Week 5 Report

Howard Jin

10/7/2023

Problem 1

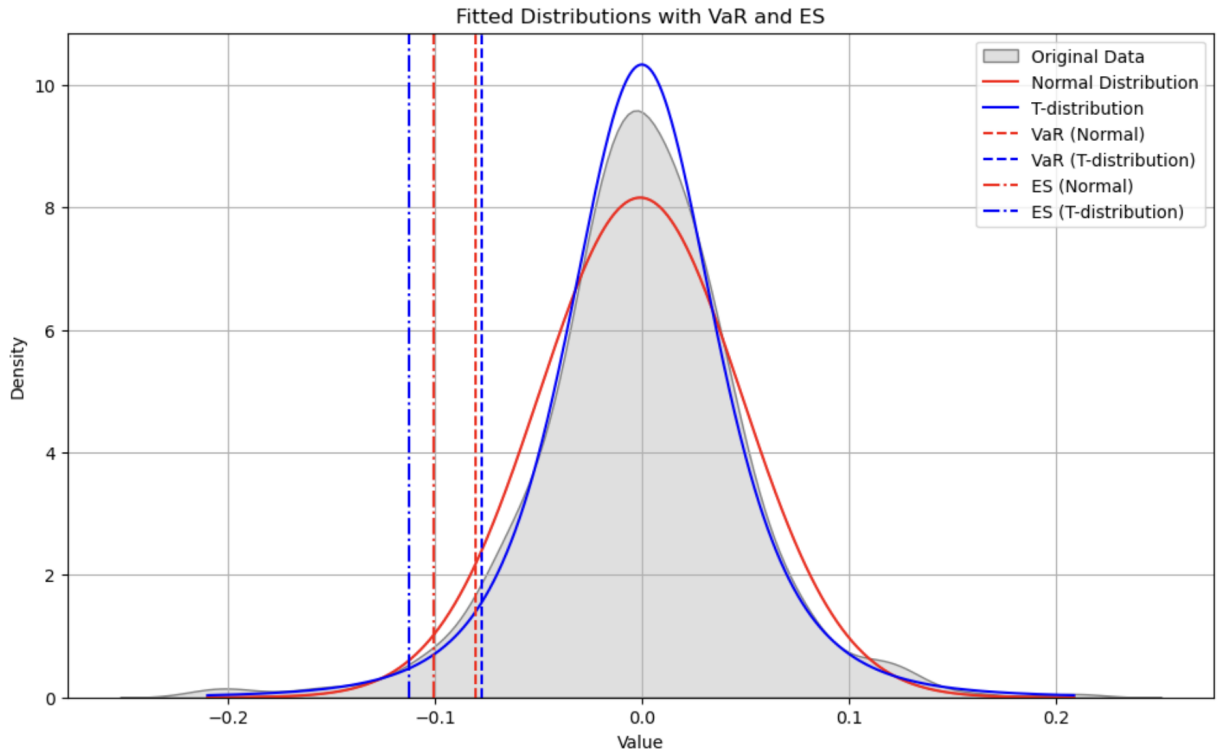
Implementation

I began with the `var_normal` function, aimed at calculating the Value-at-Risk (VaR) using normal distribution. By determining the mean and standard deviation of our dataset, the function then simulates potential future returns, subsequently identifying the VaR as the threshold value below which a specified proportion of returns is expected to fall. In parallel, the `var_t` function was designed to compute VaR but under the t-distribution framework. This function, utilizing Maximum Likelihood Estimation, fits the data to a t-distribution and then similarly simulates future returns to determine the VaR. To delve deeper into potential extreme losses, the `calculate_es` function was implemented. It takes the VaR and the simulated returns as inputs and computes the Expected Shortfall (ES) — the average loss expected to occur in the worst-case scenarios beyond the VaR.

