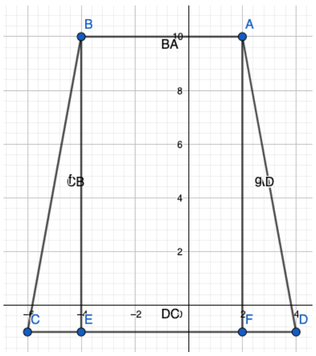
The Quadrilateral Detective

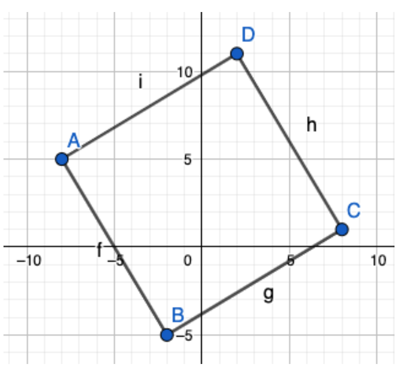
Case 2



Quadrilateral: Isosceles Trapezoid
$ABCD$ is a quadrilateral, as it has four sides.
AB and CD are parallel, as they have the same slope of 0.
BC and AD have a length of $5\sqrt{5}$ (distance formula)
$ABCD$ is an **isosceles trapezoid** as it has $\cong$ bases (BC, AD) and one pair of parallel sides (CD, AB).

Area: 88 units^2
I added auxiliary lines BE and AF.
The area of rectangle $ABEF$ is $6 * 11$, or 66.
The area of triangles CEB and AFD are 22.
Adding these numbers results in **88**, which is the area of this trapezoid.

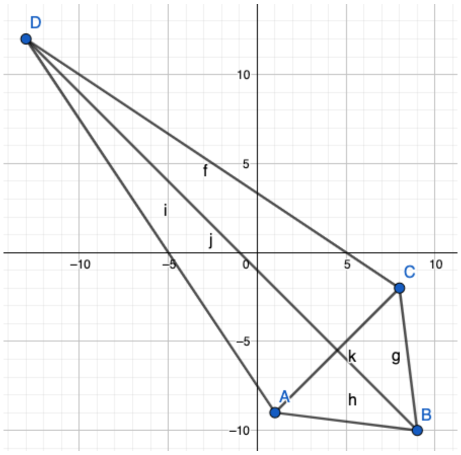
Case 3



Quadrilateral: Square
All line segments' lengths are $\sqrt{136}$ (distance formula)
AB and CD have slopes of $-5/3$ (slope formula)
BC and DA have slopes of $3/5$ (slope formula)
All angles are right angles, as perpendicular lines have negative reciprocal slopes.
$ABCD$ is a square as it has 4 right angles and 4 congruent angles.

Area: 136 units^2
$\sqrt{136} * \sqrt{136} = \mathbf{136}$

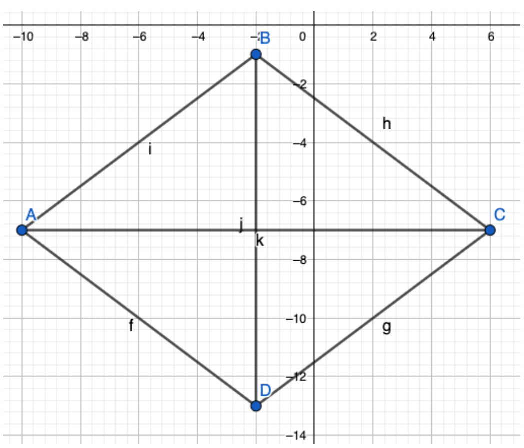
Case 4



Quadrilateral: Kite
AB and BC are $\sqrt{65}$ in length (distance formula).
CD and DA are $7\sqrt{13}$ in length (distance formula).
The slope of line BD is -1 (slope formula).
The slope of line AC is 1 (slope formula).
Because these are negative reciprocals of each other, the diagonals (AC and BD are perpendicular).
As this shape has two sets of $\cong$ sides and perpendicular diagonals, it is a **kite**.

Area: 154 units^2
BD has a length of 31.112698, while AC has a length of 9.899495 (distance formula).
The kite has an area of **154 units^2** (kite area formula).

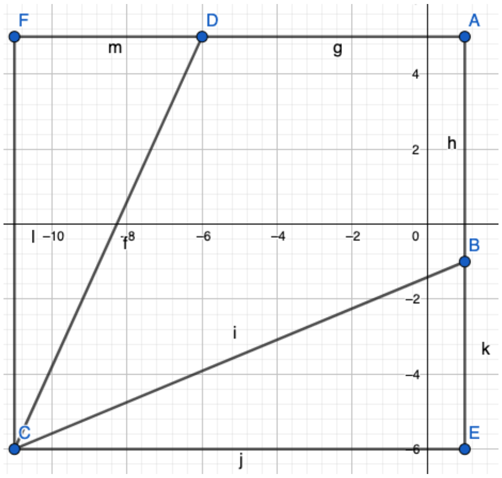
Case 5



Quadrilateral: Rhombus
All sides are $\cong$ at 10 units in length.
AB and CD have a slope of 3/4, while BC and DA have slopes of $-3/4$.
This means that AB/CD and BC/DA are parallel, as they share slopes.
As this shape has 2 parallel sets of lines and has 4 congruent sides, it is a **rhombus**.

