Simplex

The simplex Δ_k is defined by

$$\Delta_k = \left\{ p = (p_1, \dots, p_k) \in (0,1)^k : \sum\nolimits_{i=1}^{k} p_i = 1 \right\}$$

There are infinite number of vector p in Δ_k.

For example,

(1) If $X \sim$categorical distribution with $k = 3$, p could be $(0.1, 0.2, 0.7)$, $(0.5, 0.3, 0.2)$ or any vector (p_1, p_2, p_3) such that

$$\sum\nolimits_{i=1}^{k} p_i = 1 \text{ and } 0 < p_i < 1$$

(2) If $X \sim \mathrm{Poisson}(\lambda)$

$$p = (p_0, p_1, p_2, p_3, \dots) = \left(\frac{\lambda^0}{0!} e^{-\lambda}, \frac{\lambda^1}{1!} e^{-\lambda}, \frac{\lambda^2}{2!} e^{-\lambda}, \frac{\lambda^3}{3!} e^{-\lambda}, \dots \right)$$

Note that $\frac{\lambda^k}{k!} e^{-\lambda}$ is the PMF of $\mathrm{Poisson}(\lambda)$ distribution, so we have

$$p_k = \mathrm{Prob}(X = k) = \frac{\lambda^k}{k!} e^{-\lambda} \in (0,1), \text{for } k = 0,1,2, \dots$$

and

$$\sum\nolimits_{i=0}^{\infty} p_i = CDF(\infty) = 1$$

k	$k!$	$\frac{\lambda^k}{k!} e^{-\lambda}$	p(x<=k)
0	1	0.018316	0.018316
1	1	0.073263	0.091578
2	2	0.146525	0.238103
3	6	0.195367	0.433470
4	24	0.195367	0.628837
5	120	0.156293	0.785130
6	720	0.104196	0.889326
7	5040	0.059540	0.948866
8	40320	0.029770	0.978637
9	362880	0.013231	0.991868
10	3628800	0.005292	0.997160
11	39916800	0.001925	0.999085
12	479001600	0.000642	0.999726
13	6227020800	0.000197	0.999924
14	8.7178E+10	0.000056	0.999980
15	1.3077E+12	0.000015	0.999995
16	2.0923E+13	0.000004	0.999999
17	3.5569E+14	0.000001	1.000000
18	6.4024E+15	0.000000	1.000000
19	1.2165E+17	0.000000	1.000000
20	2.4329E+18	0.000000	1.000000