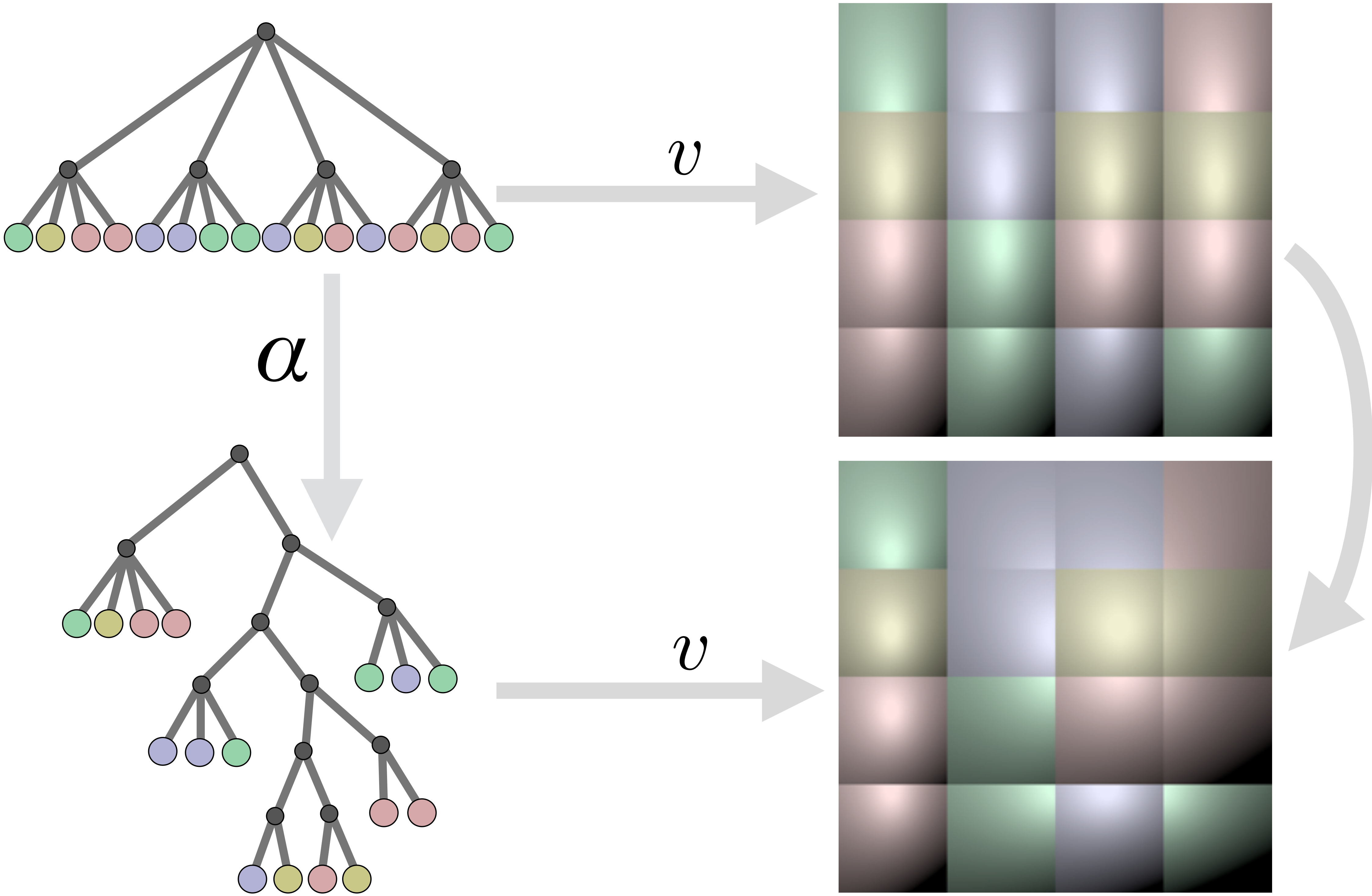
- **Expressiveness**: visualization shows all facts about data (and nothing more) [Mackinlay 1986]
- **Injectivity**: visualization preserves distinctness so viewer can invert it (read it) [Ziemkiewicz & Kosara 2009]
- If not v injective, α explicitly indicates the ambiguity:
  - α is the “**confuser**”

