電腦視覺與應用
Computer Vision and Applications

Lecture-06-1 Two-views geometry

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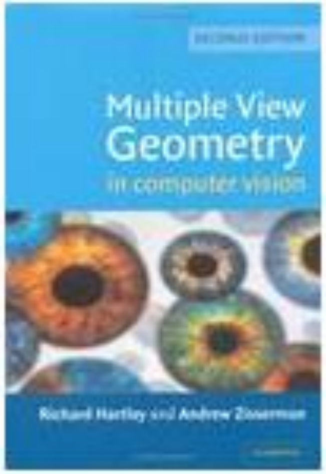
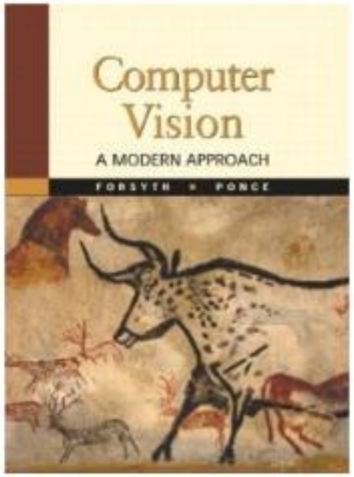
Two-views geometry

- Description for *fundamental matrix*, **F**, and *essential matrix*, **E**
- Lecture Reference at:
  - Multiple View Geometry in Computer Vision, Chapter 9, 11
  - Computer Vision A Modern Approach, Chapter 10.



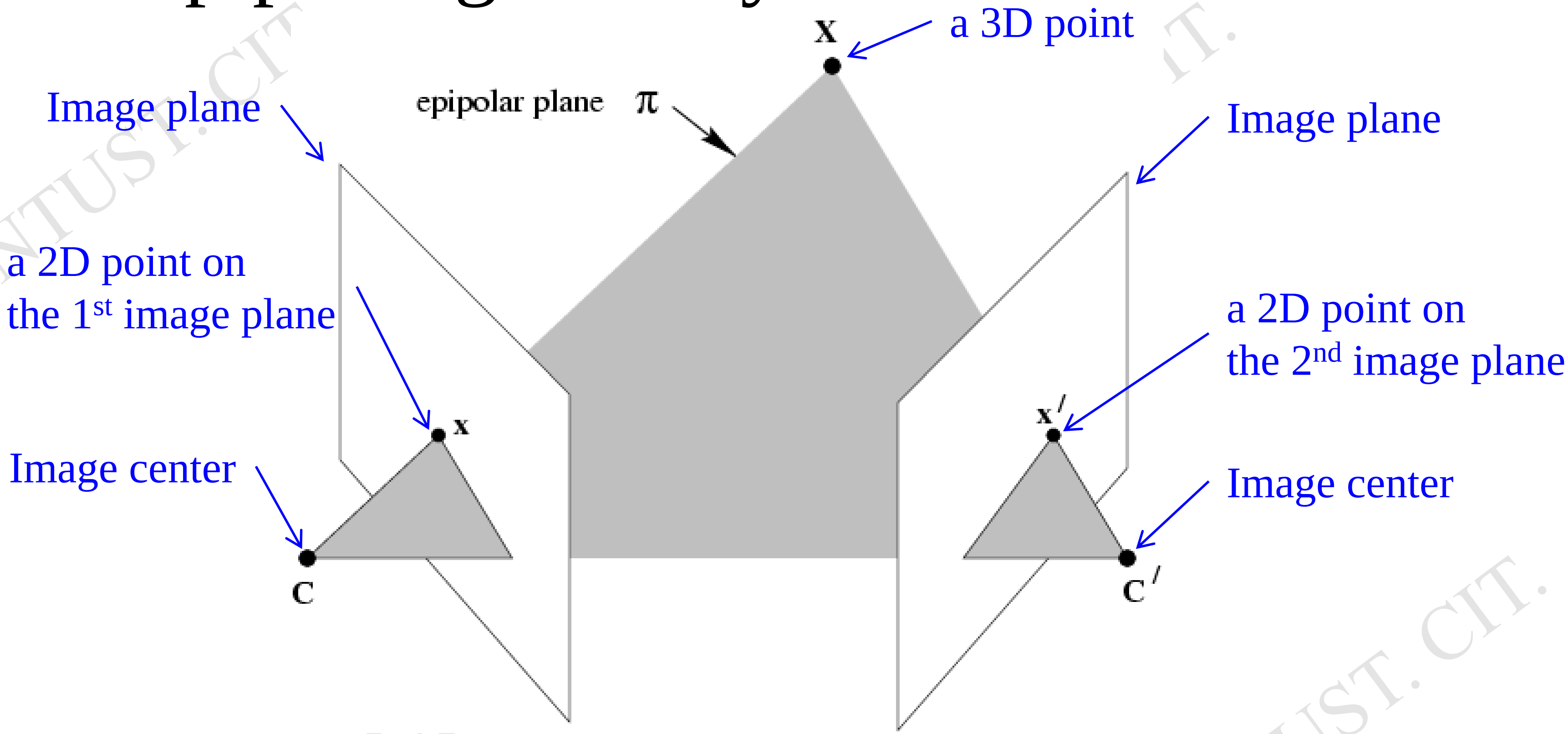


Two-views geometry – Outline

- epipole
- epipolar line
- epipolar plane

- Fundamental matrix **F**
- Essential matrix **E** → special case of **F**
- Computation for *Fundamental Matrix* **F**

The epipolar geometry

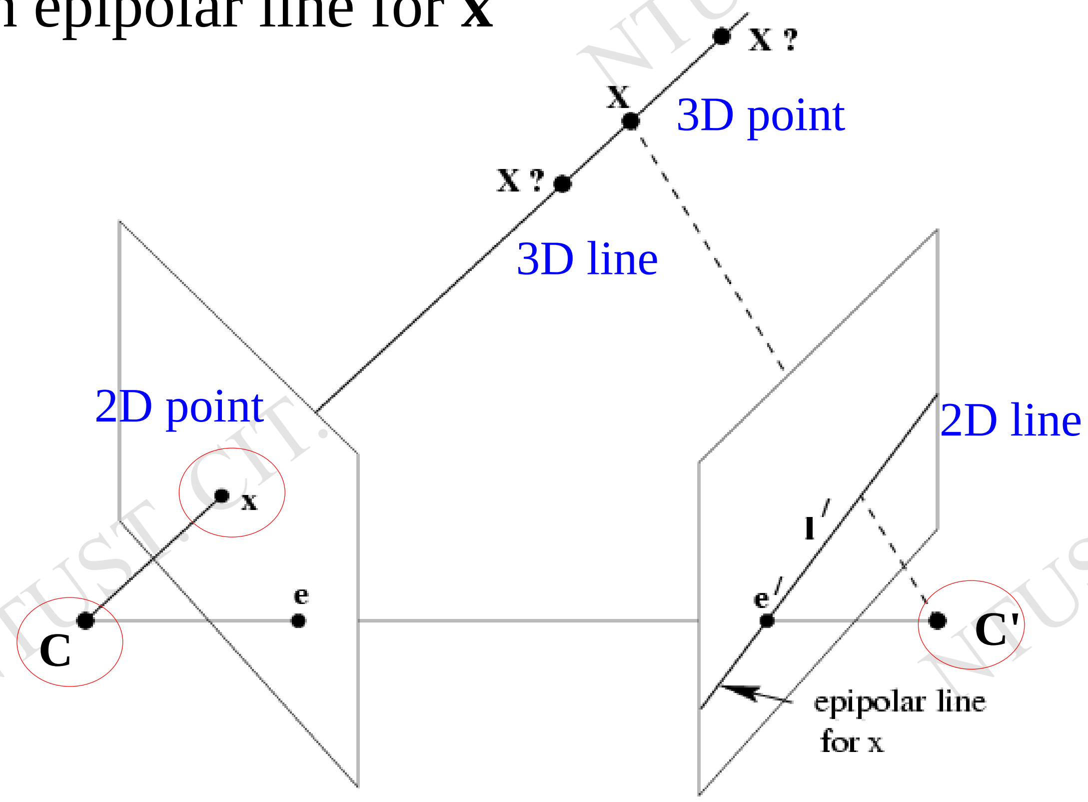


C, **C'**, **x**, **x'** and **X** are coplanar

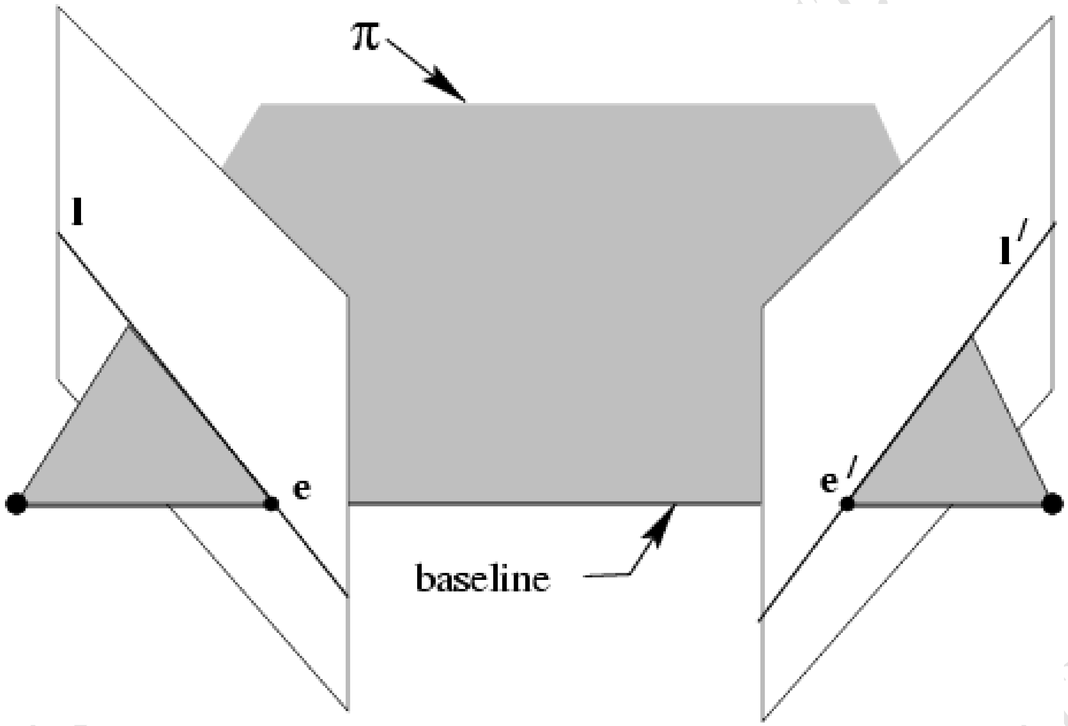
Note:
Two images may have different intrinsic parameter, be taken at either the same or different periods.

The epipolar geometry

- In case of given **c**, **c**' (says **e**, **e**' as well) and **x**?
  → define an epipolar line for **x**



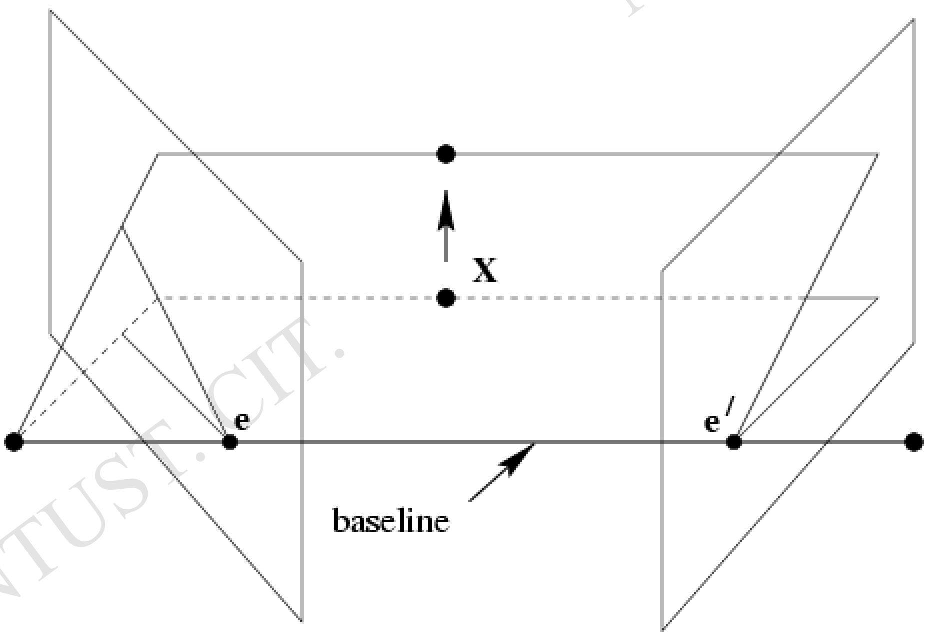
The epipolar geometry



All points on **π** project on **l** and **l'**

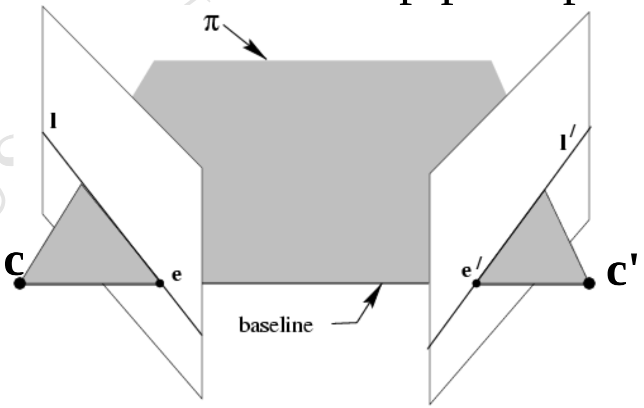
The epipolar geometry

- Family of planes **π** and lines **l** and **l'** intersection in **e** and **e'**

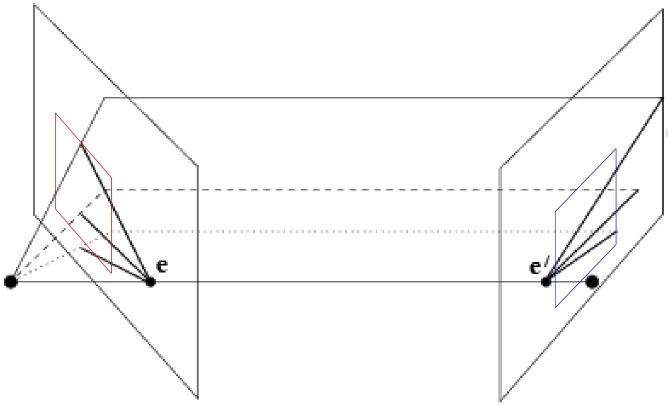


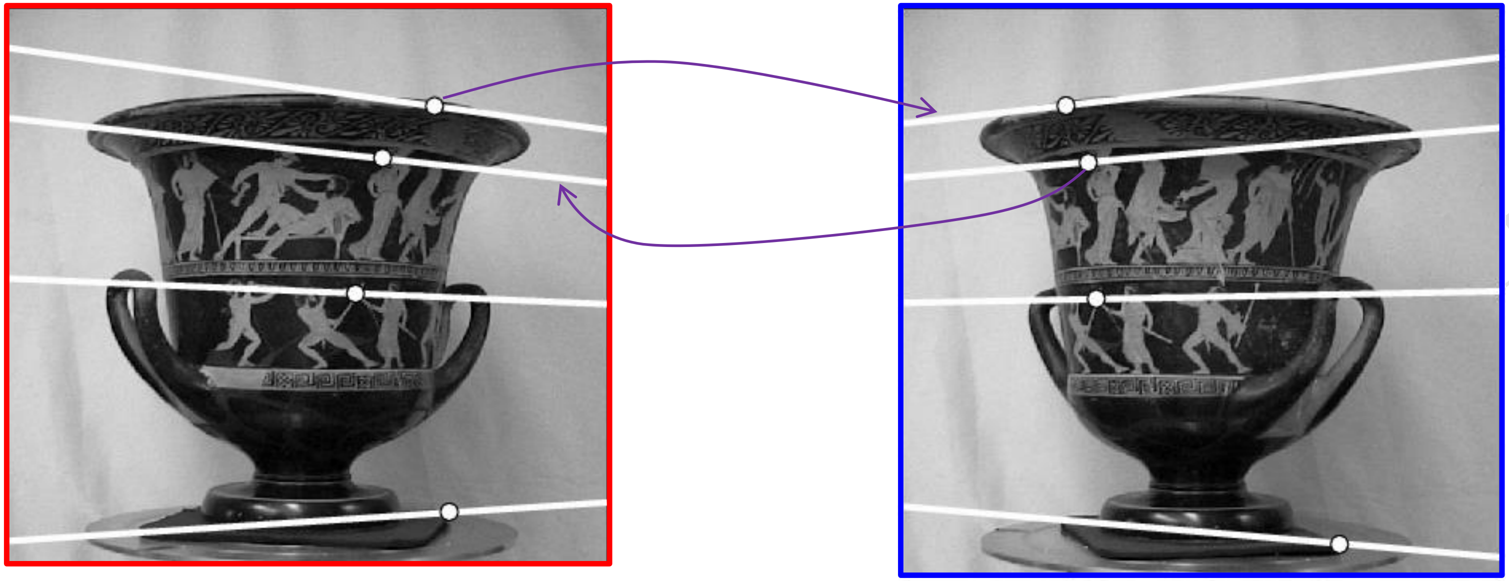
The epipolar geometry

- Summary for definition
  - **epipoles e, e'**
    = intersection of baseline with image plane
    = projection of projection center in other image
    = vanishing point of camera motion direction
  - **epipolar plane** = plane containing baseline
  - **epipolar line** = intersection of epipolar plane with image

