Week 6 Report

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Problem 1

Implementation

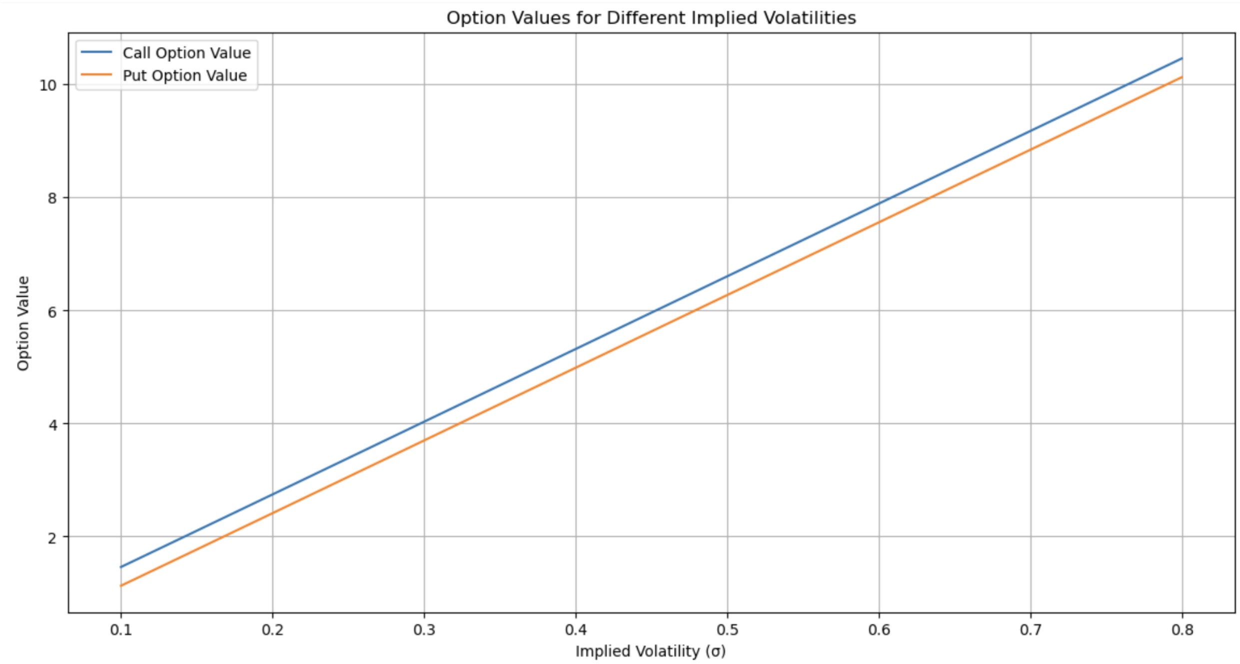
I first defined the `calculate_ttm` function, which calculates the time to maturity (TTM) for the options by determining the number of days between the current date and the expiration date and then converting this into years by dividing by 365. Then, I defined the `calculate_d1_d2` function that computes the `d1` and `d2` parameters required by the Black-Scholes formula. These parameters are functions of the current stock price, strike price, risk-free rate, time to maturity, and implied volatility. The `BS_option_values` function uses the previously defined `calculate_d1_d2` function to calculate the values of the call and put options based on the Black-Scholes model. It returns both values for a given set of input parameters.

