INF2080
Oblig 2

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Problem 1: Pumping Lemma

$$\text{L} = \{ab^nc^n \mid n \geq 0\} \cup \{a^kw \mid k \neq 1, w \in \Sigma^* \text{ does not start with an } a\}$$

Problem 1a

Use the pumping lemma to show that the language L_1 is non-regular.

$$L_1 = \{ab^nc^n \mid n \neq 0\}$$

Assume that L_1 is regular. Let p be the pumping length given by the pumping lemma. Choose s to be ab^pc^p. Because a is a member of L_1 and s has length more than p, the pumping lemma guarantees that s can be split into three pieces, $s = xyz$, where for any $i \geq 0$ the string xy^iz is in L_1.
There are three cases we need to consider to show that this result is impossible.

1. The string y consists only of bs. In this case, the string $xyyz$ has more bs than cs and so is not a member of L_1, violating contradiction 1 of the pumping lemma. This case is a contradiction.
2. The string y consists only of cs. This case also gives a contradiction. The number of cs in $xyyz$ are greater than the number of bs.
3. The string y consists of both bs and cs. If we choose $y = bc$ (the last b and first c) and pump up, we get the right amount of bs and cs, but they are in the wrong order. We get a string that looks like this: *ab...bcbc...c*. We then get that this also is a contradiction.

A contradiction is unavoidable for any variation of xyz if we assume that L_1 is regular, so L_1 is non-regular.

Problem 1b

Argue why L must then be non-regular and explain why this is not a counter-example to the pumping lemma.

As we know, the class of regular languages is closed under the intersection operation(as stated in the slides, and proven in theorem 1.25 in Sipser). If we can find a regular language and intersect it with L, and that is not regular, L is not regular. We define the language M as:

$$M = \{ab\Sigma^*\}$$

If we intersect that with L we get:

$$L \cap M = \{ab^n c^n \mid n > 0\}$$

$\{ab^n c^n \mid n > 0\}$ is non-regular (see Problem 1a).
This proves that L is non-regular.

The pumping lemma can only show if a language is non-regular, not that a language *is* regular. The language L was non-regular, but it could be pumped. That does not show that it is regular, just that it is not *certainly* non-regular.

Problem 2: Context-Free Languages

Consider the language from Problem 1:

$$L_1 = \{ab^n c^n \mid n \neq 0\}$$

Problem 2a

Construct a CFG that generates L_1.

$S \rightarrow aK$
$K \rightarrow bKc \mid \epsilon$

Problem 2b

Sketch a state diagram for a PDA that recognizes L_1.