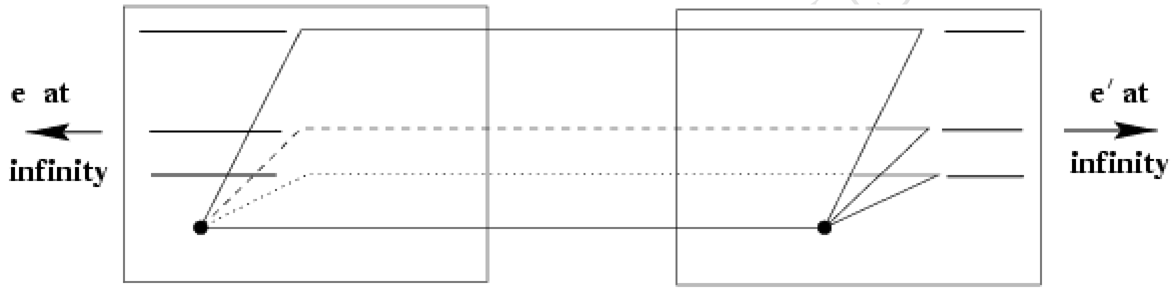


Example: converged stereo-camera

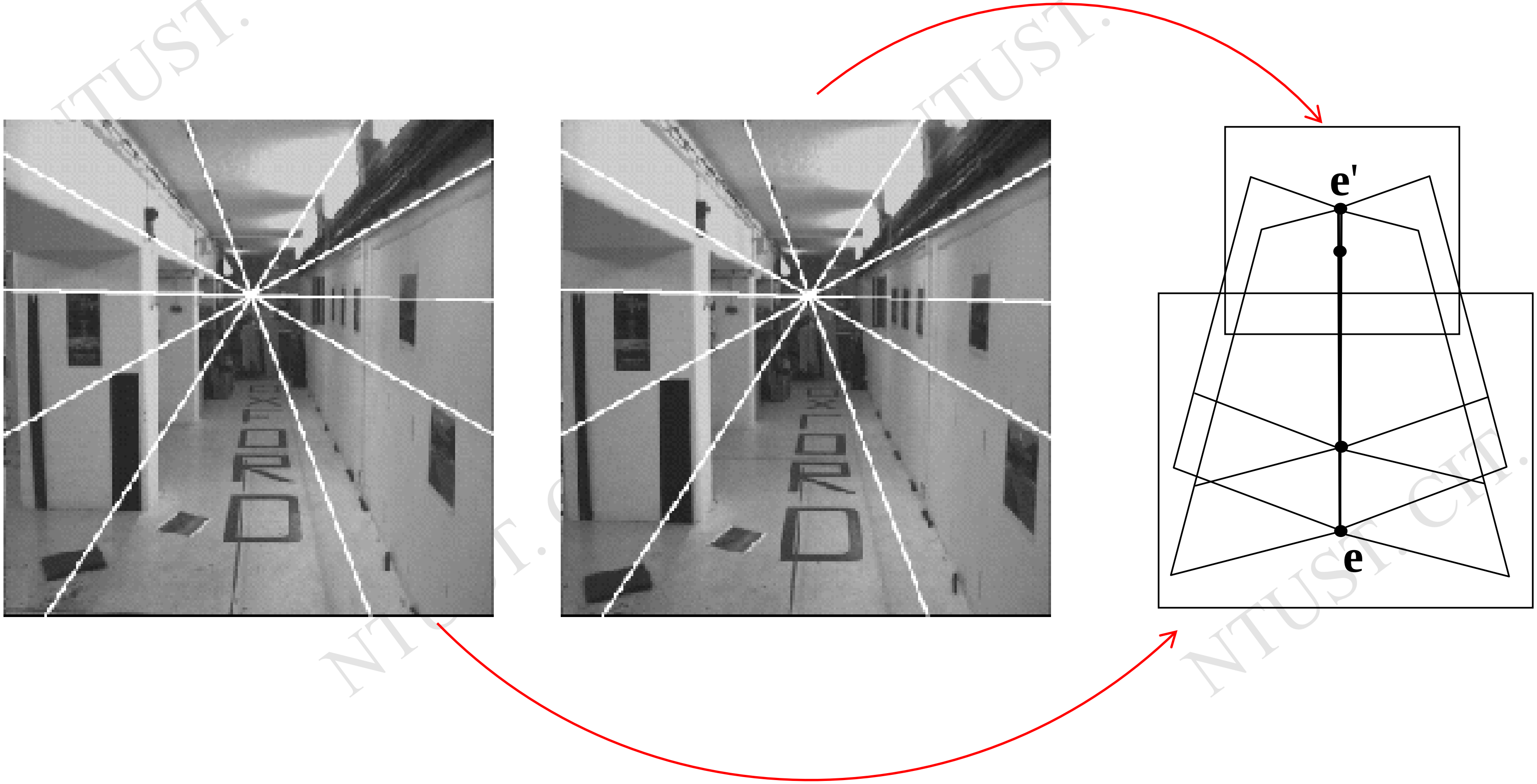




Example: motion parallel with image plane



Example: forward motion

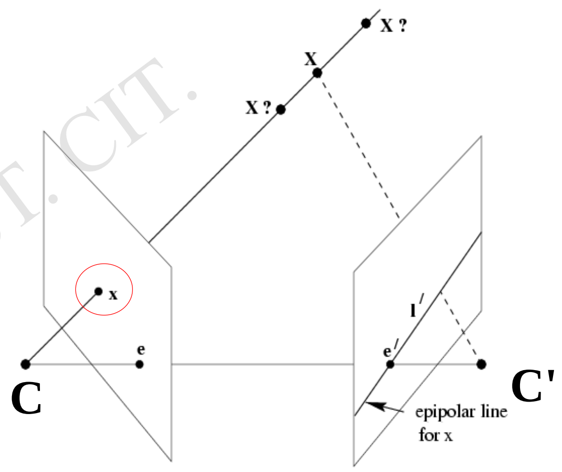


Fundamental matrix F

- Algebraic representation of epipolar geometry

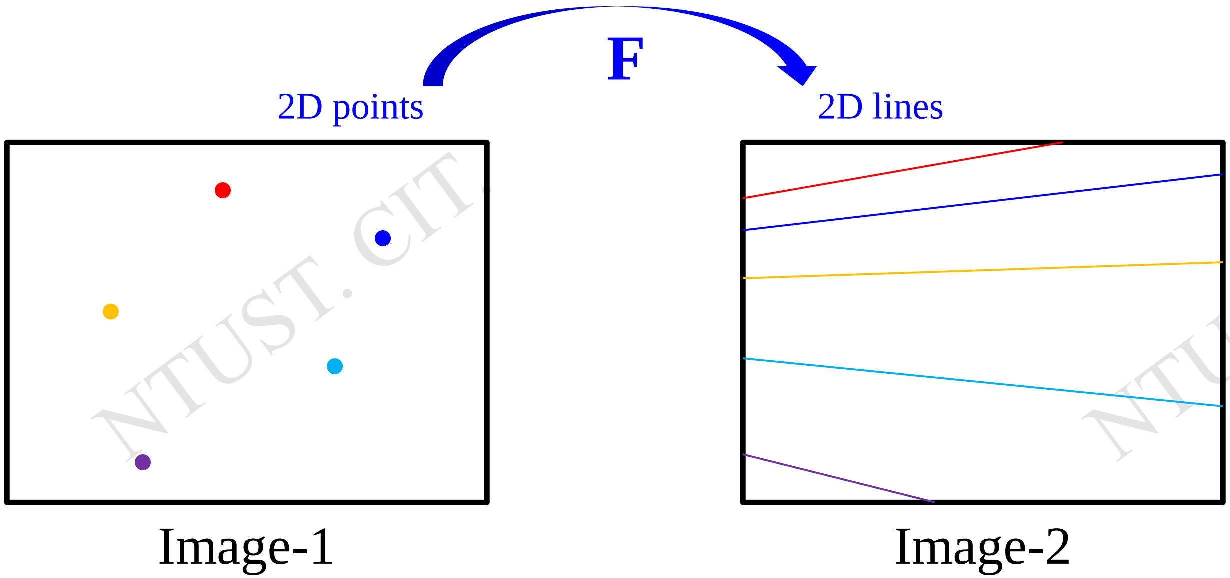
$$\mathbf{x} \mapsto \mathbf{l}'$$

- We will see that mapping is (singular) correlation (i.e. projective mapping from points to lines) represented by the fundamental matrix **F**.
- 註：已知兩張照片的**F**轉換，可從一張影像的特徵點，可以預估這些特徵點會落在另一張影像上的線上(一個點產生一條線)



Fundamental matrix F

- Points(or features) in image-1 are mapped into lines in image-2 by applying a 3x3 matrix **F.**
- Note!!! all points in image-1 are NOT necessary to be co-planar in 3D space. (different from 3x3 homography)



Fundamental matrix F

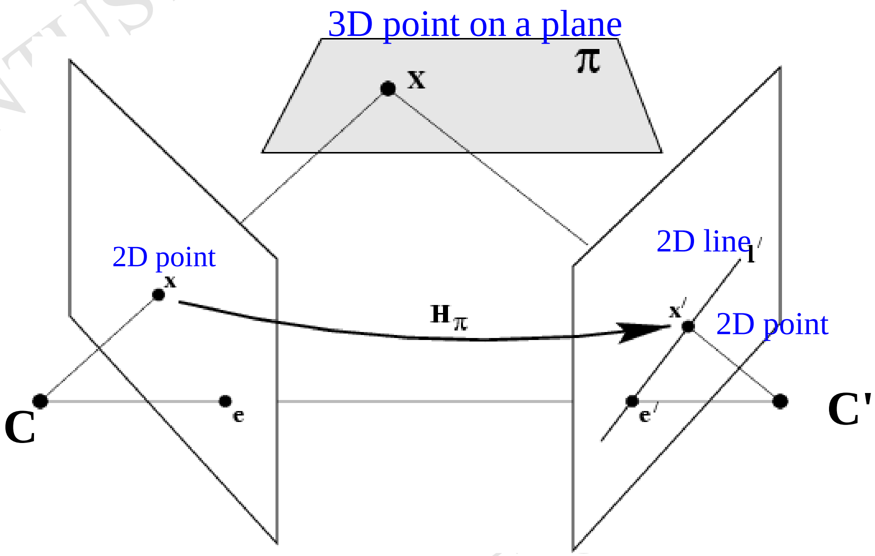
- How to determine **F** from two images?(from textbook Hartley04)
  1) Using known 3x3 Homography (3D points on one plane) and the epipole
  2) Algebraic method
  3) Using known correspondences (feature matching between two images)→most popular method in practice

Determine fundamental matrix F

1) from 3x3 homography
2) from algebraic derivation
3) from correspondence from two-views

Determine fundamental matrix F

- 1) from 3x3 homography



$$\mathbf{x}' = \mathbf{H}_\pi \mathbf{x}$$

$$\mathbf{l}' = \mathbf{e}' \times \mathbf{x}' = \mathbf{e}' \times (\mathbf{H}_\pi \mathbf{x}) = ([\mathbf{e}']_\times \mathbf{H}_\pi)\mathbf{x} = \mathbf{F}\mathbf{x}$$

epipole (vector)

epipole (matrix)

Note! notation

vector

real (實數,純量)

$$\mathbf{e}' = [e_1' \quad e_2' \quad e_3']^T$$

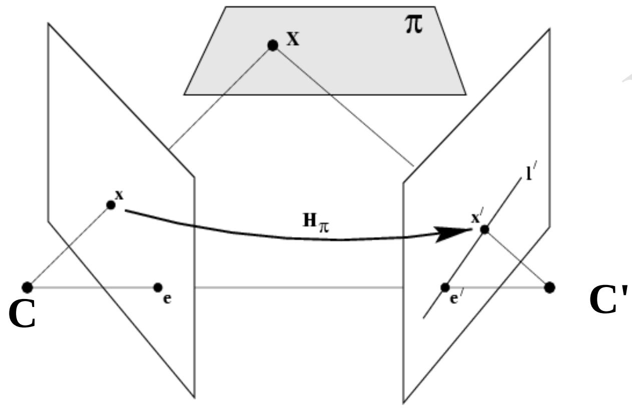
matrix (for calculation purpose)

real (實數,純量)

$$[\mathbf{e}']_\times = \begin{bmatrix} 0 & -e_3' & e_2' \\ e_3' & 0 & -e_1' \\ -e_2' & e_1' & 0 \end{bmatrix}$$

Determine fundamental matrix F

- 1) from 3x3 homography—cont.



$$\mathbf{x}' = \mathbf{H}_\pi \mathbf{x}$$ →(3x3) homography mapping
2D points mapping to 2D points

$$\mathbf{l}' = \mathbf{F}\mathbf{x}$$ →2-Dimensional Mapping

Determine fundamental matrix F

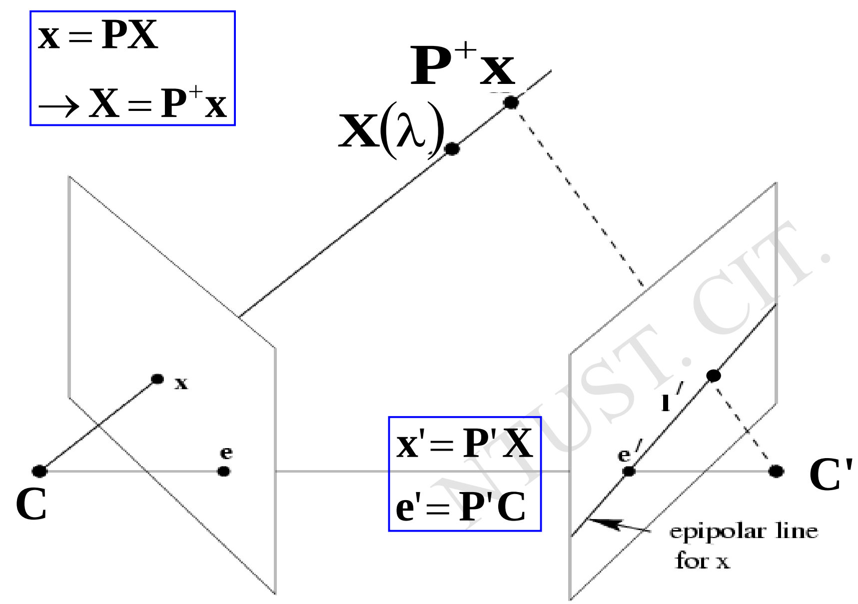
- 2) from algebraic derivation

$$\left(\mathbf{P}^{+}\mathbf{P}=\mathbf{I}\right)$$

$$\mathbf{X}(\lambda)=\mathbf{P}^{+}\mathbf{x}+\lambda\mathbf{C}$$

$$\mathbf{l}'=\mathbf{P}'\mathbf{C}\times\mathbf{P}'\mathbf{P}^{+}\mathbf{x}$$

$$\mathbf{F}=[\mathbf{e}']_{\times}\mathbf{P}'\mathbf{P}^{+}$$



This method is the same formula with the previous method, by replace $\mathbf{P}'\mathbf{P}^{+}$ with $\mathbf{H}_{\pi}$

Determine fundamental matrix F

- 3) from correspondence from two-views
  - correspondence condition
  - The fundamental matrix satisfies the condition that for any pair of corresponding points $\mathbf{x} \leftrightarrow \mathbf{x}'$ in the two images

(one point on one line could be written as:)

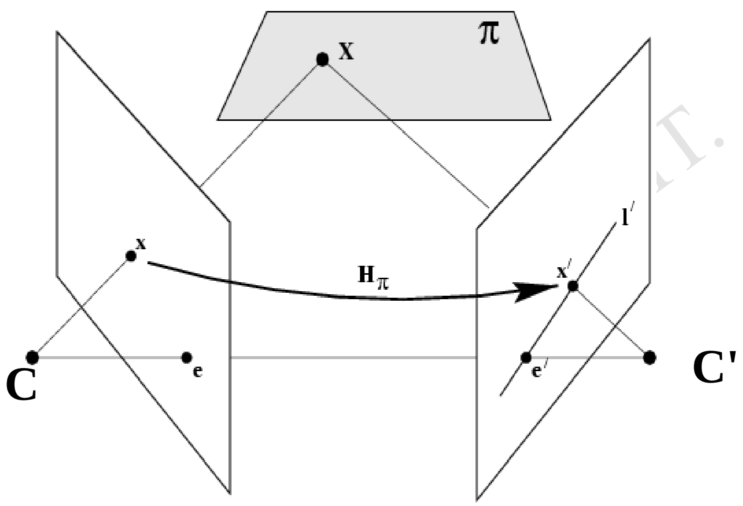
$$\mathbf{x}^T \mathbf{l} = 0 = \mathbf{l}^T \mathbf{x}$$

So, the governing equation will be

$$\boxed{\mathbf{x}'^T \mathbf{F} \mathbf{x} = 0}$$

Since we have the following equation in image-2.

$$\because \mathbf{x}'^T \mathbf{l}' = 0$$



Determine fundamental matrix F

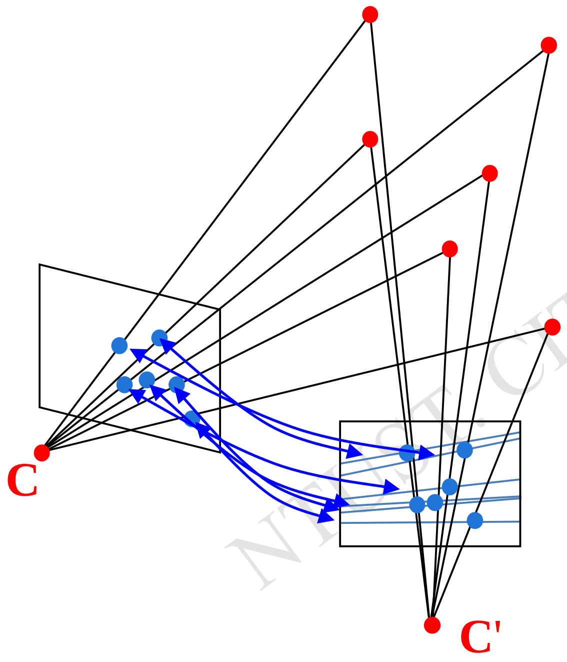
- 3) from correspondence from two-views—cont.
  - So called Weak calibration (for determining **F** in two views)

General form:

$$\mathbf{x}'^{\mathrm{T}}\mathbf{F}\mathbf{x}=0$$

Written in matrix form:

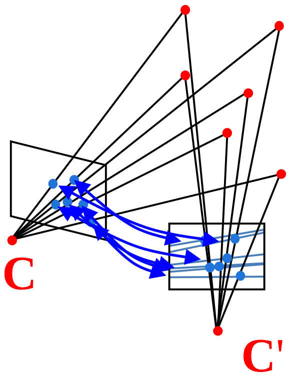
$$[u' \quad v' \quad 1]\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}=0$$



Determine fundamental matrix F

- 3) from correspondence from two-views—cont.
  - Weak calibration

$$[u' \quad v' \quad 1]\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$



$\rightarrow \quad F_{11}u'u + F_{12}u'v + F_{13}u' + F_{21}uv' + F_{22}vv' + F_{23}v' + F_{31}u + F_{32}v + F_{33} = 0$

Since **F** has 9-1 DOF, let F_{33}=1 and solving **F**.