Results

Subsequently, I visualized the original dataset, the fitted distributions, and the computed VaR and ES values for both the normal and t-distributions on a single graph:

	Distribution	VaR	ES
0	Normal	0.080970	0.102367
1	t-distribution	0.076567	0.111980



Observations

Overlaying the distribution PDFs, VaR, and ES values, several noteworthy observations arise:

- **Fit to Data**: Despite the data size being considerably large, bolstering the central limit theorem's prediction of a normal distribution, the t-distribution appears to fit the actual data more closely than the normal distribution. This suggests the presence of heavier tails or outliers in the data.
- **VaR Comparisons**: The VaR derived from the normal distribution is situated to the left of the VaR from the t-distribution. This indicates that the projected loss on a 5% worst-case scenario day under the normal distribution would surpass that under the t-distribution. Such a scenario could arise due to the differences in the critical values between the two distributions.
- **ES Comparisons**: Intriguingly, the Expected Shortfall for the normal distribution is to the right of the t-distribution's ES. This signifies that, given a loss surpassing the VaR, the expected loss under the t-distribution would be higher. This behavior can be attributed to the t-distribution's fatter tails, which accommodate more extreme values. Consequently, the mean of all these extreme values, represented by the ES, is smaller in the t-distribution.

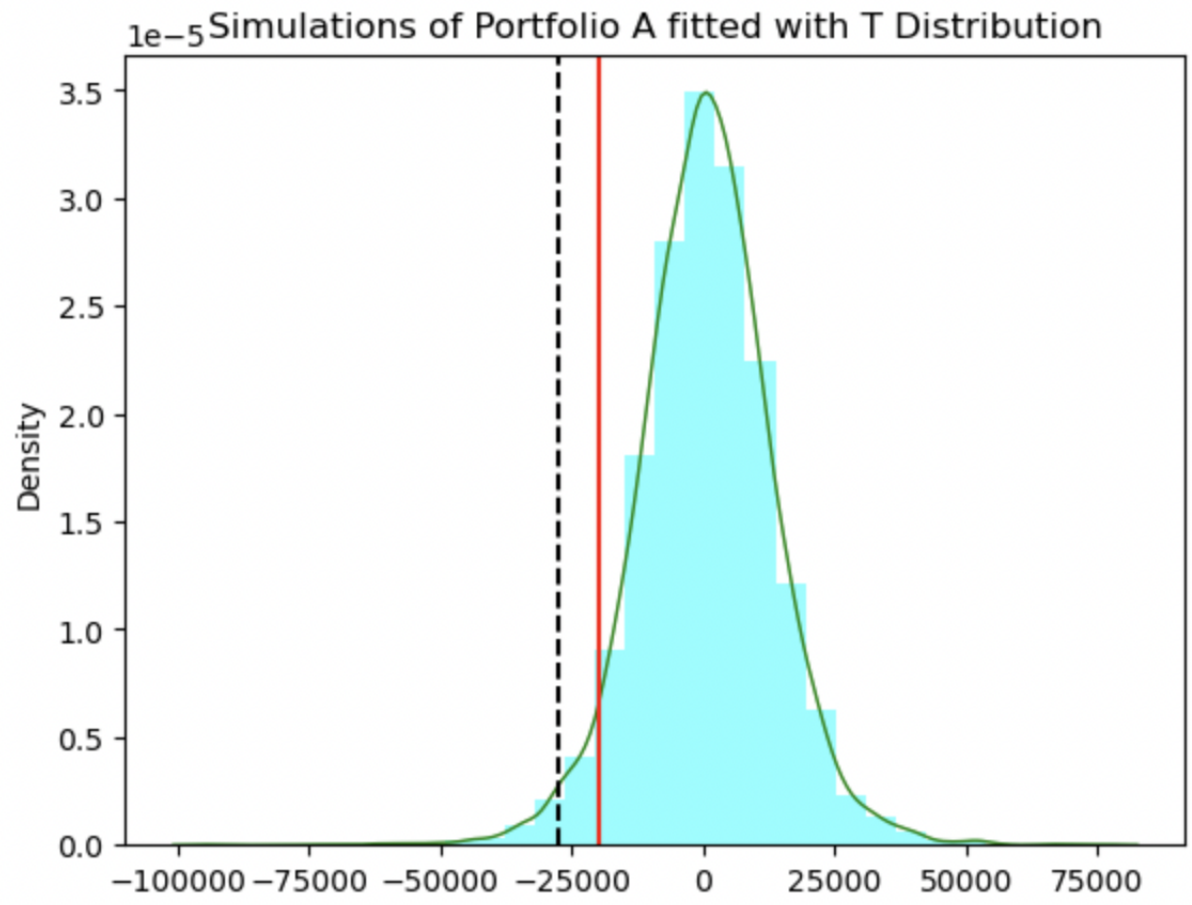
Problem 2

I developed a library called `qrm_hj`, which encompasses the `MC` and `VaR` packages. To utilize my code, you need to execute `pip install qrm_hj`. I have ensured that all functions within the library operate smoothly through comprehensive testing in my code.