Results

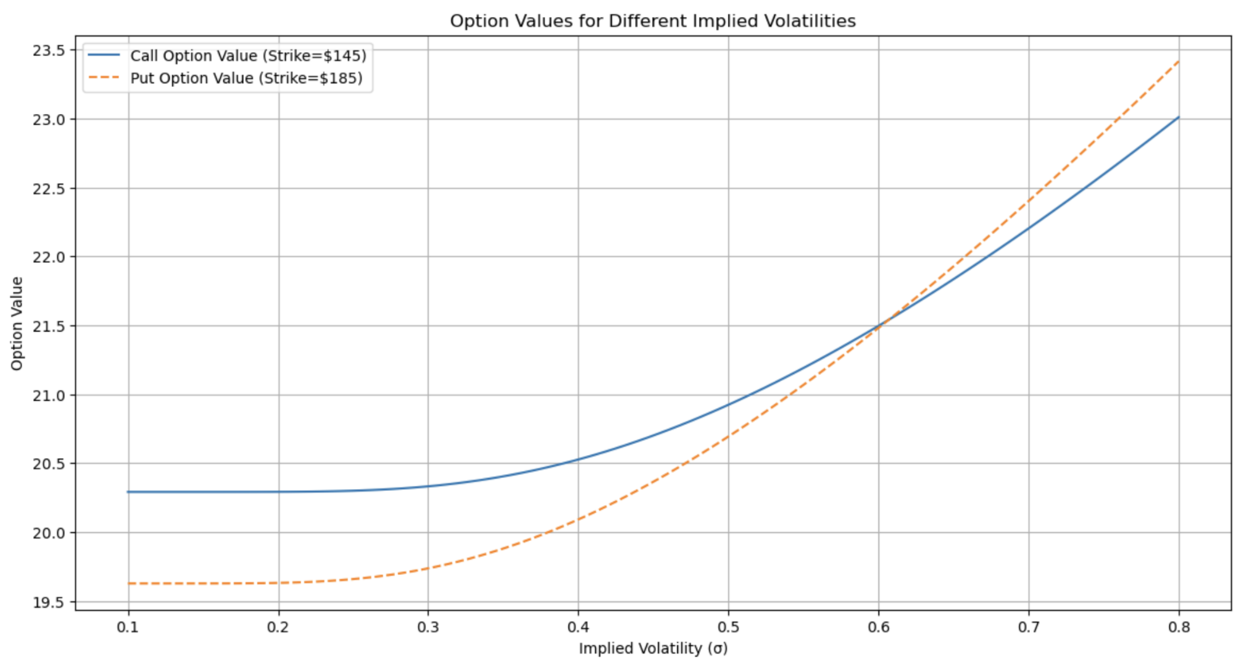
First, I calculated the time to maturity using the `calculate_ttm` function. Given the current date is 03/03/2023 and the expiration date is 03/17/2023, the ttm is around 0.0384.

```
Time to maturity:  0.038356164383561646
```

Then, the `BS_option_values` function is applied to each implied volatility in the range from 10% to 80% to calculate the call and put option values. The results are stored in a list of tuples. Notice that for simplicity, I assumed the strike price is equal to the current price.



As shown from the graph above, as volatility increases, the value of options (call and put) increase linearly. Next, I tested for strike prices different from the current price.



The updated graph shows the values of a call option with a strike price of $145 and a put option with a strike price of $185 as the implied volatility ranges from 10% to 80%.

- **Call Option (Strike=$145)**: This call option is in-the-money (ITM) because the strike price is significantly below the current stock price of $165. The value of this call option is higher at low implied volatility levels due to its substantial intrinsic value. As the implied volatility increases, the option's value also increases, but the relative increase is more subdued compared to an at-the-money option due to the already high intrinsic value.

- **Put Option (Strike=$185)**: The put option, with a strike price of $185, is out-of-the-money (OTM) and, like the call option, starts with a value that reflects only its time value. This value is initially lower because the stock price would have to move significantly for the option to become ITM. However, as the implied volatility increases, the option value increases more steeply than the ITM call option. This is because higher volatility has a greater relative impact on an OTM option, increasing the probability that the stock price could decrease below the strike price by expiration.

These graphs show the non-linear relationship between option value and implied volatility, particularly how changes in volatility can disproportionately affect ITM and OTM options. The steepness of the put option's curve at higher volatilities also underscores the leverage effect inherent in options: small changes in stock price can lead to large changes in the value of OTM options.

Certainly, here's a revised paragraph that connects the concept of supply and demand with the changes in implied volatility, as illustrated by the option pricing graph:

