Statistics with Spa  ows

Lecture 5

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Outline

- Check-in: 95CI
- T-test
- Other probability distributions you'll come across
- Conventions: how to report t-test?

What are statistics?

- We want to know if a null hypothesis is rejected
- Most of the time, we want to know if data is distributed according to what we believe it should be distributed, given what we know

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- We will test is the mean of the
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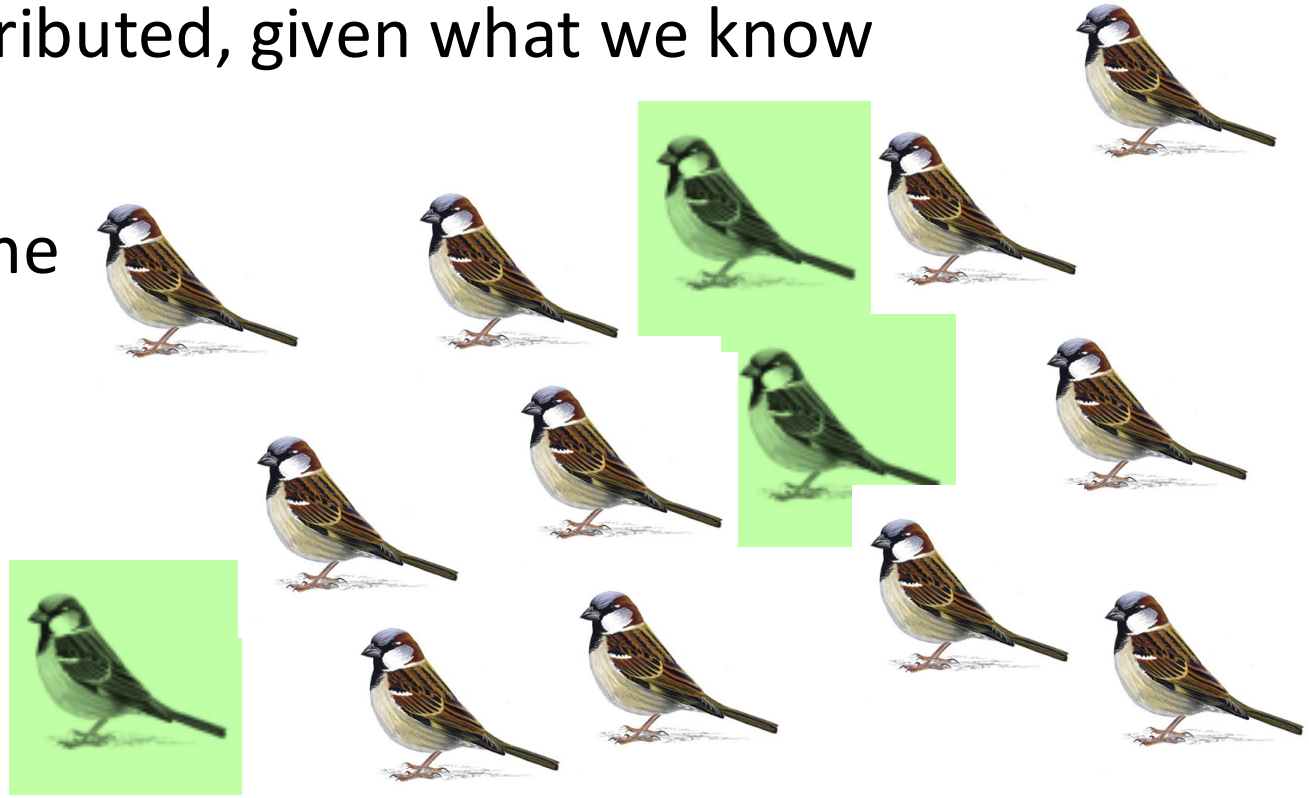
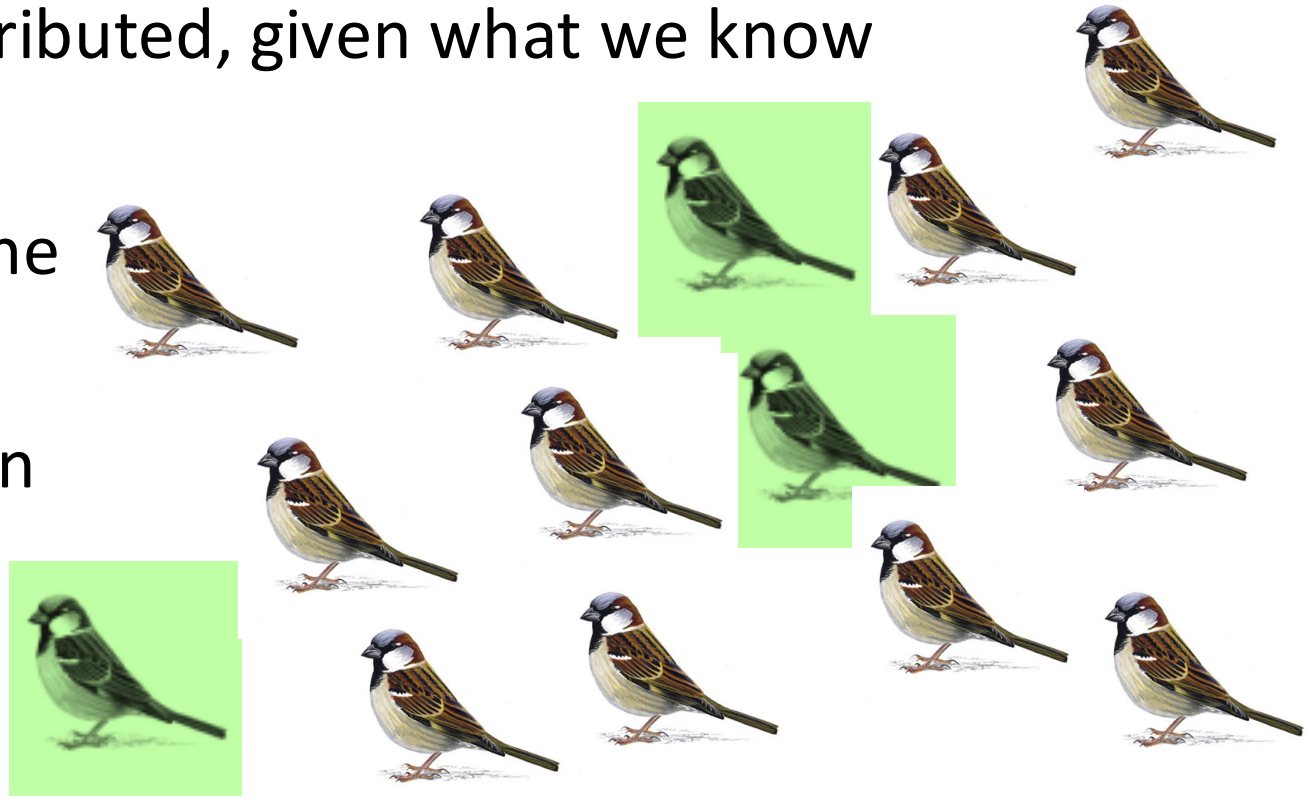


What are statistics?

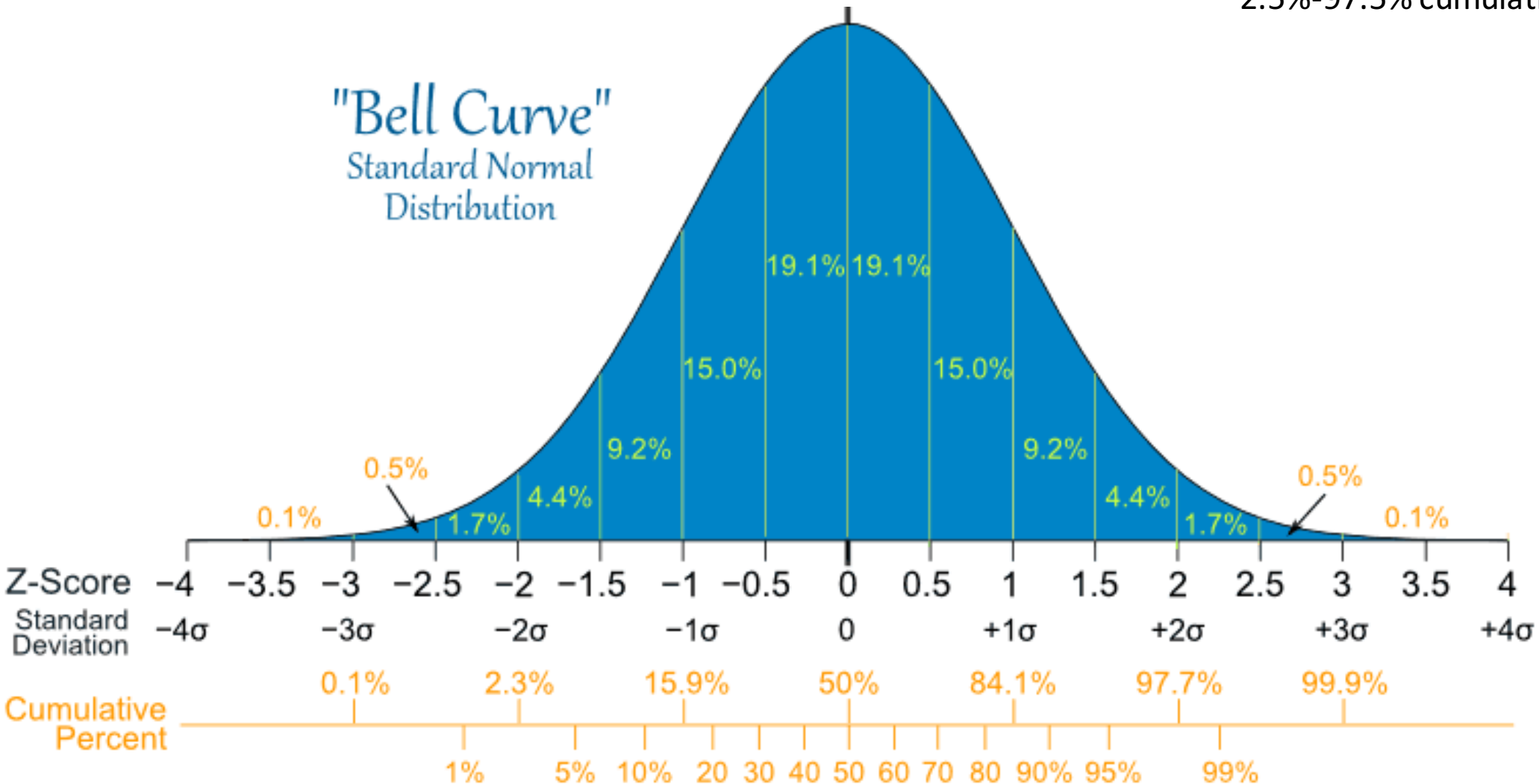
- We want to know if a null hypothesis is rejected
- Most of the time, we want to know if data is distributed according to what we believe it should be distributed, given what we know
- We will test is the mean of the
- 2001 data truthfully represents the
- complete population.
- We will test if 2001 mean is within
- a certain range of values