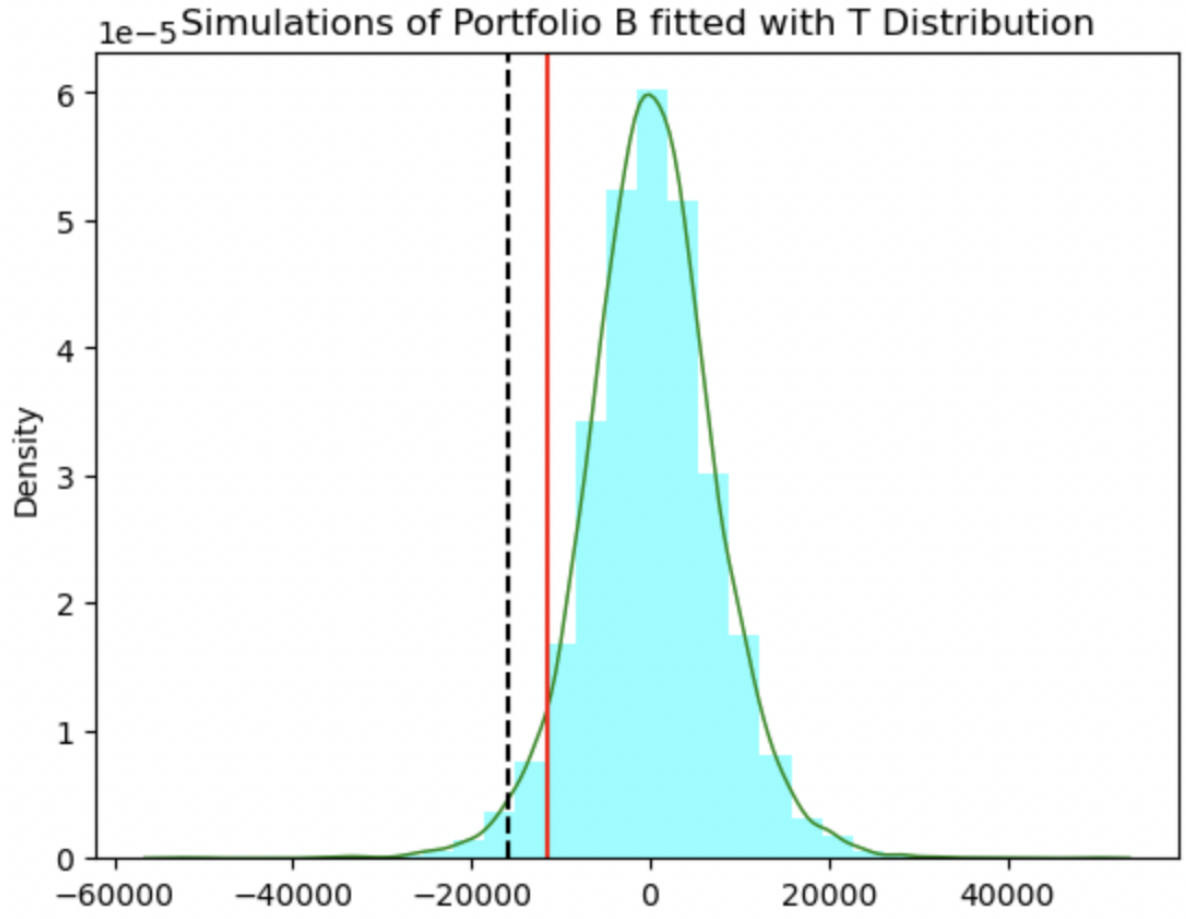
Problem 3

After fitting Generalized T Distribution to each stock, I applied the Gaussian Copula to model the returns. Here are the results:

