(Practice) Exam 1 (Modules 1-7)

Comp 682 MCS@Rice Fall 2020

Quick Answer (Answered)

1. What is the worst-case complexity for any single union operation when using weighted union and path compression? O(log2(n))
2. What is the worst-case complexity for a series of O(n) union/find operations on n (initially) disjoint sets when using weighted union and path compression? O(n log*(n))
3. What is the worst case for series of n operations on dynamically-sized arrays? NOTE: The series of operations can contain pushes. O(n)
4. What is the worst case complexity for selection sort n items? $O(n^2)$
5. What is the best case complexity for selection sort of n items? $O(n^2)$
6. What is the worst case complexity for insertion sort n items? $O(n^2)$
7. What is the best case complexity for insertion sort of n items? O(n)
8. Worst-case for Quicksort of n items? $O(n^2)$
9. Average case for quicksort of n items? O(n log2(n))
10. Worst-case for merge sort n items? O(n log2(n))
11. Average case for merge sort items? O(n log2(n))
12. Worst case for quickselect to find the n/2-smallest item in a set of n items? $O(n^2)$
13. Average case for quickselect to find the n/2-smallest item in a set of n items? O(n)
14. Worst case for quickselect to find the minimum item in a set of n items? $O(n^2)$

15. Average case for quickselect to find the minimum item in a set of n items?
    $O(n^2)$

16. What is the best algorithm for finding the minimum item in a set of n items?
    linear search : O(n)

Suppose you are looking for an item with key k in a set of n unsorted items.

17. What is the worst-case complexity to find the item (assuming it is in the set)?
    O(n)

18. What is the average-case complexity to find the item (assuming it is in the set)?
    O(n/2) = O(n)

19. What is the worst-case complexity to determine the item is not in the set? O(n)

20. What is the average-case complexity to determine the item is not in the set?
    O(n)

Suppose you are looking for an item with key k in a set of n sorted items.

21. What is the worst-case complexity to find the item (assuming it is in the set)?
    O(log2(n))

22. What is the average-case complexity to find the item (assuming it is in the set)?
    O(log2(n))

23. What is the worst-case complexity to determine the item is not in the set?
    O(log2(n))

24. What is the average-case complexity to determine the item is not in the set?
    O(log2(n))

25. What is the worst-case complexity of shellsort?
    Depends on h-sequence. For some h-sequences, worst case is $O(n^2)$.
    Best (provable) worst case is $O(n^{3/2})$

26. What is the complexity of a Knuth shuffle? (Assume getting a random number is O(1)). O(n)

27. What is the worst-case complexity of a sort shuffle (Getting a random number is O(1))? O(n log2(n))

Not-so-Quick Answers

1. Is the following (pseudo)code for program A an algorithm? Justify your answer.

```
A(x[]) {
    i=0;
    while (external_test()) {
        x[i] = 0;
        i = i+1;
    }
}
```

There are 3 cases:

1. `external_test()` does not halt —> Program A **is not** an algorithm
2. `external_test()` always returns true —> Program A **is not** an algorithm
3. `external_test()` eventually returns true —> Program A **is** an algorithm

2. What is the computational complexity of the following program? (Big Oh notation is sufficient). Assume that `X[i,j]` references the `(i,j)` component of matrix `X`. Also, assume that `N,M` are parameter describing the size of `X` as `X[N.M]`.

```
for (i=1;i<=N;i++){
     v[i] = 0;
     for (j=i;j<=M;j+=2) {
          v[i] += A[i,j]*b[j];
     }
 }
```

Be sure to give your reasons for your complexity calculation.

3. Suppose that we choose 4 as our factor in the "Big Box Store" algorithm. That is, when the stack is full we allocate 4x the amount of current storage.

What is the complexity of M operations in this case? [Big Oh answer, but justify].

4.

A. Draw the tree associated with the f array below:

i	1	2	3	4	5	6	7	8	9	10
f[i]	2	2	4	2	6	7	2	4	5	6

B. Can the above be a result of running weighted quick union? Explain why this is impossible OR ELSE give a sequence of union-find operations that produce the above table.

5. Show with a sample trace how insertion sort sorts the letters

   A Q U E S T I O N

6. Show with a sample trace how shell sort with the h-sequence 10,5,2,1 sorts the letters

   V E R Y L O N G E N T R Y

7. Using any method you like, find the Big Oh complexity of the recurrence

   $T(n) = 3T(n/2) + n^2$

8. Using any method you like, find the Big Oh complexity of the recurrence

   $T(n) = T(n/3) + T(n/2) + T(n/6) + n$

Define the pmedian of a set of n items as

$$\text{pmedian}(S) = \begin{cases} \text{median}(S) & \text{if } |S| \text{ is odd} \\ \max_{x \in S} \ x \leq \text{median}(S) & \text{if } |S| \text{ is even} \end{cases}$$

9. Suppose you could always find the pmedian for a set of n items in O(1) time. What would be the worst case performance for quicksort if you pivot on the pmedian?

10. Suppose you could always find the pmedian of a set of n items in O(n) time. What would be the worst case performance for quicksort if you pivot on the pmedian?

11. Suppose we have a k sets of various sizes. All k of the sets are sorted. Devise an efficient algorithm to merge all k sets.