Hypothesis testing

- H0 = true mean is equal to mean of 2001
- H1 = true mean is not equal to mean of 2001

We accept every mean within 95% of the distribution
That means from 2.5%-97.5% cumulative%



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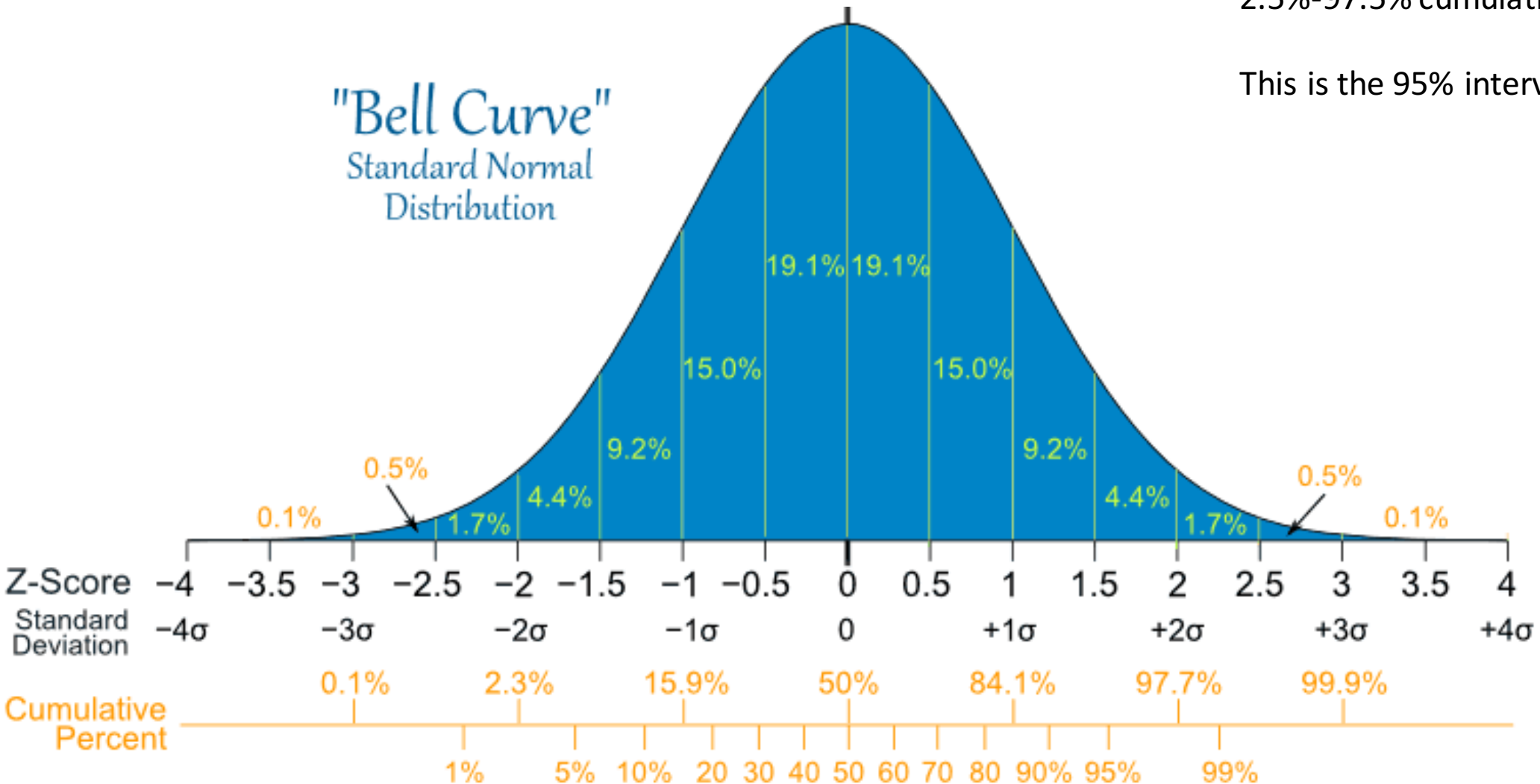

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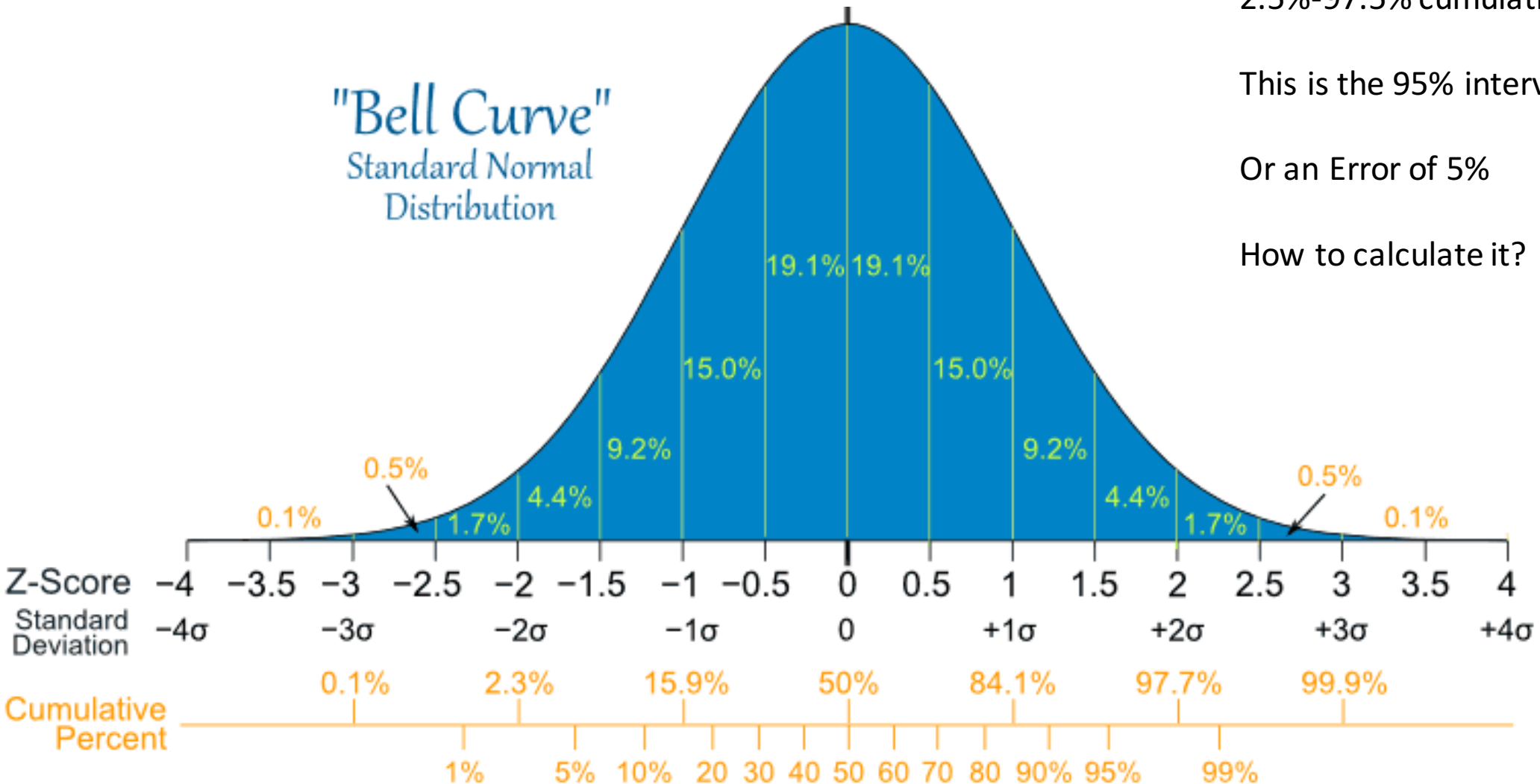
Or an Error of 5%

How to calculate it?



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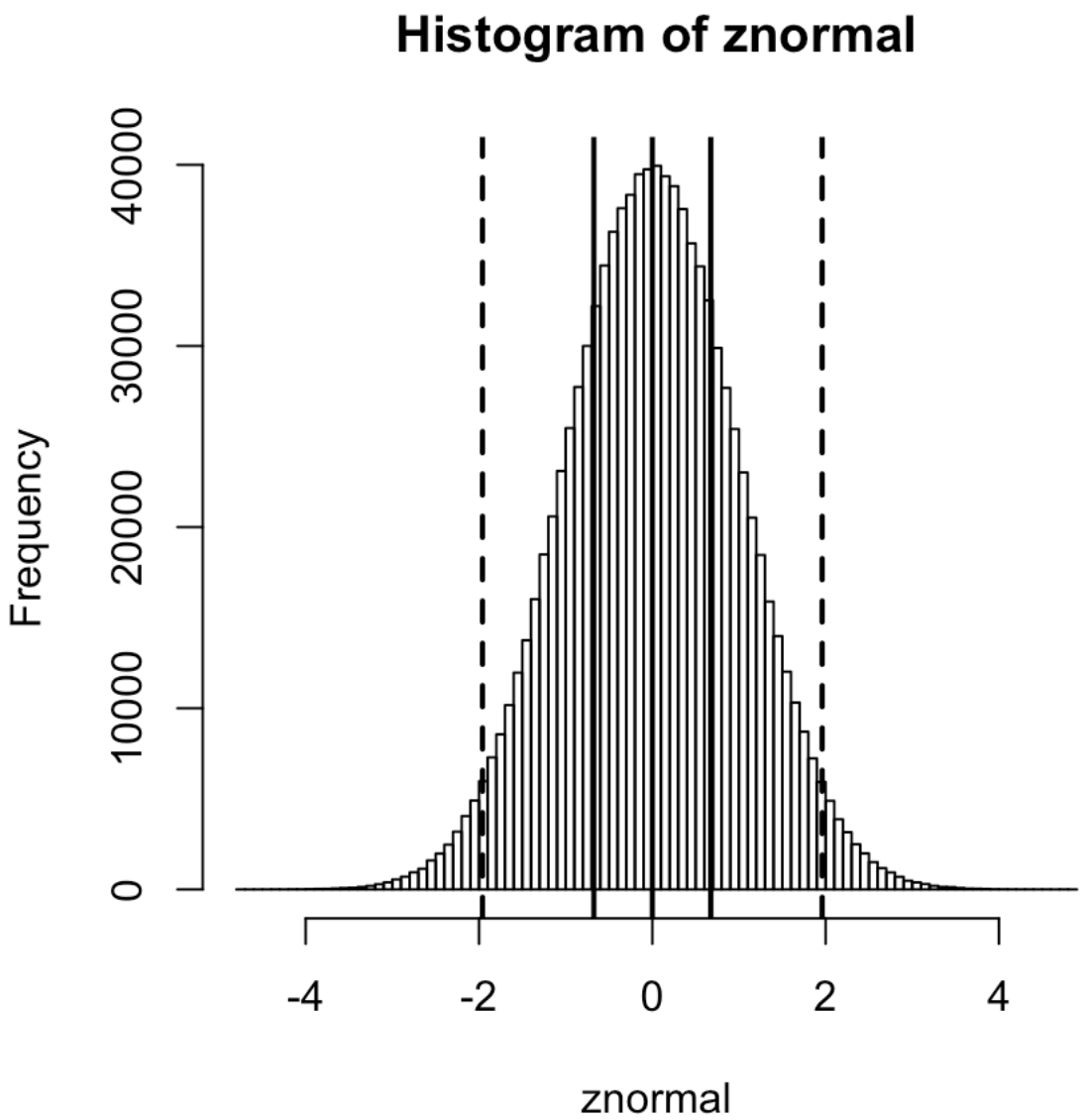
It is the mean plus/minus 1.96 times the standard deviation divided by the square root of the sample size


