Week 4 Report

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Problem 1

Given three types of price returns and assume $r_t \sim N(0, \sigma^2)$, I derived the expected value and standard deviation for each:

Classical Brownian Motion

$$P_t = P_{t-1} + r_t$$

Expected Value

$$E(P_t) = E(P_{t-1} + r_t)$$
$$E(P_t) = E(P_{t-1}) + E(r_t)$$

Since P_{t-1} is known and $r_t \sim N(0, \sigma^2) \to E(r_t) = 0$

$$\therefore E(P_t) = P_{t-1}$$

Standard Deviation

$$SD(P_t) = SD(P_{t-1} + r_t)$$

Here, assume P_{t-1} is uncorrelated with r_t

$$SD(P_t) = SD(P_{t-1}) + SD(r_t)$$

Since P_{t-1} is known and $r_t \sim N(0, \sigma^2)$

$$\therefore SD(P_t) = \sigma$$

Arithmetic Return System

$$P_t = P_{t-1}(1 + r_t)$$

Expected Value

$$E(P_t) = E(P_{t-1}(1 + r_t)) = E(P_{t-1} + P_{t-1} \times r_t) = E(P_{t-1}) + E(P_{t-1} \times r_t)$$

Here, assume that P_{t-1} is independent of r_t

$$E(P_t) = E(P_{t-1}) + E(P_{t-1}) \times E(r_t) = E(P_{t-1}) \times (1 + E(r_t))$$

Since P_{t-1} is known and $r_t \sim N(0, \sigma^2)$

$$\therefore E(P_t) = P_{t-1} \times (1 + 0) = P_{t-1}$$

Standard Deviation

Since P_{t-1} is known and $r_t \sim N(0, \sigma^2)$, then

$$Var(P_t) = Var(P_{t-1}(1 + r_t)) = P_{t-1}^2 \times Var(1 + r_t) = P_{t-1}^2 \times Var(r_t) = P_{t-1}^2 \times \sigma^2$$

By taking the square root, we get

$$\therefore SD(P_t) = P_{t-1} \times \sigma$$

Geometric Brownian Motion

$$P_t = P_{t-1}e^{r_t}$$

Expected Value

$$E(P_t) = E(P_{t-1}e^{r_t}) = P_{t-1}E(e^{r_t})$$

Since e^{r_t} is log-normally distributed

$$\therefore E(P_t) = P_{t-1}e^{\frac{\sigma^2}{2}}$$

Standard Deviation

$$SD(P_t) = SD(P_{t-1}e^{r_t}) = P_{t-1}SD(e^{r_t})$$

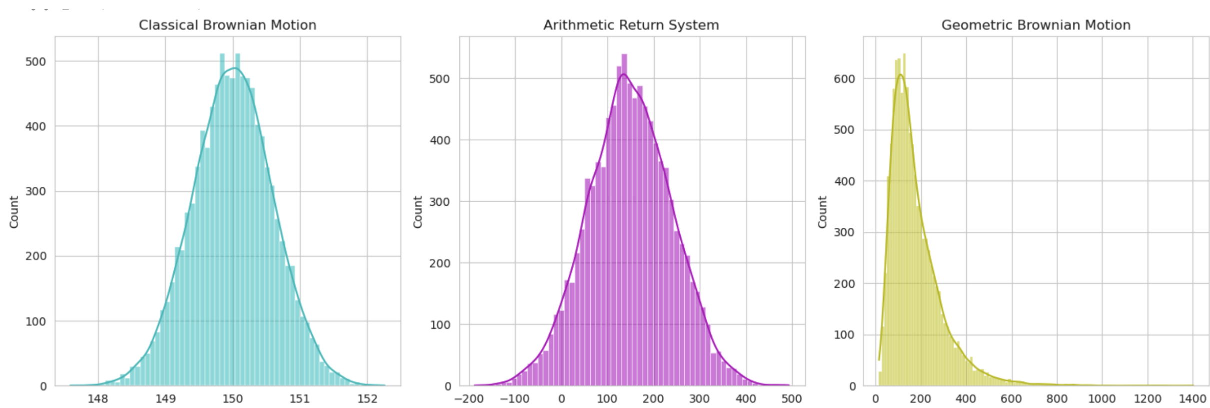
Since e^{r_t} is log-normally distributed

$$\therefore SD(P_t) = P_{t-1}\sqrt{(e^{\sigma^2} - 1)e^{\sigma^2}}$$

Given three types of price returns and assuming $r_t \sim N(0, \sigma^2)$, after setting $P_{t-1} = 150, SD(r_t) = 0.6$, we compare these theoretical results with simulated values.

Computations

```
##                  Return.Type             Metric Theoretical   Simulated
## 1 Classical Brownian Motion     Expected Value    150.0000 149.9951900
## 2  Arithmetic Return System     Expected Value    150.0000 150.5294234
## 3 Geometric Brownian Motion     Expected Value    179.5826 178.6490135
## 4 Classical Brownian Motion Standard Deviation      0.6000   0.5993011
## 5  Arithmetic Return System Standard Deviation     90.0000  89.3936950
## 6 Geometric Brownian Motion Standard Deviation    118.2152 116.6965767
```



Conclusion

For the three types of price returns examined, the following observations can be made:

1. **Classical Brownian Motion**:
   - The theoretical expected value matches the simulated mean closely, as both revolve around the value of P_{t-1}.
   - The standard deviation of the simulated results aligns with the theoretical value of σ, which is expected given that the return r_t is the only random component introduced.
2. **Arithmetic Return System**:
   - The theoretical and simulated expected values are consistent, both being P_{t-1}.
   - The standard deviation in the simulation is influenced by the magnitude of P_{t-1}, and this scaling effect is evident in the close alignment with the theoretical value of $P_{t-1} \times \sigma$.
3. **Geometric Brownian Motion**:
   - The expected value derived theoretically accounts for the exponential growth due to the return r_t. The simulated mean is in line with this value, showcasing the multiplicative nature of this return system.
   - The standard deviation displays the compounded effect of the return on the price, and the simulated results demonstrate this volatility, aligning with the theoretical calculations.

