Assignment-1

Pattern and Speech Recognition

November 10, 2016

1 Linear Algebra

1 Let M be a real symmetric matrix. Show that $\frac{x^T M x}{||x||_2^2}$ is upper and lower bounded by λ_{max} and λ_{min} (where λ_{max} and λ_{min} are the largest and smallest eigenvalues of M)(*2 points*)

2 Let M be a real symmetric matrix. Using the properties of eigendecompsition, present a method to compute M^{100} elegently (*2 points*)

3

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & x \end{bmatrix}$$

(i) Find a value for x such that the above matrix is positive definite. (*1 point*)
(ii) Find a value for x such that the above matrix is rank-2 (*1 point*)

4

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -5 & 3 & y \end{bmatrix}$$

(i) Find a value for y such that the vector $(1, 1, 1)$ is in the null space of above matrix. (*1 point*)
(ii) Find a value for y such that sum of its eigenvalues is 0. (*1 point*)

5 Find a non-trivial upper and lower bound for $||x||_1$ in terms of $||x||_\infty$ (*2 points*)

2 Probability Theory

1 In an experiment of tossing a fair die, let $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4\}$ be two events. Check whether A and B are independent. If $C = \{1, 3, 5\}$, check for independency of A and C. (*2 points*)

2 Assume 30% of computer owners use Macintosh, 50% use Windows and rest use Linux. Suppose that 65% of Mac users have succumbed to a computer virus, 82% of the Windows users get the virus and 50% of the Linux users get the virus. We select a person at random and learn that his/her system is infected with the virus. What is the probability that he/she is a Windows user? (*2 points*)

3

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{1+x} & \text{otherwise} \end{cases}$$

$$g(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{(1+x)^2} & \text{otherwise} \end{cases}$$

Check whether f and g are valid probability density functions(PDF). If it is a valid distribution, compute its mean and report your observation. (*3 points*)

4 Let X be a continuous random variable uniformly distributed between (0,1). Let $0 < a < b < 1$

$$Y = \begin{cases} 1 & \text{if } 0 < x < b \\ 0 & \text{otherwise} \end{cases}$$

and

$$Z = \begin{cases} 1 & \text{if } a < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Are Y and Z independent? Why/Why not? (*2 points*)

5 Let X and Y be two independent random variables. Show that $E[XY] = E[X]E[Y]$. Also compute the covariance between X and Y. (*2 points*)

3 Multivariable Calculus

1 Let $f(x, y) = x^3 - 3xy^2$. Find all the extermal points for the function f and report if they are local/global maxima/minima or saddle point. Also, report the maximum and minimum value f can take. (*3 points*)