# Review

14 Multiple Choice $\rightarrow$ 40 pts

3 Free Response $\rightarrow$ 60 pts

100 pts.

# Integration

no u-substitution

no trig subs.

no even/odd

yes what integrals mean w.r.t. PDFs, CDFs, probability.

yes $\int x^n = \frac{x^{n+1}}{n+1}$ $\int e^{kx} = \frac{e^{kx}}{k}$

no Integration by parts.

# Derivatives

Only basics.

# Series

know $\frac{1}{1-r} = \sum_{k=0}^{\infty} r^k$ when $|r|<1$

geometric series

# Outline What We Know

EDA
↳ Histograms
↳ Boxplots
↳ Mean, Median, Mode

EZmode

what to do w/ even #s of data points?

Unimodal
Bimodal
Multimodal
Amodal

Quartiles: $Q_1$ $Q_2$ $Q_3$

aka Median

Ex1

1 2 3 4 5 6 7

$Q_2$

1 2 3 4 | 4 5 6 7

$Q_1$

$\frac{2+3}{2} = 2.5$

$Q_3$

$\frac{5+6}{2} = 5.5$

Ex2

1 2 3 4 5 6 9 1000

$Q_2 = \frac{4+5}{2} = 4.5$

1 2 3 4    5 6 9 1000

$Q_1 = 2.5$

$Q_3 = \frac{6+9}{2} = 7.5$

↳ IQR = Inter-Quartile Range: $Q_3 - Q_1$

↳ Tukey's 5 Number Summary: Min, $Q_1$, $Q_2$, $Q_3$, Max

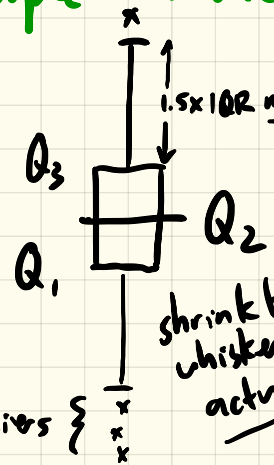
- Min … Max → Range
- $Q_1$ … $Q_3$ → IQR → Boxplot

↳ Percentiles (aka Quantiles)

(connection w/ CDF)

↳ Sample variance: $$\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

## Box plots




shrink back whisker to first actual data point.

plot

fliers/outliers {

$1.5 \times IQR$ = max possible whisker length.

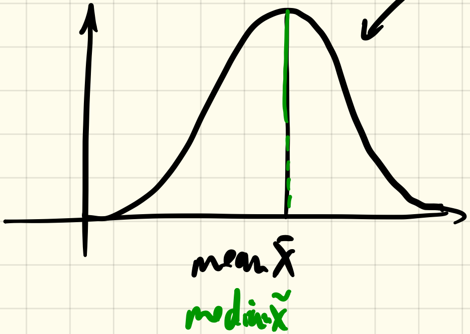
hipster constant

# Skew and you:

measure of asymmetry.

1. calc $\bar{x}$
2. calc $\tilde{x}$
3. compare and classify

v. symmetric!




$\bar{x} = \tilde{x}$

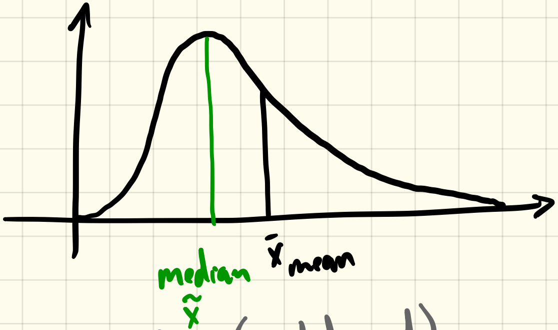
mean = median

<u>no</u> skew

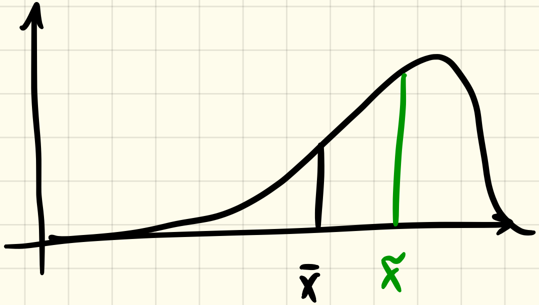
?