Unambiguity example: treemaps



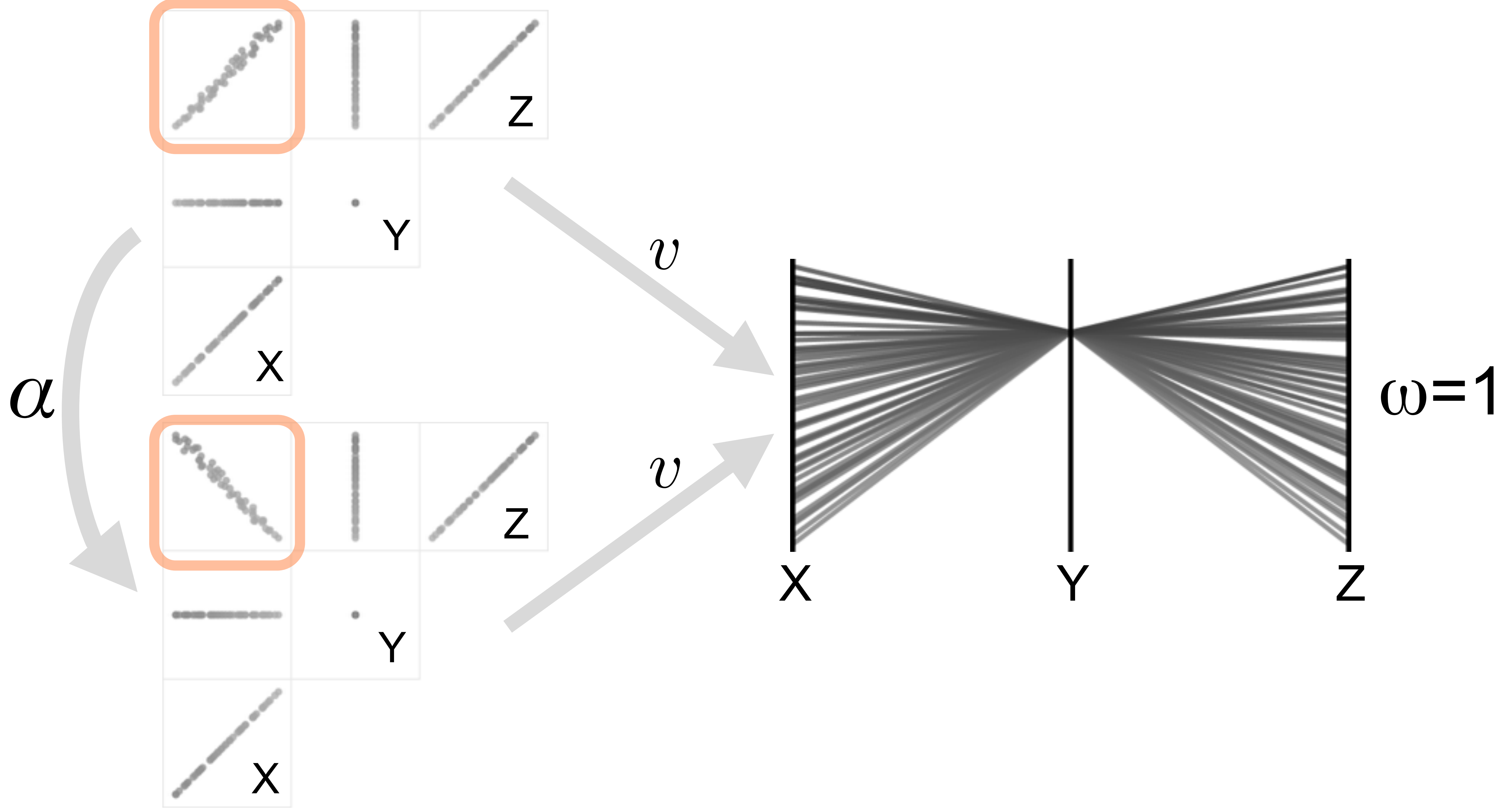
ω=1: α is
“confuser”
for treemaps

Unambiguity example: treemaps

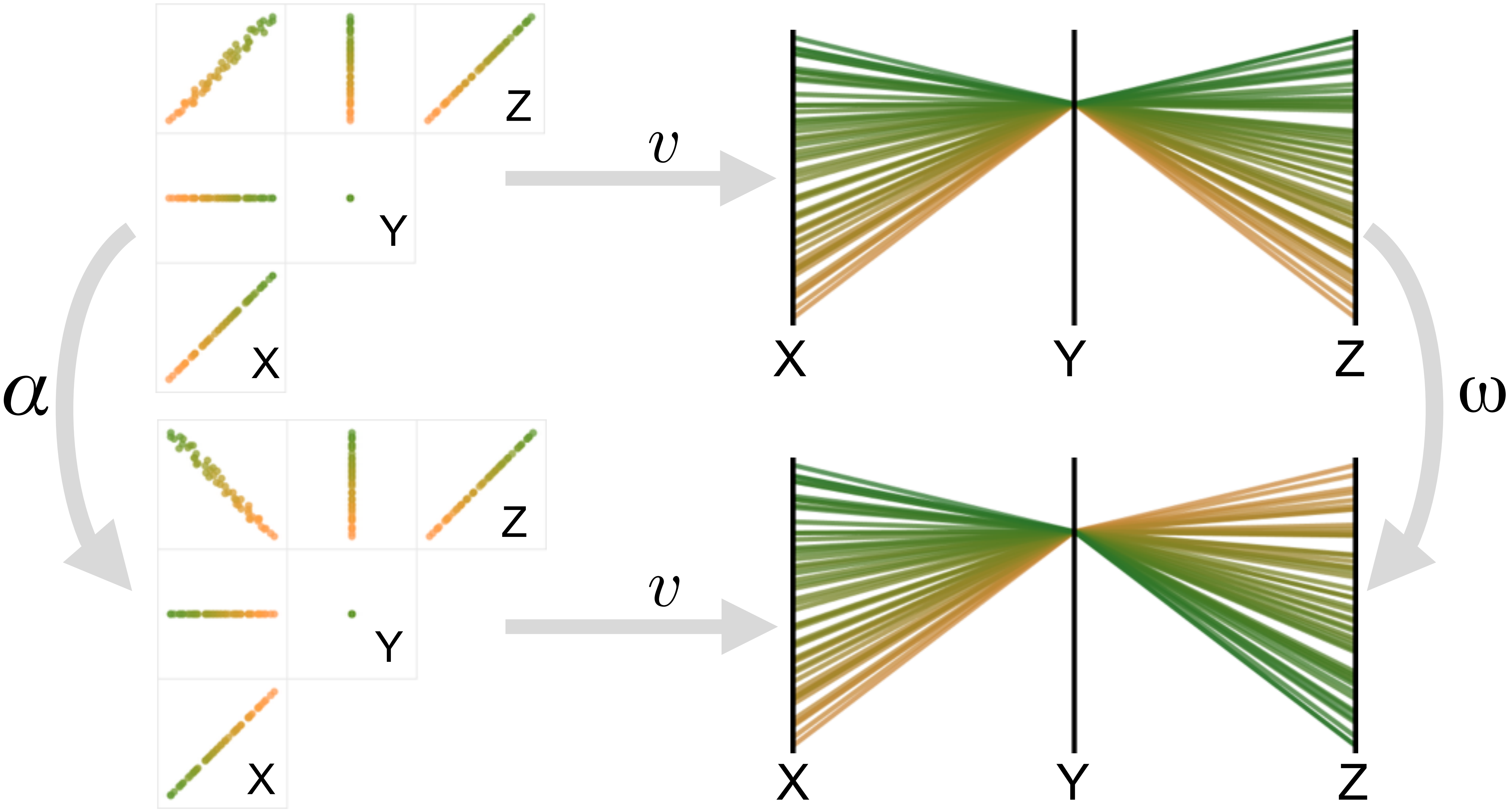


$\omega \neq 1$ with cushion treemaps [van Wijk & H. van de Wetering 1999]

Unambiguity example: parallel coordinates



Unambiguity example: parallel coordinates



3. Principle of Representation Invariance

Visualization is invariant w.r.t changes in data representation

If α=1, then ω=1.

$$
\begin{tikzcd}
 & R \arrow[r, "v"'] \arrow[dd, "h"] & V \arrow[dd, "\omega"] \\
D \arrow[ur, "r_1"] \arrow[dr, "r_2"] & & \\
(\alpha = 1) & R \arrow[r, "v"] & V
\end{tikzcd}
$$

- Underlying **data** D ≠ **representation** R of data
  - sets as lists, eigenvectors as vectors
- **Invariantive**: Scale of measurement (nominal, ordinal, interval, ratio) limits permissible statistics [Stevens 1946]
- If change h in representation is visible (ω≠1):
  - h is the "**hallucinator**"