In summary, the simulations provide a practical verification of the theoretical derivations. The consistency between the theoretical and simulated results underscores the reliability of the models in representing the behavior of price returns under the given assumptions.

Problem 2

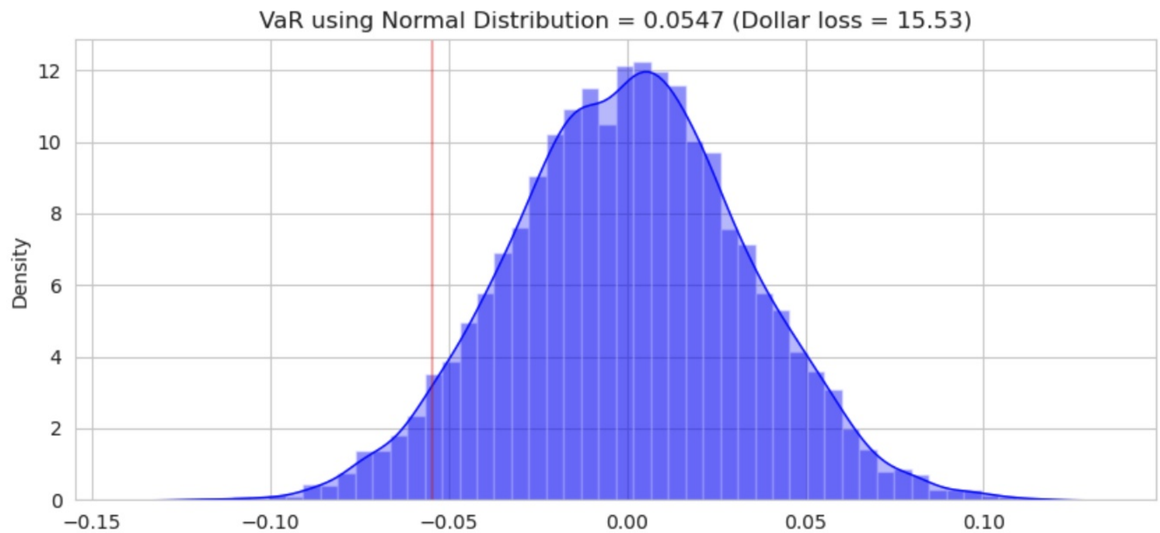
I implemented the function `return_calculate()`, aiming to compute the returns of all stocks listed in `DailyPrices.csv`. This function also allow users to choose their preferred method of return calculation, be it logarithmic or arithmetic.

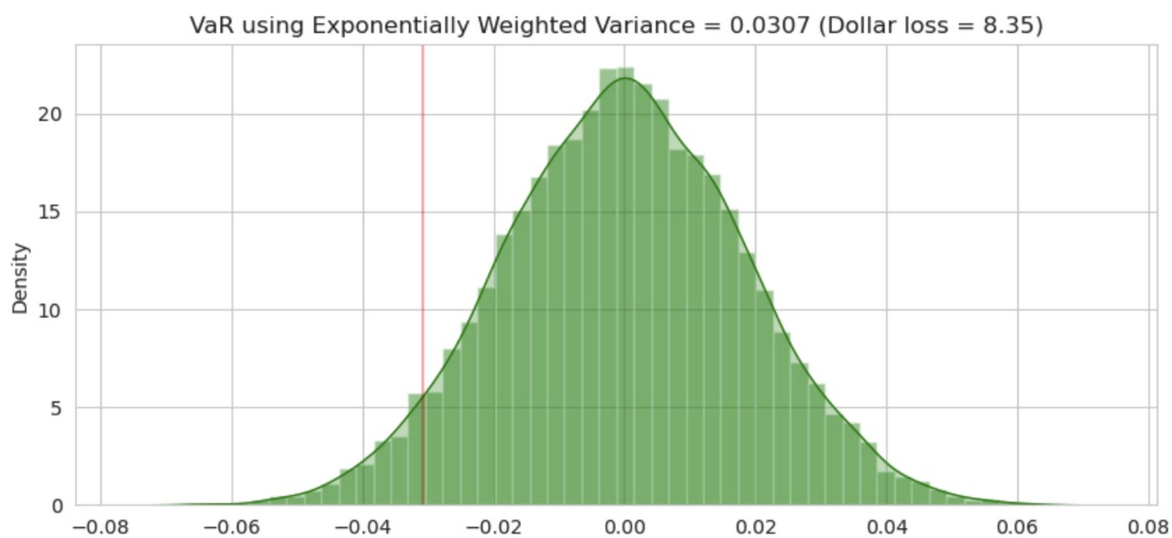
With our foundation set, I embarked on a deep dive into META's returns. Recognizing the significance of a centralized distribution, I adjusted the returns of META by subtracting its mean, ensuring that the adjusted returns had a mean of zero.