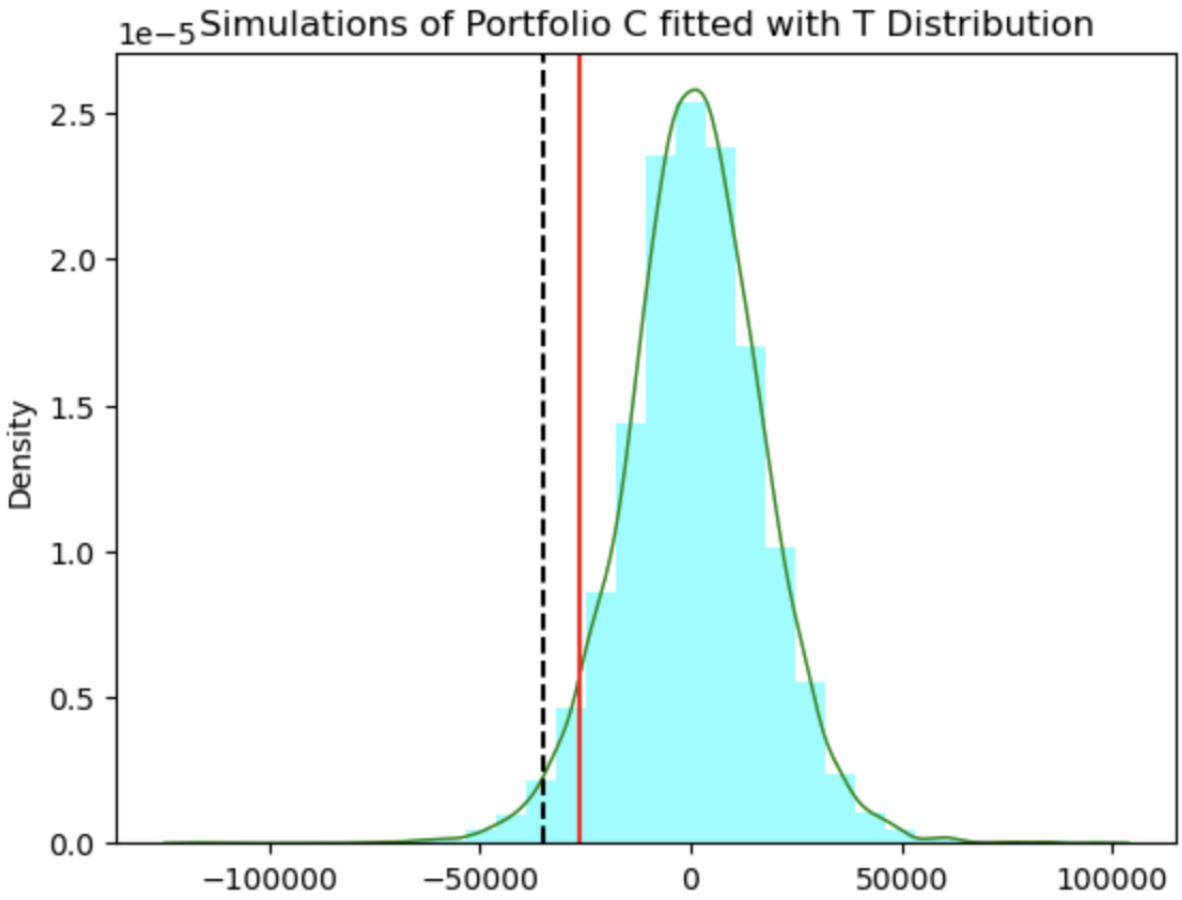
```
Porfolio A
Price:  [1089316.15994]
VaR:  19742.18655994873
ES:  27699.546692216223
```