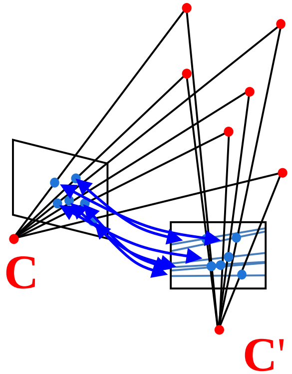
In matrix form→

$$[u'u \quad u'v \quad u' \quad uv' \quad vv' \quad v' \quad u \quad v]\begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{bmatrix} = -1$$

8 unknowns, and one correspondence gives one constraint. It needs at least 8 correspondences.

Determine fundamental matrix F

- 3) from correspondence from two-views—cont.
  - Weak calibration



$$\mathbf{x}'^{\mathrm{T}}\,\mathbf{F}\mathbf{x} = 0$$

$$\begin{bmatrix} u_1'u_1 & u_1'v_1 & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 \\ u_2'u_2 & u_2'v_2 & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 \\ u_3'u_3 & u_3'v_3 & u_3' & u_3v_3' & v_3v_3' & v_3' & u_3 & v_3 \\ u_4'u_4 & u_4'v_4 & u_4' & u_4v_4' & v_4v_4' & v_4' & u_4 & v_4 \\ u_5'u_5 & u_5'v_5 & u_5' & u_5v_5' & v_5v_5' & v_5' & u_5 & v_5 \\ u_6'u_6 & u_6'v_6 & u_6' & u_6v_6' & v_6v_6' & v_6' & u_6 & v_6 \\ u_7'u_7 & u_7'v_7 & u_7' & u_7v_7' & v_7v_7' & v_7' & u_7 & v_7 \\ u_8'u_8 & u_8'v_8 & u_8' & u_8v_8' & v_8v_8' & v_8' & u_8 & v_8 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

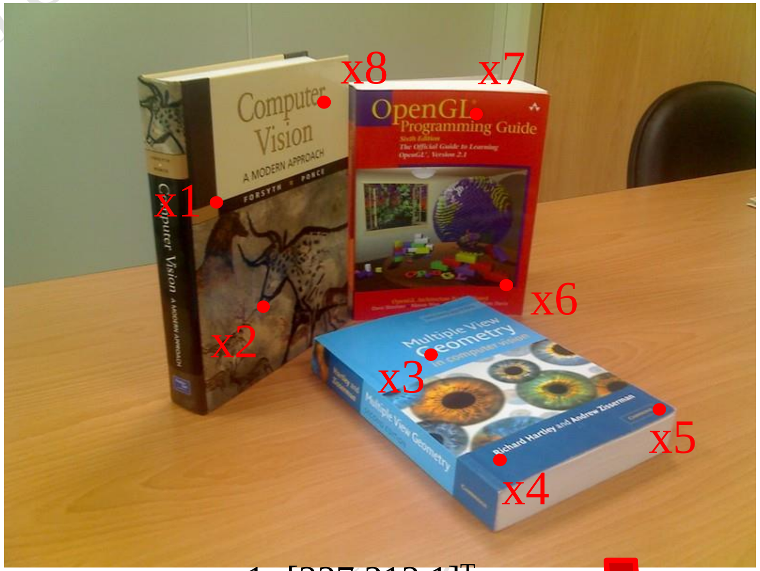
Solve **F** by taking an inverse operation to the above equation.
If you get more than 8 correspondences, least-square method or SVD may be used.
NOTE!! Since the order in every column is very different. The 1st column has values around 10^4~10^6, but the 3th column is 10^2~10^3. Without normalized, Least-Square-Method may yield poor results.

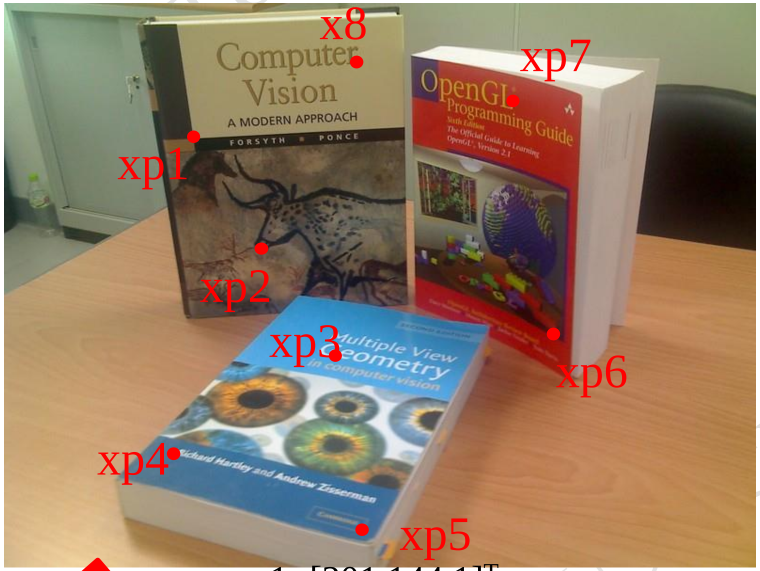
Property of the fundamental matrix F

- **F** is the unique 3x3 rank 2 matrix that satisfies $\mathbf{x}'^T\mathbf{F}\mathbf{x}=0$ for all $\mathbf{x}\leftrightarrow\mathbf{x}'$
  1) **Transpose:** if **F** is fundamental matrix for the pair of cameras (**P**, **P'**), then $\mathbf{F}^T$ is fundamental matrix for (**P'**, **P**)
  2) **Epipolar lines:** $\mathbf{l}'=\mathbf{F}\mathbf{x}$ & $\mathbf{l}=\mathbf{F}^T\mathbf{x}'$
  3) **Epipoles:** on all epipolar lines, thus $\mathbf{e}'^T\mathbf{F}\mathbf{x}=0, \forall\mathbf{x} \Rightarrow \mathbf{e}'^T\mathbf{F}=0$, similarly $\mathbf{F}\mathbf{e}=0$
  4) **F** has 7 DOF , i.e. 3x3-1(homogeneous)-1(rank2)
  5) **F** is a correlation, projective mapping from a point **x** to a line $\mathbf{l}'=\mathbf{F}\mathbf{x}$ (not a proper correlation, i.e. not invertible)

Determine fundamental matrix F

- correspondence from two-views—example





$x1=[227,212,1]^T$
$x2=[275,322,1]^T$
$x3=[449,370,1]^T$
$x4=[525,481,1]^T$
$x5=[699,432,1]^T$
$x6=[535,298,1]^T$
$x7=[498,118,1]^T$
$x8=[339,106,1]^T$

F?

Note! This is **NOT** 2D point to point mapping

$xp1=[201,144,1]^T$
$xp2=[275,261,1]^T$
$xp3=[349,369,1]^T$
$xp4=[182,479,1]^T$
$xp5=[380,562,1]^T$
$xp6=[584,351,1]^T$
$xp7=[542,108,1]^T$
$xp8=[373,64,1]^T$

Determine fundamental matrix F

- correspondence from two-views—example

$$\begin{bmatrix} u_1'u_1 & u_1'v_1 & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 \\ u_2'u_2 & u_2'v_2 & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 \\ u_3'u_3 & u_3'v_3 & u_3' & u_3v_3' & v_3v_3' & v_3' & u_3 & v_3 \\ u_4'u_4 & u_4'v_4 & u_4' & u_4v_4' & v_4v_4' & v_4' & u_4 & v_4 \\ u_5'u_5 & u_5'v_5 & u_5' & u_5v_5' & v_5v_5' & v_5' & u_5 & v_5 \\ u_6'u_6 & u_6'v_6 & u_6' & u_6v_6' & v_6v_6' & v_6' & u_6 & v_6 \\ u_7'u_7 & u_7'v_7 & u_7' & u_7v_7' & v_7v_7' & v_7' & u_7 & v_7 \\ u_8'u_8 & u_8'v_8 & u_8' & u_8v_8' & v_8v_8' & v_8' & u_8 & v_8 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

x1=$[227,212,1]^T$
x2=$[275,322,1]^T$
x3=$[449,370,1]^T$
x4=$[525,481,1]^T$
x5=$[699,432,1]^T$
x6=$[535,298,1]^T$
x7=$[498,118,1]^T$
x8=$[339,106,1]^T$

xp1=$[201,144,1]^T$
xp2=$[275,261,1]^T$
xp3=$[349,369,1]^T$
xp4=$[182,479,1]^T$
xp5=$[380,562,1]^T$
xp6=$[584,351,1]^T$
xp7=$[542,108,1]^T$
xp8=$[373,64,1]^T$

```
A=[
xp1(1)*x1(1) xp1(1)*x1(2) xp1(1) x1(1)*xp1(2) x1(2)*xp1(2) xp1(2) x1(1) x1(2);
xp2(1)*x2(1) xp2(1)*x2(2) xp2(1) x2(1)*xp2(2) x2(2)*xp2(2) xp2(2) x2(1) x2(2);
xp3(1)*x3(1) xp3(1)*x3(2) xp3(1) x3(1)*xp3(2) x3(2)*xp3(2) xp3(2) x3(1) x3(2);
xp4(1)*x4(1) xp4(1)*x4(2) xp4(1) x4(1)*xp4(2) x4(2)*xp4(2) xp4(2) x4(1) x4(2);
xp5(1)*x5(1) xp5(1)*x5(2) xp5(1) x5(1)*xp5(2) x5(2)*xp5(2) xp5(2) x5(1) x5(2);
xp6(1)*x6(1) xp6(1)*x6(2) xp6(1) x6(1)*xp6(2) x6(2)*xp6(2) xp6(2) x6(1) x6(2);
xp7(1)*x7(1) xp7(1)*x7(2) xp7(1) x7(1)*xp7(2) x7(2)*xp7(2) xp7(2) x7(1) x7(2);
xp8(1)*x8(1) xp8(1)*x8(2) xp8(1) x8(1)*xp8(2) x8(2)*xp8(2) xp8(2) x8(1) x8(2)]
```

```
F =

    0.0000   -0.0000   -0.0007
   -0.0000    0.0000    0.0105
   -0.0011   -0.0093    1.0000
```

Determine fundamental matrix F

- correspondence from two-views—example, cont.

$$\mathbf{l'} = \mathbf{Fx}$$

F =

$$\begin{matrix} 0.0000 & -0.0000 & -0.0007 \\ -0.0000 & 0.0000 & 0.0105 \\ -0.0011 & -0.0093 & 1.0000 \end{matrix}$$

lp1 =	lp2 =	lp3 =	lp4 =	lp5 =	lp6 =	lp7 =	lp8 =
-0.0009	-0.0012	-0.0010	-0.0012	-0.0007	-0.0007	-0.0002	-0.0005
0.0098	0.0101	0.0089	0.0089	0.0073	0.0078	0.0071	0.0083
-1.2267	-2.3044	-2.9449	-4.0631	-3.8003	-2.3701	-0.6527	-0.3641

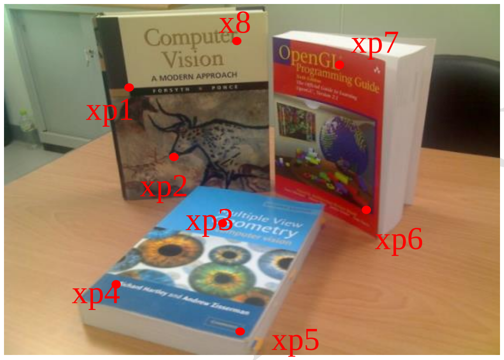
Determine fundamental matrix F

- correspondence from two-views—example, cont.

$$\mathbf{l} = \mathbf{F}^T \mathbf{x}'$$

```
>> F'

ans =

  0.0000  -0.0000  -0.0011
 -0.0000   0.0000  -0.0093
 -0.0007   0.0105   1.0000
```

l1 =	l2 =	l3 =	l4 =	l5 =	l6 =	l7 =	l8 =
-0.0019	-0.0027	-0.0035	-0.0046	-0.0050	-0.0029	-0.0011	-0.0010
-0.0091	-0.0086	-0.0082	-0.0072	-0.0073	-0.0090	-0.0102	-0.0100
2.3599	3.5309	4.6076	5.8837	6.6067	4.2450	1.7302	1.3944



Determine fundamental matrix F

- correspondence from two-views—example, cont.
  - Evaluation for error, for example check $\mathbf{l}'^{T}\mathbf{x}'$ or $\mathbf{l}^{T}\mathbf{x}$ or $\mathbf{x}'^{T}\mathbf{F}\mathbf{x}$

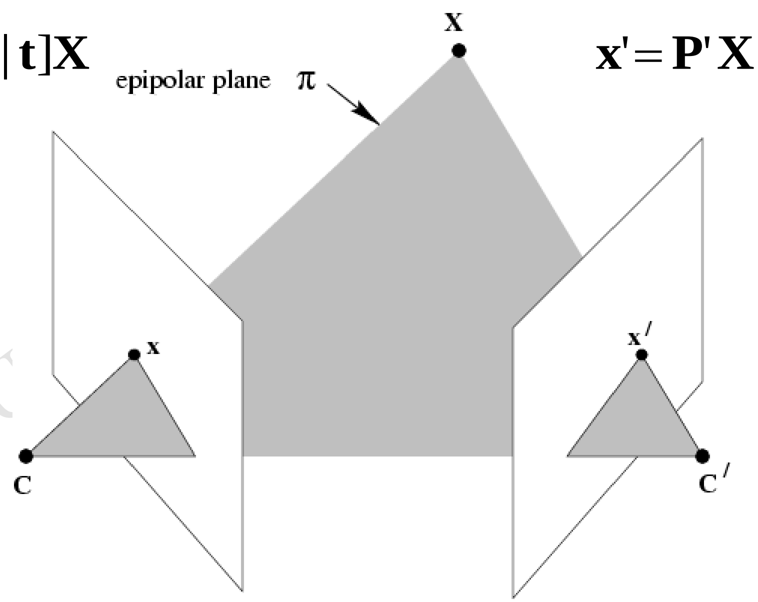
	$\mathbf{l}'^{T}\mathbf{x}'$	$\mathbf{l}^{T}\mathbf{x}$	$\mathbf{x}'^{T}\mathbf{F}\mathbf{x}$
1)	-2.4425e-015	-2.6645e-015	-2.6645e-015
2)	4.5741e-014	-3.5527e-015	-3.5527e-015
3)	-1.2390e-013	-4.4409e-015	-4.4409e-015
4)	-5.3291e-015	-5.3291e-015	-5.3291e-015
5)	-3.5527e-015	-3.5527e-015	-3.5527e-015
6)	-3.5527e-015	-3.5527e-015	-3.5527e-015
7)	-8.8818e-016	-8.8818e-016	-8.8818e-016
8)	-1.6653e-015	-1.5543e-015	-1.5543e-015

Essential matrix E

- The *essential matrix* is the specialization of the *fundamental matrix* to the case of normalized image coordinate. Historically, the *essential matrix* was introduced before the *fundamental matrix*, and the *fundamental matrix* may be thought of as the generalization of the *essential matrix* in which the assumption of calibrated cameras is removed.

$$\mathbf{x} = \mathbf{P}\mathbf{X} = \mathbf{K}[\mathbf{R} \mid \mathbf{t}]\mathbf{X}$$

$$\mathbf{x}' = \mathbf{P}'\mathbf{X} = \mathbf{K}'[\mathbf{R}' \mid \mathbf{t}']\mathbf{X}$$



Essential matrix E

- Normalized coordinates:
- Consider the image without **K** effect.