Histogram of znormal
Frequency
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10000
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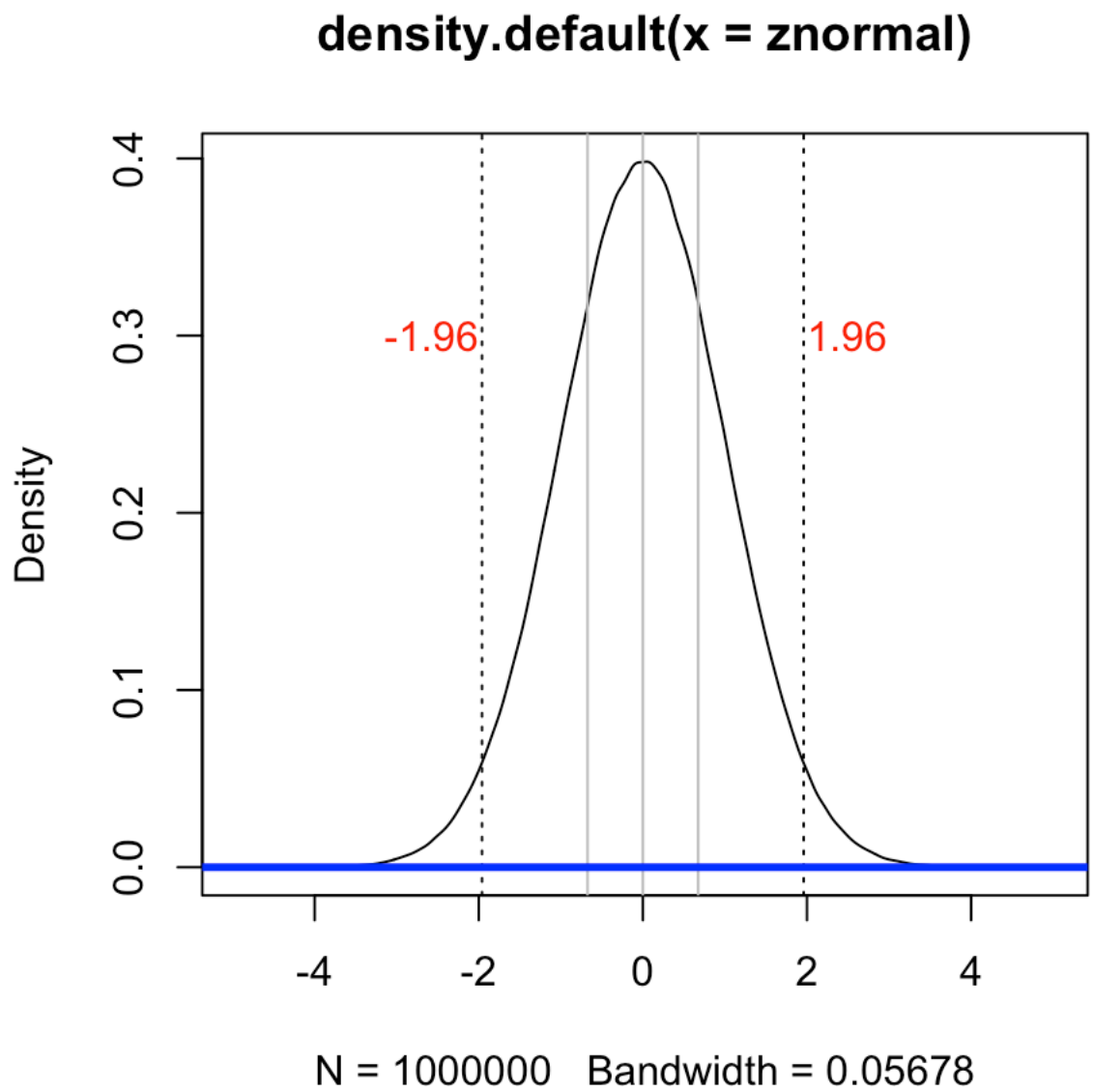
density.default(x = znormal)
Density
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N = 1000000 Bandwidth = 0.05678

Looks familiar?

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Rule-of-thumb:
Twice the SE!

$$se = \sqrt{\frac{s^2}{n}}$$

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Remember:

	Tarsus	Tarsus 2001
Variance	0.74	0.72
Standard deviation	0.86	0.85
N	1685	168
Standard error	0.02	0.07
Mean	18.52	18.19

Remember:

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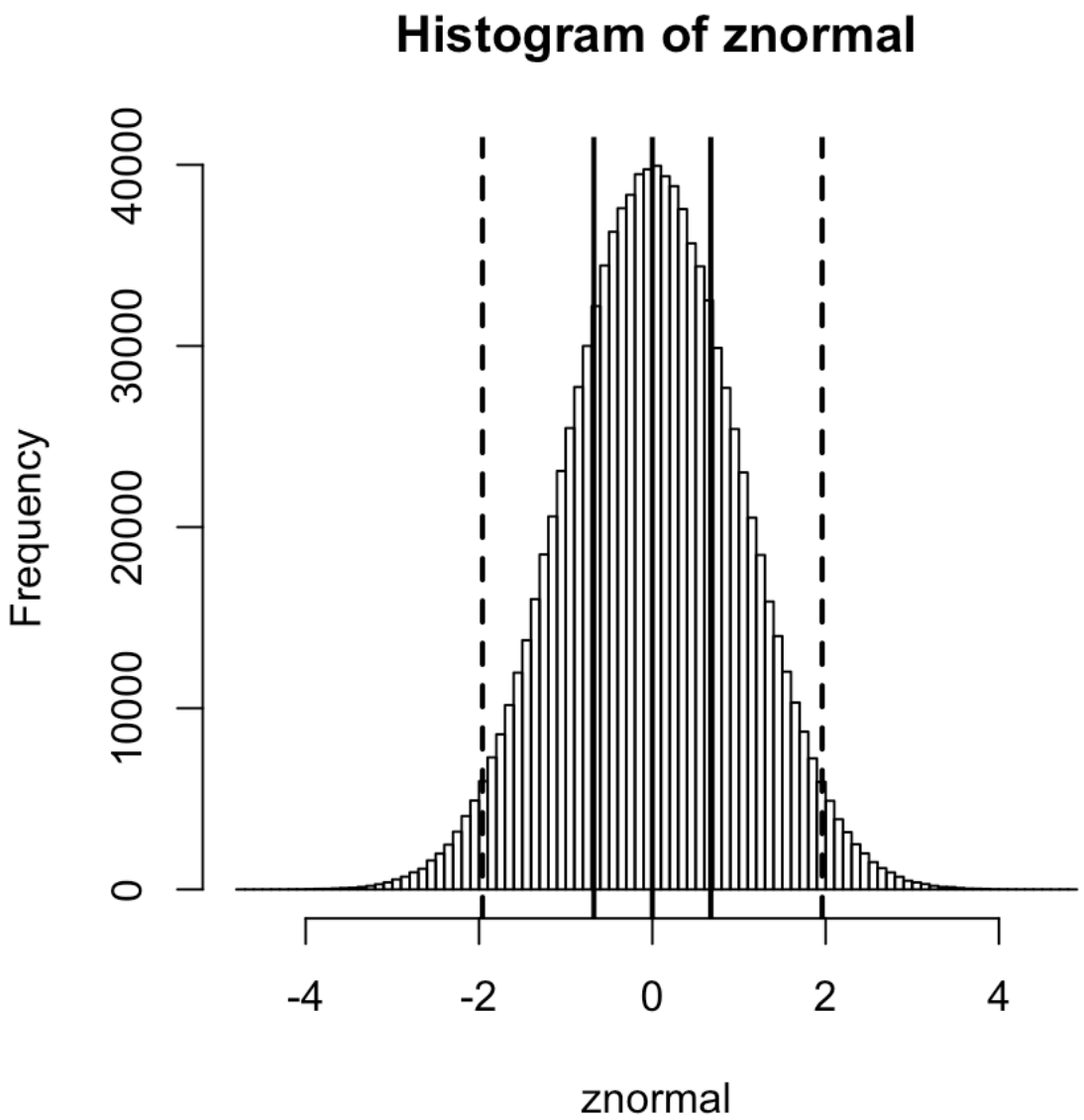
Is the sample from 2001 representative of the whole population?