?

?




$\bar{x} > \tilde{x}$ (rightward) skew
positive




$\bar{x} < \tilde{x}$
negative
(leftward) skew

# PMF PDF CDF

tells me prob. of an outcome.
either: lookup table (see Q about O'Flaherty Fam)
— or —
Formula
- Binomial
- Poisson
- geom.

Probability Mass Function - Discrete RVs.

" Density Function - Cont. RVs.

Cumulative Distribution Function: both

Prob. Density at a particular outcome.

→ Prob of any specific value = 0

Only get prob > 0 when we integrate, i.e,

$P(a \leq X \leq b)$

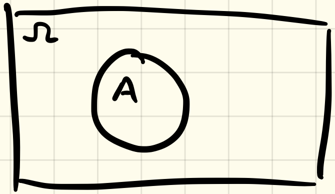
# Samples

- sample frame
- types of sample
  - census
  - systematic
  - stratified
  - uniform
- variable of interest

# Random Variables, Events

$\Omega$ = space of all possible outcomes.

$$P(\Omega) = 1$$




$$P(A) + P(A^c) = 1$$

since

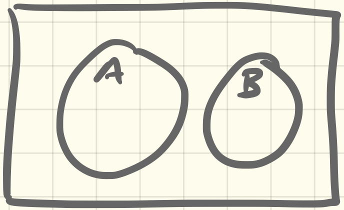
$$A \cup A^c = \Omega$$

$\cup$, $\cap$, Independence, Disjoint, Complement

# Prob of Union of two Events

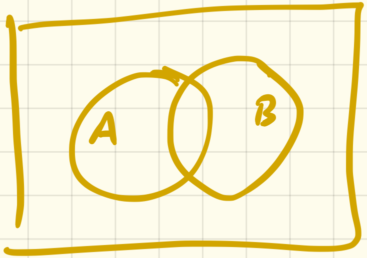
$$P(A \underset{\text{or}}{\cup} B) = P(A) + P(B)$$

only if A and B disjoint




What if A, B are not disjoint?

$$P(A \underset{\text{or}}{\cup} B) = P(A) + P(B) - \underbrace{P(A \underset{\text{and}}{\cap} B)}_{\text{adjustment required since A and B are not disjoint.}}$$




disjoint $\longleftrightarrow$ mutually exclusive

Q: can both A and B be true at once?
(happen)

# Independence (canonical example: flip 2 coins)

$$P(A \mid B) = P(A)$$

(outcome of B is totally irrelevant to A... A is independent of B)

$$\Updownarrow$$

$$P(B \mid A) = P(B)$$

$$\Updownarrow$$

$$P(A \cap B) = P(A)P(B)$$

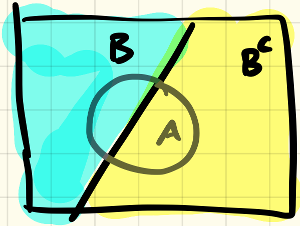
$$\Updownarrow$$

A, B Independent

good example of non-independence

- if we know a 5 has been drawn, the future draws from deck of cards will be affected.

# LTP




$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

Often use LTP to rewrite the denom. In Bayes' Rule Probs.

## Product Rule

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$$

$$P(Y)P(X|Y) = P(X \cap Y) = P(Y|X)P(X)$$

$$P(Y)P(X|Y) = P(Y|X)P(X)$$

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

Bayes

OH today till 5

OH weds 4-6

Slides to be posted.