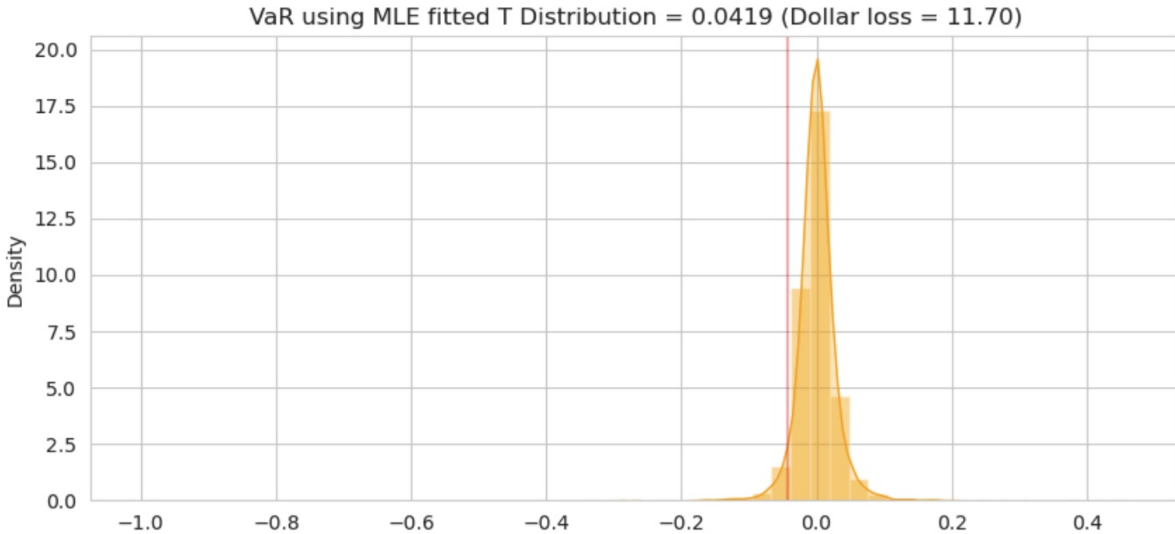
Then, five different methods are employed to calculate the Value at Risk (VaR) for META :

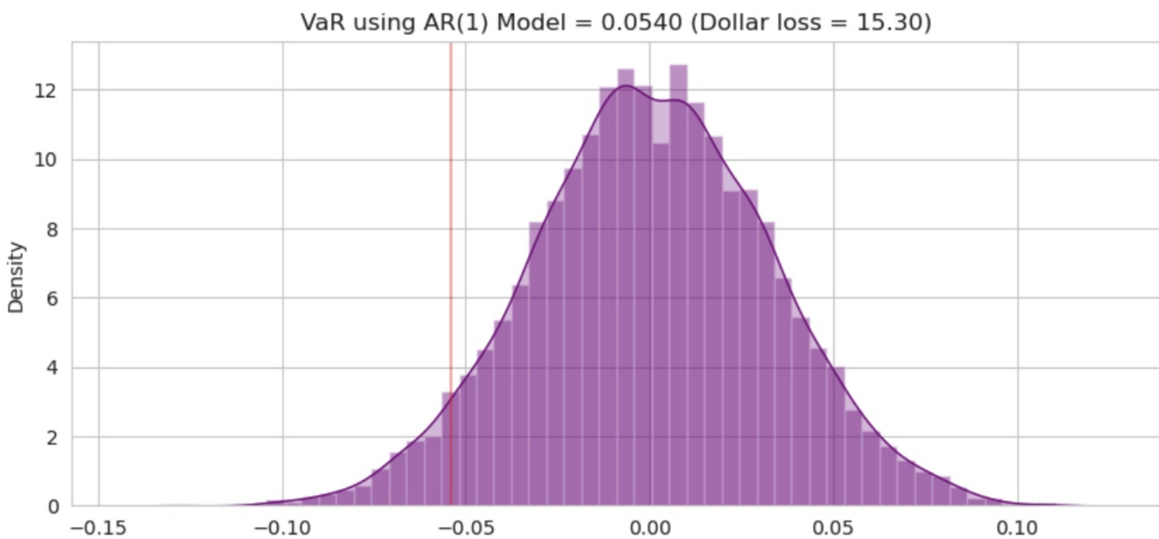
1. **Normal Distribution**: A foundamental approach assuming symmetrically distributed returns.
2. **Exponentially Weighted Variance**: A dynamic method that gives more prominence to recent returns, adapting to current market scenarios.
3. **MLE fitted T Distribution**: A flexible method adept at capturing the tails of the return distribution, often more accurately than the normal distribution.
4. **AR(1) Model**: Tailored for time series data, this method captures the inherent autocorrelations in returns, reflecting the influence of past events on present returns.
5. **Historic Simulation**: A pragmatic approach deriving VaR directly from past returns without the constraints of distributional assumptions.