Does the mean of 2001 fall within all possible means of the true distribution? We allow an error of 5%.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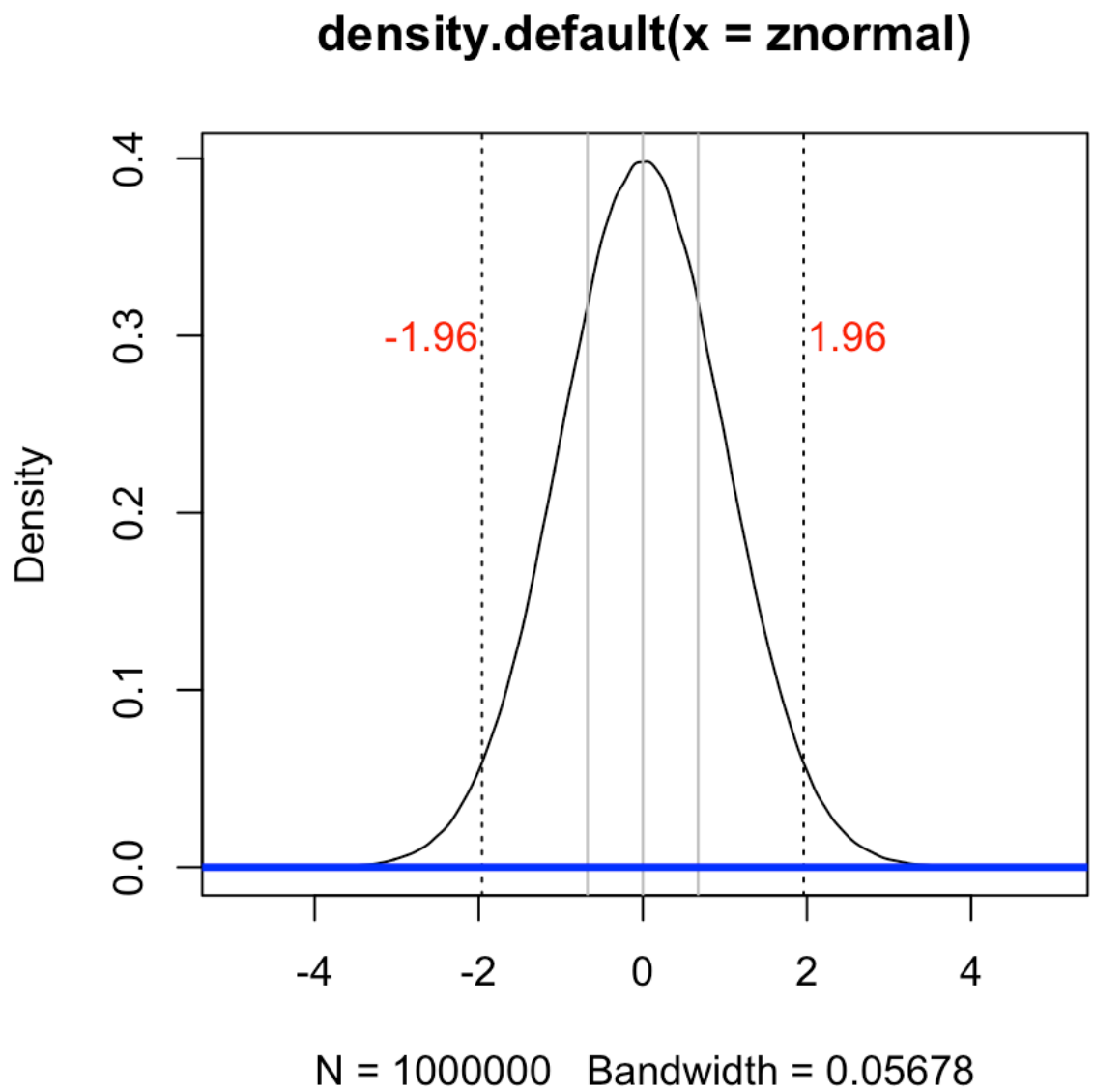
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- Let’s test this!

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	Tarsus	Tarsus 2001
N	1685	168
Standard error		0.07
Mean	18.52	18.19
~95%CI Mean+- (2*SE)	DOES NOT SPAN 18.52	18.05 – 18.33

NO! We reject H_0!

T-test

- H0 = true mean is equal to 18.5
- H1 = true mean is not equal to 18.5

$$t_{\hat{\beta}} = \frac{\hat{\beta} - \beta_0}{\mathrm{s.e.}(\hat{\beta})}$$

- Sample size is 168, thus *df* = 167

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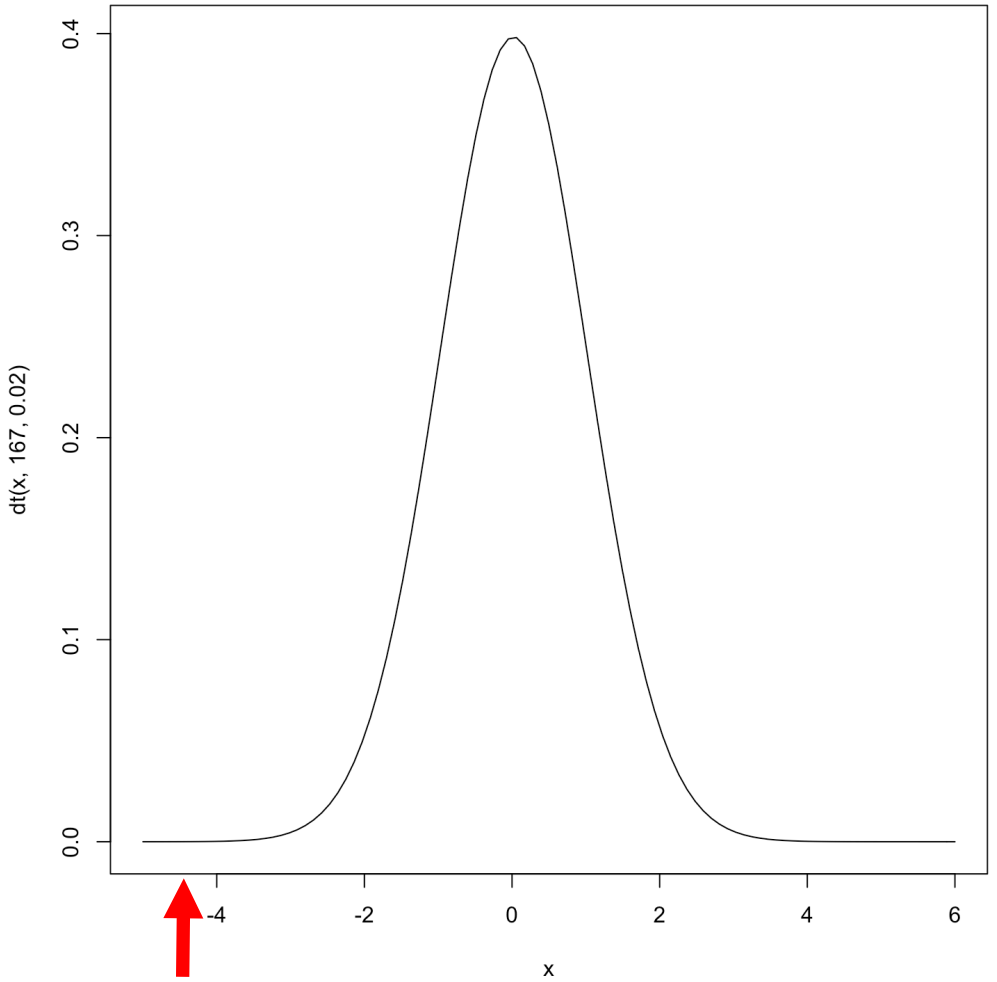

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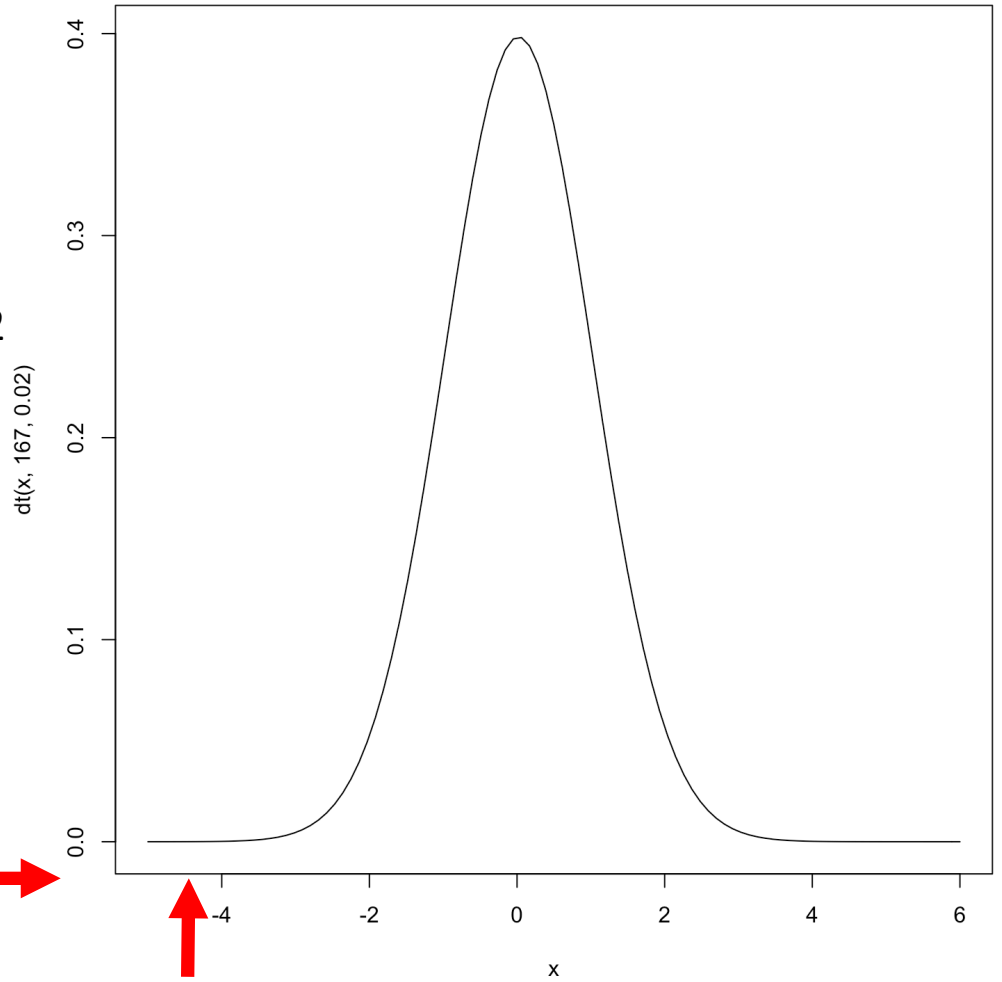

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- Sample size is 168, thus *df* = 167

p-value

TABLE of CRITICAL VALUES for STUDENT'S *t* DISTRIBUTIONS

Column headings denote probabilities (α) ***above*** tabulated values.