# Expected Value

- If I draw $n$ draws from R.V. $X$, and I compute $\bar{x}$, then $E[X] = \bar{x}$ as $n \to \infty$

## Discrete

$$\left.\begin{array}{ll} \text{outcomes:} & 1+2,\ 1+4,\ 1+19 \\ \text{Probs:} & 1/3,\ \frac{1}{2},\ \frac{1}{6} \end{array}\right] \text{PMF}$$

$$\sum_i a_i \Pr(a_i) = 2\left(\frac{1}{3}\right) + 4\left(\frac{1}{2}\right) + 19\left(\frac{1}{6}\right) = E[X]$$

linearity of expectation

$$E[X+1] = E[X] + 1 \qquad E[aX+b] = aE[X]+b$$

# Ex

$P(x=1) = \frac{1}{2}$

$P(x=2) = \frac{1}{4}$

$P(x=3) = \frac{1}{8}$

$P(x=4) = \frac{1}{8}$

1 compute $E[X]$

$1 \frac{1}{2} + 2 \frac{1}{4} + 3 \frac{1}{8} + 4 \frac{1}{8}$

$\frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{2} = \frac{3}{2} + \frac{3}{8} = \frac{12}{8} + \frac{3}{8} = \frac{15}{8}$

$1 \, 7/8$

2 Compute $E[Y]$ where
$Y = 6X - \pi$

$E[Y] = E[6X - \pi] = 6E[X] - \pi$

$= 6\left(\frac{15}{8}\right) - \pi$

Recipe

$$\sum \text{outcome value} \times \overset{\text{(prob)}}{\text{weight}}$$

discrete

$$\sum_i a_i f(a_i)$$

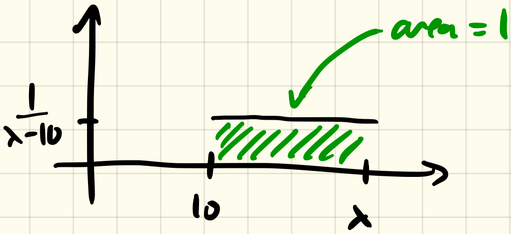
$$\int \text{outcome value} \times \text{prob density}$$

continuous

$$\int x f(x)\, dx$$

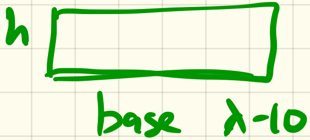
Compute E.V. of

unif(10, λ) where λ>10




$$\int_{10}^{\lambda} x \frac{1}{\lambda-10}\, dx$$

$$a^2-b^2=(a-b)(a+b)$$




base × h = 1

$(\lambda-10)\times h = 1$

$$h = \frac{1}{\lambda-10}$$

$$\frac{1}{\lambda-10}\int_{10}^{\lambda} x\, dx = \frac{1}{\lambda-10} \left.\frac{x^2}{2}\right|_{x=10}^{x=\lambda}$$

$$= \frac{1}{\lambda-10}\left(\frac{\lambda^2}{2}-\frac{10^2}{2}\right)$$

$$= \frac{1}{2}\cancel{\frac{1}{\lambda-10}}\cancel{(\lambda-10)}(\lambda+10) = \boxed{\frac{\lambda+10}{2}}$$

$f(x) = ax^2 \quad -1 \le x \le 1, \quad 0$ otherwise.

Is there a value of a that makes f(x) a <u>valid PDF</u>?
If so, find it. If not, 'splain why not?

① $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$P(\Omega) = 1$

② $f(x) \geq 0 \ \forall x$

$\forall$ forall

\forall

② says $a \geq 0$

① $a\int_{-1}^{1} x^2\,dx = 1$ set it

$\frac{2}{3}a = 1$ so $\boxed{a = \frac{3}{2}}$

$a \left.\frac{x^3}{3}\right|_{-1}^{1} = a\left(\frac{1^3}{3} - \frac{(-1)^3}{3}\right) = a\left(\frac{1}{3} - -\frac{1}{3}\right)$