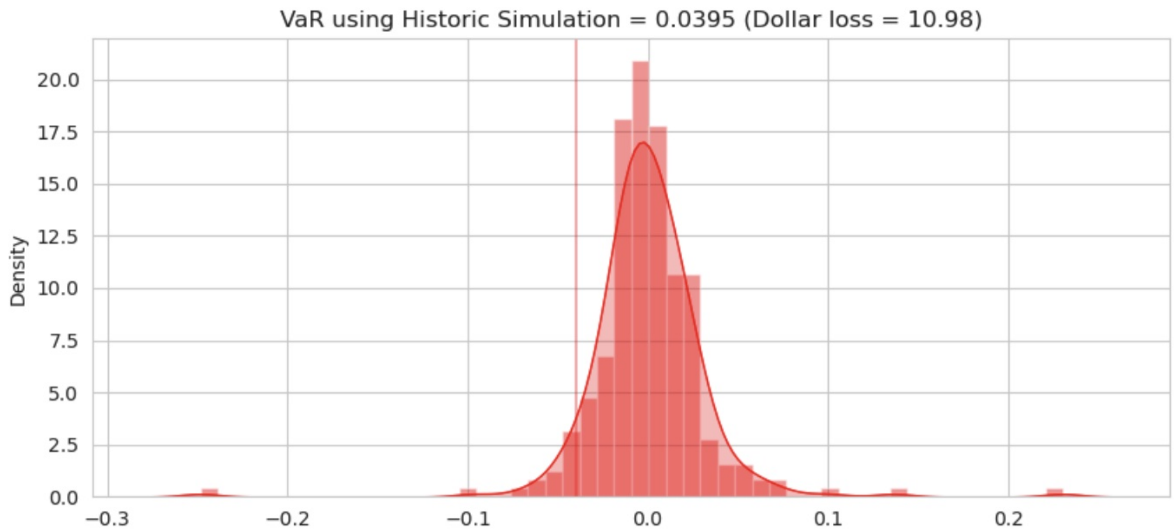
Results






VaR using MLE fitted T Distribution = 0.0419 (Dollar loss = 11.70)
Density
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4


VaR using AR(1) Model = 0.0540 (Dollar loss = 15.30)
Density
12
10
8
6
4
2
0
−0.15
−0.10
−0.05
0.00
0.05
0.10




```
{'Normal Distribution': 15.528271676723222,
 'Exponentially Weighted Variance': 8.353937224636637,
 'MLE fitted T Distribution': 11.704354714629357,
 'AR(1) Model': 15.302343687257153,
 'Historic Simulation': 10.9823813585034}
```

Conclusion

Upon examination of the Value at Risk (VaR) values for META returns, several insightful observations can be made:

- **Historic Simulation**: With a VaR of 10.98, this method, which derives its results directly from past returns, offers a middle-ground perspective on risk. This reflects historical trends without making assumptions about return distributions.
- **Normal Distribution** and **AR(1) Model**: These methods, yielding VaRs of 15.53 and 15.30 respectively, present a higher risk perspective. The close proximity of their results suggests that both the basic statistical properties and the time-series properties of the META returns perceive similar levels of risk.
- **MLE fitted T Distribution**: This approach, with a VaR of 11.70, provides a risk assessment that's slightly above the historic simulation. Its ability to adapt the distribution based on data makes it a relatively robust method, suggesting that the actual distribution of returns might have heavier tails than a normal distribution.
- **Exponentially Weighted Variance**: Remarkably, this method reports the lowest VaR at 8.35. Given its methodology, which places greater weight on recent returns, this suggests that the most recent returns might be less volatile or that the older returns were more extreme, thereby pulling down the VaR when recent data is emphasized.

In summary, the choice of VaR calculation methodology can lead to varied interpretations of risk. The MLE fitted T distribution appears to offer a balanced view, suggesting a distribution that's not strictly normal. The particularly low VaR from the exponentially weighted variance method could signal a period of relative stability in recent times.

Problem 3

I used three methods to calculate VaR:

Monte Carlo Simulation:

Assumptions

1. **Return Distribution**: Returns can follow any distribution, but often the assumption is that they follow a normal distribution.
2. **Correlation**: Assumes that the relationship between asset returns, captured by the covariance or correlation matrix, remains constant throughout the simulation period.
3. **Stochastic Process**: Assumes that the returns' evolution follows a certain stochastic process, often geometric Brownian motion for stock prices.

Historical Simulation:

Assumptions

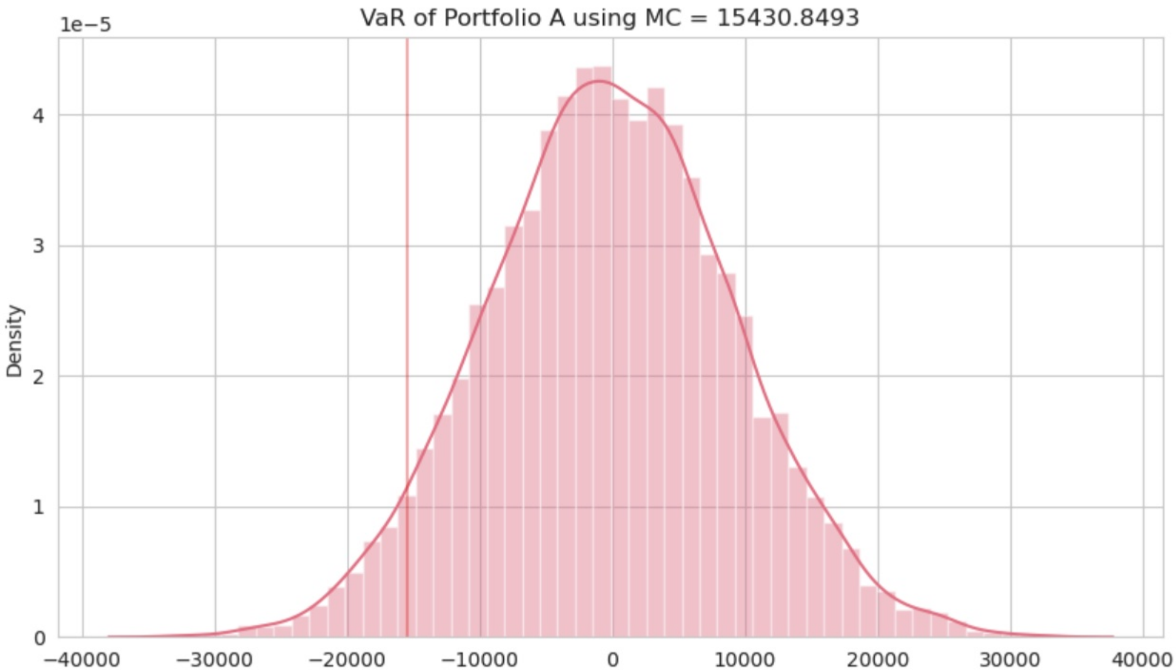
1. **Past as a Predictor**: Assumes that the past is a good predictor of the future. This method directly resamples from the past returns to generate the distribution.
2. **No Normality Assumption**: Does not make any assumptions about the return distribution.
3. **Static Relationships**: Assumes that the relationships between asset returns remain the same as in the historical data.

