A complex impedance meter

Group 11

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March 2023

1 Introduction

Impedance measurement is a crucial process for determining the impedance of a component in a circuit, which is essential for various engineering applications. While ideal components are purely resistive, capacitive, or inductive, real-life components can differ from the ideal ones due to factors such as the effect of leads and other parasitic elements. For example, the equivalent circuit of the capacitor is shunt or series. Therefore, various methodologies have been introduced for measuring impedance accurately.

These methodologies take into account the non-ideal characteristics of components and are designed to minimize the influence of these parasitic elements. Some commonly used techniques for impedance measurement include I-V method, bridge method, auto-balancing bridge method and so on.

The I-V method is a simple technique to implement for measuring the electrical characteristics of a Device Under Test (DUT), such as a diode or transistor. Figure 1 demonstrates the basic idea of the method, where the DUT, denoted as Z_x, is in series with a reference resistor, R. Using the formula $\frac{V_2}{R}$, the value of Z_x can be easily calculated. The circuit is not complex and is commonly used, especially for probing-type test needs. However, the operating frequency range of the method is limited by the transformer used in the probe. To overcome this limitation, the auto-balancing bridge method is often used as an alternative. This method provides greater flexibility in terms of the frequency range that can be tested.

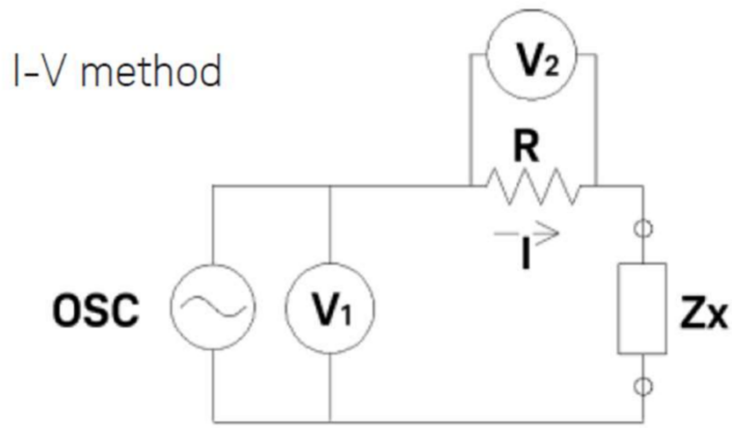


Figure 1: The I-V Method

The auto-balancing bridge method is considered to be one of the most accurate and common methods of impedance measurement. Figure 2 shows the basic circuit of an auto-balancing bridge. The operational amplifier has an infinitely large gain and negative feedback to maintain the voltage difference between the positive and negative terminals at zero volts. Since $V_x = 0$, it can be assumed that the voltages across Z_1 and Z_2 are v_1 and v_2, respectively. Z_2 is the reference resistor with a known value, and based on this, the current I through Z_2 can be calculated. The impedance of Z_1, which is the DUT, can then be measured. When the circuit is balanced, meaning the impedance of Z_2 is equal to that of the DUT and the voltage across them is equal, the measurement is most accurate. Compared to the I-V method, the auto-balancing bridge method offers wider frequency coverage from low frequency to high frequency and higher accuracy over a wide range of impedance measurement. However, it should be noted that the highest frequency range available is limited, and the most applicable frequency range is from $20Hz$ to $110MHz$.

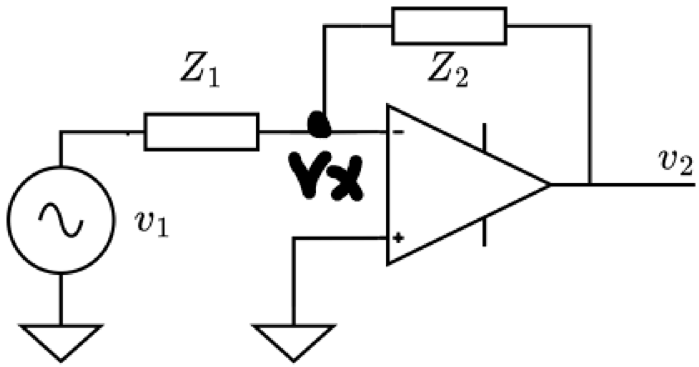


Figure 2: The Basic Circuit of ABB

In this case, we need to design and build an instrument to do impedance measurement. The specification is:

1. Impedance Magnitude Range: 10Ω to $1M\Omega$
2. Impedance Phase: Impedance phase accurate to 5 degrees
3. Frequency Range: Frequency range at least 100Hz – 100kHz

Following our group discussion, we have decided to design a circuit using the fundamental auto-balancing bridge as a starting point to meet the specifications. Additionally, we plan to incorporate some advanced functions into the circuit design.

