Universität des Saarlandes
FR 4.7 Sprach- und Signalverarbeitung

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Pattern and Speech Recognition

Exercise Sheet 2

PCA and Numerical Computation

Deadline: 22.11.2016, 23:56

PCA

Exercise 2.1 *(2 + 2 + 2 = 6 points)*

In this exercise we will exploit the PCA in order to compress a 2-dimensional dataset into a 1-dimensional set. Do not forget to normalize your data such that the mean becomes 0, when taking the eigenvalue approach as described in the lecture for part a) and b).

a) Consider the following dataset consisting of 4 2-dimensional vectors:

$$\mathbf{x}^{(1)} = (1,1)^T,\ \mathbf{x}^{(2)} = (2,2)^T,\ \mathbf{x}^{(3)} = (3,1)^T,\ \mathbf{x}^{(4)} = (4,1)^T.$$

Compress this dataset to a 1-dimensional set using the PCA i.e. derive the encoder function $f(\mathbf{x}) = \mathbf{D}^T \cdot \mathbf{x}$ as defined in the lecture. Then apply f to the dataset in order to compress it.

b) Now consider the set:

$$\mathbf{x}^{(1)} = (-1,1)^T,\ \mathbf{x}^{(2)} = (-2,2)^T,\ \mathbf{x}^{(3)} = (-1,3)^T,\ \mathbf{x}^{(4)} = (-1,4)^T.$$

As in part a) compress this set by deriving the encoder function f and apply it to the set.

c) For both the parts a) and b) sketch the corresponding datasets in a separate figure. Also include the reconstructed vectors into the corresponding figures. Explain the values of the reconstructed vectors.

Numerical Computation

Exercise 2.2 *(2 + 2 = 4 points)*

In the lecture you encountered the function

$$\text{softmax}(\mathbf{x})_i = \frac{\exp(x_i)}{\sum\limits_{j=1}^{n} \exp(x_j)}$$

and learned about numerical issues that might arise when computing this function on a computer which can only compute this up to a certain precision.

a) Name the proposed approach to circumvent the mentioned over- and underflows and show why this approach prevents the stated problems.
Hint: Consider cases where the entries of $\mathbf{x}$ *are in the same range of some extreme.*

b) The proposed approach takes care of some over- and underflows. However using this scheme still over- or underflows can occur. State where we might still might encounter problems and give an example which makes softmax useless even when using the proposed scheme. I.e. your computation using softmax would lead to an undefined result ($-\infty$ or ∞).

Exercise 2.3 *(2 + 2 + 1 = 5 points)*

Consider the following function

$$f(x, y) = 20 \cdot x^2 + \frac{1}{4} \cdot y^2.$$

a) Implement a python script `gradient_descent.py` which tries to find the minimum of f using the method of gradient descent as discussed in the lecture. As a starting point use $\mathbf{x} = (-2, 4)^T$ and as learning rate $\epsilon = 0.04$.

b) In which way does your implemented script find the minimum of f? Draw a sketch or plot of f (e.g. as a contour plot) and indicate which intermediate points $\mathbf{x}$ are computed to find the minimum. Is this path optimal? Justify your answer and explain why the path looks like that.

c) What happens if you change the learning rate to $\epsilon = 0.1$? Briefly describe and explain your observations.

Exercise 2.4 *(3 + 1 = 4 points)*

Consider the function f from exercise 3.

a) Implement a python script `newton.py` that finds the minimum of f by incorporating the Hessian matrix and using newtons method as discussed in the lecture.

b) Compare the path of finding the minimum to the one from exercise 3.

Exercise 2.5 *(1 points)*

Design a meaningful stopping criterion for stopping the iterations from exercise 3 or 4.

Submission instructions

The following instructions are mandatory. If you are not following them, tutors can decide to not correct your exercise.

Submission architecture

You have to generate a **single ZIP file** respecting the following architecture:

```
tutorial2_<matriculation_nb1>_<matriculation_nb2>_<matriculation_nb3>
|
+--- source
|      |
|      +------- file 1
|      +------- file 2
|      +------- ...
+--- rapport.pdf
+--- README.txt
```

where

- **source** contains the source code of your project,
- **rapport.pdf** is the report where you present your solution with **the explanations** and the plots,
- **README** which contains group member informations (name, matriculation numbers and emails) and a **clear** explanation about how to compile and run your source code

The ZIP filename has to be :

`tutorial2_<matriculation_nb1>_<matriculation_nb2>_<matriculation_nb3>.zip`

Some hints

We advice you to follow the following guidelines in order to avoid problems :

- Avoid building complex systems. The exercises are simple enough.
- Do not include any executables in your submission, as this will cause the e-mail server to reject it.

Grading

Send your assignment to the tutor who is responsible of your group:

- Merlin Köhler s9mnkoeh@stud.uni-saarland.de
- Yelluru Gopal Goutam goutamyg@lsv.uni-saarland.de
- Ahmad Taie ataie@lsv.uni-saarland.de

The email subject should start with `[PSR TUTORIAL 2]`