Area: 96 units^2
$BD = 12$ and $AC = 16$ (distance formula).
The rhombus has an area of **96 units^2** (rhombus area formula).

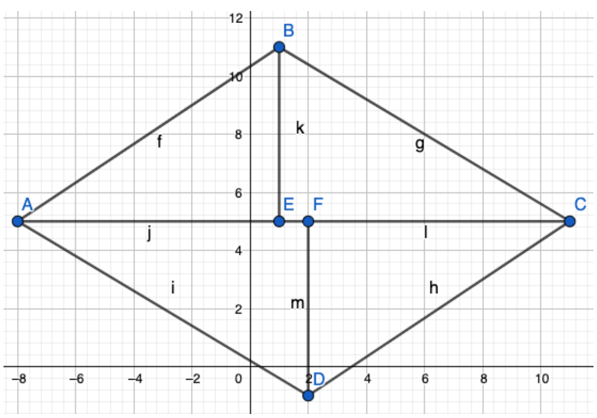
Case 6



Quadrilateral: Parallelogram
AD, BC, CD, and DA all have different lengths and slopes.
This shape does not meet all requirements of any special quadrilaterals, therefore it is a plain **quadrilateral**.

Area: 74.5 units^2
Rectangle $FACE$ has an area of 132 (11x12)
Calculating the area of triangle FDC results in 27.5, while the area of triangle CEB is 30.
Taking away the areas of these triangles to find the true area of the given shape, we get **74.5** units^2.

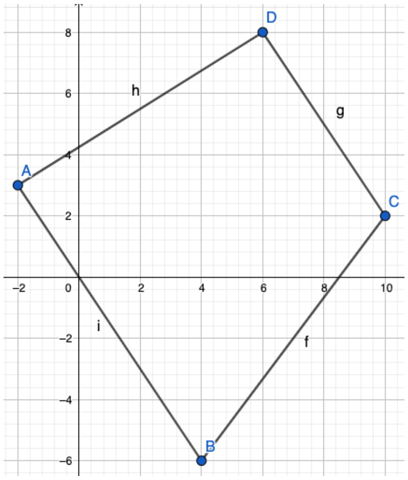
Case 7



Quadrilateral: Parallelogram
AB and DC have slopes of $2/3$, while BC and DA have slopes of $-3/5$.
Because they have the same slopes, AB/DC and BC/DA are parallel.
As $ABCD$ has 4 sides and has 2 sets of parallel lines, it is a **parallelogram**.

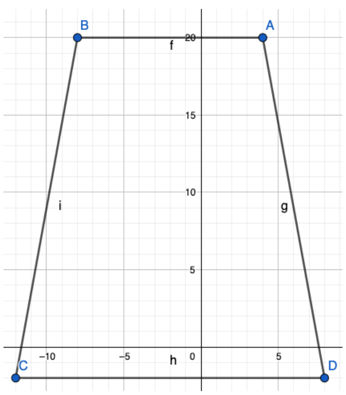
Area: 114 units^2
Triangles BAE and FCD have the area of 27, and triangles BEC and FAD have an area of 30.
Adding these areas up, the total area of this parallelogram is 114 units^2.

Case 8



Quadrilateral: Trapezoid
DC and AB have the same slope of $-3/2$
$ABCD$ is a **trapezoid** because it has 1 set of parallel lines (DC and AB) and is a quadrilateral.

Case 9



Quadrilateral: Isosceles Trapezoid
A: (4, 20) B: (-8, 20) C: (-12, -1) D: (8, -2)
Through the distance formula, the length of AB is 12 units, the lengths of BC and AD are $10\sqrt{5}$, and the length of CD is 20.
In Case 2's shape, the length of AB was 6 units, the lengths of BC and AD were $5\sqrt{5}$, and the length of CD was 10.
Here, we can see that these lengths all share the same ratio between Case 2 and 9's shapes: 1 : 2.
This means that dilation would cause these two shapes to perfectly match.
AB and CD are parallel, as they have the same slope of 0.
In addition, $ABCD$ is an **isosceles trapezoid**, just like Case 2, as it has $\cong$ bases (BC, AD) and one pair of parallel sides (CD, AB).
Because all these sides share the same ratios and class of quadrilateral, these shapes are similar.