2 Design and Build of Analogue Interface

Below is the figure for the analogure interface design.

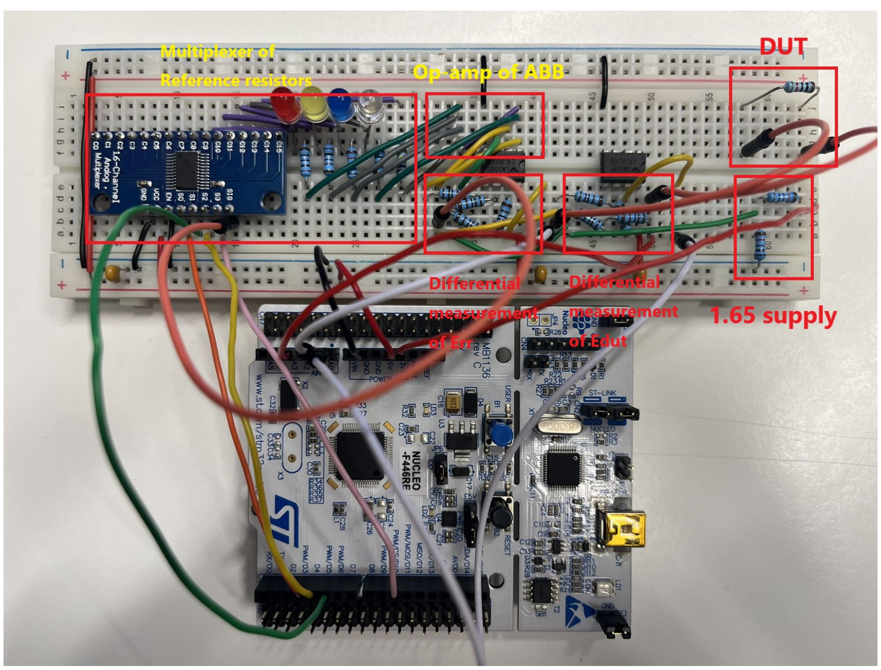


Figure 3: The Circuit of Our Analogue Interface

The red blocks labeled **differential measurement of E_{rr} and E_{dut}** in the diagram refer to the subtractors. To minimize errors, subtractors are utilized to obtain the potential difference between the voltage across the DUT and the voltage across the reference resistors. This enables calculation of the voltage across the DUT and reference resistors accurately.

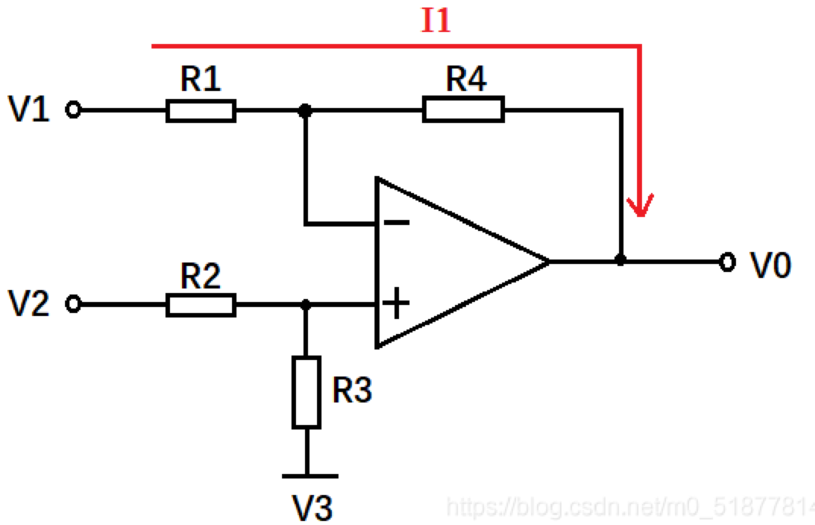


Figure 4: The Circuit for the Subtractor[1]