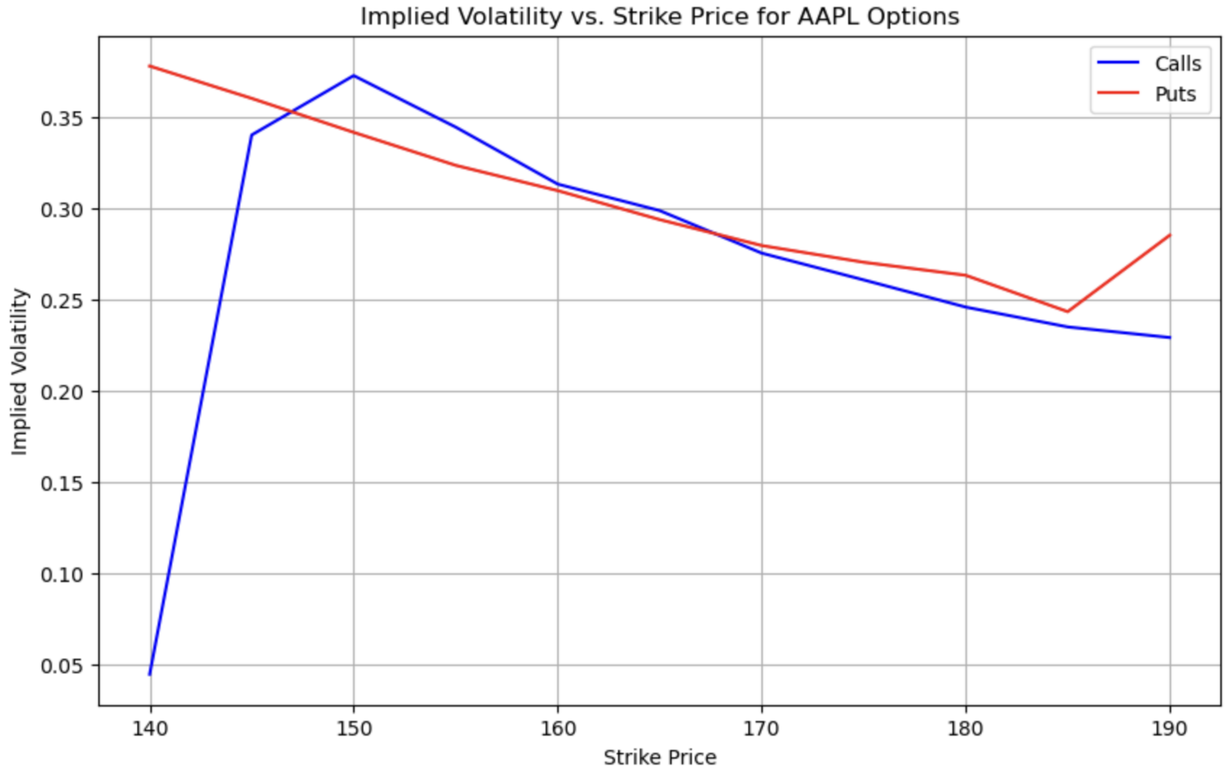
When a particular asset experiences a surge in demand that surpasses available supply, its price typically escalates. This uptrend in price is mirrored in the options market through an increase in implied volatility. The rationale behind this is straightforward: as traders compete to establish their stakes in the asset by driving its price upward, there emerges a shared expectation of more pronounced price swings in the future. Such expectations fuel the implied volatility, a proxy for the market's forecast of the asset's potential fluctuation. On the flip side, when the market is saturated with an asset, surpassing the interest of buyers, the price begins to wane. This decline is often accompanied by a decrease in implied volatility, signaling a market consensus of diminished variability in future prices. The graph of option values against implied volatilities encapsulates this phenomenon, reflecting the intrinsic connection between market sentiment, as expressed through supply and demand, and the perceived risk of price changes, as captured by implied volatility.

Problem 2

In this question, I first computed the implied volatilities by finding the roots of the functions defined as the discrepancies between the calculated values of AAPL options and their market prices, with the outcomes as follows:

	Stock	Expiration	Type	Strike	Last Price	Implied Volatility
0	AAPL	12/15/2023	Call	140	30.95	0.044813
1	AAPL	12/15/2023	Call	145	26.74	0.340517
2	AAPL	12/15/2023	Call	150	22.80	0.373077
3	AAPL	12/15/2023	Call	155	18.40	0.344828
4	AAPL	12/15/2023	Call	160	14.15	0.313586
5	AAPL	12/15/2023	Call	165	10.55	0.299043
6	AAPL	12/15/2023	Call	170	7.21	0.275727
7	AAPL	12/15/2023	Call	175	4.63	0.261042
8	AAPL	12/15/2023	Call	180	2.67	0.246144
9	AAPL	12/15/2023	Call	185	1.40	0.235265
10	AAPL	12/15/2023	Call	190	0.69	0.229456
11	AAPL	12/15/2023	Put	140	0.61	0.378308
12	AAPL	12/15/2023	Put	145	0.92	0.360580
13	AAPL	12/15/2023	Put	150	1.36	0.341955
14	AAPL	12/15/2023	Put	155	2.00	0.323863
15	AAPL	12/15/2023	Put	160	2.99	0.310012
16	AAPL	12/15/2023	Put	165	4.32	0.294111
17	AAPL	12/15/2023	Put	170	6.16	0.279894
18	AAPL	12/15/2023	Put	175	8.68	0.270720
19	AAPL	12/15/2023	Put	180	11.83	0.263578
20	AAPL	12/15/2023	Put	185	15.30	0.243641
21	AAPL	12/15/2023	Put	190	20.16	0.285541

Next, I plotted the implied volatility vs the strike price for Puts and Calls:



The shape of the implied volatility curve against the strike price of options can reveal much about market sentiments and expectations:

Call Options:

- **Downward Sloping**: A decrease in implied volatility with higher strike prices indicates a typical volatility skew, suggesting higher risk associated with in-the-money options.
- **Market Dynamics**: This skew often results from investors' risk aversion, increased stock volatility during price declines (leverage effect), and higher demand for protective puts influencing call prices.

