Jeffreys prior for $\mathrm{Ber(p)}$ distribution:

Jeffreys prior is a non-informative prior distribution for a parameter space; its density function is proportional to the square root of the determinant of the Fisher information matrix:

$$\pi_J(p) \propto \sqrt{\det[\mathrm{I(p)}]}$$

For a $\mathrm{Ber(p)}$ distribution:

$L_1(p) = p^x(1-p)^{1-x}$
$l_1(p) = \ln[p^x(1-p)^{1-x}] = x\ln(p) + (1-x)\log(1-p)$

There are two ways to obtain the fisher information, $Var[l_1'(p)]$ or $-E[l_1''(p)]$.

Given that

$$l_1'(p) = \frac{x}{p} - \frac{1-x}{1-p}$$

and

$$l_1''(p) = -\frac{x}{p^2} - \frac{1-x}{(1-p)^2}$$

We can get

$$Var[l_1'(p)] = Var\left[\frac{x}{p} - \frac{1-x}{1-p}\right] = Var\left(\frac{x}{p} - \frac{1}{1-p} + \frac{x}{1-p}\right) = Var\left(\frac{x}{p} + \frac{x}{1-p} + constant\right)$$
$$= Var\left[\frac{1}{p(1-p)}x\right] = \left[\frac{1}{p(1-p)}\right]^2 Var(x) = \frac{p(1-p)}{[p(1-p)]^2} = \frac{1}{p(1-p)}$$

and

$$-E[l_1''(p)] = -E\left[-\frac{x}{p^2} - \frac{1-x}{(1-p)^2}\right] = -\left[-\frac{E(x)}{p^2} - \frac{1-E(x)}{(1-p)^2}\right] = \frac{p}{p^2} + \frac{1-p}{(1-p)^2} = \frac{1}{p} + \frac{1}{1-p}$$
$$= \frac{1}{p(1-p)}$$

The two method leads to the same result:

$$\text{Fisher Information} = \frac{1}{p(1-p)}$$

and

$$\sqrt{\det[\mathrm{I(p)}]} = \sqrt{\frac{1}{p(1-p)}} = p^{-0.5}(1-p)^{-0.5} = p^{0.5-1}(1-p)^{0.5-1}$$

So,

$$\pi_J(p) \propto \mathrm{Beta}(0.5,0,5)$$