2D points on an image

2D points on a normalized image

$$\mathbf{x} = \mathbf{P}\mathbf{X} = \mathbf{K}[\mathbf{R} \mid \mathbf{t}]\mathbf{X} \quad \Rightarrow \quad \hat{\mathbf{x}} = \mathbf{K}^{-1}\mathbf{x} = \mathbf{K}^{-1}\mathbf{P}\mathbf{X} = [\mathbf{R} \mid \mathbf{t}]\mathbf{X}$$

$$\mathbf{x}' = \mathbf{P}'\mathbf{X} = \mathbf{K}'[\mathbf{R}' \mid \mathbf{t}']\mathbf{X} \quad \Rightarrow \quad \hat{\mathbf{x}}' = \mathbf{K}'^{-1}\mathbf{x}' = \mathbf{K}'^{-1}\mathbf{P}'\mathbf{X} = [\mathbf{R}' \mid \mathbf{t}']\mathbf{X}$$

$\mathbf{P} = \mathbf{K}[\mathbf{R} \mid \mathbf{t}]$ →general camera matrix

$\mathbf{K}^{-1}\mathbf{P} = [\mathbf{R} \mid \mathbf{t}]$ →normalized camera matrix

$\boxed{\hat{\mathbf{x}}'^{\mathrm{T}}\mathbf{E}\hat{\mathbf{x}} = 0}$ similar to fundamental matrix format

$$\left.\begin{array}{l} \because \mathbf{x}'^{\mathrm{T}}\mathbf{F}\mathbf{x} = 0 \\ \hat{\mathbf{x}} = \mathbf{K}^{-1}\mathbf{x} \\ \hat{\mathbf{x}}' = \mathbf{K}'^{-1}\mathbf{x}' \end{array}\right\} (\mathbf{K}'\hat{\mathbf{x}}')^{\mathrm{T}}\mathbf{F}(\mathbf{K}\hat{\mathbf{x}}) = 0 \Rightarrow \hat{\mathbf{x}}'^{\mathrm{T}}(\mathbf{K}'^{\mathrm{T}}\mathbf{F}\mathbf{K})\hat{\mathbf{x}} = 0 \Rightarrow \mathbf{E} = \mathbf{K}'^{\mathrm{T}}\mathbf{F}\mathbf{K}$$

Essential matrix E

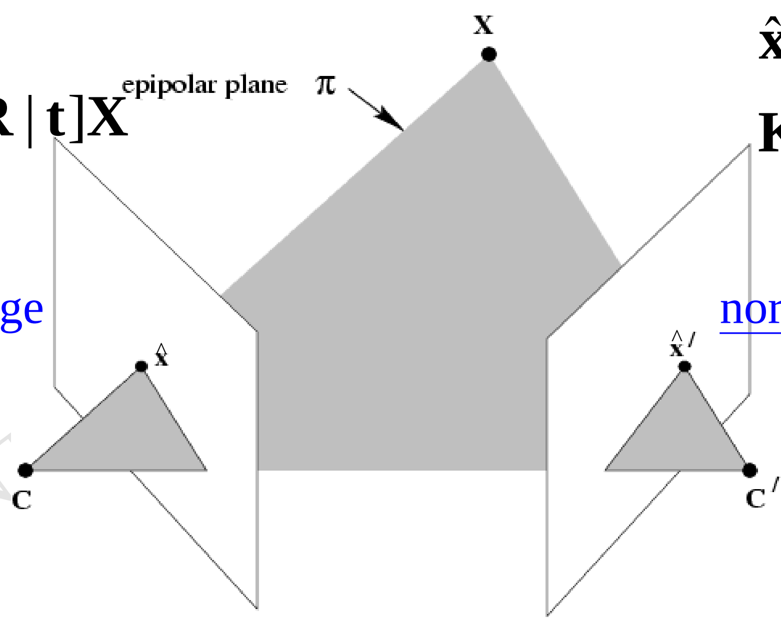
- Normalized coordinates:
- Consider the image without **K** effect.

$$\hat{\mathbf{x}} = \mathbf{K}^{-1}\mathbf{x} = \mathbf{K}^{-1}\mathbf{P}\mathbf{X} = [\mathbf{R} \mid \mathbf{t}]\mathbf{X}$$

$$\hat{\mathbf{x}}' = \mathbf{K}'^{-1}\mathbf{x}' = \mathbf{K}'^{-1}\mathbf{P}'\mathbf{X} = [\mathbf{R}' \mid \mathbf{t}']\mathbf{X}$$

normalized image

normalized image



Essential matrix E –example

- Continue the previous example:

F =

0.0000	-0.0000	-0.0007
-0.0000	0.0000	0.0105
-0.0011	-0.0093	1.0000

Assume we have intrinsic parameter of image 1&2:

K=[
857.249077 0.000000 402.813609
0.000000 866.660878 250.492920
0.000000 0.000000 1.000000]

Essential matrix can be determined by $\mathbf{E} = \mathbf{K}'^{T}\mathbf{F}\mathbf{K}$

Intrinsic parameter of **image-2**

Intrinsic parameter of **image-1**

Fundamental matrix from image-1 to image-2

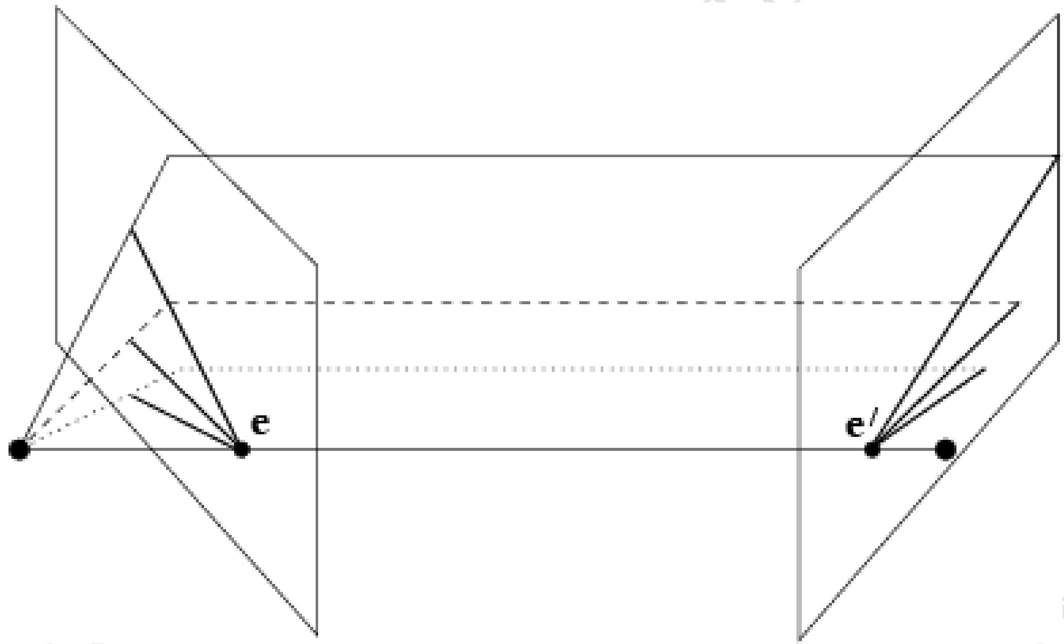
```
>> E=K'*F*K
```

E =

1.2509	-2.0515	-0.6399
-5.9498	4.1481	7.4831
-2.0853	-7.8357	0.0815

Epipolar geometry

- Review:

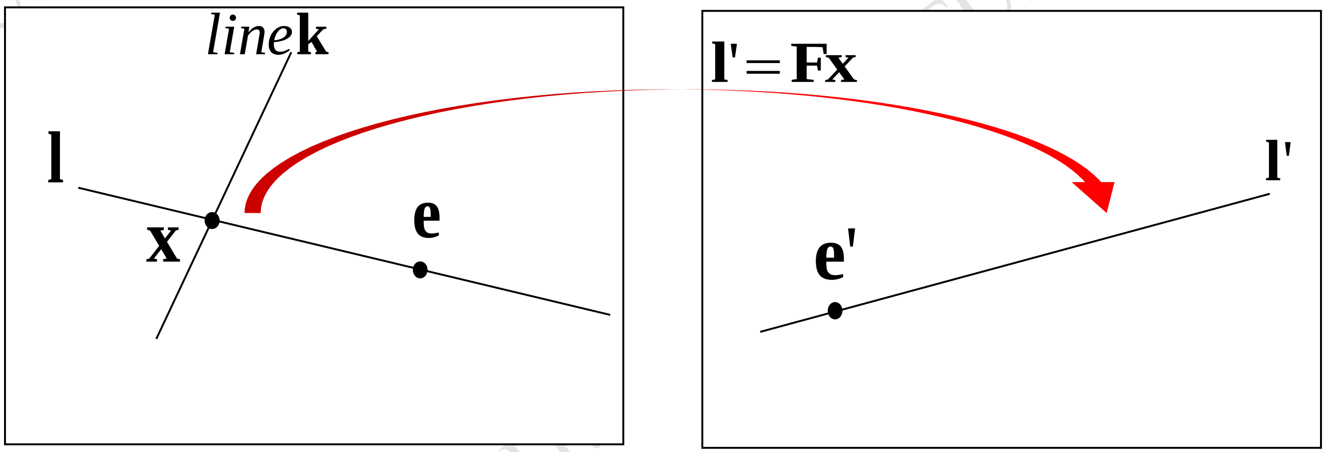


$$\mathbf{l} = \mathbf{F}^{\mathrm{T}}\mathbf{x}'$$

$$\mathbf{l}' = \mathbf{F}\mathbf{x}$$

Epipolar geometry-epipolar line homography

- **l**, **l'** epipolar lines in left and right images.



Suppose **l** and **l'** are corresponding epipolar lines, and **k** is ANY "line" NOT passing through the epipole **e**, then **l** and **l'** are related by

$$\mathbf{l}' = \mathbf{F}[\mathbf{k}]_\times \mathbf{l} \quad \rightarrow \quad \because \mathbf{k}^T\mathbf{e} \neq 0, \quad \mathbf{e}^T\mathbf{e} \neq 0$$

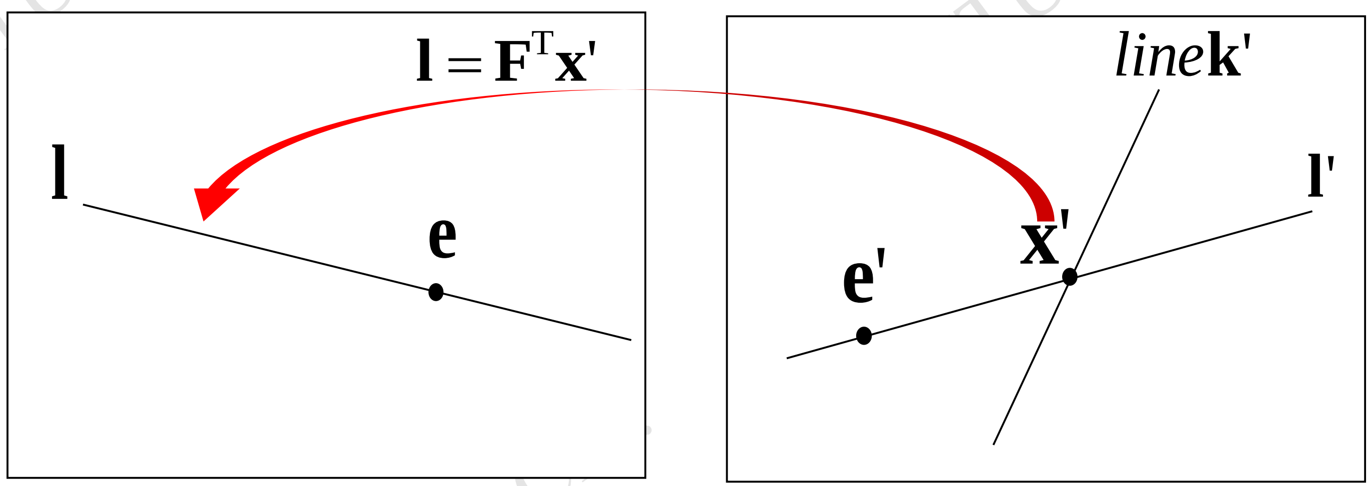
$$\boxed{\mathbf{l}' = \mathbf{F}[\mathbf{e}]_\times \mathbf{l}}$$

→Note: **e,** here, means the LINE for convenience not passing through the epipole **e**. So, we say **e** is one choice of line **k**.

Hartley04, sec.9.2.5

Epipolar geometry-epipolar line homography

- $\mathbf{l}$, $\mathbf{l}'$ epipolar lines in left and right images.

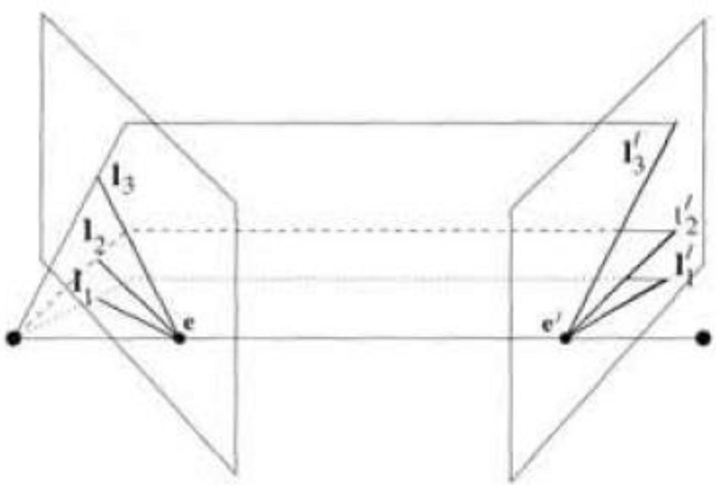


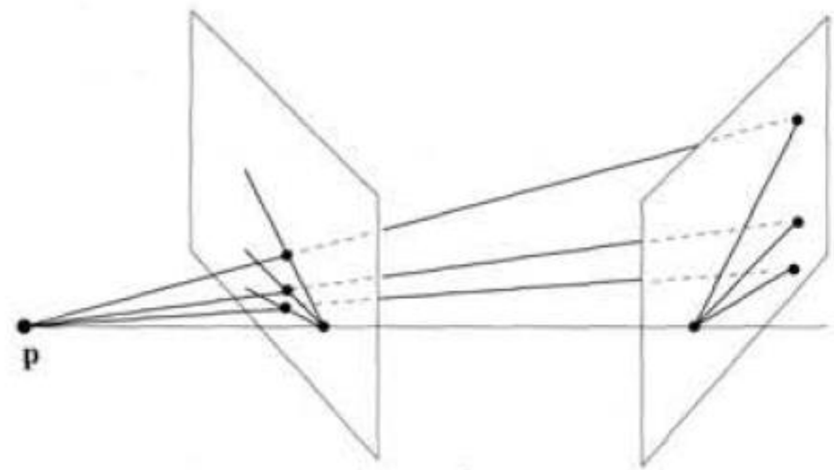
$$\underbrace{\mathbf{l}' = \mathbf{F}[\mathbf{e}]_{\times}}\mathbf{l} \qquad \mathbf{l} = \underbrace{\mathbf{F}^{\mathrm{T}}[\mathbf{e}']_{\times}}\mathbf{l}'$$