The gain of the subtractor can be adjusted by varying the resistors. The circuit is capable of providing higher gain by selecting appropriate resistor values. Figure 4 illustrates the circuit diagram of a subtractor, where assuming $R1 = R2 = R1'$ and $R3 = R4 = R2'$, the negative terminal is connected to V' and the positive terminal is connected to V''. The current passing through $R1$ and $R4$ is denoted as $I1$.[1]

$$V'' = V' = \left(\frac{V2 - V3}{R1' + R2'} \times R2'\right) + V3 \tag{1}$$

$$V'' = \frac{V2R2' + V3R1'}{R1' + R2'} \tag{2}$$

$$\frac{V1 - V'}{R1'} = \frac{V' - V0}{R2'} = I1 \tag{3}$$

$$V0 = (V2 - V1)\,\frac{R2'}{R1'} + V3 \tag{4}$$

Based on the formulas above, we can see the ratio of $R2'$ and $R1'$ can determine the gain.

In auto-balancing bridge, when the voltage across the reference resistors and DUT is approximately equal, the impedance of DUT is also the impedance of the reference resistor. However, the reference resistors should be variable to achieve this. As a result, the multiplexer is used to change the resistors automatically instead of manually work. LED light can indicate which reference resistors is connected with the circuit. The specification for impedance magnitude is 10Ω to $1M\Omega$, so the reference resistors we choose are: $R_{ref} = 100, 1K, 10K, 100K, 1M$, just as 3.

Turning to the specification of the frequency range, we need to measure the impedance of the device under test at different frequencies. The frequency sweep can be performed in different ways, such as linear frequency sweep, logarithmic frequency sweep, or arbitrary frequency sweep. We decide to use logarithmic frequency. In a logarithmic frequency sweep, the frequency increases or decreases logarithmically with time[2]. Then we plot the magnitude and phase response of the DUT as a function of frequency. This will give us a clear understanding of the behavior of the DUT over a range of frequencies.

For the sine wave generated by the DAC in the microcontroller, the frequency is calculated by the formula: $\frac{84MHz}{n \times LUT\_size}$, 84MHz is the clock frequency, n is to define the various LUT period values, and LUT_size represents the size of the array used to generate sine wave, which is 256 in this case. As a result, if we want to change the frequency, we can change n. Based on the formula, a table which displays the relation between n and f is below. Theoretically, Then n decreases from 3250 to approximately to 0.3 logarithmicly, which means the frequency increases from 100 to 1MHz. However, the function $sync\_sample\_blocking(uint32\_t * buffer, int buffer\_size)$ (from the sample code) used to input data from DAC and read data to ADC has limitations. The $buffer\_size$ must be integer, so it is hard for our instrument to reach the 1MHz frequency range.

```
for (int i = 0; i < 11; i++){
    int freq = expf(log(start_freq) - i * log(2));
```

Figure 5: The Logarithmic Decreasing With Code

The reason to use $i < 11$ and $ln(2)$ is that:

$$\log_{10} \frac{3250}{3} = \log_{10} 1083 \tag{5}$$

$$2^{10} = 1024 < 1083 < 2^{11} = 2048 \tag{6}$$

$$3^6 = 729 < 1083 < 3^7 = 2187 \tag{7}$$

n	f(Hz)
3250	100
325	1K
32	10K
3	100K
0.3	1M

Table 1: The Relation Between n And f

Now the analogue interface and the function of the microcontroller is finished. Here is the table about the input and the output of our design. 5V and 3.3V is supply voltage for opamp and offset. The Edut and Err is the output signal which detected by the ADC module in the microcontroller.