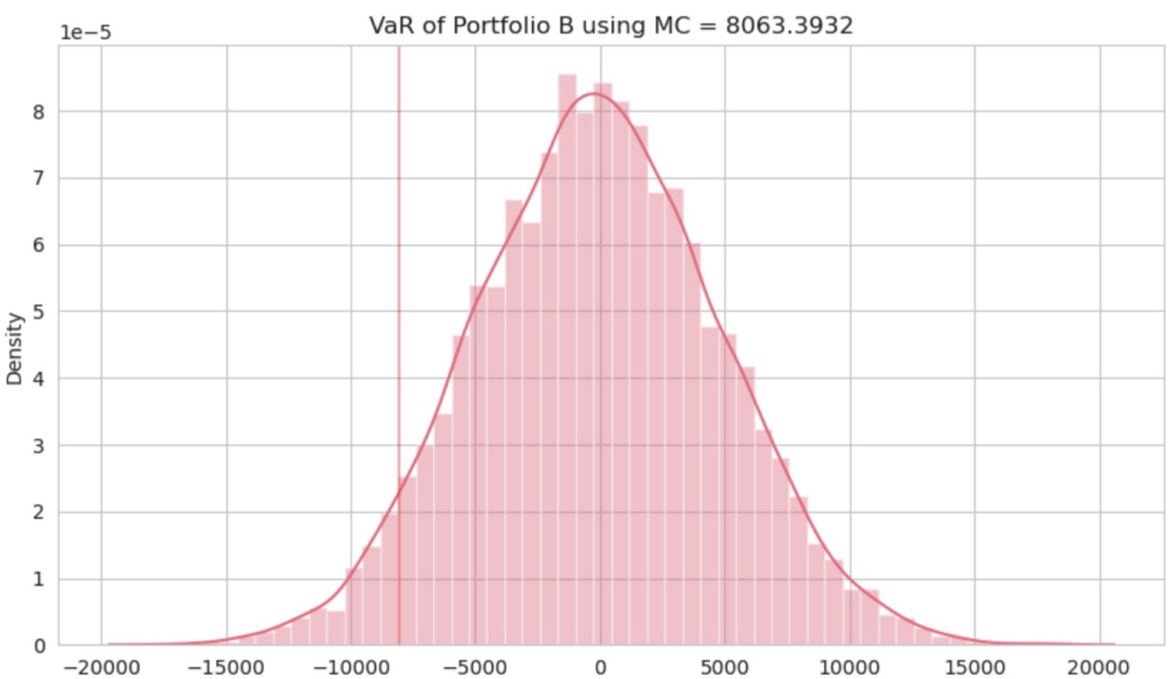
Delta-Normal:

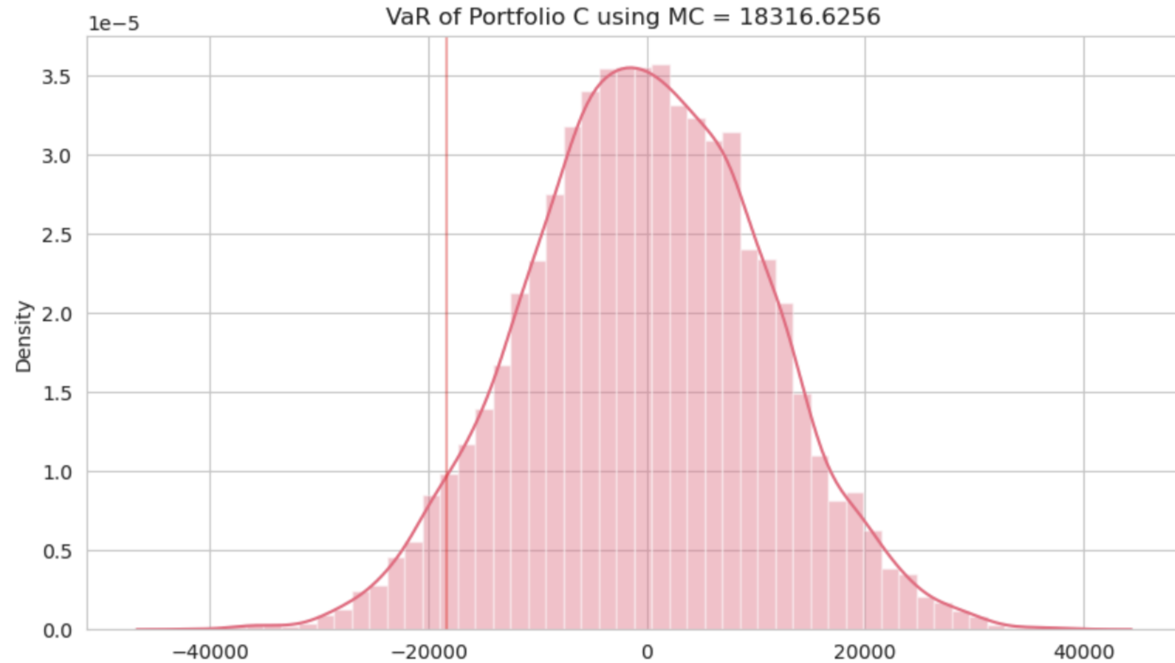
Assumptions

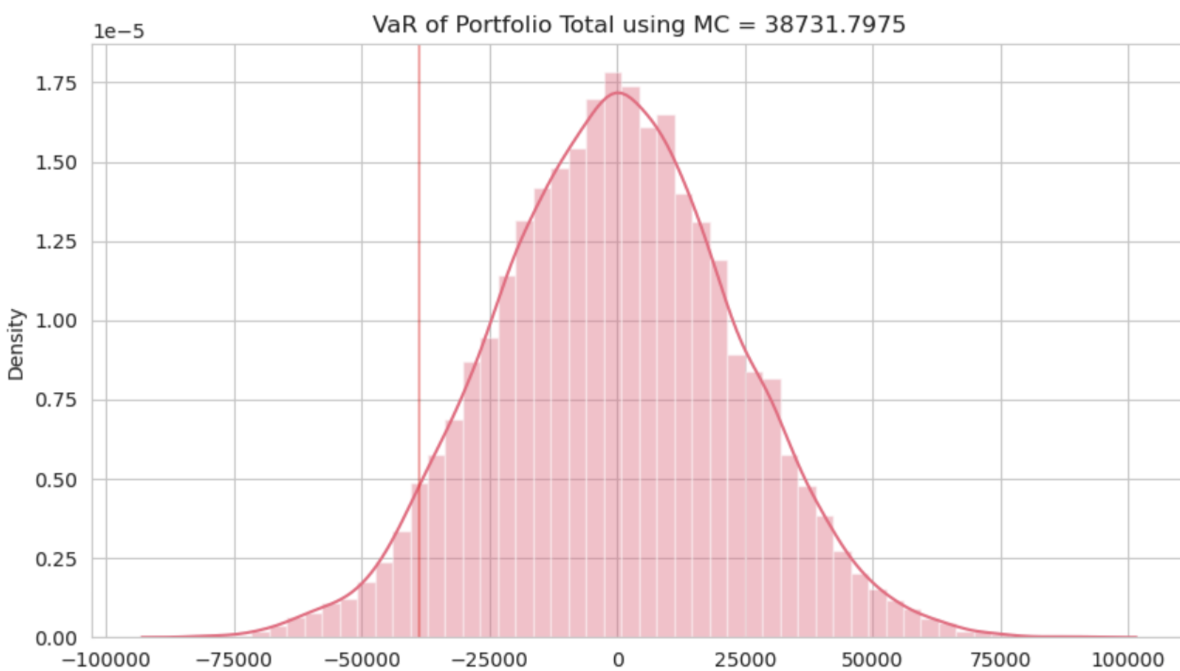
1. **Normality**: Assumes that asset returns are normally distributed.
2. **Linearity**: Assumes that the portfolio's value change is linearly dependent on the individual asset returns.
3. **Constant Volatility and Correlation**: Assumes that the volatility and correlation between assets remain constant.

Results


VaR of Portfolio A using MC = 15430.8493
1e−5
Density
0
1
2
3
4
−40000
−30000
−20000
−10000
0
10000
20000
30000
40000


VaR of Portfolio B using MC = 8063.3932
1e−5
Density
0
1
2
3
4
5
6
7
8
−20000
−15000
−10000
−5000
0
5000
10000
15000
20000


VaR of Portfolio C using MC = 18316.6256
1e−5
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Density
−40000
−20000
0
20000
40000