A homography between two lines

Hartley04, sec.9.2.5

Epipolar geometry-epipolar line homography



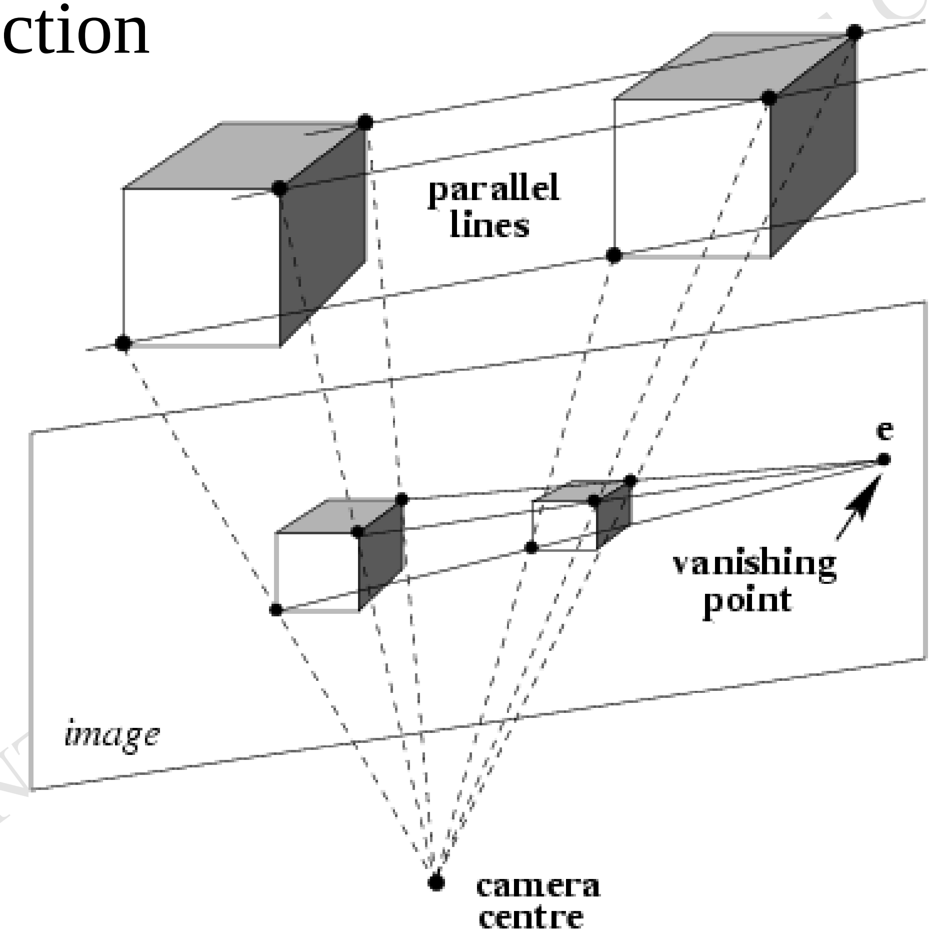


p is any point on the baseline

Hartley04, sec.9.2.5

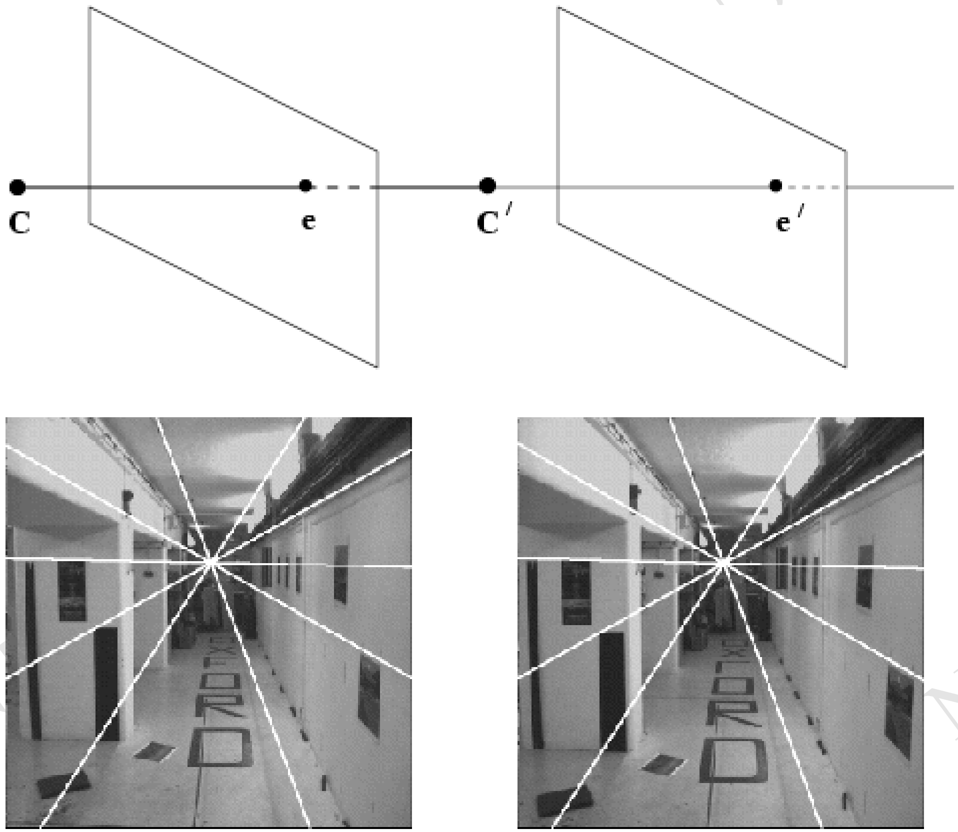
Fundamental matrix for pure translation

- Side direction

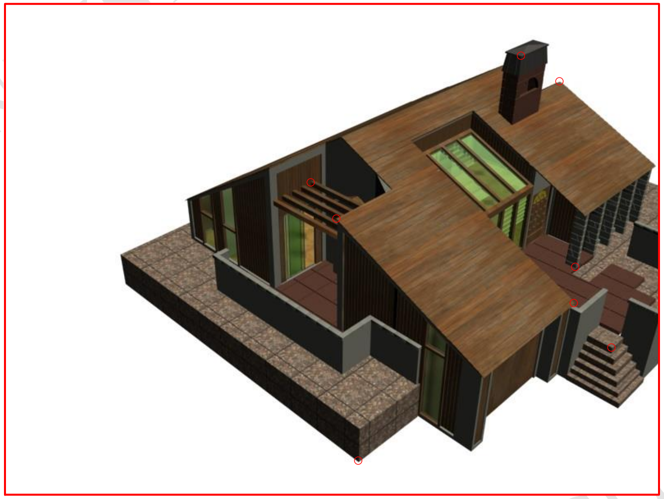


Fundamental matrix for pure translation

- Forward and backward

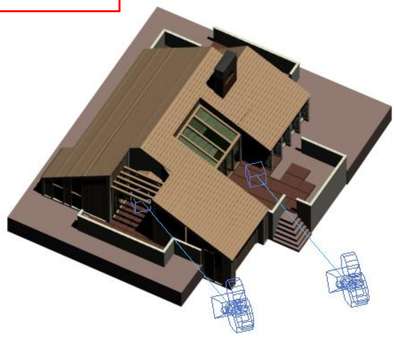


Fundamental matrix—example1



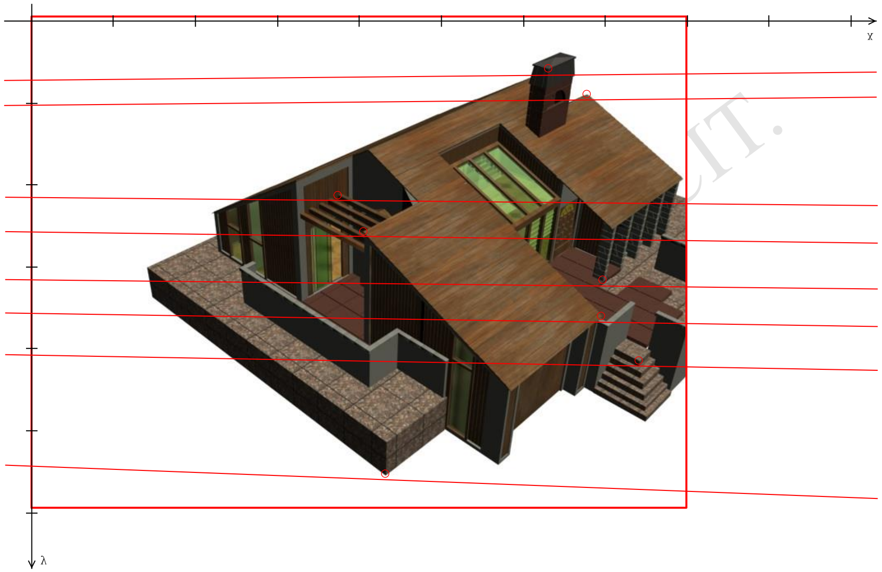
$x1=[375,219,1]^T$
$x2=[405,263,1]^T$
$x3=[433,560,1]^T$
$x4=[630,66,1]^T$
$x5=[678,96,1]^T$
$x6=[698,323,1]^T$
$x7=[696,367,1]^T$
$x8=[741,421,1]^T$

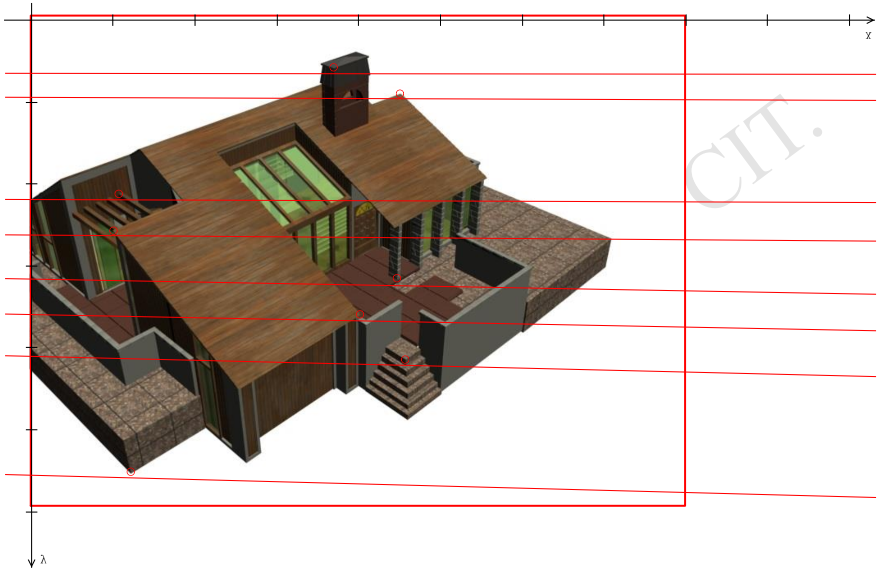
$xp1=[108,219,1]^T$
$xp2=[100,263,1]^T$
$xp3=[123,559,1]^T$
$xp4=[370,65,1]^T$
$xp5=[452,96,1]^T$
$xp6=[448,324,1]^T$
$xp7=[403,367,1]^T$
$xp8=[458,421,1]^T$



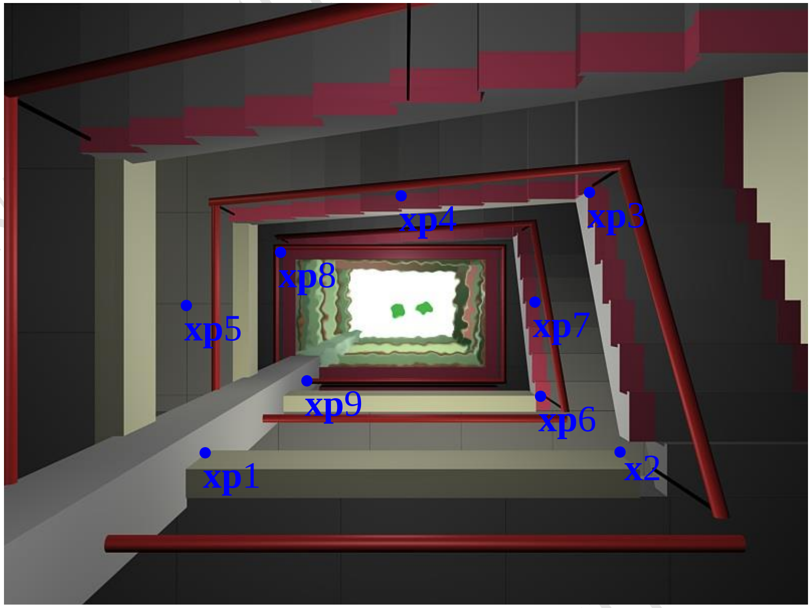
Fundamental matrix—example1, cont.

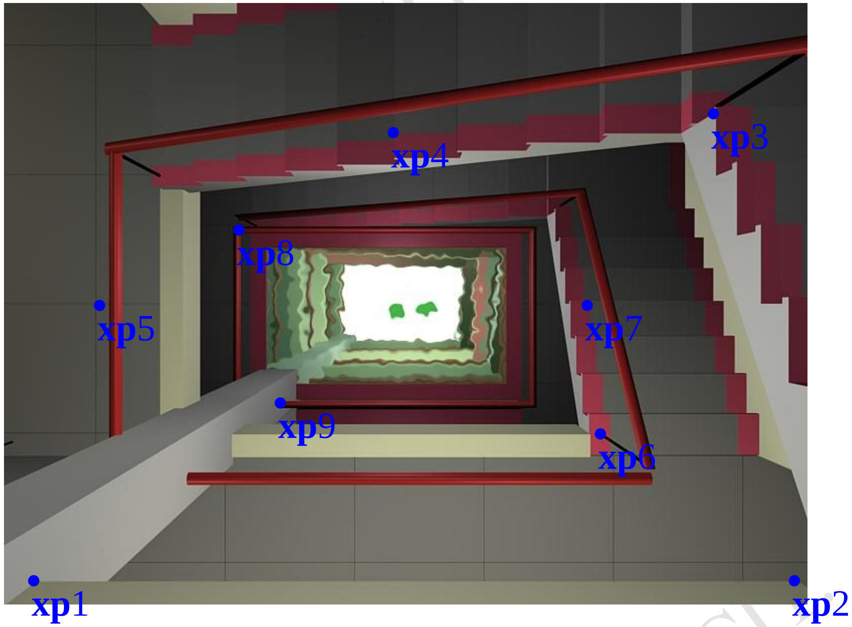

```
A=[
xp1(1)*x1(1) xp1(1)*x1(2) xp1(1) xp1(2)*x1(1) xp1(2)*x1(2) xp1(2) x1(1) x1(2) ;
xp2(1)*x2(1) xp2(1)*x2(2) xp2(1) xp2(2)*x2(1) xp2(2)*x2(2) xp2(2) x2(1) x2(2) ;
xp3(1)*x3(1) xp3(1)*x3(2) xp3(1) xp3(2)*x3(1) xp3(2)*x3(2) xp3(2) x3(1) x3(2) ;
xp4(1)*x4(1) xp4(1)*x4(2) xp4(1) xp4(2)*x4(1) xp4(2)*x4(2) xp4(2) x4(1) x4(2) ;
xp5(1)*x5(1) xp5(1)*x5(2) xp5(1) xp5(2)*x5(1) xp5(2)*x5(2) xp5(2) x5(1) x5(2) ;
xp6(1)*x6(1) xp6(1)*x6(2) xp6(1) xp6(2)*x6(1) xp6(2)*x6(2) xp6(2) x6(1) x6(2) ;
xp7(1)*x7(1) xp7(1)*x7(2) xp7(1) xp7(2)*x7(1) xp7(2)*x7(2) xp7(2) x7(1) x7(2) ;
xp8(1)*x8(1) xp8(1)*x8(2) xp8(1) xp8(2)*x8(1) xp8(2)*x8(2) xp8(2) x8(1) x8(2) ];
d=[-1 -1 -1 -1 -1 -1 -1 -1]';
f=inv(A)*d;
F=[f(1:3)';f(4:6)';f(7:8)' 1]
```





Fundamental matrix—example2

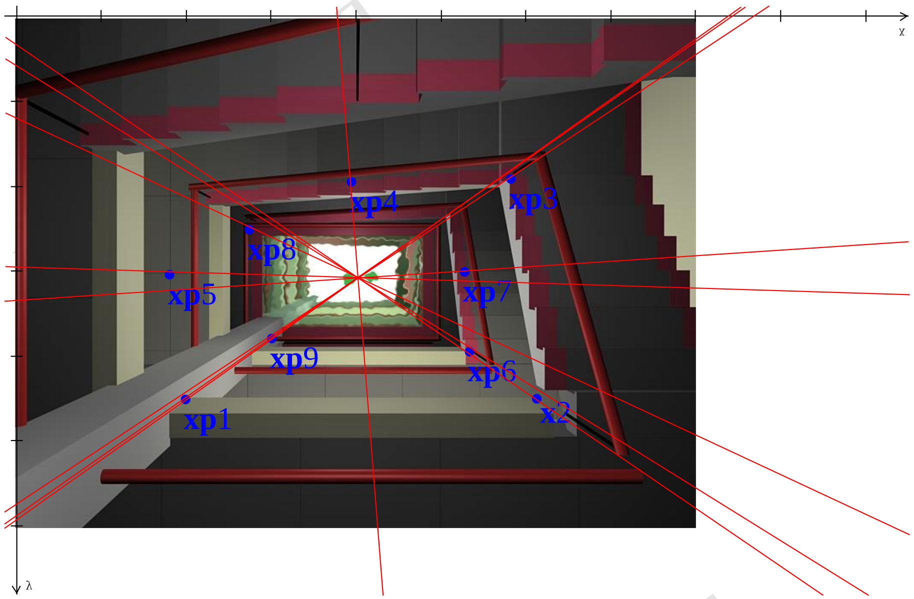


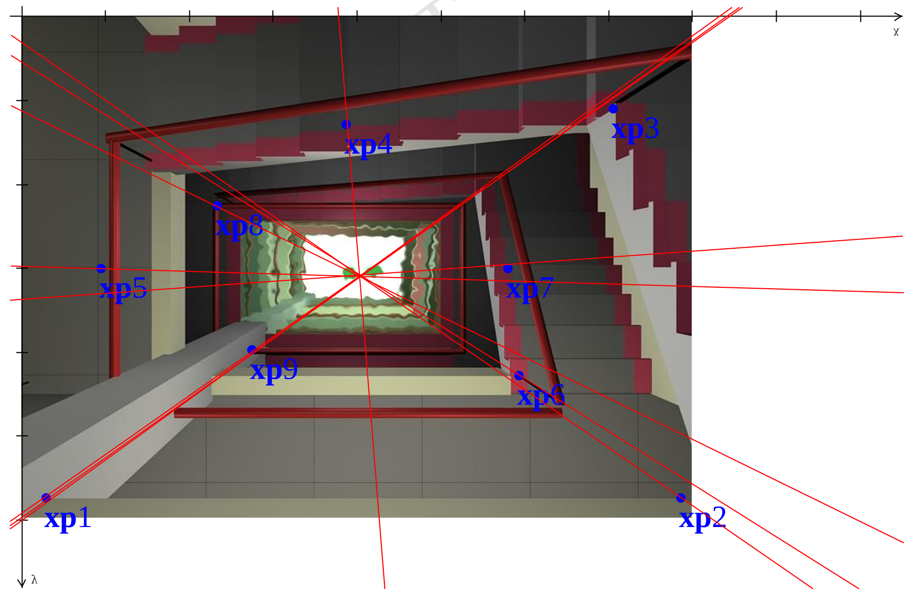


$x1=[207,446,1]^T$
$x2=[605,446,1]^T$
$x3=[586,182,1]^T$
$x4=[393,191,1]^T$
$x5=[182,301,1]^T$
$x6=[535,390,1]^T$
$x7=[532,299,1]^T$
$x8=[274,246,1]^T$
$x9=[303,377,1]^T$

$xp1=[34,577,1]^T$
$xp2=[790,577,1]^T$
$xp3=[705,105,1]^T$
$xp4=[389,131,1]^T$
$xp5=[92,300,1]^T$
$xp6=[592,428,1]^T$
$xp7=[581,297,1]^T$
$xp8=[236,230,1]^T$
$xp9=[275,401,1]^T$

Fundamental matrix—example2, cont.