Input	5V	3.3V	GND	Vin(Generated by STM32 DAC)	DUT
Output	Edut	Err	N/A	N/A	N/A

Table 2: Input and Output List

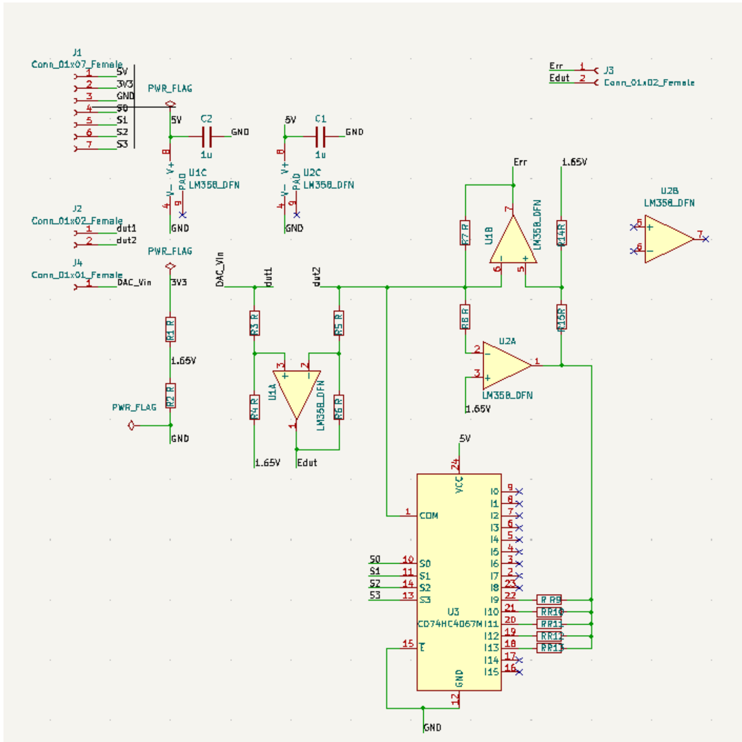


Figure 6: The Schematic of design

3 Algorithm Implementation and Data Processing

After the analogue interface is built, we began to pay attention on the function of the microcontroller. Firstly, the Vin, a varying frequency sinusoidal wave, is generated by the sine wave array and then DAC of the microcontroller. Secondly, the voltage of Edut and Err is detected and sent to the ADC of the microcontroller. Then all the data is logged and processed by the microcontroller. This section will explain the algorithm to improve the accuracy of the measurement.

3.1 Error Reduced of The Circuit

Calibration is done by measuring two standards: an open circuit and a short circuit. For DUT, we can open circuit at first and to measure the impedance. In this case, the impedance should be infinitely large ideally but in real circuit, the impedance is not. Then we can connect two terminals of DUT with a wire directly. The impedance should be 0 but in real circuit there are impedance in the wire. In our design, the accuracy and precision is not so high. After discussion, our team decide to neglect the error and try to improve the accuracy of the result by Discrete Fourier Transform (DFT).

3.2 Goertzel Algorithm

Digital signal processing is essential for improving measurement accuracy. During lectures and classes, an alternative method was recommended for computing a single Discrete Fourier Transform (DFT) bin: the Goertzel algorithm (1958). This algorithm provides an efficient way to calculate individual frequency components in a signal without having to compute the full DFT. The code for implementing the filter is not complex.

The pseudo-code of the Goertzel Algorithm is demonstrated in Algorithm 1[3].

Algorithm 1 DFT based on Goertzel Algorithm

```
%1 Variable Initialization
Initialize sine wave array x
Initialize parameter S0, S1, S2
Initialize complex number y
Size of array n = 256
%2 Recursive Calculation of Complex y
while (i < n) do S0=X[i]+2*cos(2*PI*f/fs)*S1-S2,y = S0-EXP(-I*2*PI*f/fs)*S1, S2=S1,S1=S0 end
Recursive variable:
%3 Magnitude and Phase from a sine wave array
Return complex variable y;
```

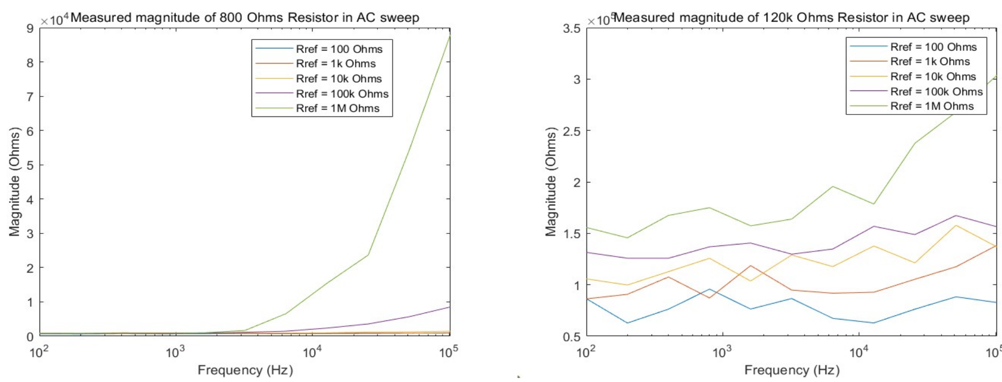
4 Measurement and Analysis of the Results

In the preceding sections, we introduced and explained the digital and analog interfaces of our product. In this section, we will use our product to perform measurements to test the design. We will begin by comparing the series and shunt equivalent circuits.