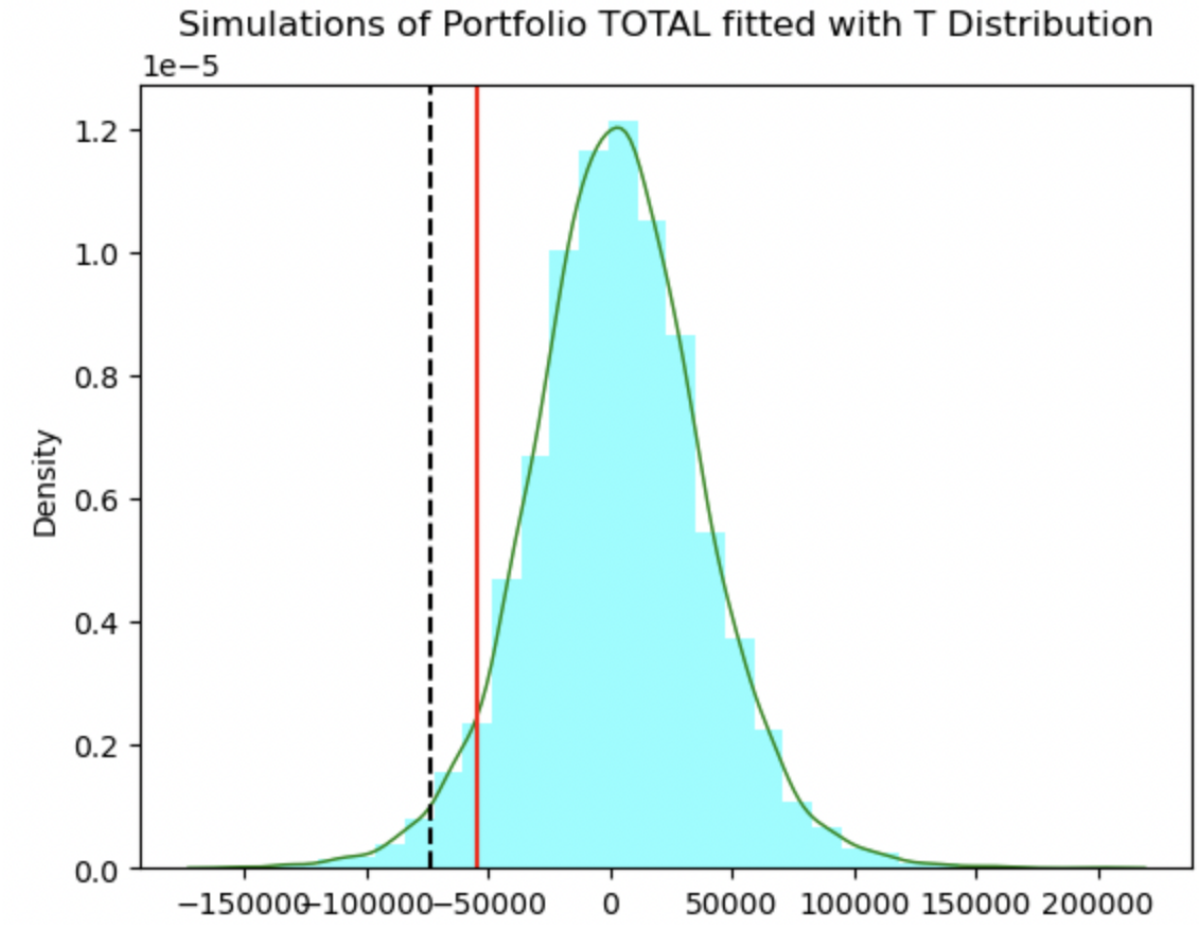
```
Porfolio B
Price:  [574542.40515]
VaR:  11572.827944431032
ES:  15915.441790277131
```



Porfolio C
Price: [1387409.50752]
VaR: 26107.838425595623
ES: 35066.29156664601



```
Porfolio TOTAL
Price:  [3051268.07261]
VaR:  54767.35397159837
ES:  73668.96059071671
```



Here are the VaR results from Problem 3 from Week 4:

Portfolio	Monte Carlo	Historical Simulation	Delta-Normal
A	15430.849312	17065.300954	15426.968017
B	8063.393224	10983.463847	8082.572402
C	18316.625616	22186.519226	18163.291619
Total	38731.797514	47618.778376	38941.375729

Observations

We observe that VaRs determined by the Generalized T distribution are akin to those derived from Historical Simulation, especially when compared to Delta VaR and Monte Carlo VaR. This suggests that the Generalized T distribution provides a closer representation of actual historical performances. This might be due to the fact that while normal distributions often underestimate the likelihood of extreme events, the T distribution, with its fat tail, accounts for these outliers similarly to real-world scenarios. However, given the constraints of historical data and recognizing that the Generalized T distribution is still a theoretical assumption, there are noticeable disparities between VaRs based on Generalized T assumptions and those based on Historical VaRs.