d.f.	0.40	0.25	0.10	0.05	0.04	0.025	0.02	0.01	0.005	0.0025	0.001	0
1	0.325	1.000	3.078	6.314	7.916	12.706	15.894	31.821	63.656	127.321	318.289	6
2	0.289	0.816	1.886	2.920	3.320	4.303	4.849	6.965	9.925	14.089	22.328	3
3	0.277	0.765	1.638	2.353	2.605	3.182	3.482	4.541	5.841	7.453	10.214	1
4	0.271	0.741	1.533	2.132	2.333	2.776	2.999	3.747	4.604	5.598	7.173	
5	0.267	0.727	1.476	2.015	2.191	2.571	2.757	3.365	4.032	4.773	5.894	
6	0.265	0.718	1.440	1.943	2.104	2.447	2.612	3.143	3.707	4.317	5.208	
7	0.263	0.711	1.415	1.895	2.046	2.365	2.517	2.998	3.499	4.029	4.785	
8	0.262	0.706	1.397	1.860	2.004	2.306	2.449	2.896	3.355	3.833	4.501	
9	0.261	0.703	1.383	1.833	1.973	2.262	2.398	2.821	3.250	3.690	4.297	
10	0.260	0.700	1.372	1.812	1.948	2.228	2.359	2.764	3.169	3.581	4.144	
11	0.260	0.697	1.363	1.796	1.928	2.201	2.328	2.718	3.106	3.497	4.025	
12	0.259	0.695	1.356	1.782	1.912	2.179	2.303	2.681	3.055	3.428	3.930	
13	0.259	0.694	1.350	1.771	1.899	2.160	2.282	2.650	3.012	3.372	3.852	
14	0.258	0.692	1.345	1.761	1.887	2.145	2.264	2.624	2.977	3.326	3.787	
15	0.258	0.691	1.341	1.753	1.878	2.131	2.249	2.602	2.947	3.286	3.733	
16	0.258	0.690	1.337	1.746	1.869	2.120	2.235	2.583	2.921	3.252	3.686	
17	0.257	0.689	1.333	1.740	1.862	2.110	2.224	2.567	2.898	3.222	3.646	
18	0.257	0.688	1.330	1.734	1.855	2.101	2.214	2.552	2.878	3.197	3.610	
19	0.257	0.688	1.328	1.729	1.850	2.093	2.205	2.539	2.861	3.174	3.579	
20	0.257	0.687	1.325	1.725	1.844	2.086	2.197	2.528	2.845	3.153	3.552	
21	0.257	0.686	1.323	1.721	1.840	2.080	2.189	2.518	2.831	3.135	3.527	
22	0.256	0.686	1.321	1.717	1.835	2.074	2.183	2.508	2.819	3.119	3.505	
23	0.256	0.685	1.319	1.714	1.832	2.069	2.177	2.500	2.807	3.104	3.485	
24	0.256	0.685	1.318	1.711	1.828	2.064	2.172	2.492	2.797	3.091	3.467	
25	0.256	0.684	1.316	1.708	1.825	2.060	2.167	2.485	2.787	3.078	3.450	
26	0.256	0.684	1.315	1.706	1.822	2.056	2.162	2.479	2.779	3.067	3.435	
27	0.256	0.684	1.314	1.703	1.819	2.052	2.158	2.473	2.771	3.057	3.421	
28	0.256	0.683	1.313	1.701	1.817	2.048	2.154	2.467	2.763	3.047	3.408	
29	0.256	0.683	1.311	1.699	1.814	2.045	2.150	2.462	2.756	3.038	3.396	
30	0.256	0.683	1.310	1.697	1.812	2.042	2.147	2.457	2.750	3.030	3.385	
31	0.256	0.682	1.309	1.696	1.810	2.040	2.144	2.453	2.744	3.022	3.375	
32	0.255	0.682	1.309	1.694	1.808	2.037	2.141	2.449	2.738	3.015	3.365	
33	0.255	0.682	1.308	1.692	1.806	2.035	2.138	2.445	2.733	3.008	3.356	
34	0.255	0.682	1.307	1.691	1.805	2.032	2.136	2.441	2.728	3.002	3.348	
35	0.255	0.682	1.306	1.690	1.803	2.030	2.133	2.438	2.724	2.996	3.340	
36	0.255	0.681	1.306	1.688	1.802	2.028	2.131	2.434	2.719	2.990	3.333	
37	0.255	0.681	1.305	1.687	1.800	2.026	2.129	2.431	2.715	2.985	3.326	
38	0.255	0.681	1.304	1.686	1.799	2.024	2.127	2.429	2.712	2.980	3.319	
39	0.255	0.681	1.304	1.685	1.798	2.023	2.125	2.426	2.708	2.976	3.313	
40	0.255	0.681	1.303	1.684	1.796	2.021	2.123	2.423	2.704	2.971	3.307	
60	0.254	0.679	1.296	1.671	1.781	2.000	2.099	2.390	2.660	2.915	3.232	
80	0.254	0.678	1.292	1.664	1.773	1.990	2.088	2.374	2.639	2.887	3.195	
100	0.254	0.677	1.290	1.660	1.769	1.984	2.081	2.364	2.626	2.871	3.174	
120	0.254	0.677	1.289	1.658	1.766	1.980	2.076	2.358	2.617	2.860	3.160	
140	0.254	0.676	1.288	1.656	1.763	1.977	2.073	2.353	2.611	2.852	3.149	
160	0.254	0.676	1.287	1.654	1.762	1.975	2.071	2.350	2.607	2.847	3.142	
180	0.254	0.676	1.286	1.653	1.761	1.973	2.069	2.347	2.603	2.842	3.136	
200	0.254	0.676	1.286	1.653	1.760	1.972	2.067	2.345	2.601	2.838	3.131	
250	0.254	0.675	1.285	1.651	1.758	1.969	2.065	2.341	2.596	2.832	3.123	
inf	0.253	0.674	1.282	1.645	1.751	1.960	2.054	2.326	2.576	2.807	3.090	

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t = -4.42

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37	0.255	0.681	1.305	1.687	1.800	2.026	2.129	2.431	2.715	2.985	3.326	
38	0.255	0.681	1.304	1.686	1.799	2.024	2.127	2.429	2.712	2.980	3.319	
39	0.255	0.681	1.304	1.685	1.798	2.023	2.125	2.426	2.708	2.976	3.313	
40	0.255	0.681	1.303	1.684	1.796	2.021	2.123	2.423	2.704	2.971	3.307	
60	0.254	0.679	1.296	1.671	1.781	2.000	2.099	2.390	2.660	2.915	3.232	
80	0.254	0.678	1.292	1.664	1.773	1.990	2.088	2.374	2.639	2.887	3.195	
100	0.254	0.677	1.290	1.660	1.769	1.984	2.081	2.364	2.626	2.871	3.174	
120	0.254	0.677	1.289	1.658	1.766	1.980	2.076	2.358	2.617	2.860	3.160	
140	0.254	0.676	1.288	1.656	1.763	1.977	2.073	2.353	2.611	2.852	3.149	
160	0.254	0.676	1.287	1.654	1.762	1.975	2.071	2.350	2.607	2.847	3.142	
180	0.254	0.676	1.286	1.653	1.761	1.973	2.069	2.347	2.603	2.842	3.136	
200	0.254	0.676	1.286	1.653	1.760	1.972	2.067	2.345	2.601	2.838	3.131	
250	0.254	0.675	1.285	1.651	1.758	1.969	2.065	2.341	2.596	2.832	3.123	
inf	0.253	0.674	1.282	1.645	1.751	1.960	2.054	2.326	2.576	2.807	3.090	

T-test

- H0 = true mean is equal to 18.5
- H1 = true mean is not equal to 18.5

$$t_{\hat{\beta}} = \frac{\hat{\beta} - \beta_0}{\text{s. e.}(\hat{\beta})}$$

$$\frac{18.19 - 18.5}{0.07}$$

t = -4.42

- Sample size is 168, thus *df* = 167

p-value

TABLE of CRITICAL VALUES for STUDENT'S *t* DISTRIBUTIONS

Column heading denote probabilities (α) ***above*** tabulated values.

d.f.	0.40	0.25	0.10	0.05	0.04	0.025	0.02	0.01	0.005	0.0025	0.001	0
1	0.325	1.000	3.078	6.314	7.916	12.706	15.894	31.821	63.656	127.321	318.289	6
2	0.289	0.816	1.886	2.920	3.320	4.303	4.849	6.965	9.925	14.089	22.328	3
3	0.277	0.765	1.638	2.353	2.605	3.182	3.482	4.541	5.841	7.453	10.214	1
4	0.271	0.741	1.533	2.132	2.333	2.776	2.999	3.747	4.604	5.598	7.173	
5	0.267	0.727	1.476	2.015	2.191	2.571	2.757	3.365	4.032	4.773	5.894	
6	0.265	0.718	1.440	1.943	2.104	2.447	2.612	3.143	3.707	4.317	5.208	
7	0.263	0.711	1.415	1.895	2.046	2.365	2.517	2.998	3.499	4.029	4.785	
8	0.262	0.706	1.397	1.860	2.004	2.306	2.449	2.896	3.355	3.833	4.501	
9	0.261	0.703	1.383	1.833	1.973	2.262	2.398	2.821	3.250	3.690	4.297	
10	0.260	0.700	1.372	1.812	1.948	2.228	2.359	2.764	3.169	3.581	4.144	
11	0.260	0.697	1.363	1.796	1.928	2.201	2.328	2.718	3.106	3.497	4.025	
12	0.259	0.695	1.356	1.782	1.912	2.179	2.303	2.681	3.055	3.428	3.930	
13	0.259	0.694	1.350	1.771	1.899	2.160	2.282	2.650	3.012	3.372	3.852	
14	0.258	0.692	1.345	1.761	1.887	2.145	2.264	2.624	2.977	3.326	3.787	
15	0.258	0.691	1.341	1.753	1.878	2.131	2.249	2.602	2.947	3.286	3.733	
16	0.258	0.690	1.337	1.746	1.869	2.120	2.235	2.583	2.921	3.252	3.686	
17	0.257	0.689	1.333	1.740	1.862	2.110	2.224	2.567	2.898	3.222	3.646	
18	0.257	0.688	1.330	1.734	1.855	2.101	2.214	2.552	2.878	3.197	3.610	
19	0.257	0.688	1.328	1.729	1.850	2.093	2.205	2.539	2.861	3.174	3.579	
20	0.257	0.687	1.325	1.725	1.844	2.086	2.197	2.528	2.845	3.153	3.552	
21	0.257	0.686	1.323	1.721	1.840	2.080	2.189	2.518	2.831	3.135	3.527	
22	0.256	0.686	1.321	1.717	1.835	2.074	2.183	2.508	2.819	3.119	3.505	
23	0.256	0.685	1.319	1.714	1.832	2.069	2.177	2.500	2.807	3.104	3.485	
24	0.256	0.685	1.318	1.711	1.828	2.064	2.172	2.492	2.797	3.091	3.467	
25	0.256	0.684	1.316	1.708	1.825	2.060	2.167	2.485	2.787	3.078	3.450	
26	0.256	0.684	1.315	1.706	1.822	2.056	2.162	2.479	2.779	3.067	3.435	
27	0.256	0.684	1.314	1.703	1.819	2.052	2.158	2.473	2.771	3.057	3.421	
28	0.256	0.683	1.313	1.701	1.817	2.048	2.154	2.467	2.763	3.047	3.408	
29	0.256	0.683	1.311	1.699	1.814	2.045	2.150	2.462	2.756	3.038	3.396	
30	0.256	0.683	1.310	1.697	1.812	2.042	2.147	2.457	2.750	3.030	3.385	
31	0.256	0.682	1.309	1.696	1.810	2.040	2.144	2.453	2.744	3.022	3.375	
32	0.255	0.682	1.309	1.694	1.808	2.037	2.141	2.449	2.738	3.015	3.365	
33	0.255	0.682	1.308	1.692	1.806	2.035	2.138	2.445	2.733	3.008	3.356	
34	0.255	0.682	1.307	1.691	1.805	2.032	2.136	2.441	2.728	3.002	3.348	
35	0.255	0.682	1.306	1.690	1.803	2.030	2.133	2.438	2.724	2.996	3.340	
36	0.255	0.681	1.306	1.688	1.802	2.028	2.131	2.434	2.719	2.990	3.333	
37	0.255	0.681	1.305	1.687	1.800	2.026	2.129	2.431	2.715	2.985	3.326	
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- We consider $p<0.05$ (5%) as statistically significant
- That's a convention, there is no real reason for why that's better than $p<0.04$ or 0.06