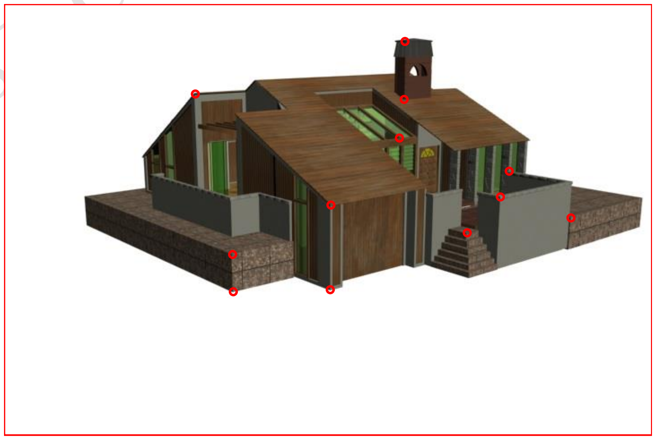


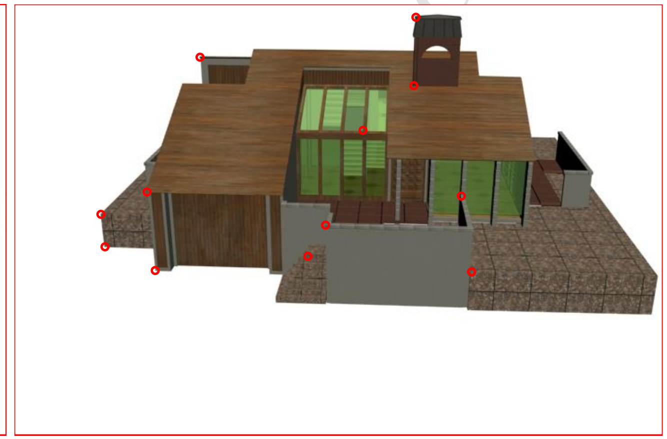
Solve it by OpenCV
F=[-0.000001 0.000648 -0.199148
-0.000636 -0.000002 0.256521
0.197009 -0.260813 1.000000]

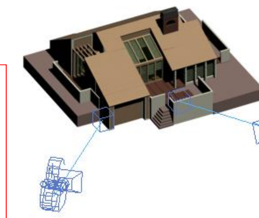
$l_1^T=$	-0.1700	-0.2399	142.2416
$l_2^T=$	-0.1708	0.2500	-8.3143
⋮	0.1295	0.1958	-112.4646
	0.1133	-0.0090	-42.8643
	0.0061	-0.2018	59.6347
	-0.0758	0.1219	-7.1046
	0.0075	0.1151	-38.5183
	0.0505	-0.1083	13.0009
$l_9^T=$	-0.0583	-0.0834	49.0992

$l'^T_1=$	0.0897	0.1240	-74.5417
$l'^T_2=$	0.0893	-0.1292	3.8678
⋮	-0.0818	-0.1165	68.9793
	-0.0758	0.0062	28.6093
	-0.0043	0.1402	-41.6491
	0.0530	-0.0845	4.6827
	-0.0059	-0.0824	27.8257
	-0.0400	0.0818	-9.1795
$l'^T_9=$	0.0448	0.0631	-37.6328

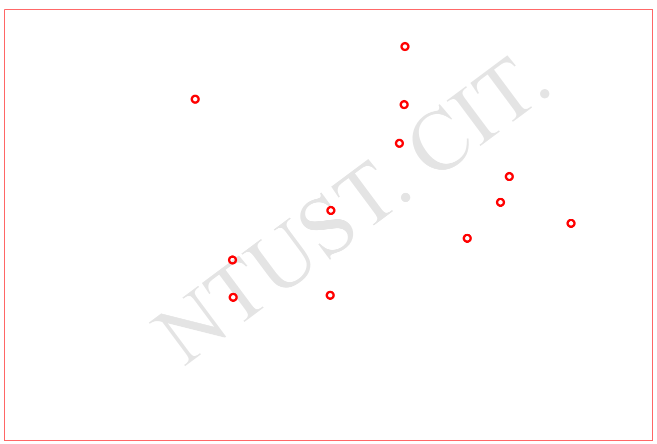
Fundamental matrix—example3

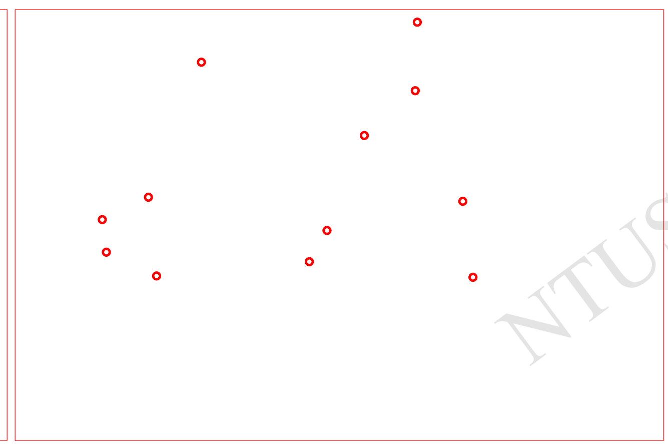






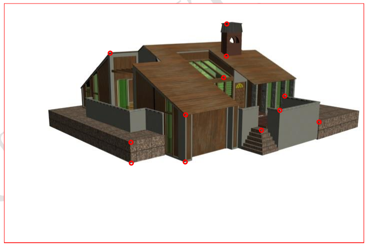
$x1=[211,99,1]^T$
$x2=[252,278,1]^T$
$x3=[253,320,1]^T$
$x4=[362,318,1]^T$
$x5=[362,223,1]^T$
$x6=[514,255,1]^T$
$x7=[630,238,1]^T$
$x8=[551,214,1]^T$
$x9=[561,185,1]^T$
$x10=[437,148,1]^T$
$x11=[444,105,1]^T$
$x12=[445,39,1]^T$





$xp1=[205,57,1]^T$
$xp2=[95,233,1]^T$
$xp3=[99,270,1]^T$
$xp4=[156,296,1]^T$
$xp5=[146,210,1]^T$
$xp6=[325,279,1]^T$
$xp7=[507,296,1]^T$
$xp8=[345,246,1]^T$
$xp9=[496,211,1]^T$
$xp10=[386,140,1]^T$
$xp11=[442,90,1]^T$
$xp12=[445,12,1]^T$

Fundamental matrix—example3, cont.



normalized

$x1=[211,99,1]^T$
$x2=[252,278,1]^T$
$x3=[253,320,1]^T$
$x4=[362,318,1]^T$
$x5=[362,223,1]^T$
$x6=[514,255,1]^T$
$x7=[630,238,1]^T$
$x8=[551,214,1]^T$
$x9=[561,185,1]^T$
$x10=[437,148,1]^T$
$x11=[444,105,1]^T$
$x12=[445,39,1]^T$

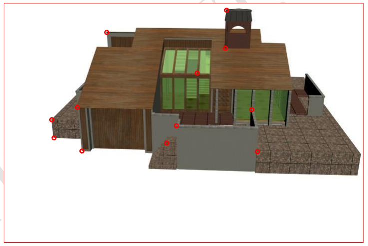
	A	B	C	D	E	F	G	H	I	J
1	x						length		nx	
2	211	99		-207.5	-102.8333		231.5836		-2.03549	-1.00875
3	252	278		-166.5	76.16667		183.0945		-1.63329	0.747163
4	253	320		-165.5	118.1667		203.3559		-1.62348	1.159165
5	362	318		-56.5	116.1667		129.178		-0.55424	1.139545
6	362	223		-56.5	21.16667		60.33471		-0.55424	0.207636
7	514	255		95.5	53.16667		109.3021		0.936814	0.521542
8	630	238		211.5	36.16667		214.57		2.074725	0.35478
9	551	214		132.5	12.16667		133.0574		1.299769	0.11935
10	561	185		142.5	-16.83333		143.4908		1.397864	-0.16513
11	437	148		18.5	-53.83333		56.92344		0.181477	-0.52808
12	444	105		25.5	-96.83333		100.1346		0.250144	-0.94989
13	445	39		26.5	-162.8333		164.9756		0.259954	-1.59733
14	Average									
15	418.5	201.83333		0	3.27E-13		144.1667			

$$\mathbf{T}=\begin{bmatrix} \frac{\sqrt{2}}{144.1667} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{144.1667} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -418.5 \\ 0 & 1 & -201.8333 \\ 0 & 0 & 1 \end{bmatrix}$$

T =

0.0098	0	-4.1053
0	0.0098	-1.9799
0	0	1.0000

Fundamental matrix—example3, cont.



$xp1=[205,57,1]^T$
$xp2=[95,233,1]^T$
$xp3=[99,270,1]^T$
$xp4=[156,296,1]^T$
$xp5=[146,210,1]^T$
$xp6=[325,279,1]^T$
$xp7=[507,296,1]^T$
$xp8=[345,246,1]^T$
$xp9=[496,211,1]^T$
$xp10=[386,140,1]^T$
$xp11=[442,90,1]^T$
$xp12=[445,12,1]^T$

normalized

	A	B	C	D	E	F	G	H	I	J
1	xp						length		nxp	
2	205	57		-98.9167	-138		169.789596		-0.83388	-1.16336
3	95	233		-208.917	38		212.344469		-1.76119	0.320345
4	99	270		-204.917	75		218.210541		-1.72747	0.63226
5	156	296		-147.917	101		179.109855		-1.24696	0.851443
6	146	210		-157.917	15		158.627468		-1.33126	0.126452
7	325	279		21.08333	84		86.6054672		0.177735	0.708131
8	507	296		203.0833	101		226.812346		1.712019	0.851443
9	345	246		41.08333	51		65.4892379		0.346338	0.429937
10	496	211		192.0833	16		192.748559		1.619287	0.134882
11	386	140		82.08333	-55		98.8062428		0.691973	-0.46366
12	442	90		138.0833	-105		173.470479		1.16406	-0.88516
13	445	12		141.0833	-183		231.070351		1.189351	-1.54271
14	Average									
15	303.9167	195		6.44E-13	0		167.757051			

$$\mathbf{T}' = \begin{bmatrix} \frac{\sqrt{2}}{167.757051} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{167.757051} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -303.9167 \\ 0 & 1 & -195 \\ 0 & 0 & 1 \end{bmatrix}$$

TP =

0.0084	0	-2.5621
0	0.0084	-1.6439
0	0	1.0000

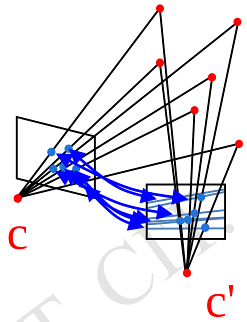
Fundamental matrix—example3, cont.

- In this example, we have 12 correspondences (over-determine than 8), solve it by SVD.

$$\mathbf{x}'^{\mathrm{T}}\mathbf{F}\mathbf{x} = 0 \rightarrow [u' \quad v' \quad 1]\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$

$$\rightarrow F_{11}u'u + F_{12}u'v + F_{13}u' + F_{21}uv' + F_{22}vv' + F_{23}v' + F_{31}u + F_{32}v + F_{33} = 0$$

$$[u'u \quad u'v \quad u' \quad uv' \quad vv' \quad v' \quad u \quad v \quad 1]\begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$



Fundamental matrix—example3, cont.

- NOTE! Data is normalized! So far, we are determining $\hat{\mathbf{F}}$

```
A=[
nxp1(1)*nx1(1) nxp1(1)*nx1(2) nxp1(1) nxp1(2)*nx1(1) nxp1(2)*nx1(2) nxp1(2) nx1(1) nx1(2) 1;
nxp2(1)*nx2(1) nxp2(1)*nx2(2) nxp2(1) nxp2(2)*nx2(1) nxp2(2)*nx2(2) nxp2(2) nx2(1) nx2(2) 1;
nxp3(1)*nx3(1) nxp3(1)*nx3(2) nxp3(1) nxp3(2)*nx3(1) nxp3(2)*nx3(2) nxp3(2) nx3(1) nx3(2) 1;
nxp4(1)*nx4(1) nxp4(1)*nx4(2) nxp4(1) nxp4(2)*nx4(1) nxp4(2)*nx4(2) nxp4(2) nx4(1) nx4(2) 1;
nxp5(1)*nx5(1) nxp5(1)*nx5(2) nxp5(1) nxp5(2)*nx5(1) nxp5(2)*nx5(2) nxp5(2) nx5(1) nx5(2) 1;
nxp6(1)*nx6(1) nxp6(1)*nx6(2) nxp6(1) nxp6(2)*nx6(1) nxp6(2)*nx6(2) nxp6(2) nx6(1) nx6(2) 1;
nxp7(1)*nx7(1) nxp7(1)*nx7(2) nxp7(1) nxp7(2)*nx7(1) nxp7(2)*nx7(2) nxp7(2) nx7(1) nx7(2) 1;
nxp8(1)*nx8(1) nxp8(1)*nx8(2) nxp8(1) nxp8(2)*nx8(1) nxp8(2)*nx8(2) nxp8(2) nx8(1) nx8(2) 1;
nxp9(1)*nx9(1) nxp9(1)*nx9(2) nxp9(1) nxp9(2)*nx9(1) nxp9(2)*nx9(2) nxp9(2) nx9(1) nx9(2) 1;
nxp10(1)*nx10(1) nxp10(1)*nx10(2) nxp10(1) nxp10(2)*nx10(1) nxp10(2)*nx10(2) nxp10(2) nx10(1) nx10(2) 1;
nxp11(1)*nx11(1) nxp11(1)*nx11(2) nxp11(1) nxp11(2)*nx11(1) nxp11(2)*nx11(2) nxp11(2) nx11(1) nx11(2) 1;
nxp12(1)*nx12(1) nxp12(1)*nx12(2) nxp12(1) nxp12(2)*nx12(1) nxp12(2)*nx12(2) nxp12(2) nx12(1) nx12(2) 1]
```

```
[U,S,V]=svd(A)
```

```
>> Fh=[V(1:3,9)';V(4:6,9)';V(7:9,9)']

Fh =
```

$$\hat{\mathbf{F}} = \begin{matrix} 0.0058 & 0.0290 & -0.0303 \\ 0.0353 & -0.0197 & -0.7377 \\ 0.1644 & 0.6520 & 0.0134 \end{matrix}$$

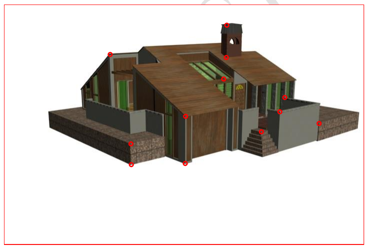
Denormalized for **F**

```
>> F=TP'*Fh*T

F =
```

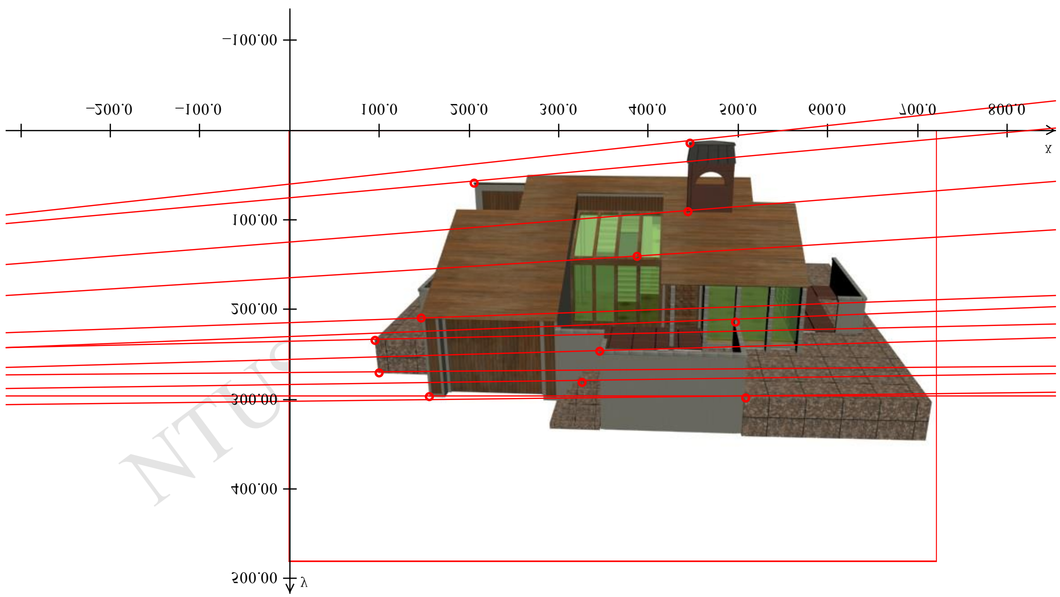
$$\mathbf{F} = \mathbf{T}'^{T} \hat{\mathbf{F}} \mathbf{T} = \begin{matrix} 0.0000 & 0.0000 & -0.0009 \\ 0.0000 & -0.0000 & -0.0071 \\ 0.0009 & 0.0060 & -0.2791 \end{matrix}$$

Fundamental matrix—example3, cont.