Invariance example: alpha-blended marks

Data: **set** of locations of taxi pickups & drop-offs

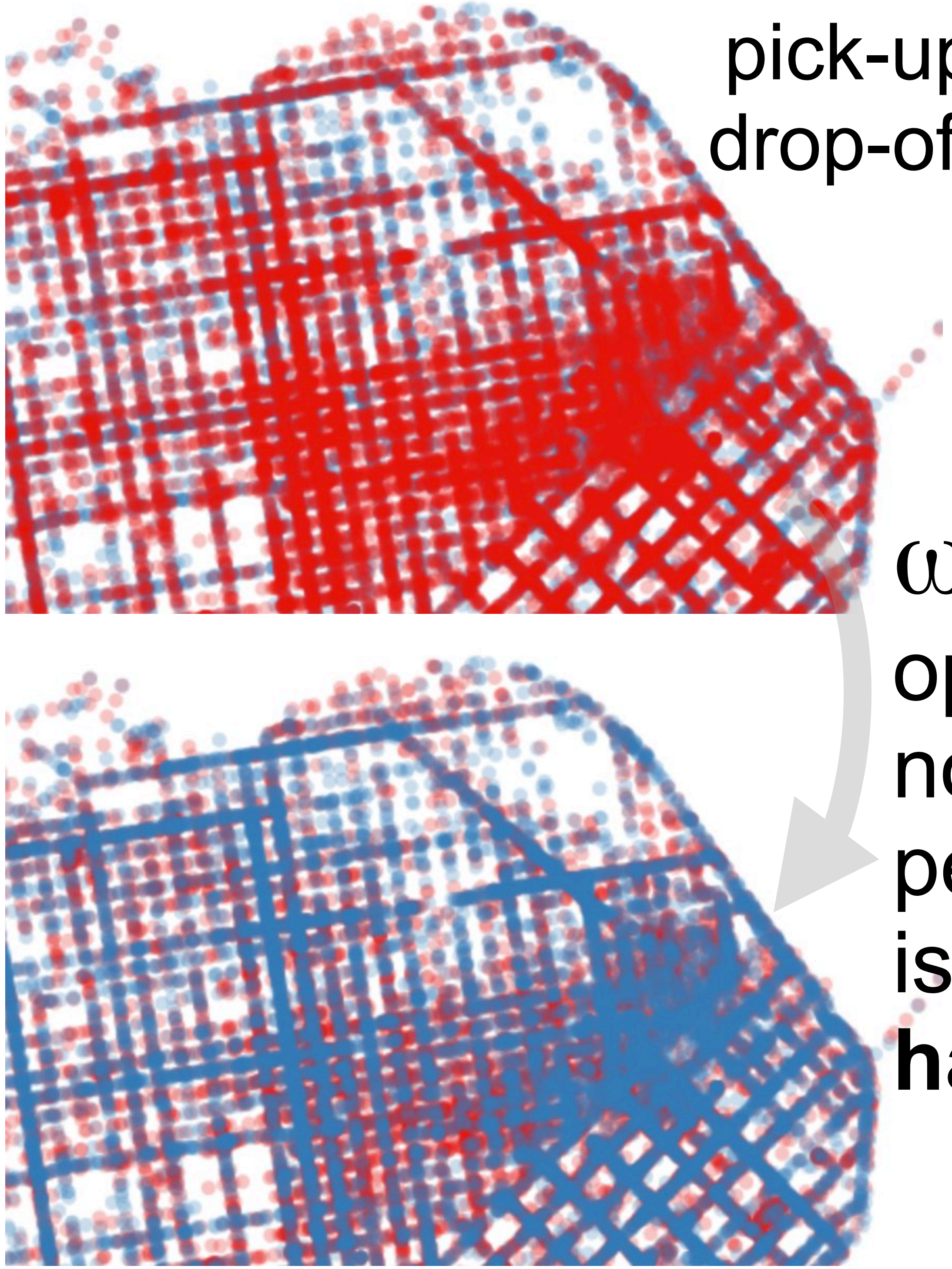
Representation: **list** of locations

D (α=1)