Put Options:

- **Upward Sloping**: An increase in implied volatility with lower strike prices reflects a reverse skew or smile, indicating a premium for downside protection.
- **Market Dynamics**: Bearish sentiment, event risk anticipation, and the market's pricing of extreme outcomes (fat tails) contribute to this pattern.

Combined View:

- **Skew and Smile**: A skew in calls and a smile in puts together suggest a market wary of downside risks but also aware of potential extreme price movements.
- **Market Dynamics**: Imbalances in market orders, non-constant volatility perceptions, and the pricing of jump risks influence this complex shape.

Deviations from Theory:

- The Black-Scholes model's constant volatility and normal distribution assumptions often fail to match real-market conditions, as evidenced by the skew and smile patterns.

Conclusion:

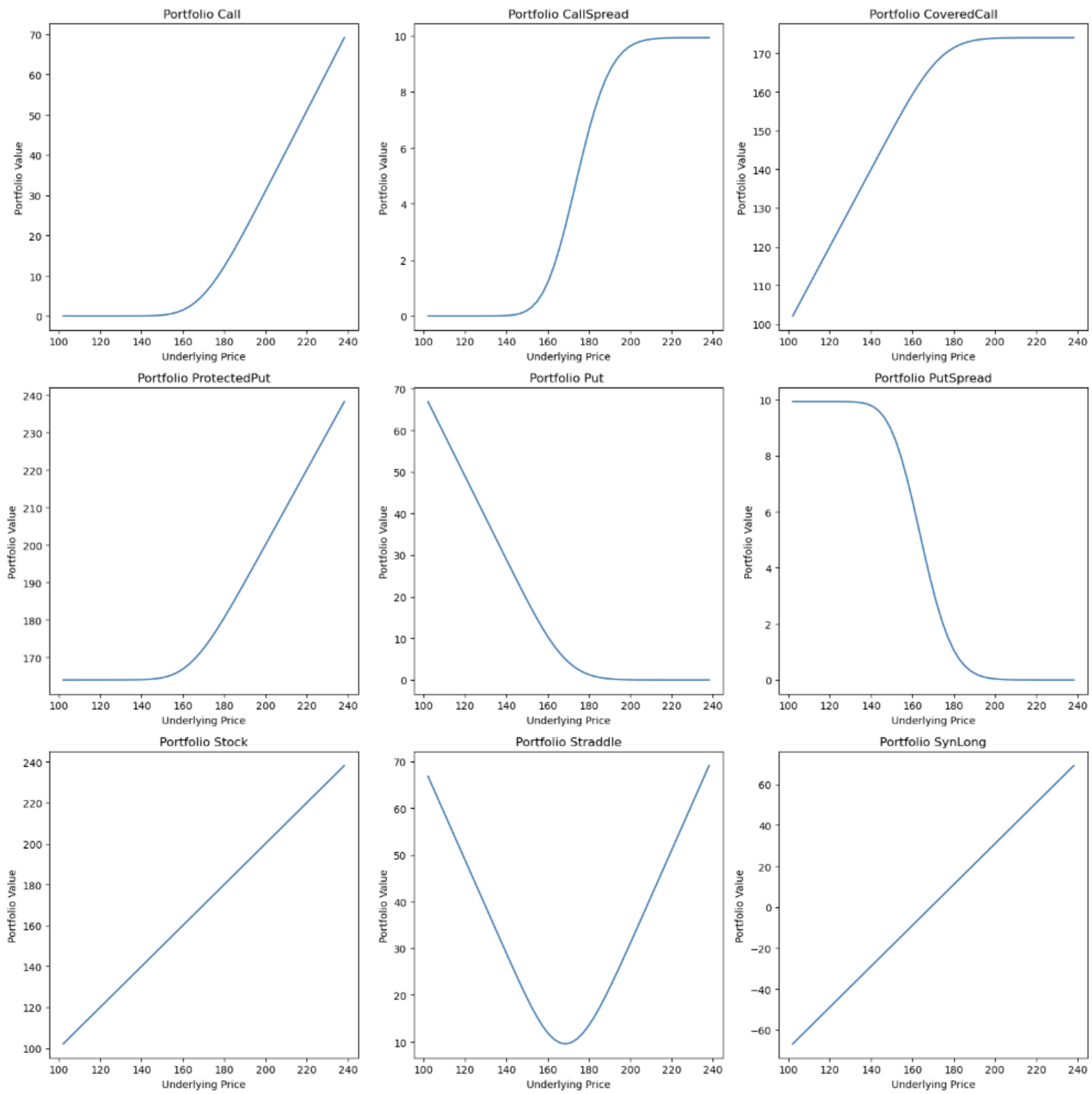
The implied volatility curves for calls and puts provide insights into market behavior, highlighting the need for nuanced approaches to option pricing and risk management beyond traditional models.