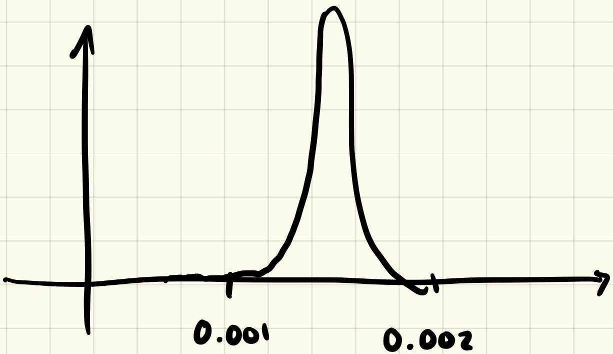
$= \frac{2}{3}a$

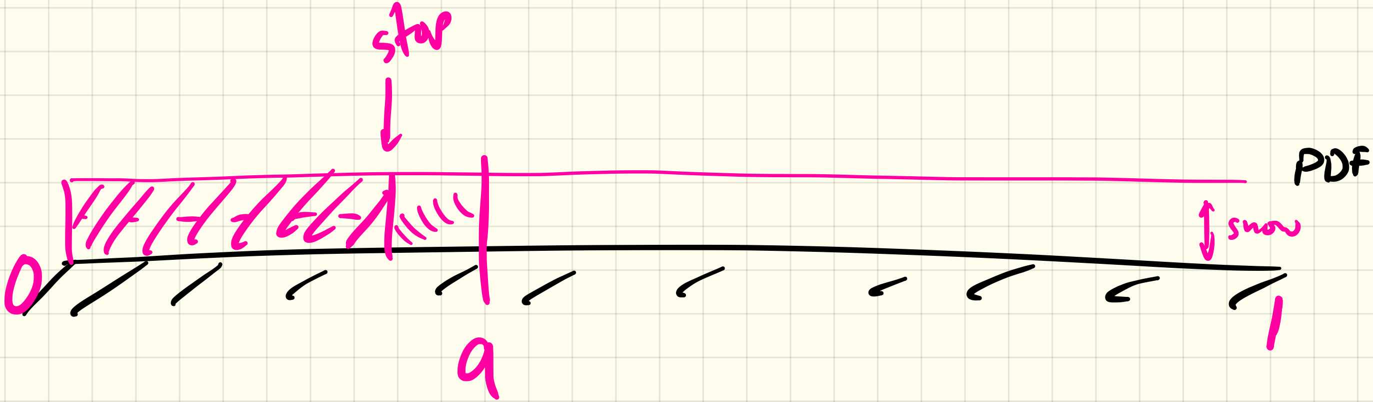
A PDF can be $>1$, $f(x) > 1$

but a PMF cannot.