$R \xrightarrow{v}$

permute list

$R \xrightarrow{v}$



pick-up
drop-off

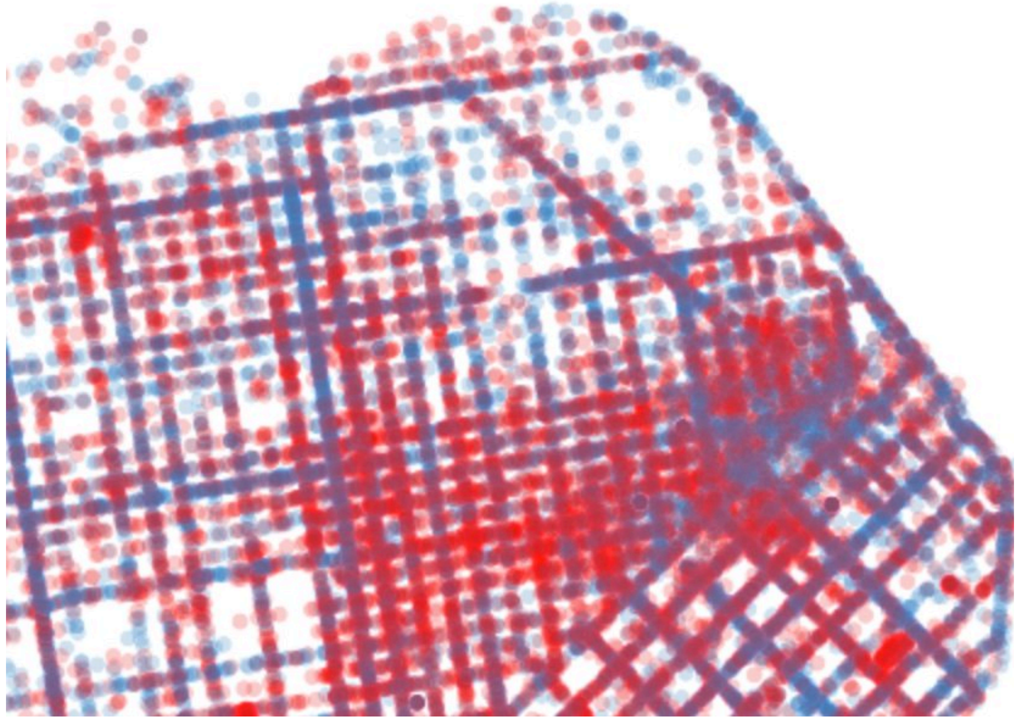
ω≠1 : “over” operator does not commute: permutation is a **hallucinator**

Invariance example: alpha-blended marks

Data: **set** of locations of taxi pickups & drop-offs

Representation: **list** of locations

$$\begin{tikzcd} & R \arrow[dd, "\text{permute list}"] \arrow[dr, "v"] & \\ D \ (\alpha=1) \arrow[ur] \arrow[dr] & & \bullet \\ & R \arrow[ur, "v"] & \end{tikzcd}$$



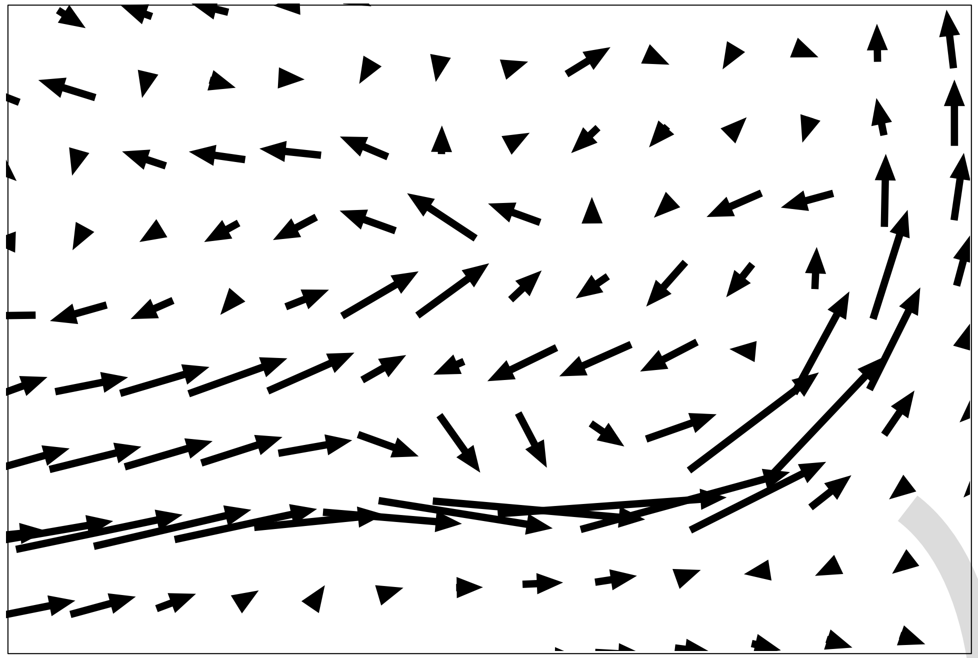
ω=1 with **order-invariant** (commutative) compositing

