Wald test statistic for Bernoulli distribution

The general form of W statistic is

$$W = \frac{\hat{\theta} - \theta}{\sqrt{var(\hat{\theta})}}$$

For a given sample, we genially plug in the estimated $var(\hat{\theta})$ to the denominator, i.e.,

$$W = \frac{\hat{\theta} - \theta}{\sqrt{\widehat{var}(\hat{\theta})}}$$

In the following hypothesis test, $X \sim Ber(p)$, where p is unknown.

$$H_0 : p = p_0$$
$$H_1 : p \neq p_0$$

The Maximum likelihood estimator of p is

$$\hat{p} = \overline{\mathrm{X}}_{\mathrm{n}}$$

The sample variance of $\hat{p}$ is

$$\widehat{var}(\hat{p}) = \widehat{var}(\bar{X}_n) = \widehat{var}\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\widehat{var}\left(\sum_{i=1}^{n} X_i\right) = \frac{n}{n^2}\widehat{Var}(X_i)$$
$$= \frac{\hat{p}(1-\hat{p})}{n} = \frac{\bar{X}_n(1-\bar{X}_n)}{n}$$

So the observed W statistic in a given sample is

$$W = \frac{\hat{p} - p_0}{\sqrt{\widehat{\mathrm{var}}(\hat{p})}} = \frac{\overline{\mathrm{X}}_{\mathrm{n}} - p_0}{\sqrt{\frac{\overline{\mathrm{X}}_{\mathrm{n}}(1-\overline{\mathrm{X}}_{\mathrm{n}})}{\mathrm{n}}}} = \frac{\sqrt{n}(\bar{X}_n - p_0)}{\sqrt{\bar{X}_n(1-\bar{X}_n)}}$$