4.1 Measurement of simple resistors

Firstly, we decided to use 5 reference resistors to measure 800Ω and $120k\Omega$ to test whether our product is acceptable. Below is the magnitude and phase plot for 800Ω and $120K\Omega$ with 5 reference resistors over 100Hz to 100KHz. Based on these graphs, we can speculate that for low frequency range our design which is much more accurate than that of high frequency range.

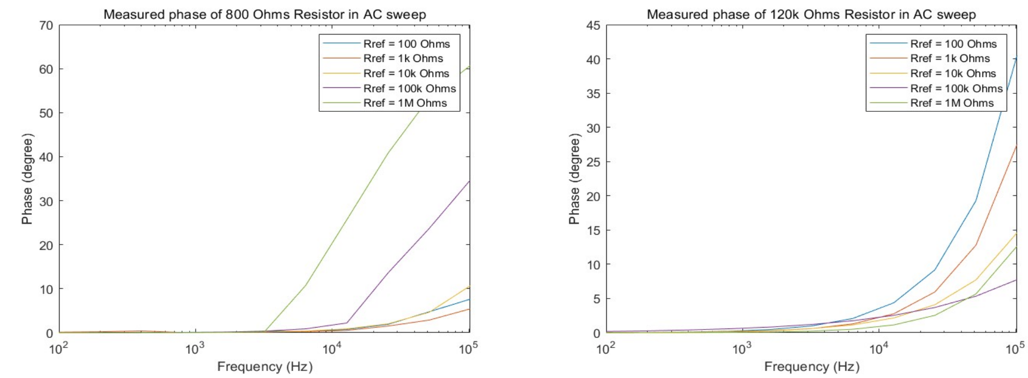
Furthermore, it is evident that the measurement results will be most accurate when the impedance of the reference resistors is equal to that of the device under test (DUT).



(a) Magnitude of 800Ω

(b) Magnitude of $120K\Omega$

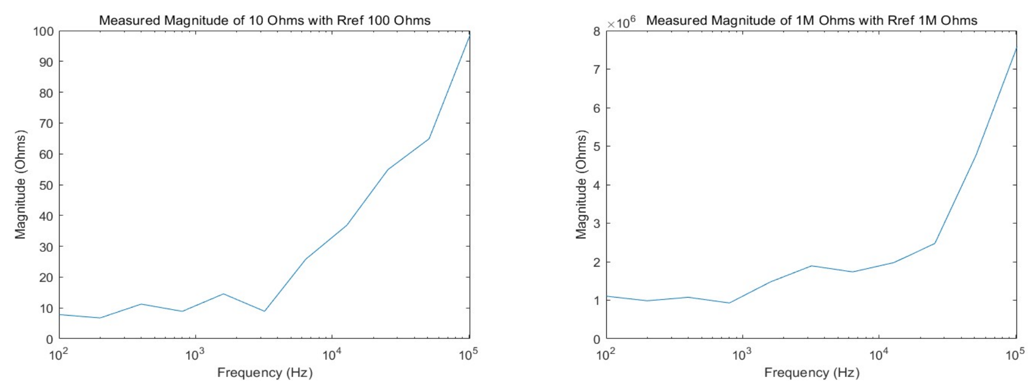
Figure 7: Magnitude of Our Auto-Balancing Bridge



(a) Phase of 800Ω (b) Phase of $120K\Omega$

Figure 8: Phase of Our Auto-Balancing Bridge

We have successfully verified that our design is functional, and when the value of reference resistor equals to the impedance of DUT, the result is the most acceptable, and we will proceed to test the impedance magnitude range of our device by measuring 10Ω and $1M\Omega$. These measurements will determine whether the device meets the specifications.



(a) Magnitude of 10Ω (b) Magnitude of $1M\Omega$

Figure 9: Magnitude of Our Auto-Balancing Bridge

Our circuit was able to achieve the specified impedance magnitude range, as demonstrated by our measurements of 1Ω and $1M\Omega$, particularly at low frequencies. However, at high frequencies, we observed inaccurate measurements of the impedance magnitude for 10Ω and $1M\Omega$. We suspect that this is due to the presence of inductive components in the resistors, which affected the accuracy of the measurements.