Invariance example: quiver plot

Data: underlying continuous vector field

Representation: **grid** of **sampled** vectors

R $\xrightarrow{v}$

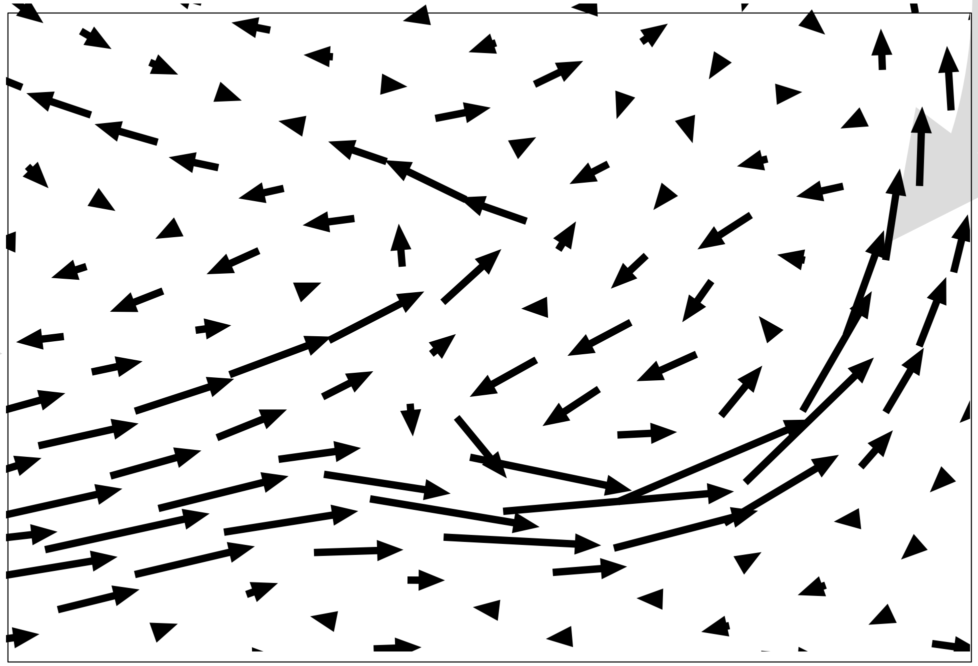


D

(α=1)