VaR of Portfolio Total using MC = 38731.7975
1e−5
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Density
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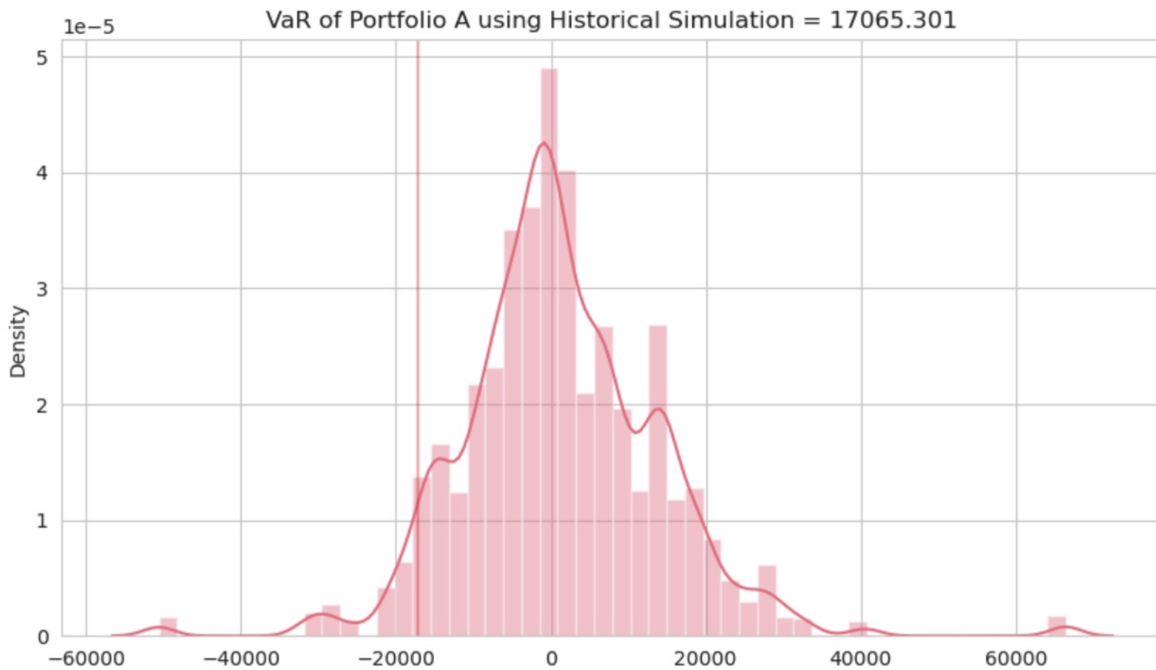
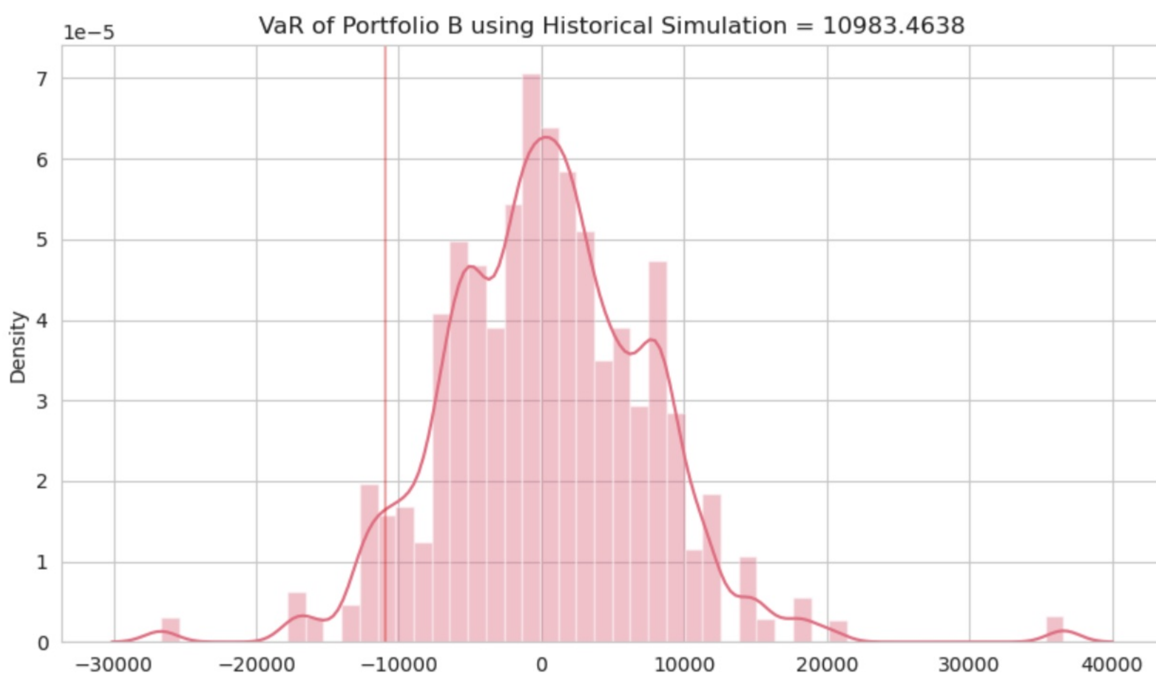
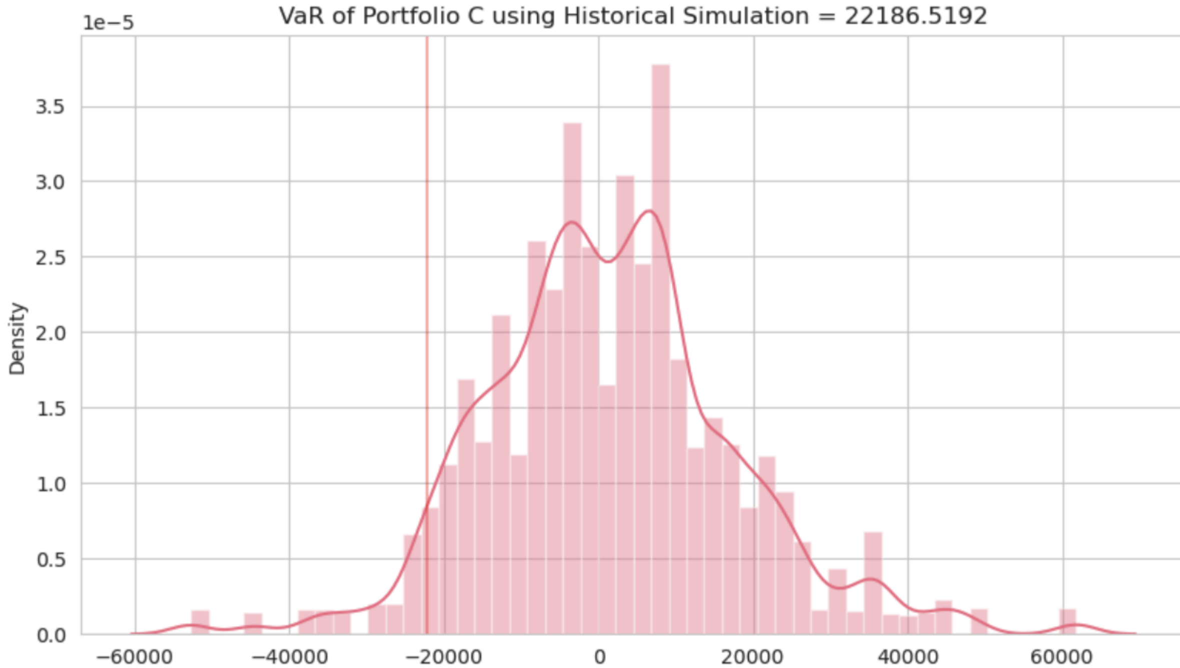
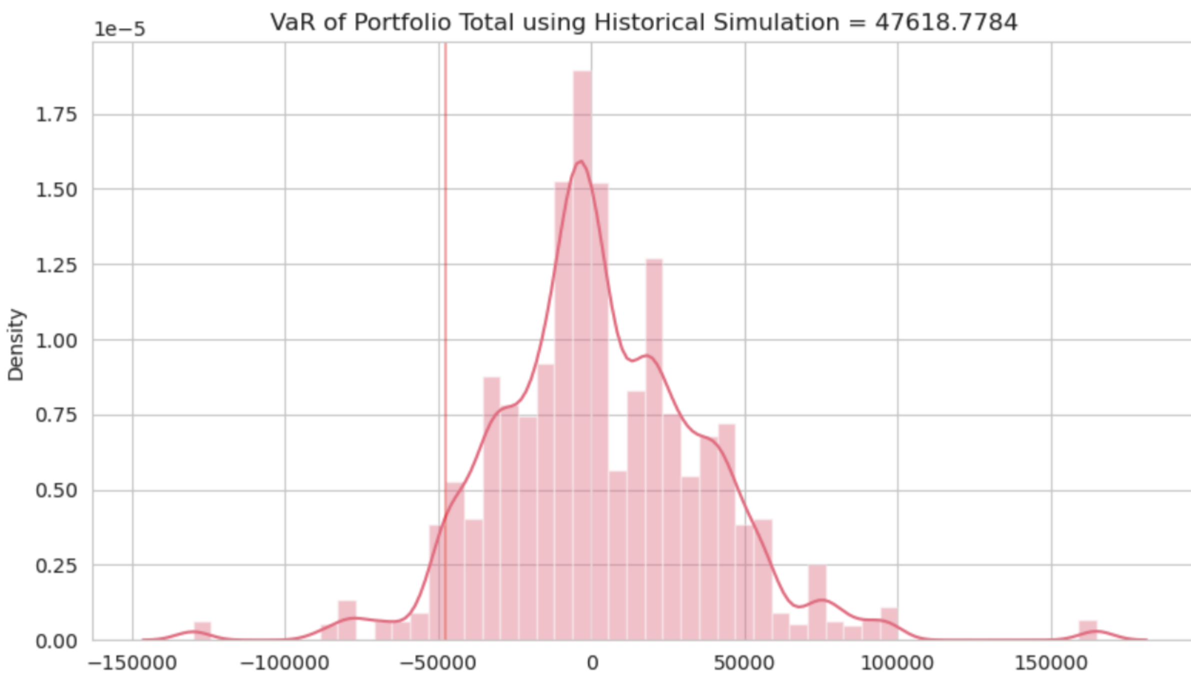
−75000
−50000
−25000
0
25000
50000
75000
100000


