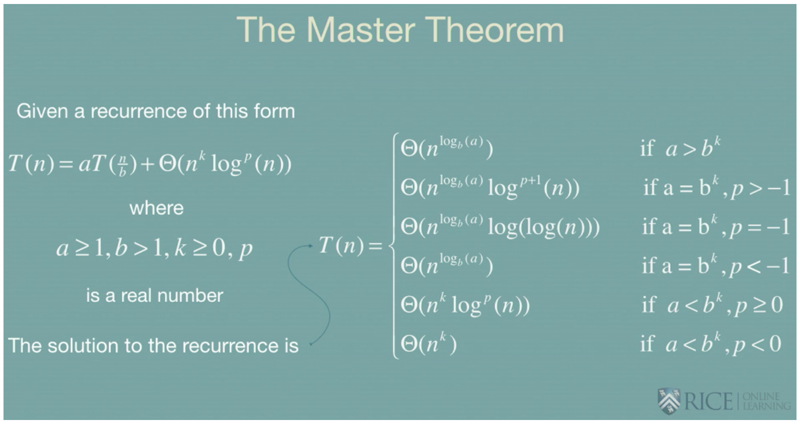

1) $T(n) = 3T(n/2) + n^2$. $a = 3, b = 2, k = 2, p = 0$. So by the master theorem case where $a < b^k, p >= 0$, $\theta(n^k log^p(n) = \theta(n^2 log^0(n)) = \theta(n^2)$.

2) $T(n) = 4T(n/2) + n^2$. $a = 4, b = 2, k = 2, p = 0$. So by the master theorem case where $a = b^k, p >= -1$, $\theta(n^{log_b(a)} log^{p+1}(n)) = \theta(n^2 log(n))$.

3) $T(n) = T(n/2) + n^2$. $a = 1, b = 2, k = 2, p = 0$. So by the master theorem case where $a < b^k, p >= 0$, $\theta(n^k log^p(n)) = \theta(n^2)$

4) $T(n) = 16T(n/4) + n$. $a = 16, b = 4, k = 1, p = 0$. So by the master theorem case where $a > b^k$, $\theta(n^{log_4(16)}) = \theta(n^2)$.

5) $T(n) = 2T(n/2)+nlog(n)$. $a = 2, b = 2, k = 1, p = 1$. So by the master theorem case where $a = b^k, p > -1$, $\theta(n^{log_b(a)} log^{p+1}(n) = \theta(nlog^2(n))$.

6) $T(n) = 2T(n/2) + n/log(n)$. $a = 2, b = 2, k = 1, p = -1$. So by the master theorem case where $a = b^k, p = -1$ $\theta(n^{log_b(a)} log(log(n))) = \theta(nlog(log(n))$.

7) $T(n) = 2T(n/4)+n^{0.51}$. $a = 2, b = 4, k = 0.51, p = 0$. So by the master theorem case where $a < b^k, p >= 0$. $\theta(n^k log^p(n)) = \theta(n^{0.51})$.

8) $T(n) = 6T(n/3)+n^2 log(n)$. $a = 6, b = 3, k = 2, p = 1$. So by the master theorem case where $a < b^k, p >= 0$. $\theta(n^k log^p(n)) = \theta(n^2 log(n))$.

9) $T(n) = 7T(n/3) + n^2$. $a = 7, b = 3, k = 2, p = 0$. So by the master theorem case where $a < b^k, p >= 0$. $\theta(n^k log^p(n)) = \theta(n^2)$.

10) $T(n) = \sqrt{2}T(n/2) + log(n)$. $a = \sqrt{2}, b = 2, k = 0, p = 1$. So by the master theorem case where $a > b^k$, $\theta(n^{log_b(a)}) = \theta(n^{log_2(\sqrt{2})}) = \theta(\sqrt{n})$.

11) $T(n) = 3T(n/3) + 1, T(1) = 2$. We make the guess $T(n)$ is something like summation of powers of 3. $3^k + 3^{k-1} + ... + 3 + 1, n = 3^k$ To deal with the base case $T(1) = 2$, we put that 2 in the first term. So we get $2*3^k + 3^{k-1} + ... + 3 + 1 = 2*3^k + (3^k - 1)/(3-1) = (4*3^k + 3^k - 1)/2 = (5*3^k - 1)/2 = (5n-1)/2 = \theta(n)$.

we prove by induction that $T(n) = (5n - 1)/2$:
base case $T(1) = (5 * 1 - 1)/2 = 2$ which is correct.

We assume it's true for n/3 and show that it's true for n. $T(n) = 3T(n/3) + 1 = 3(5(n/3) - 1)/2 + 1 = (5n - 1)/2$ Thus, $T(n) = (5n - 1)/2$.

12) $T(n) = nT(n-1), T(2) = 2$. we guess that $T(n) = n!$, which is pretty easy to see in this case.

We prove by induction that $T(n) = n!$ base case $T(2) = 2! = 2$ which is correct.
We assume it's true for n and show it is correct for n+1.
$T(n+1) = (n+1)T((n+1)-1) = (n+1)T(n) = (n+1)n! = (n+1)!$ Thus, $T(n) = (n+1)!$.

13) Using recursive substitution:
$T(9n/10) = T(81n/100) + T(9n/100) + 9n/10$
$T(n/10) = T(9n/100) + T(n/100) + n/10$
$T(n) = T(9n/10) + T(n/10) + n = T(81n/100) + T(9n/100) + 9n/10 + T(9n/100) + T(n/100) + n/10 + n$

$T(n) = T(81n/100) + 2T(9n/100) + T(n/100) + 2n$
$T(81n/100) = T(9^3n/1000) + T(81n/1000) + 81n/100$
$T(n/100) = T(9n/1000) + T(n/1000) + n/100$ $T(n) = T(9^3n/1000) + T(81n/1000) + 81n/100 + T(9n/100) + T(9n/1000) + T(n/1000) + n/100$
Therefore, $T(n) = \theta(nlog_{10/9}(n))$

14) We can guess that the complexity is $\theta(nlog_{1/a}(n))$ or $\theta(nlog_{1/(1-a)}(n))$ (depending on which a or $1-a$ is greater). We just assume $a > (1-a) without loss of generality.$
So $T(n) = cnlog_{1/a}(n)$, where c is a constant. We want to show $T(n) <= cnlog_{1/a}(n)$
$T(n) = T(an) + T((1-a)n) + n$
$<= c(an)log_{1/a}(an) + c(1-a)nlog_{1/a}((1-a)n) + n$
$<= cnlog_{1/a}(n)$
We can do something similar for the lower bound.
Thus, $\theta(nlog_{1/a}(n)) when (a > 1-a)$