- Find epipolar lines in the 2nd image for points of 1st image



>> F*x1	>> F*x2	>> F*x3	>> F*x4	>> F*x5	>> F*x6	>> F*x7	>> F*x8	>> F*x9	>> F*x10	>> F*x11	>> F*x12
ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =
-0.0006	-0.0002	-0.0001	-0.0000	-0.0002	-0.0001	-0.0001	-0.0002	-0.0002	-0.0004	-0.0005	-0.0006
-0.0067	-0.0068	-0.0069	-0.0066	-0.0064	-0.0060	-0.0057	-0.0059	-0.0058	-0.0061	-0.0060	-0.0059
0.5025	1.6102	1.8624	1.9482	1.3798	1.7077	1.7100	1.4955	1.3310	0.9984	0.7474	0.3534

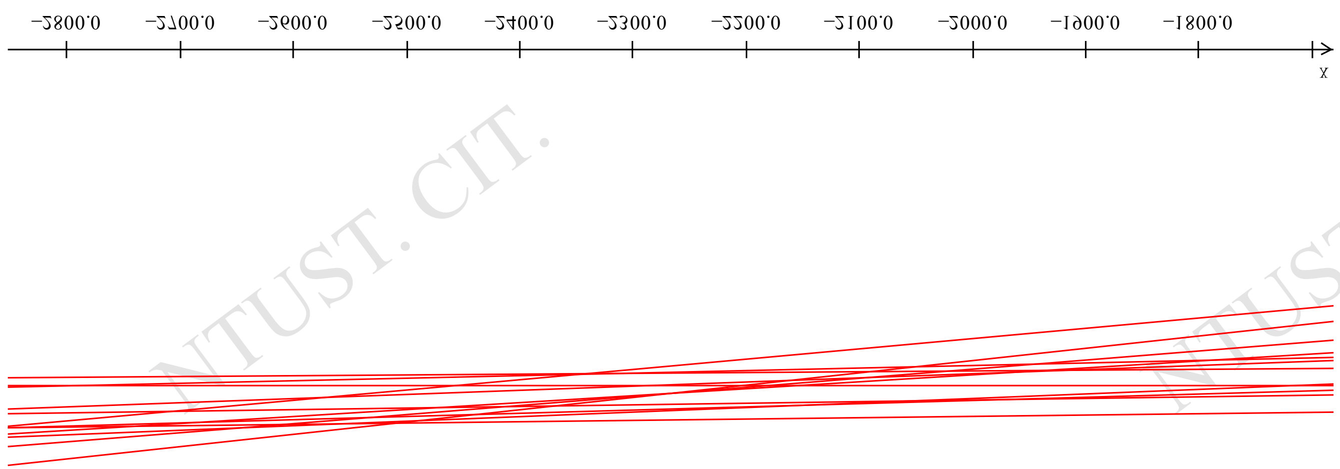


Fundamental matrix—example3, cont.

- Estimate the error, by $\mathbf{x'}^{T}\mathbf{Fx}$

>> xp1'*F*x1	>> xp2'*F*x2	>> xp3'*F*x3	>> xp4'*F*x4	>> xp5'*F*x5	>> xp6'*F*x6	>> xp7'*F*x7	>> xp8'*F*x8	>> xp9'*F*x9	>> xp10'*F*x10	>> xp11'*F*x11	>> xp12'*F*x12
ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =
-5.3137e-004	0.0045	-0.0042	0.0020	-0.0020	-5.4760e-004	4.2649e-004	-2.6209e-004	-5.8426e-004	0.0023	-0.0018	7.5518e-004

Correspondences 2 has large error than others, let remove them then re-calculate (see next slide)



Fundamental matrix—example3, cont.

```
A=[
nxp1(1)*nx1(1) nxp1(1)*nx1(2) nxp1(1) nxp1(2)*nx1(1) nxp1(2)*nx1(2) nxp1(2) nx1(1) nx1(2) 1;
nxp3(1)*nx3(1) nxp3(1)*nx3(2) nxp3(1) nxp3(2)*nx3(1) nxp3(2)*nx3(2) nxp3(2) nx3(1) nx3(2) 1;
nxp4(1)*nx4(1) nxp4(1)*nx4(2) nxp4(1) nxp4(2)*nx4(1) nxp4(2)*nx4(2) nxp4(2) nx4(1) nx4(2) 1;
nxp5(1)*nx5(1) nxp5(1)*nx5(2) nxp5(1) nxp5(2)*nx5(1) nxp5(2)*nx5(2) nxp5(2) nx5(1) nx5(2) 1;
nxp6(1)*nx6(1) nxp6(1)*nx6(2) nxp6(1) nxp6(2)*nx6(1) nxp6(2)*nx6(2) nxp6(2) nx6(1) nx6(2) 1;
nxp7(1)*nx7(1) nxp7(1)*nx7(2) nxp7(1) nxp7(2)*nx7(1) nxp7(2)*nx7(2) nxp7(2) nx7(1) nx7(2) 1;
nxp8(1)*nx8(1) nxp8(1)*nx8(2) nxp8(1) nxp8(2)*nx8(1) nxp8(2)*nx8(2) nxp8(2) nx8(1) nx8(2) 1;
nxp9(1)*nx9(1) nxp9(1)*nx9(2) nxp9(1) nxp9(2)*nx9(1) nxp9(2)*nx9(2) nxp9(2) nx9(1) nx9(2) 1;
nxp10(1)*nx10(1) nxp10(1)*nx10(2) nxp10(1) nxp10(2)*nx10(1) nxp10(2)*nx10(2) nxp10(2) nx10(1) nx10(2) 1;
nxp11(1)*nx11(1) nxp11(1)*nx11(2) nxp11(1) nxp11(2)*nx11(1) nxp11(2)*nx11(2) nxp11(2) nx11(1) nx11(2) 1;
nxp12(1)*nx12(1) nxp12(1)*nx12(2) nxp12(1) nxp12(2)*nx12(1) nxp12(2)*nx12(2) nxp12(2) nx12(1) nx12(2) 1]
```

```
[U,S,V]=svd(A)
Fh=[V(1:3,9)';V(4:6,9)';V(7:9,9)']
F=TP'*Fh*T
```

>> xp1'*F*x1	>> xp3'*F*x3	>> xp4'*F*x4	>> xp5'*F*x5	>> xp6'*F*x6	>> xp7'*F*x7	>> xp8'*F*x8	>> xp9'*F*x9	>> xp10'*F*x10	>> xp11'*F*x11	>> xp12'*F*x12
ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =
-9.0214e-005	9.6499e-004	-0.0019	2.3422e-004	0.0025	-4.8966e-004	-7.3015e-004	6.9723e-005	-0.0015	8.8603e-004	5.1497e-005

Fundamental matrix—example3, cont.

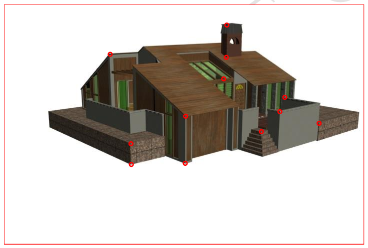
```
A=[
nxp1(1)*nx1(1) nxp1(1)*nx1(2) nxp1(1) nxp1(2)*nx1(1) nxp1(2)*nx1(2) nxp1(2) nx1(1) nx1(2) 1;
nxp3(1)*nx3(1) nxp3(1)*nx3(2) nxp3(1) nxp3(2)*nx3(1) nxp3(2)*nx3(2) nxp3(2) nx3(1) nx3(2) 1;
nxp4(1)*nx4(1) nxp4(1)*nx4(2) nxp4(1) nxp4(2)*nx4(1) nxp4(2)*nx4(2) nxp4(2) nx4(1) nx4(2) 1;
nxp5(1)*nx5(1) nxp5(1)*nx5(2) nxp5(1) nxp5(2)*nx5(1) nxp5(2)*nx5(2) nxp5(2) nx5(1) nx5(2) 1;
nxp7(1)*nx7(1) nxp7(1)*nx7(2) nxp7(1) nxp7(2)*nx7(1) nxp7(2)*nx7(2) nxp7(2) nx7(1) nx7(2) 1;
nxp8(1)*nx8(1) nxp8(1)*nx8(2) nxp8(1) nxp8(2)*nx8(1) nxp8(2)*nx8(2) nxp8(2) nx8(1) nx8(2) 1;
nxp9(1)*nx9(1) nxp9(1)*nx9(2) nxp9(1) nxp9(2)*nx9(1) nxp9(2)*nx9(2) nxp9(2) nx9(1) nx9(2) 1;
nxp10(1)*nx10(1) nxp10(1)*nx10(2) nxp10(1) nxp10(2)*nx10(1) nxp10(2)*nx10(2) nxp10(2) nx10(1) nx10(2) 1;
nxp11(1)*nx11(1) nxp11(1)*nx11(2) nxp11(1) nxp11(2)*nx11(1) nxp11(2)*nx11(2) nxp11(2) nx11(1) nx11(2) 1;
nxp12(1)*nx12(1) nxp12(1)*nx12(2) nxp12(1) nxp12(2)*nx12(1) nxp12(2)*nx12(2) nxp12(2) nx12(1) nx12(2) 1]
```

```
[U,S,V]=svd(A)
Fh=[V(1:3,9)';V(4:6,9)';V(7:9,9)']
F=TP'*Fh*T
```

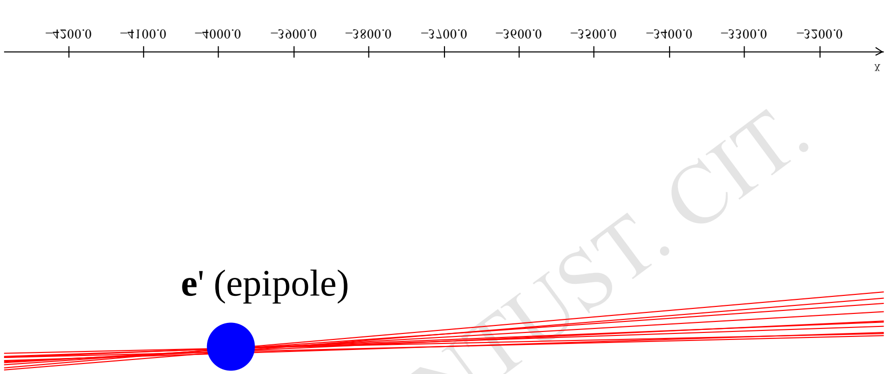
>> xp1'*F*x1	>> xp3'*F*x3	>> xp4'*F*x4	>> xp5'*F*x5	>> xp7'*F*x7	>> xp8'*F*x8	>> xp9'*F*x9	>> xp10'*F*x10	>> xp11'*F*x11	>> xp12'*F*x12
ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =
-4.4279e-005	-1.5922e-004	1.1958e-004	2.4719e-004	1.0935e-004	-2.8128e-004	-1.6777e-005	-1.5967e-004	2.4016e-004	-5.5050e-005

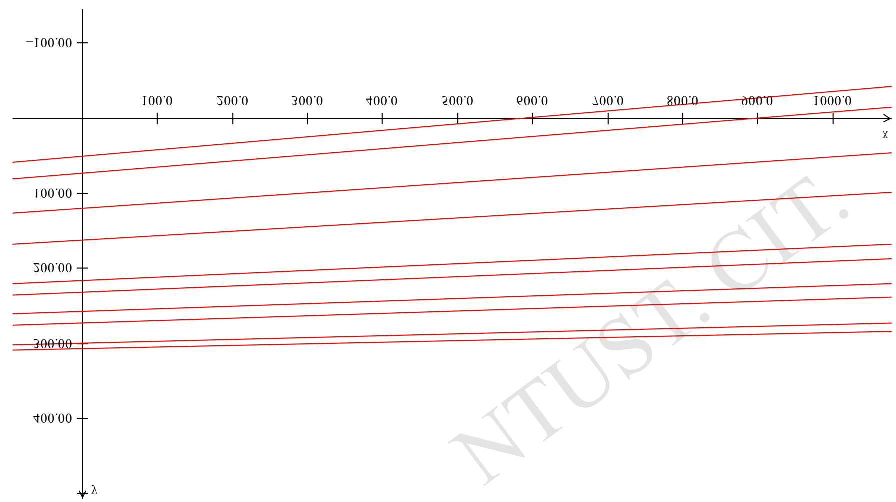
Fundamental matrix—example3, cont.

Find epipolar lines in the 2nd image for points of 1st image (10 correspondences)



>> F*x1	>> F*x3	>> F*x4	>> F*x5	>> F*x7	>> F*x8	>> F*x9	>> F*x10	>> F*x11	>> F*x12
ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =
0.0006	0.0002	0.0002	0.0003	0.0001	0.0002	0.0002	0.0004	0.0004	0.0005
0.0067	0.0071	0.0067	0.0065	0.0056	0.0058	0.0057	0.0060	0.0059	0.0058
-0.4978	-1.9467	-2.0165	-1.4073	-1.7067	-1.4929	-1.3145	-0.9833	-0.7129	-0.2904





Fundamental matrix—example3, cont.

```
A=[
nxp1(1)*nx1(1) nxp1(1)*nx1(2) nxp1(1) nxp1(2)*nx1(1) nxp1(2)*nx1(2) nxp1(2) nx1(1) nx1(2) 1;
nxp3(1)*nx3(1) nxp3(1)*nx3(2) nxp3(1) nxp3(2)*nx3(1) nxp3(2)*nx3(2) nxp3(2) nx3(1) nx3(2) 1;
nxp4(1)*nx4(1) nxp4(1)*nx4(2) nxp4(1) nxp4(2)*nx4(1) nxp4(2)*nx4(2) nxp4(2) nx4(1) nx4(2) 1;
nxp5(1)*nx5(1) nxp5(1)*nx5(2) nxp5(1) nxp5(2)*nx5(1) nxp5(2)*nx5(2) nxp5(2) nx5(1) nx5(2) 1;
nxp7(1)*nx7(1) nxp7(1)*nx7(2) nxp7(1) nxp7(2)*nx7(1) nxp7(2)*nx7(2) nxp7(2) nx7(1) nx7(2) 1;
nxp9(1)*nx9(1) nxp9(1)*nx9(2) nxp9(1) nxp9(2)*nx9(1) nxp9(2)*nx9(2) nxp9(2) nx9(1) nx9(2) 1;
nxp10(1)*nx10(1) nxp10(1)*nx10(2) nxp10(1) nxp10(2)*nx10(1) nxp10(2)*nx10(2) nxp10(2) nx10(1) nx10(2) 1;
nxp11(1)*nx11(1) nxp11(1)*nx11(2) nxp11(1) nxp11(2)*nx11(1) nxp11(2)*nx11(2) nxp11(2) nx11(1) nx11(2) 1;
nxp12(1)*nx12(1) nxp12(1)*nx12(2) nxp12(1) nxp12(2)*nx12(1) nxp12(2)*nx12(2) nxp12(2) nx12(1) nx12(2) 1]
```

```
[U,S,V]=svd(A)
Fh=[V(1:3,9)';V(4:6,9)';V(7:9,9)']
F=TP'*Fh*T
```

>> xp1'*F*x1	>> xp3'*F*x3	>> xp4'*F*x4	>> xp5'*F*x5	>> xp7'*F*x7	>> xp9'*F*x9	>> xp10'*F*x10	>> xp11'*F*x11	>> xp12'*F*x12
ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =
-1.2016e-005	1.7811e-005	-2.4047e-005	1.6801e-006	2.8201e-005	-5.3142e-005	3.7243e-005	3.0845e-006	1.1849e-006

Fundamental matrix—example3, cont.