rotate sampling grid

R $\xrightarrow{v}$



ω≠1 : sampling on a rotated grid is a **hallucinator**

Invariance example: quiver plot

Data: underlying continuous vector field

Representation: **grid** of **sampled** vectors

R

rotate sampling grid

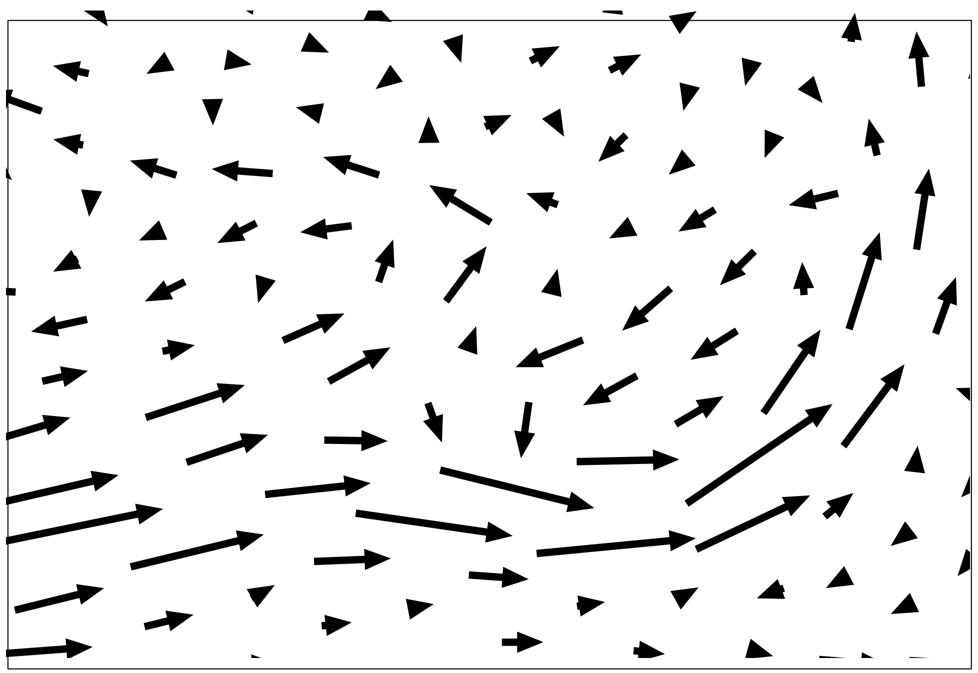
v

D

(α=1)

v

R



ω=1 with arrows optimally placed in reconstructed continuous field

Summary of 3 Principles

- Visual-Data Correspondence
  or else a **jumbler** α, or **misleader** ω

$$\begin{tikzcd} D \arrow[r, "v"] \arrow[d, "\alpha"'] & V \arrow[d, "\omega"] \\ D \arrow[r, "v"] & V \end{tikzcd}$$

- Unambiguous Data Depiction
  or else a **confuser** α

$$\begin{tikzcd} D \arrow[dr, "v"] \arrow[dd, "\alpha"'] & \\ & V \\ D \arrow[ur, "v"'] & \end{tikzcd} \quad (\omega = 1)$$

- Representation Invariance
  or else a **hallucinator** h

$$\begin{tikzcd} & R \arrow[r, "v"] \arrow[dd, "h"] & V \arrow[dd, "\omega"] \\ D \arrow[ur, "r_1"] \arrow[dr, "r_2"'] & & \\ & R \arrow[r, "v"] & V \end{tikzcd} \quad (\alpha = 1)$$

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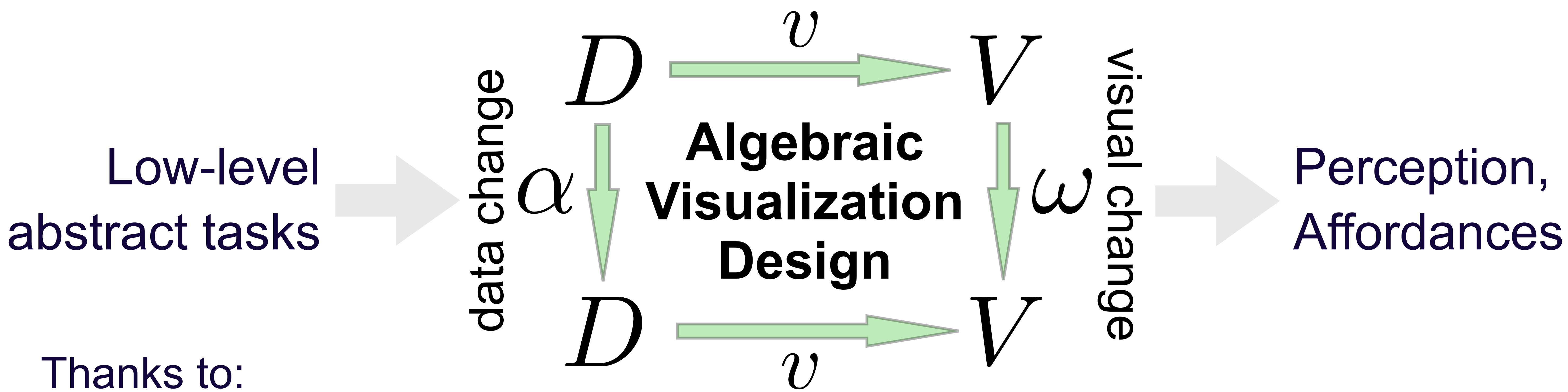

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- 2009 Dagstuhl Scientific Visualization Seminar 09251

Web page for paper, these slides, and eventually more:

- http://AlgebraicVis.net

and on Twitter! @algebraicvis