Problem 3

Implementation

The `black_scholes` function computes prices for European-style options using the underlying stock price, strike price, time to maturity, risk-free rate, dividend yield, volatility, and option type. A companion function, `simulate_portfolio_values`, leverages this to assess portfolio valuations across a spectrum of stock prices. It multiplies the derived option values—or the stock prices in the case of equities—by holdings, then aggregates by portfolio. After setting up the market parameters and assuming a uniform implied volatility, we simulate stock prices between 60% to 140% of the current level. The results, plotted for each portfolio, reveal distinct risk and reward profiles: the linear payoff for single options, capped gains for covered calls, protected downsides for puts, leveraged returns for synthetic longs, and the pronounced "V" shapes for straddles, each shape delineating the strategy's response to stock price movements.

Results



Regarding the discussion of the shapes of each graph:

- **Call and Put Portfolios**: These graphs typically show a hockey stick shape, with the call (put) portfolio increasing (decreasing) in value as the underlying price moves above (below) the strike price.
- **CallSpread and PutSpread Portfolios**: The spreads involve buying and selling options with different strikes, which creates a payoff that is limited both on the upside and downside, resulting in a tent-shaped or inverted tent-shaped graph.
- **Covered Call Portfolio**: This graph shows a rising value as the stock price increases, but it flattens out at the strike price of the written call option, reflecting the capped upside potential.
- **Protected Put Portfolio**: Here, the value decreases with the stock price but levels off once the put option's protection kicks in, indicating a floor in the portfolio value.
- **SynLong Portfolio**: This portfolio mimics a long stock position using options, so the graph would typically look similar to that of a long stock position but can show exaggerated moves due to leverage.
- **Straddle Portfolio**: A straddle portfolio graph is shaped like a "V" or an inverted "V", showing significant gains when the stock price moves significantly away from the strike price, either up or down.

Each portfolio's shape reflects the combination of options and/or stock positions, showing how their values are expected to change with the underlying asset's price movements.

After simulating AAPL returns 10 days ahead, we obtained the following results in terms of mean, VaR, and ES:

Portfolio	Mean	VaR	ES
Call	0.413354	16.360143	19.603306
CallSpread	-0.174533	1.950891	2.993261
CoveredCall	0.055617	1.868837	3.426501
ProtectedPut	0.578513	16.647456	20.241670
Put	0.085788	0.331660	0.335255
PutSpread	0.068352	0.200116	0.202589
Stock	0.535874	17.441673	21.771266
Straddle	0.499141	15.039281	17.186994
SynLong	0.327566	17.681005	22.019618

Call and Put Options: The Call portfolio shows a positive mean, indicating profitability, with moderate risk as evidenced by the Value at Risk (VaR) and Expected Shortfall (ES) metrics. Similarly, the Put portfolio, while showing a lower mean return, also exhibits lower risk. These strategies benefit from directional moves in the stock price—calls profit from upward moves, while puts gain from downward trends.

Call Spread and Put Spread: The negative mean for the Call Spread suggests limited success in the current market conditions, but the lower VaR and ES values highlight a reduction in risk compared to the basic Call and Put options. The Put Spread has a positive mean with very low risk, indicating a conservative strategy that aims to protect against downside risk while providing some upside potential.

Covered Call and Protected Put: The Covered Call has a low mean return with moderate risk, while the Protected Put shows a higher mean return but also a higher risk, suggesting more aggressive protection against a decline in the stock's price.

Synthetic Long (SynLong) and Stock: The Synthetic Long position is designed to mimic the payoff of a long stock position using options. The data indicates that SynLong offers twice the mean return compared to the Stock portfolio, which suggests a leveraged position. However, this comes at the cost of increased risk, as both the VaR and ES are higher for SynLong, making it a high-risk, high-reward strategy. The Stock portfolio has the highest mean return but also the highest risk, which is typical for equity investments.

Straddle: The Straddle's risk metrics are lower compared to the Call, Put, and Stock portfolios, indicating a comparatively lower risk profile. The positive mean return suggests that the portfolio has been successful in capitalizing on market volatility.