Turning to the specifications of phase, we compare the simulation and measured results and find that most of them are in 5 degrees difference.

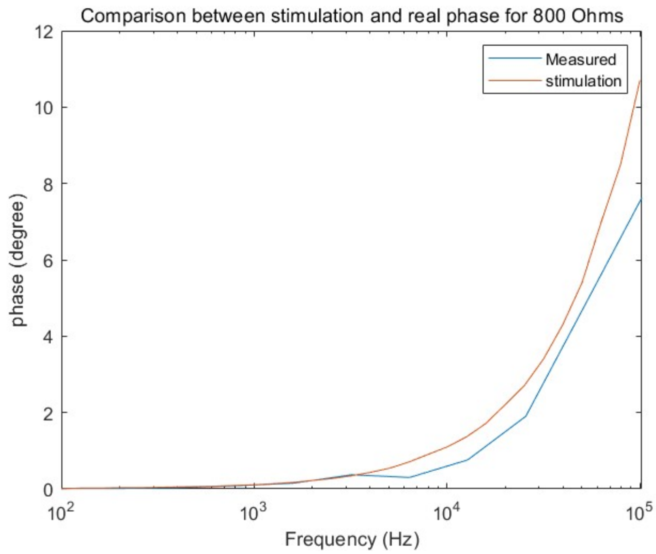
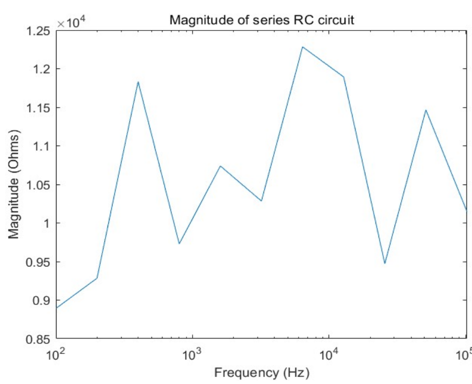


Figure 10: The Comparison Between Simulation and Real Phase

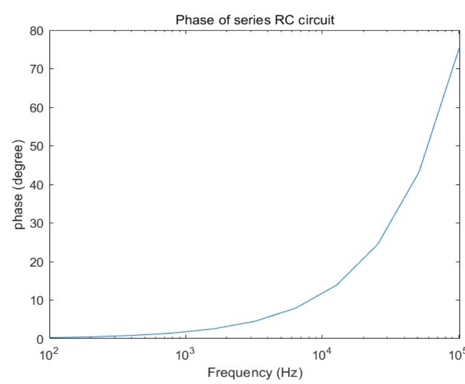
4.2 Measurement of Equivalent Model of The Capacitor

There are two types of equivalent model for the capacitors, series and shunt model. Diagrams below display the magnitude and phase of the RC model, which is more complex than that of only one simple resistors. The capacitor we used is $10\mu F$ and the resistor is $10k\Omega$.

For series model, the total impedance is: $R+\frac{1}{jwC}$. Based on this, for low frequency, the magnitude should be approximately at $\left|\frac{1}{j\omega C}\right| = 10K + \left|\frac{1}{j\times 2\pi 100\times 10\times 10^{-6}}\right| = 10159.15\Omega$. For high frequency, the magnitude should be approximately at $10k\Omega$.



(a) Magnitude of series model

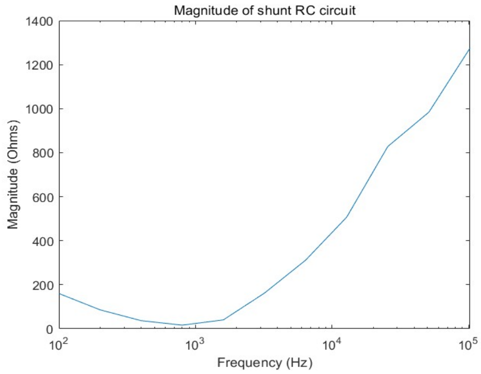


(b) Phase of series model

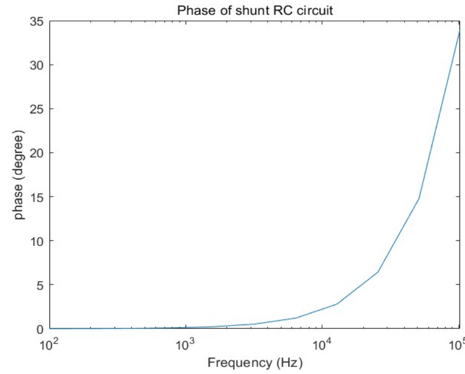
Figure 11: Measurement of series RC model

The diagrams show that series RC model could be measured.The magnitude is approximately at $10k\Omega$.