T-test – hypothesis testing

- H0 = true mean is equal to 18.5
- H1 = true mean is not equal to 18.5

- `d1<-subset(d, d$YEAR==2001)`
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data:  d1$Tarsus
t = -4.7719, df = 167, p-value = 3.961e-06
alternative hypothesis: true mean is not equal to 18.5
95 percent confidence interval:
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sample estimates:
mean of x
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	Tarsus	Tarsus 2001
N	1685	168
Standard error		0.07
Mean	18.52	18.19
~95%CI Mean+- (2*SE)	DOES NOT SPAN 18.52	18.05 – 18.33

We can also use t to test for differences in mean:

- H0 = males and females have equal mean
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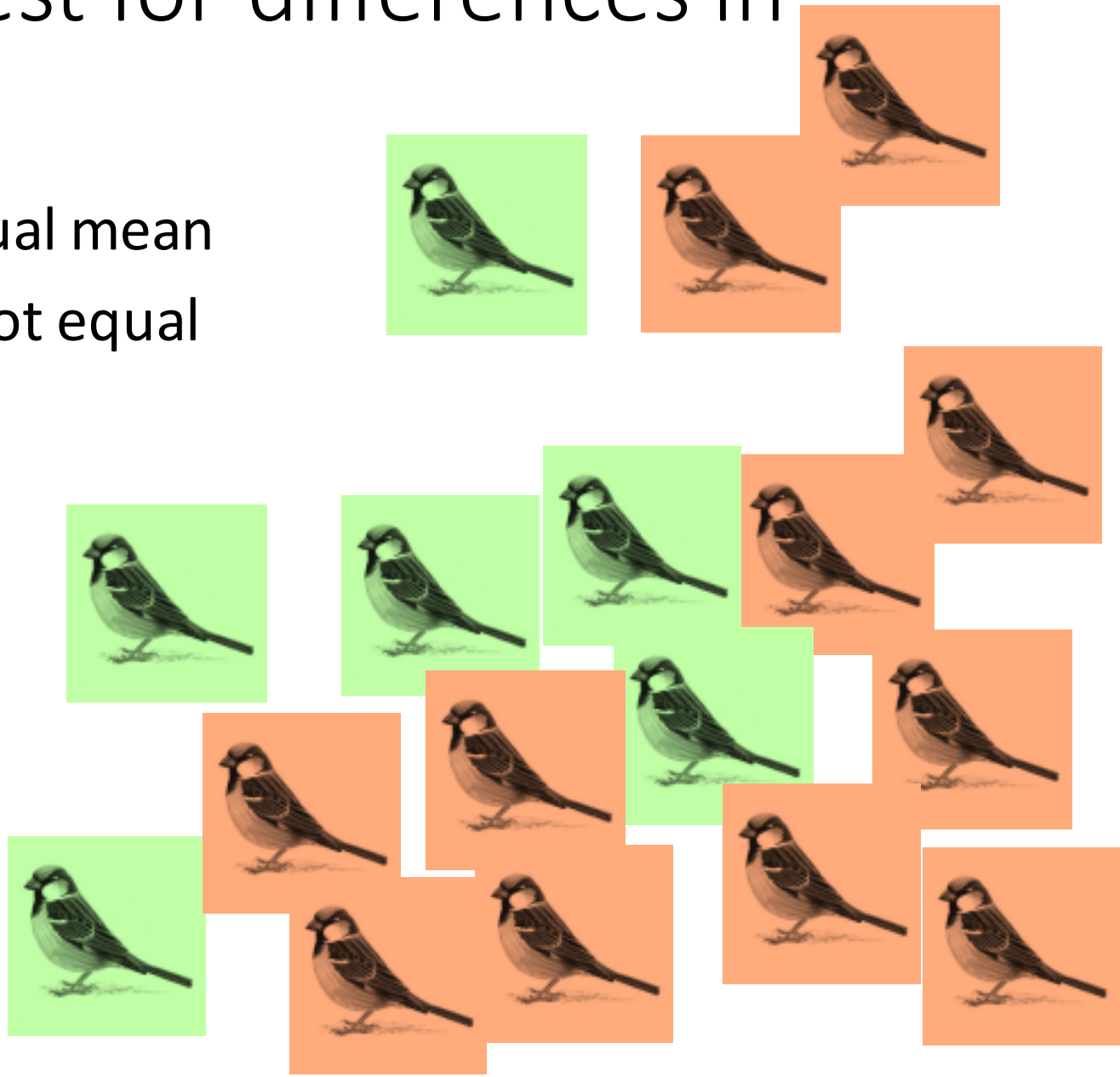

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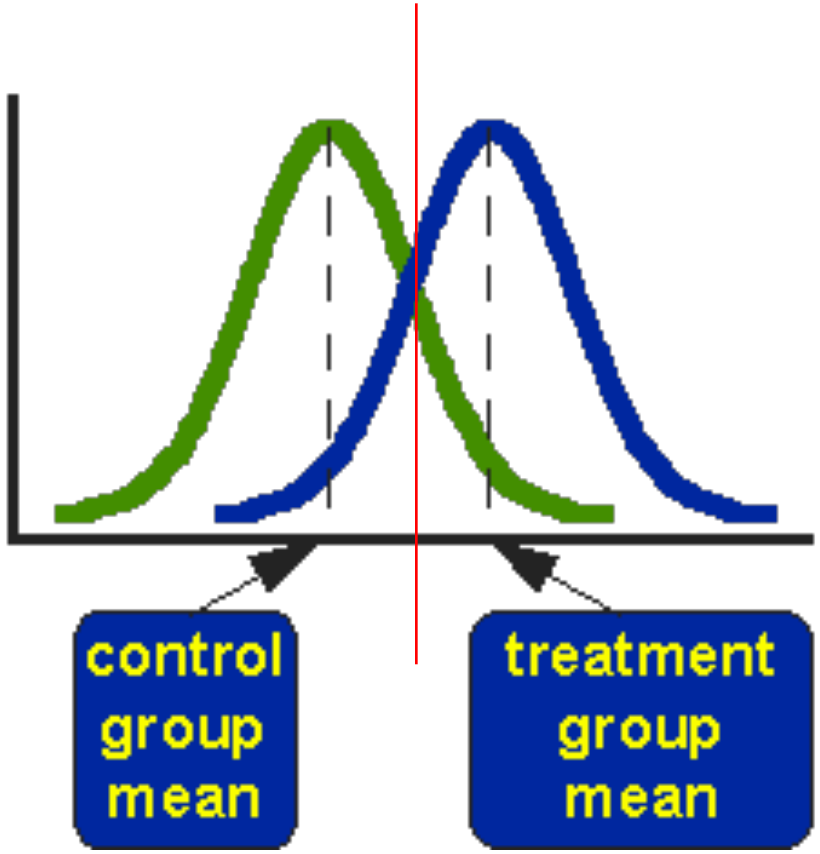

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        Welch Two Sample t-test

data:  d1$Tarsus by d1$Sex
t = 1.2257, df = 139.07, p-value = 0.2224
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
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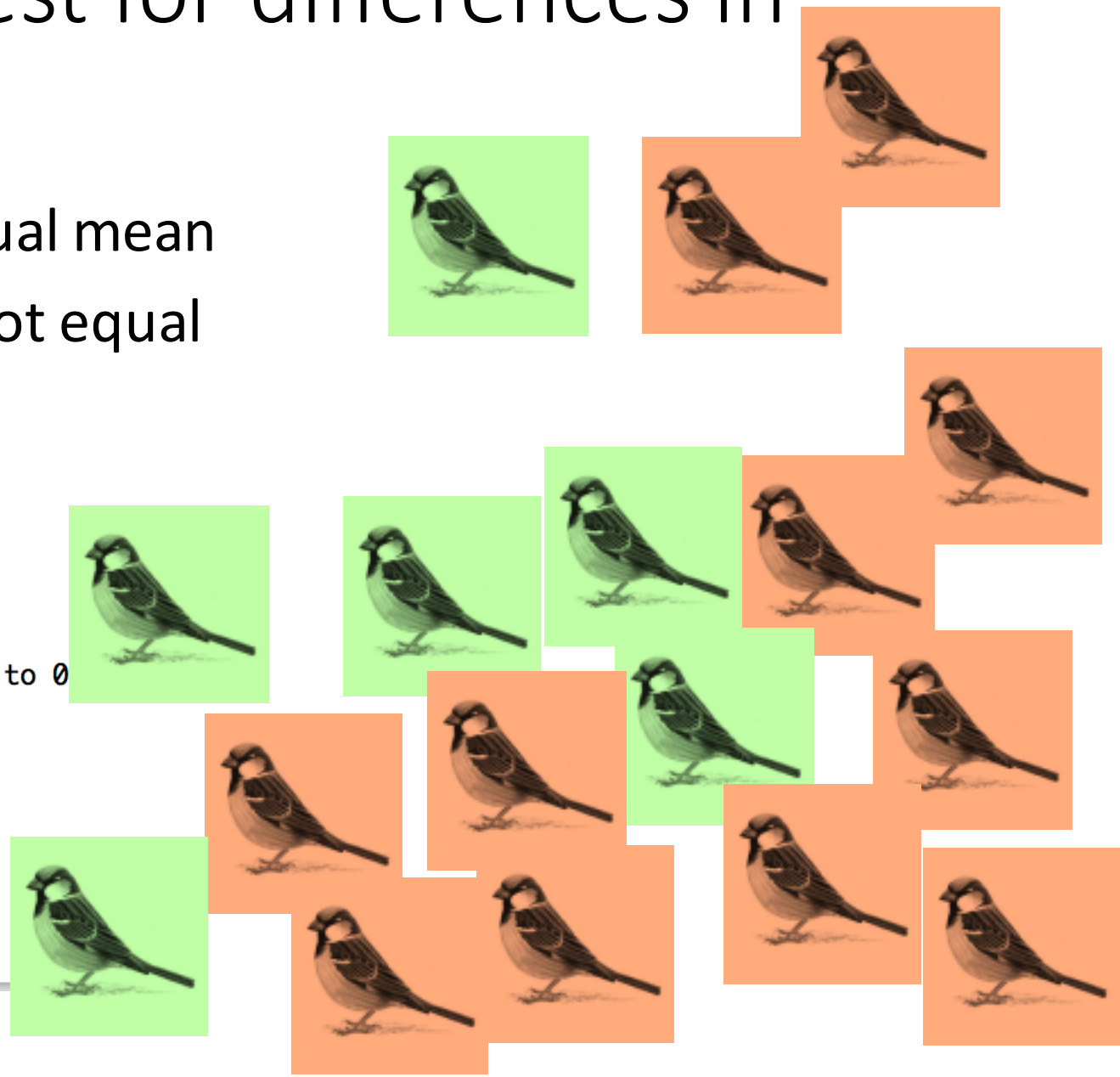

