Expectation and variance of y in terms of the canonical parameter

The canonical form the pdf is given by

$$f_\theta(y) = \exp\left[\frac{y\theta - b(\theta)}{\phi} + c(y,\phi)\right]$$

(1) The log-likelihood is

$$l(\theta) = \frac{y\theta - b(\theta)}{\phi} + c(y,\phi) = 0$$

So

$$E\left[\frac{\partial l(\theta)}{\partial \theta}\right] = E\left[\frac{y - b'(\theta)}{\phi}\right] = 0$$

So

$$E(y) = b'(\theta)$$

(2) The two expression of the Fisher Information of θ^{MLE} are equal:

$$Var\left[\frac{\partial l(\theta)}{\partial \theta}\right] = -E\left[\frac{\partial^2 l(\theta)}{\partial \theta^2}\right]$$

On the left hand side

$$Var\left[\frac{\partial l(\theta)}{\partial \theta}\right] = E\left(\left[\frac{\partial l(\theta)}{\partial \theta}\right]^2\right) - \left(E\left[\frac{\partial l(\theta)}{\partial \theta}\right]\right)^2 = E\left(\left[\frac{\partial l(\theta)}{\partial \theta}\right]^2\right) - 0$$

so

$$E\left(\left[\frac{\partial l(\theta)}{\partial \theta}\right]^2\right) = -E\left[\frac{\partial^2 l(\theta)}{\partial \theta^2}\right]$$

$$\left[\frac{y - b'(\theta)}{\phi}\right]^2 + \frac{-b''(\theta)}{\phi} = \left[\frac{y - E(y)}{\phi}\right]^2 - \frac{b''^{(\theta)}}{\phi} = 0$$

so

$$\frac{Var(y)}{\phi^2} = \frac{b''^{(\theta)}}{\phi}$$

i.e.,

$$Var(y) = b''^{(\theta)}\phi$$