ERROR Comparison: residual error (for equation or line)

```
A=[
nxp1(1)*nx1(1) nxp1(1)*nx1(2) nxp1(1) nxp1(2)*nx1(1) nxp1(2)*nx1(2) nxp1(2) nx1(1) nx1(2) 1;
nxp2(1)*nx2(1) nxp2(1)*nx2(2) nxp2(1) nxp2(2)*nx2(1) nxp2(2)*nx2(2) nxp2(2) nx2(1) nx2(2) 1;
nxp3(1)*nx3(1) nxp3(1)*nx3(2) nxp3(1) nxp3(2)*nx3(1) nxp3(2)*nx3(2) nxp3(2) nx3(1) nx3(2) 1;
nxp4(1)*nx4(1) nxp4(1)*nx4(2) nxp4(1) nxp4(2)*nx4(1) nxp4(2)*nx4(2) nxp4(2) nx4(1) nx4(2) 1;
nxp5(1)*nx5(1) nxp5(1)*nx5(2) nxp5(1) nxp5(2)*nx5(1) nxp5(2)*nx5(2) nxp5(2) nx5(1) nx5(2) 1;
nxp6(1)*nx6(1) nxp6(1)*nx6(2) nxp6(1) nxp6(2)*nx6(1) nxp6(2)*nx6(2) nxp6(2) nx6(1) nx6(2) 1;
nxp7(1)*nx7(1) nxp7(1)*nx7(2) nxp7(1) nxp7(2)*nx7(1) nxp7(2)*nx7(2) nxp7(2) nx7(1) nx7(2) 1;
nxp8(1)*nx8(1) nxp8(1)*nx8(2) nxp8(1) nxp8(2)*nx8(1) nxp8(2)*nx8(2) nxp8(2) nx8(1) nx8(2) 1;
nxp9(1)*nx9(1) nxp9(1)*nx9(2) nxp9(1) nxp9(2)*nx9(1) nxp9(2)*nx9(2) nxp9(2) nx9(1) nx9(2) 1;
nxp10(1)*nx10(1) nxp10(1)*nx10(2) nxp10(1) nxp10(2)*nx10(1) nxp10(2)*nx10(2) nxp10(2) nx10(1) nx10(2) 1;
nxp11(1)*nx11(1) nxp11(1)*nx11(2) nxp11(1) nxp11(2)*nx11(1) nxp11(2)*nx11(2) nxp11(2) nx11(1) nx11(2) 1;
nxp12(1)*nx12(1) nxp12(1)*nx12(2) nxp12(1) nxp12(2)*nx12(1) nxp12(2)*nx12(2) nxp12(2) nx12(1) nx12(2) 1]

[U,S,V]=svd(A)
Fh=[V(1:3,9)';V(4:6,9)';V(7:9,9)']
F=TP'*Fh*T

lp1=F*x1;lp1=lp1./sqrt(lp1(1)^2+lp1(2)^2);
……..
```

Error estimation function

xp1'*lp1	xp2'*lp2	xp3'*lp3	xp4'*lp4	xp5'*lp5	xp6'*lp6	xp7'*lp7	xp8'*lp8	xp9'*lp9	xp10'*lp10	xp11'*lp11	xp12'*lp12
ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =
-0.0795	0.6574	-0.6152	0.2976	-0.3051	-0.0908	0.0753	-0.0448	-0.1011	0.3848	-0.3072	0.1278

Euclidean distance

>> xp1'*F*x1	>> xp2'*F*x2	>> xp3'*F*x3	>> xp4'*F*x4	>> xp5'*F*x5	>> xp6'*F*x6	>> xp7'*F*x7	>> xp8'*F*x8	>> xp9'*F*x9	>> xp10'*F*x10	>> xp11'*F*x11	>> xp12'*F*x12
ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =
-5.3137e-004	0.0045	-0.0042	0.0020	-0.0020	-5.4760e-004	4.2649e-004	-2.6209e-004	-5.8426e-004	0.0023	-0.0018	7.5518e-004

$\mathbf{x'^T F x}$

Fundamental matrix computation

- Short summary
  - Why error occurs ?
    - Since the real camera is NOT the perfect pin-hole camera model

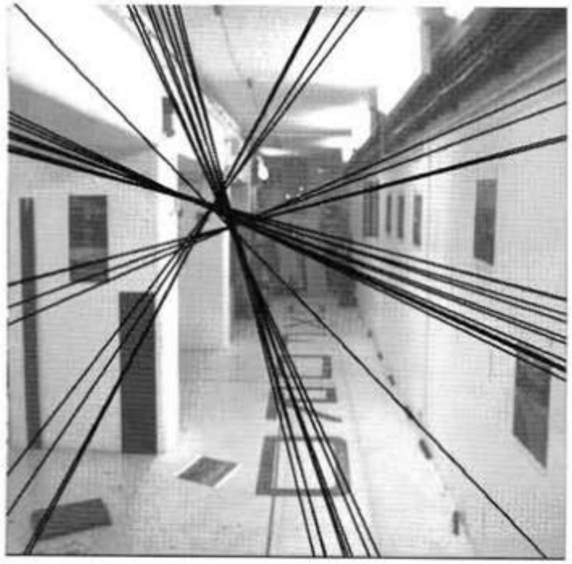
    →undistort images or avoid lens distortion in practice.
    - The image has physical limits in resolution and capacity.

    →earn more budget? Subpixel? Interpolation / super-resolution
    - Numerical issue. Currently, in computer vision field, less textbooks will teach you the *Analytical Solution*. Iteration error, round-off error, truncated error,
    - Matching error (correspondences) & measurement uncertainty

Fundamental matrix (enforcing singularity)

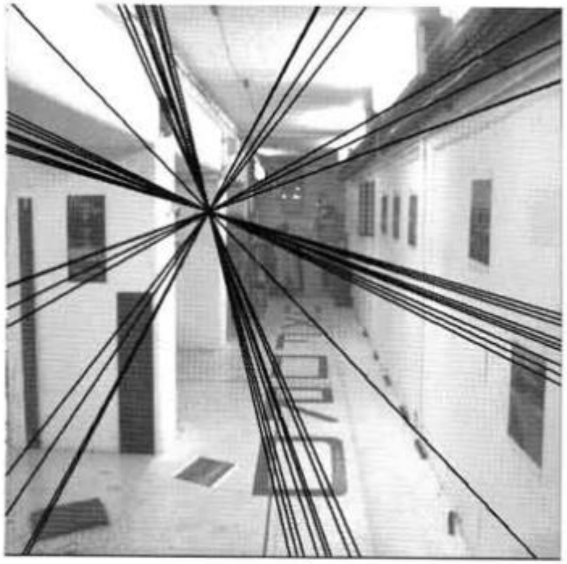
$$\mathbf{F} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}} = \mathbf{U}\begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & \boxed{t} \end{bmatrix}\mathbf{V}^{\mathrm{T}} \Rightarrow \mathbf{F}' = \mathbf{U}\mathbf{S}'\mathbf{V}^{\mathrm{T}} = \mathbf{U}\begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & \boxed{0} \end{bmatrix}\mathbf{V}^{\mathrm{T}}$$

The effect of a non-singular fundamental matrix



a

A singular fundamental matrix



b

Fundamental matrix (enforcing singularity)

- Example

Recall the previous example again.

Determine the normalized fundamental matrix $\hat{\mathbf{F}}$
Then, enforcing the singularity for $\hat{\mathbf{F}}$ to have $\hat{\mathbf{F}}'$
Denoramalized by $\hat{\mathbf{F}}'$ instead of $\hat{\mathbf{F}}$.

```
//SAMPLE CODE in MATLAB
A=[nxp1(1)*nx1(1) nxp1(1)*nx1(2) nxp1(1) nxp1(2)*nx1(1) nxp1(2)*nx1(2) nxp1(2) nx1(1) nx1(2) 1;
…….
nxp12(1)*nx12(1) nxp12(1)*nx12(2) nxp12(1) nxp12(2)*nx12(1) nxp12(2)*nx12(2) nxp12(2) nx12(1) nx12(2) 1]

[U,S,V]=svd(A)
f=[V(1:3,9)';V(4:6,9)';V(7:9,9)']

[Uf,Sf,Vf]=svd(f)
```

$$\Rightarrow \begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & t \end{bmatrix} = \begin{matrix} 0.7413 & 0 & 0 \\ 0 & 0.6712 & 0 \\ 0 & 0 & \boxed{0.0031} \end{matrix}$$

replace it by 0.0

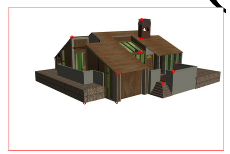
```
Sf(3,3)=0;

FP=Uf*Sf*Vf'
F=TP'*FP*T
```

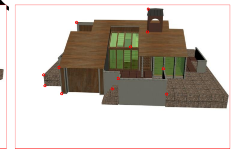
$$\mathbf{F} = \mathbf{T}'^{\mathrm{T}} \hat{\mathbf{F}}' \mathbf{T} = \begin{matrix} 0.0000 & 0.0000 & -0.0010 \\ 0.0000 & -0.0000 & -0.0071 \\ 0.0008 & 0.0060 & -0.2523 \end{matrix}$$

(let's redraw epipolar lines for image2, next slide)

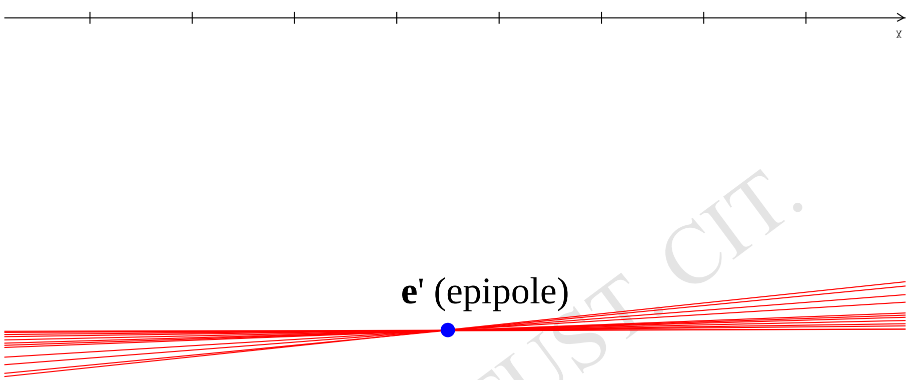
Fundamental matrix (enforcing singularity)

- Example—cont.





>> F*x1	>> F*x2	>> F*x3	>> F*x4	>> F*x5	>> F*x6	>> F*x7	>> F*x8	>> F*x9	>> F*x10	>> F*x11	>> F*x12
ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =	ans =
-0.0006	-0.0002	-0.0001	-0.0000	-0.0002	-0.0001	-0.0000	-0.0001	-0.0002	-0.0004	-0.0005	-0.0006
-0.0067	-0.0068	-0.0069	-0.0066	-0.0064	-0.0060	-0.0057	-0.0059	-0.0058	-0.0061	-0.0060	-0.0059
0.5155	1.6236	1.8765	1.9542	1.3840	1.7012	1.6946	1.4855	1.3197	0.9956	0.7433	0.3480



Auto Fundamental matrix algorithm

Objective Compute the fundamental matrix between two images.

Algorithm

(i) **Interest points:** Compute interest points in each image.
(ii) **Putative correspondences:** Compute a set of interest point matches based on proximity and similarity of their intensity neighbourhood.
(iii) **RANSAC robust estimation:** Repeat for N samples, where N is determined adaptively as in algorithm 4.5(p121):
  (a) Select a random sample of 7 correspondences and compute the fundamental matrix F as described in section 11.1.2. There will be one or three real solutions.
  (b) Calculate the distance $d_\perp$ for each putative correspondence.
  (c) Compute the number of inliers consistent with F by the number of correspondences for which $d_\perp < t$ pixels.
  (d) If there are three real solutions for F the number of inliers is computed for each solution, and the solution with most inliers retained.

  Choose the F with the largest number of inliers. In the case of ties choose the solution that has the lowest standard deviation of inliers.
(iv) **Non-linear estimation:** re-estimate F from all correspondences classified as inliers by minimizing a cost function, e.g. (11.6), using the Levenberg–Marquardt algorithm of section A6.2(p600).
(v) **Guided matching:** Further interest point correspondences are now determined using the estimated F to define a search strip about the epipolar line.

The last two steps can be iterated until the number of correspondences is stable.

Algorithm 11.4 [Hartley04]

Enforce singularity Fundamental matrix algorithm

Objective

Find the fundamental matrix $\mathtt{F}$ that minimizes the algebraic error $\|\mathtt{A}\mathbf{f}\|$ subject to $\|\mathbf{f}\| = 1$ and $\det \mathtt{F} = 0$.

Algorithm

(i) Find a first approximation $\mathtt{F}_0$ for the fundamental matrix using the normalized 8-point algorithm 11.1. Then find the right null-vector $\mathbf{e}_0$ of $\mathtt{F}_0$.

(ii) Starting with the estimate $\mathbf{e}_i = \mathbf{e}_0$ for the epipole, compute the matrix $\mathtt{E}_i$ according to (11.4), then find the vector $\mathbf{f}_i = \mathtt{E}_i\mathbf{m}_i$ that minimizes $\|\mathtt{A}\mathbf{f}_i\|$ subject to $\|\mathbf{f}_i\| = 1$. This is done using algorithm A5.6(p595).

(iii) Compute the algebraic error $\boldsymbol{\epsilon}_i = \mathtt{A}\mathbf{f}_i$. Since $\mathbf{f}_i$ and hence $\boldsymbol{\epsilon}_i$ is defined only up to sign, correct the sign of $\boldsymbol{\epsilon}_i$ (multiplying by minus 1 if necessary) so that $\boldsymbol{\epsilon}_i^\mathsf{T}\boldsymbol{\epsilon}_{i-1} > 0$ for $i > 0$. This is done to ensure that $\boldsymbol{\epsilon}_i$ varies smoothly as a function of $\mathbf{e}_i$.

(iv) The previous two steps define a mapping $\mathbb{R}^3 \to \mathbb{R}^9$ mapping $\mathbf{e}_i \mapsto \boldsymbol{\epsilon}_i$. Now use the Levenberg–Marquardt algorithm (section A6.2(p600)) to vary $\mathbf{e}_i$ iteratively so as to minimize $\|\boldsymbol{\epsilon}_i\|$.

(v) Upon convergence, $\mathbf{f}_i$ represents the desired fundamental matrix.

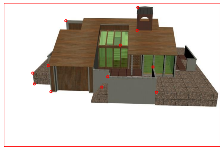
Algorithm 11.1 [Hartley04]

Fundamental matrix (enforcing singularity)

- Of course, openCV provides *findFundamentalMat function*
- Parameters
  - points1 – Array of N points from the first image. The point coordinates should be floating-
  - point (single or double precision).
  - points2 – Array of the second image points of the same size and format as points1 .
  - method – Method for computing a fundamental matrix.
    - – CV_FM_7POINT for a 7-point algorithm. $N = 7$
    - – CV_FM_8POINT for an 8-point algorithm. $N \geq 8$
    - – CV_FM_RANSAC for the RANSAC algorithm. $N \geq 8$
    - – CV_FM_LMEDS for the LMedS algorithm. $N \geq 8$

Fundamental matrix using openCV

- *findFundamentalMat*

(RANSAC)
F=
-0.000219 -0.000913 0.292220
0.000103 -0.000245 0.737529
-0.142952 -0.450960 1.000000

(LMEDS, Levenberg-Marquardt)
F=
-0.000219 -0.000913 0.292220
0.000103 -0.000245 0.737529
-0.142952 -0.450960 1.000000

$$\mathbf{F} = \mathbf{U}\underbrace{\mathbf{S}}\mathbf{V}^{\mathrm{T}}$$

1.331905 0.000000 0.000000
0.000000 0.281374 0.000000
0.000000 0.000000 0.000000

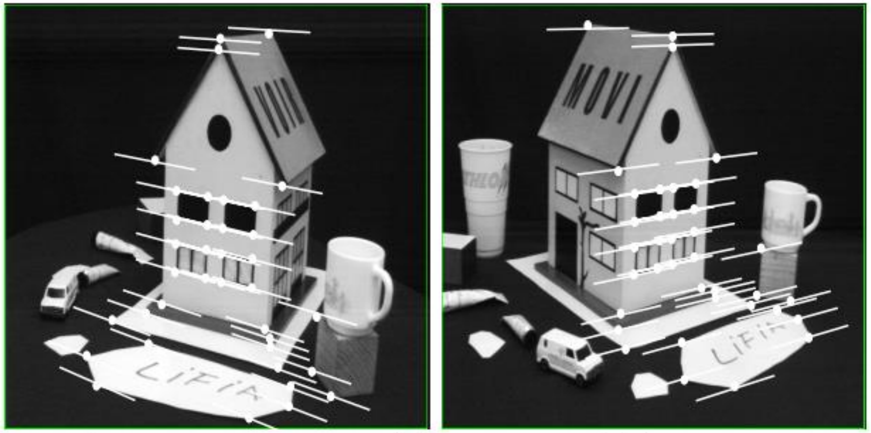
$$\mathbf{e}' = \mathbf{l}_1' \times \mathbf{l}_2' = [\mathbf{F}\mathbf{x}_1] \times [\mathbf{F}\mathbf{x}_2] = \begin{matrix} -552.206970 \\ 217.436905 \\ 1.000000 \end{matrix}$$

$$\mathbf{e} = \mathbf{l}_1 \times \mathbf{l}_2 = [\mathbf{F}^{\mathrm{T}}\mathbf{x}_1'] \times [\mathbf{F}^{\mathrm{T}}\mathbf{x}_2'] = \begin{matrix} -4089.085693 \\ 1298.432373 \\ 1.000000 \end{matrix}$$

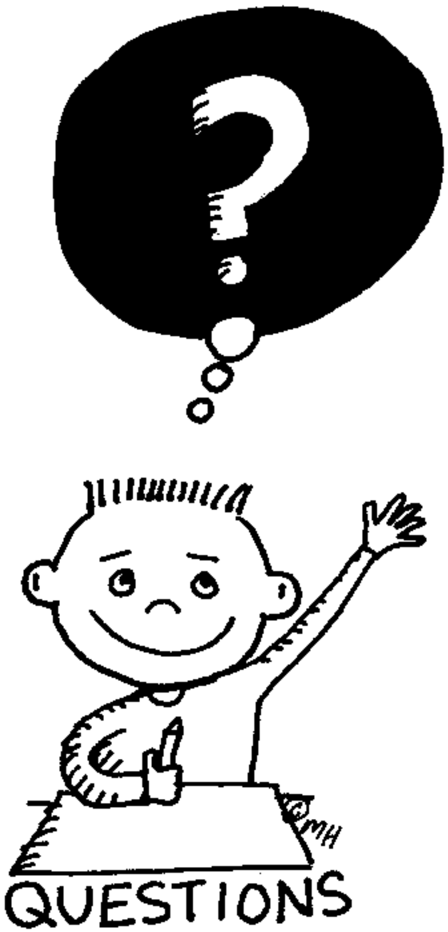
Comparison of different methods

(a)

(b)



	Linear Least Squares	[Hartley, 1995]	[Luong *et al.*, 1993]
Av. Dist. 1	2.33 pixels	0.92 pixel	0.86 pixel
Av. Dist. 2	2.18 pixels	0.85 pixel	0.80 pixel



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