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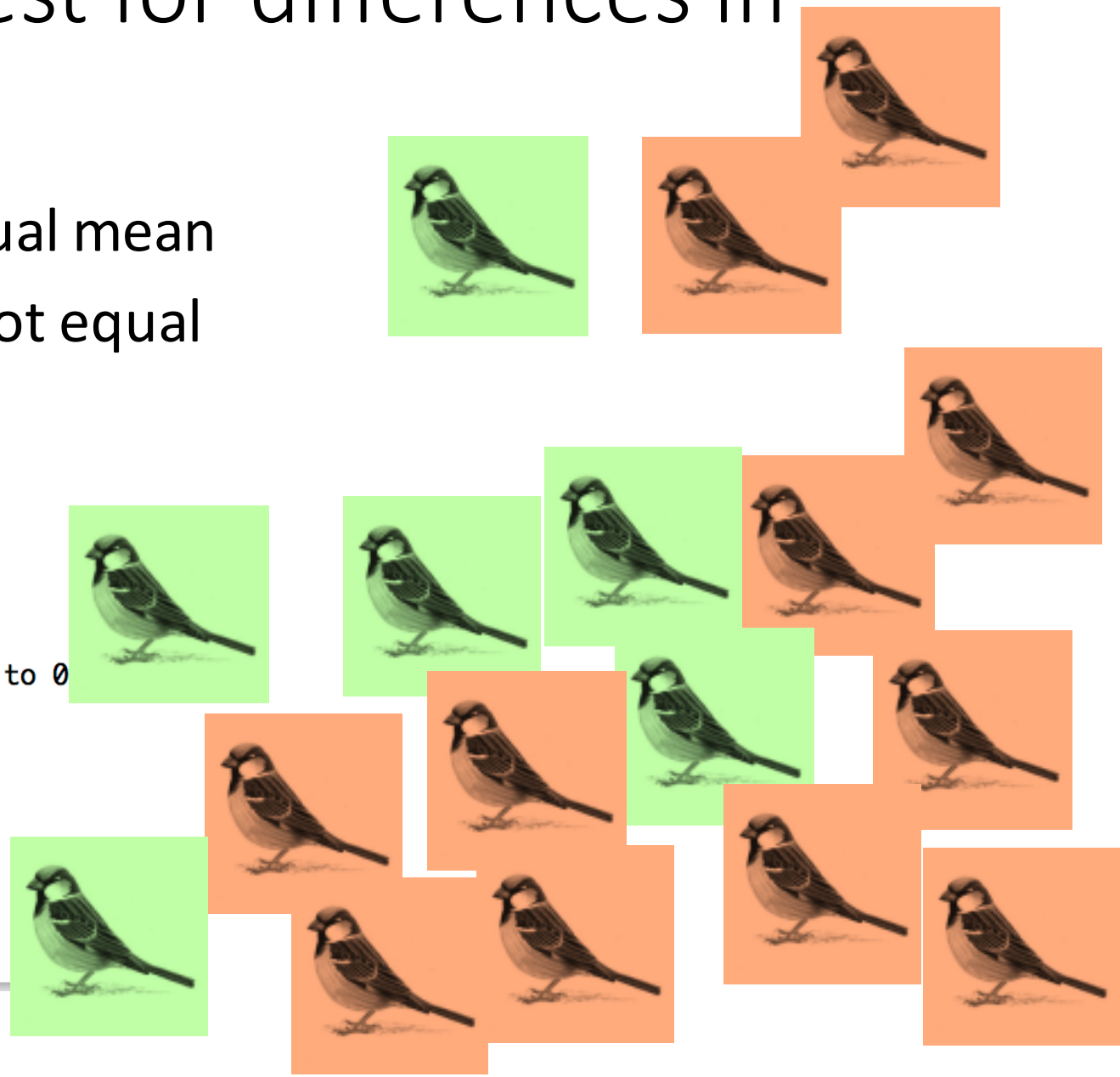

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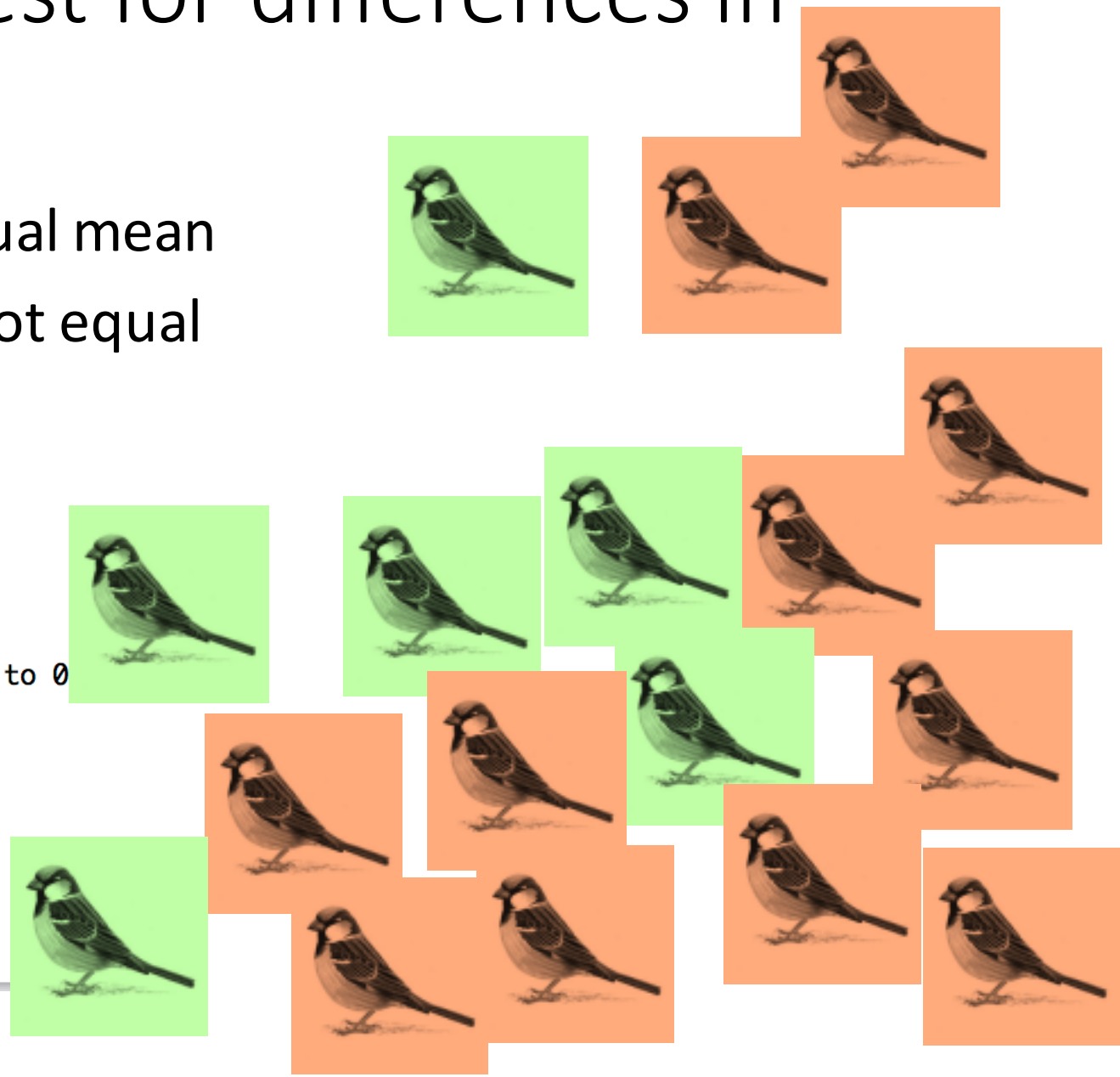

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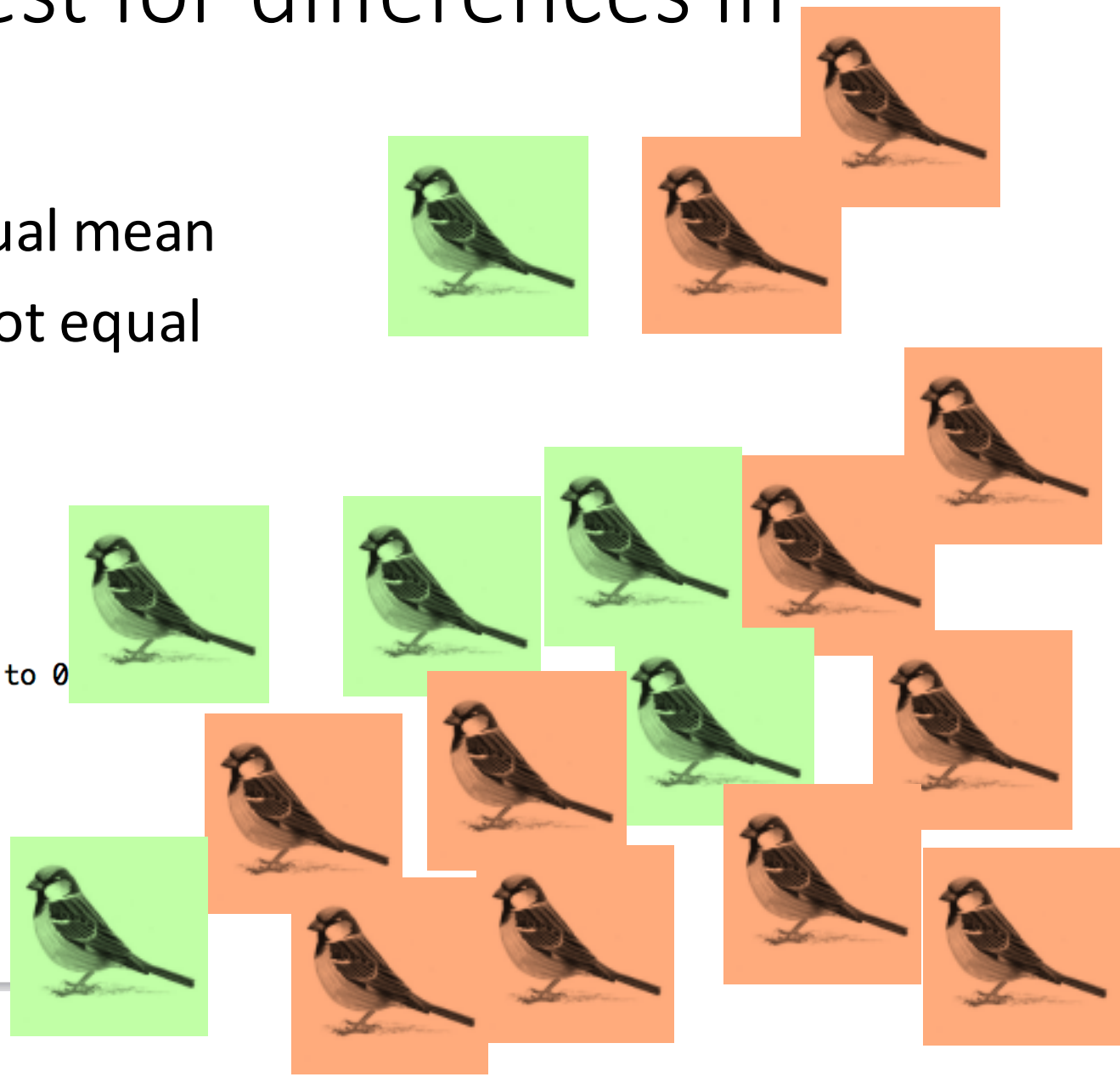