For shunt model, when it is low frequency range, the impedance we calculated is 1571.6Ω, and for high frequency range is 1.54Ω. The diagrams below show the magnitude and phase of the shunt model. Our instrument cannot work perfectly for the shunt equivalent circuit of the capacitor.



(a) Magnitude of shunt model



(b) Phase of shunt model

Figure 12: Measurement of shunt RC model

5 Bills and Cost of the Design

Name	Quantity	Price
CD74HC4067	1	1.738
LM358	2	0.34
Resistor	12	N/A
Capacitor	3	N/A
STM32F446 Nucleo-64	1	30
		32.07

Table 3: The Cost of The Components

6 Team Role Contribution and Time Line

There are 2 members in our team. For this design, the timeline of this task is long, the workload is heavy and the coursework is overall difficult. Effective collaboration is crucial for the success of this project. This design involves integrating software and hardware, and therefore requires strong communication and coordination between team members.

Regarding the division of tasks for this coursework, Bolin Mai is primarily responsible for implementing the algorithmic portion, which includes the Goertzel Algorithm and determining the frequency sweeping approach to ensure better results across the frequency range. Meanwhile, Houzhe Wang is primarily responsible for the analogue interface, including the implementation of a subtractor to facilitate differential measurement, as well as writing the code for the microcontroller to control the multiplexer and adjust the reference resistors.

In addition, we not only work on our individual tasks, but also collaborate to address any challenges that arise. For instance, after a team meeting, we decided to implement an auto-balancing bridge as the foundation of the circuit. Furthermore, we worked together to integrate our separate parts seamlessly.

The figure below is the timeline of this task:

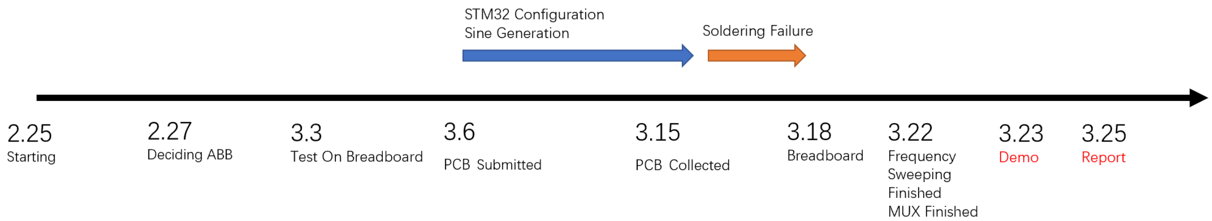


Figure 13: The Timeline of The Coursework

7 Reflection and Future Improvement

Our team has gained a lot of valuable experience from this coursework. This was the first time we had to combine DSP algorithms with an analog interface, which required us to develop skills in controlling pins and other hardware components. Overall, the coursework provided us with a challenging and rewarding opportunity to understand both the digital and analog aspects of the system.

To further improve our design, we recommend implementing some new methodologies in the future. One of improvements to increase the accuracy of the measurement is the Goertzel Algorithm, where adding more cycles can enhance its accuracy and reliability.

Meanwhile, the function we used to send and read data throguh DAC and ADC is $sync\_sample\_blocking(uint32\_t * buffer, intbuffer\_size)$, which is based on sample code, can only use integers. This limits the range of frequency sweeping. One approach to improve the frequency sweeping part is to use more advanced chips that allow for easier adjustment of the frequency, rather than relying on step changes.

References

[1] "Introduction to subtractors." https://blog.csdn.net/m0_51877814/article/details/111626857.

[2] "Introduction to subtractors." https://blog.csdn.net/qq_33559992/article/details/113710091.

[3] "Goertzel algorithm generalized to non-integer multiples of fundamental frequency." https://asp-eurasipjournals.springeropen.com/articles/10.1186/1687-6180-2012-56.