If a PMF had a prob. that was $>1$, then

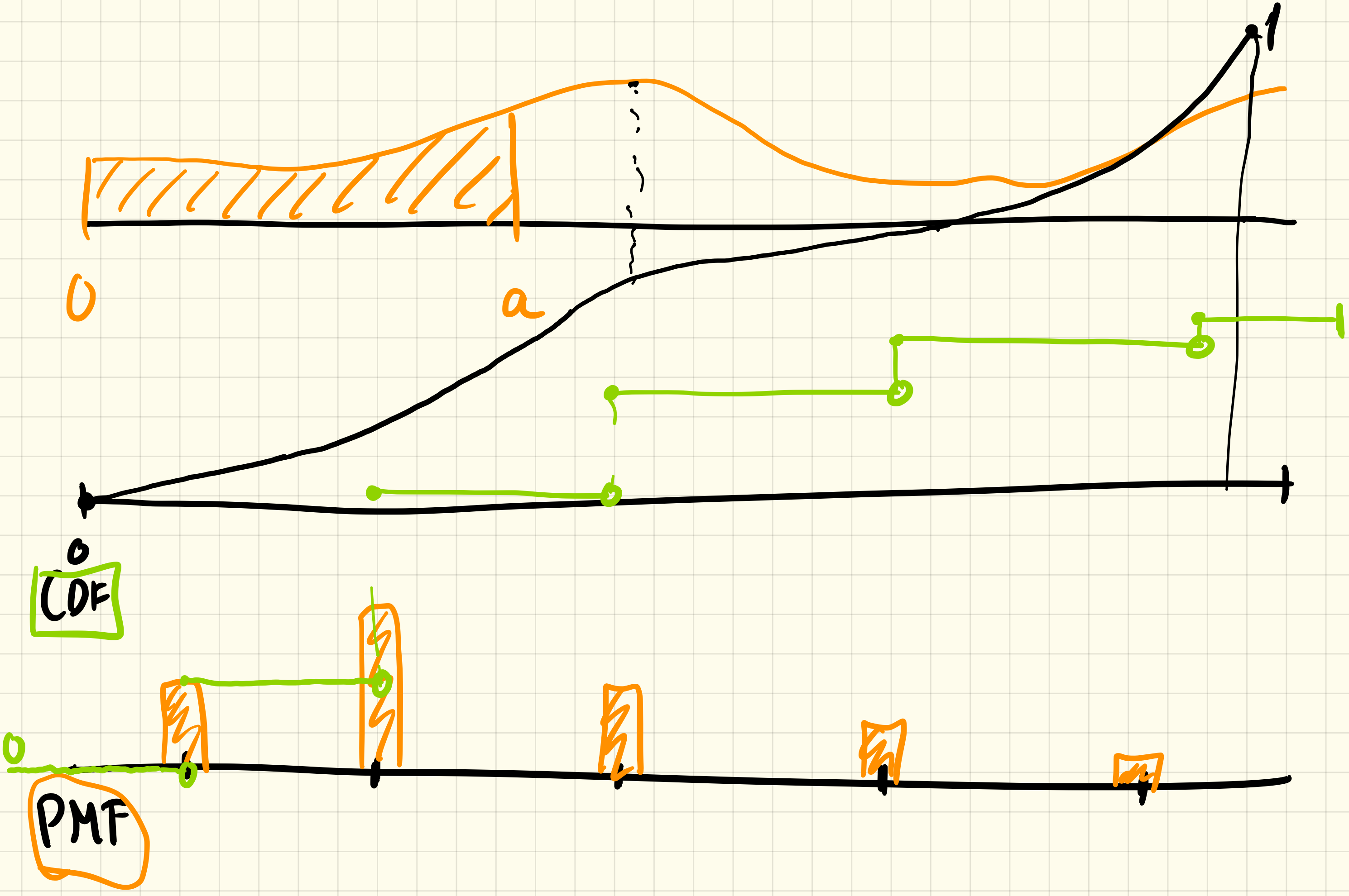
$\sum_i f(a_i) > 1$ which means, f not a PMF! ↯





stop
PDF
snow
0
a
1


1
0
CDF



1
0
a
0
CDF
0
PMF

Ber(p) - Coin Flip

$E[Ber(p)] = 1 \cdot p + 0(1-p)$
$= p$

$Var(Ber(p)) = p(1-p)$

Throw a dart at 1x1 dartboard that has a circle on it, $r = \frac{1}{2}$. Do I hit inside the circle?

Poisson Next →

Add n Ber(p)'s together

Repeat Ber(p) until outcome is a 1.

Repeat Ber(p) until r 1s

**Bin(n, p)**

If I throw 12 darts at that board, how many will hit in the circle? could be 0, could be 12. R.V. between [0,12]

**Geo(p)**

How many darts till I hit the bullseye? $[1, \infty)$

set r=1

**Neg. Binom(r, p)**

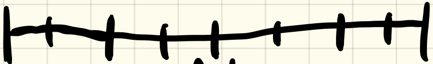
How many trials until the $r^{th}$ bullseye? $[r, \infty)$

# Poisson

**How many BLAHs occur in $\Delta t$ if avg rate $\lambda$ is constant?**

Assume we have a fixed avg. rate at which events occur.

(rain drops)



$\Delta t$

rate $\lambda \ \frac{\text{events}}{\text{time}}$

subdivide and subdivide until $\Pr(\text{event in } \frac{\Delta t}{n})$ is a Bernoulli, let $n \to \infty$

$$P(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

p

Limit of Binomial$\left(n, \frac{\lambda}{n}\right)$

time between events is $\text{Exp}(\lambda)$