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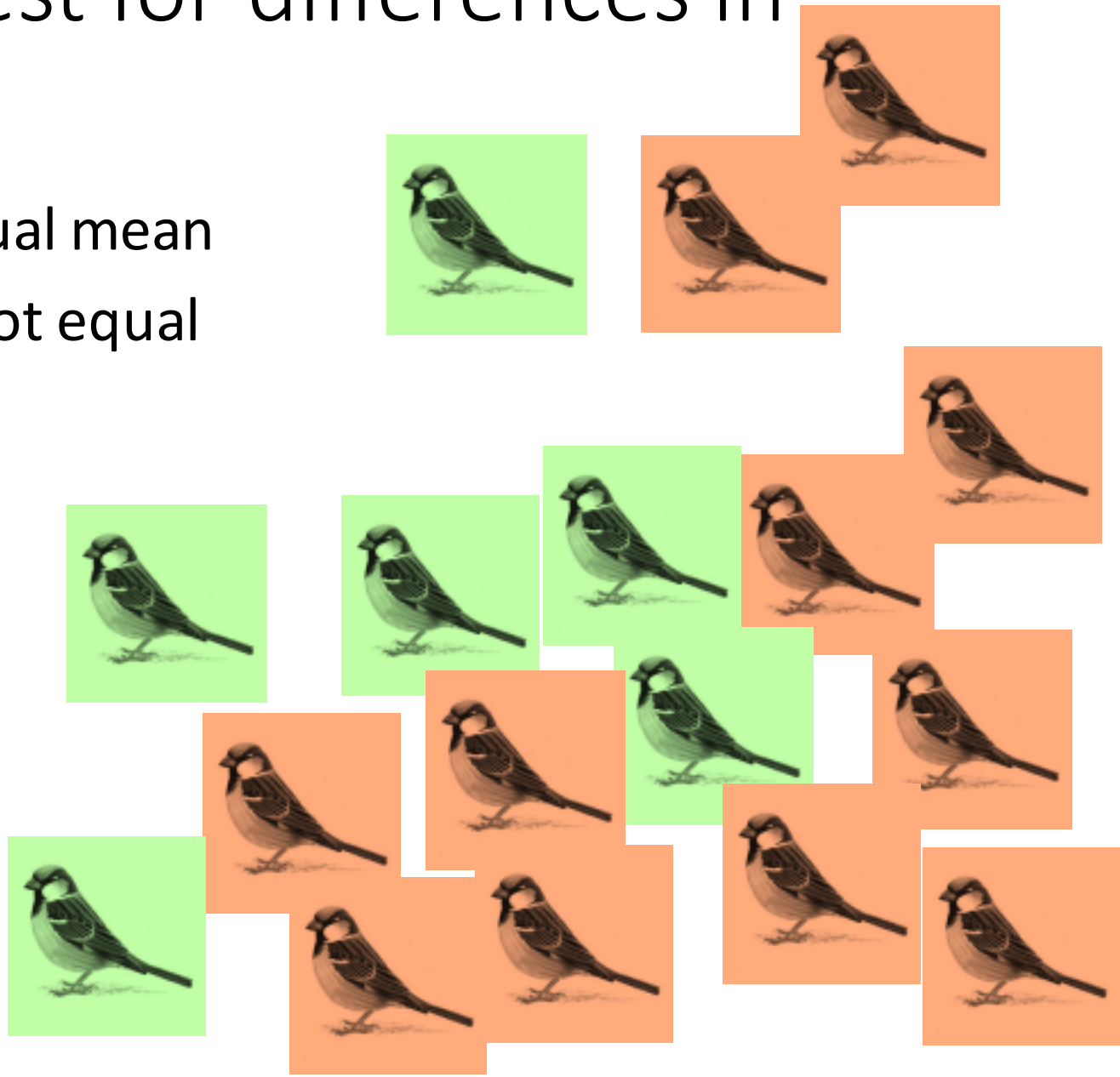
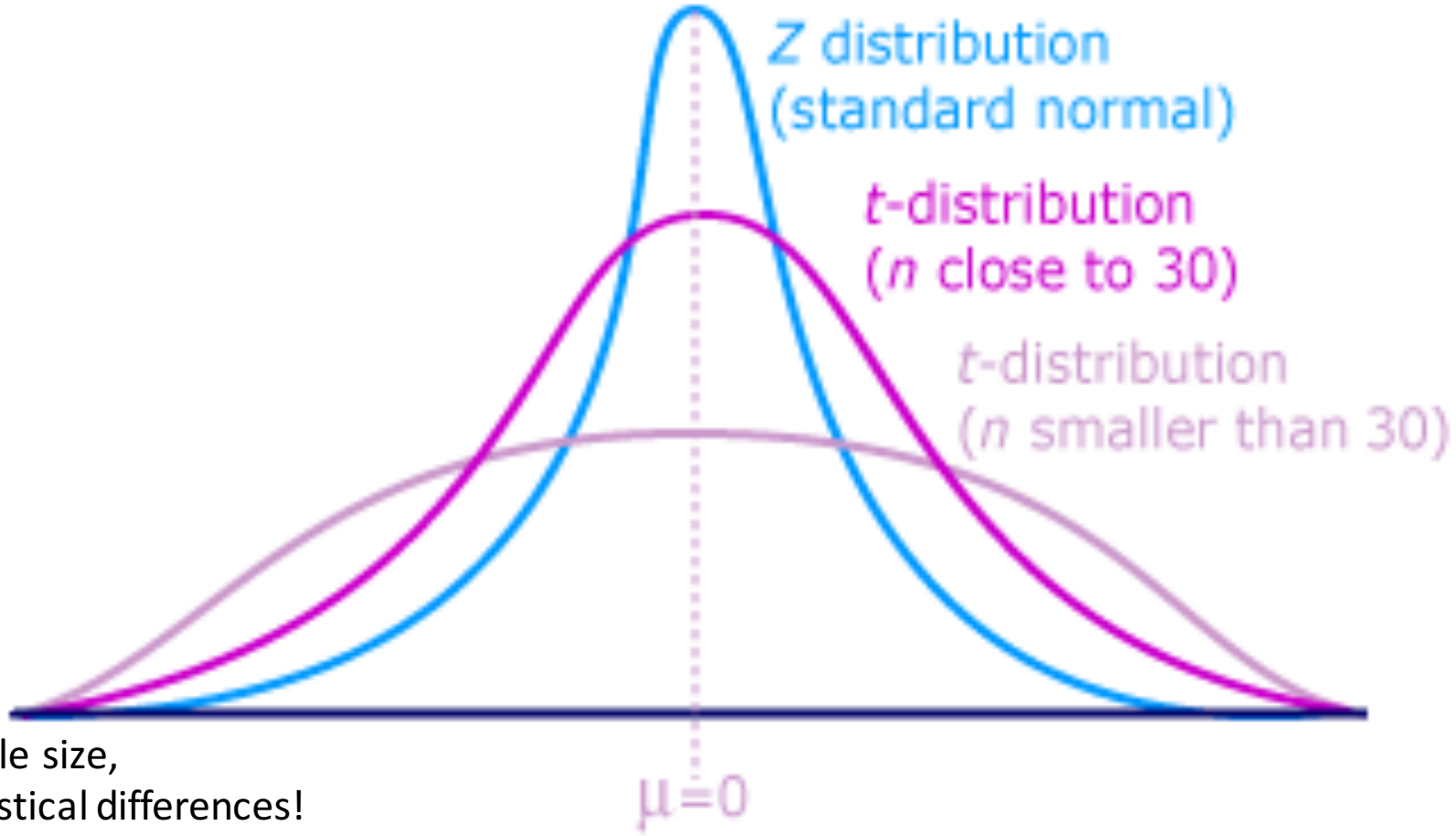


t-distribution: dependent on degrees of freedom

$$t_{\hat{\beta}} = \frac{\hat{\beta} - \beta_0}{\text{s. e.}(\hat{\beta})}$$



Means that the bigger the sample size,
the more likely we will find statistical differences!

What are all these funny distributions anyways?

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- Pot with infinite number of balls inside
- Draw one at random, writing on ball = random variable
- Basis for statistical models

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- Distributions describes a set of objects that are not identified, or that have no information, data summaries
- We *fit* a distribution to data *y* and then model *y'* dependent on *X* with errors *e*

Other distributions that we will come across

- Z-scores (normal dist, with variance=1 and stdev=1)
- Students’t
- Chi-square
  - Probability distribution of a variable that is the **square of values from a normal distribution.**
  - Positive (bounded by zero and infinity)
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- F-distribution
  - Probability distribution of a variable that is the ratio of two independent chi-square variables, each divided by its *df*
  - Positive (bounded by zero and infinity)
  - Used to test hypotheses about ratios of variances

Convention – reality check

- How to report results from a t-test?

```
> t.test(d1$Tarsus~d1$Sex,na.rm=TRUE)

        Welch Two Sample t-test

data:  d1$Tarsus by d1$Sex
t = 1.2257, df = 139.07, p-value = 0.2224
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.1012318  0.4314949
sample estimates:
mean in group 0 mean in group 1
       18.27763        18.11250

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Male and female tarsi did not differ in size between male and females (mean: 18.18, two sample t-test: t=1.23, df=139, p<0.22).

($t_{df=139}$=1.23, p<0.001).

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```

- In a table

Variable	Mean females±SE	N females	Mean males±SE	N males	t	df	p
Tarsus	18.27±0.09		18.11±0.13		1.23	139	0.22
Wing							
Ect							

DO IT NOW– HO 5

- Test if wing length in 2001 differs from the grand-total mean in wing length
- Test if male and female wing length differ in 2001
- Test if male and female wing length differ in the full dataset
- Report in a table, don’t forget the N’s!

- Report in text

Exercise – discussion

- What did you notice happened when you took smaller samples?
- Why did the precision go down?
- How many sparrows do you have to sample to get the correct answer?