VaR of Portfolio A using Historical Simulation = 17065.301
1e−5
Density
5
4
3
2
1
0
−60000
−40000
−20000
0
20000
40000
60000


VaR of Portfolio B using Historical Simulation = 10983.4638
1e−5
Density
7
6
5
4
3
2
1
0
−30000
−20000
−10000
0
10000
20000
30000
40000


VaR of Portfolio C using Historical Simulation = 22186.5192
1e−5
Density
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−60000
−40000
−20000
0
20000
40000
60000


VaR of Portfolio Total using Historical Simulation = 47618.7784
1e−5
Density
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
−150000
−100000
−50000
0
50000
100000
150000

Portfolio	Monte Carlo	Historical Simulation	Delta-Normal
A	15430.849312	17065.300954	15426.968017
B	8063.393224	10983.463847	8082.572402
C	18316.625616	22186.519226	18163.291619
Total	38731.797514	47618.778376	38941.375729

Conclusion

- Monte Carlo Simulation: This approach uses simulated price paths to calculate potential losses. The VaR values obtained using the Monte Carlo method are relatively close to those from the Historical Simulation for portfolios A and B, but there's a noticeable difference for portfolio C and the total portfolio.
- Historical Simulation: This method directly uses historical data to estimate potential future losses. For all portfolios, the VaR values from the Historical Simulation are higher than the Monte Carlo and Delta-Normal methods.
- Delta-Normal Method: This approach assumes that returns are normally distributed. The VaR values using this method are fairly close to those obtained via the Monte Carlo approach.

To conclude, while the Delta-Normal and Monte Carlo methods provided relatively similar values, the Historical Simulation yielded the highest VaR estimates. The potential reason might be that the Historical Simulation method often yields the highest VaR estimates because it directly uses historical data to model potential future price changes, without making any assumptions about the underlying distribution of returns. This means that if there were extreme events or "tail events" in the historical data, they will be incorporated into the VaR calculation. In contrast, other methods, like Delta-Normal, assume returns follow a specific distribution (like the normal distribution), which might underestimate extreme events. Hence, Historical Simulation captures the actual observed extremes from the past, potentially leading to higher VaR estimates compared to methods that smooth out or ignore these extremes.