

Exercise Sheet 11

Hidden Markov Models

Deadline: 06.02.2017, 10:00

Exercise 11.1

(5+5+5=15 points)

Consider a HMM with 3 states ($K = 3$) and 2 output symbols 1,2, with the transition matrix:

$$A = \begin{pmatrix} 0.5 & 0 & 0 \\ 0.3 & 0.6 & 0 \\ 0.2 & 0.4 & 1 \end{pmatrix}$$

where $A_{i,j} = P(\pi_{t+1} = i | \pi_t = j)$. The emission matrix is (first row corresponds to 1, second row to 2):

$$B = \begin{pmatrix} 0.7 & 0.4 & 0.8 \\ 0.3 & 0.6 & 0.2 \end{pmatrix}$$

where $B_{i,j} = P(x_t = i | \pi_t = j)$. The initial probabilities are 0.9, 0.1, 0 for the three states respectively. Given that the observed sequence is $x_1 = 1, x_2 = 2, x_3 = 1$:

- Compute $P(x_1, x_2, x_3)$.
- Compute $P(\pi_1 | x_1, x_2, x_3)$ for all states.
- Find the most likely hidden state sequence.

Perform all the computations by hand, i.e. there is no implementation required here.

Exercise 11.2

(3+2=5 points)

Consider an HMM representation of a coin tossing experiment. Assume a 3-state model (corresponding to 3 different coins) with the same structure as in exercise 1. The emission matrix is given by

$$B = \begin{pmatrix} 0.5 & 0.75 & 0.25 \\ 0.5 & 0.25 & 0.75 \end{pmatrix}$$

where all the first row corresponds to H and the second row to T. All state transition have probabilities equal to $\frac{1}{3}$. Assume initial state probabilities of $\frac{1}{3}$.

- You observe the sequence: O=H H H H T H T T T T. What is the most likely state sequence and its probability (likelihood)?
- What is the probability that the observation sequence came entirely from state 1?

Submission instructions

The following instructions are mandatory. If you are not following them, tutors can decide to not correct your exercise.

Submission architecture

You have to generate a **single ZIP file** respecting the following architecture:

```
tutorial11_<matriculation1>_<matriculation2>_<matriculation3>
|
+--- source
|     |
|     +----- file 1
|     +----- file 2
|     +----- ...
+--- rapport.pdf
+--- README.txt
```

where

- **source** contains the source code of your project,
- **rapport.pdf** is the report where you present your solution with **the explanations** and the plots,
- **README** which contains group member informations (name, matriculation numbers and emails) and a **clear** explanation about how to compile and run your source code

The ZIP filename has to be :

```
tutorial11_<matriculation1>_<matriculation2>_<matriculation3>.zip
```

Some hints

We advice you to follow the following guidelines in order to avoid problems :

- Avoid building complex systems. The exercises are simple enough.
- Do not include any executables in your submission, as this will cause the e-mail server to reject it.

Grading

Send your assignment to the tutor who is responsible of your group:

- Merlin Köhler s9mnkoeh@stud.uni-saarland.de
- Yelluru Gopal Goutam goutamyg@lsv.uni-saarland.de
- Ahmad Taie ataie@lsv.uni-saarland.de

If you are assigned to different tutorials send your assignment to the tutor to whom most of you are assigned to.

The email subject should start with [PSR TUTORIAL 11]