Name______ Student ID □□□□□□□□□□□□ School______ Major/Class______ Seat No.______

(DONNOT WRITE YOUR ANSWER IN THIS AREA)

Seal Line Seal Line Seal Line

WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

SCUT Final Exam

Mathematical Analysis I Exam Paper B（2019-2020-1）

Notice:

1. **Make sure that you have filled the form on the left side of seal line.**
2. **Write your answers on the exam paper.**
3. **This is a close-book exam.**
4. **The exam with full score of 100 points lasts 120 minutes.**

Question No.	I	II	III	IV	Sum
Score					

Score

I. Please fill the correct answers in the following blanks. ($4' \times 5 = 20'$)

1. $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{2n}\right)^n =$ ________________.

2. If $e^y \sin x = x + xy$, then $dy =$ ________________________.

3. The inflection points of the curve $f(x) = \dfrac{1}{1+x^2}$ are____________________________.

4. Suppose f is continuous with the property that $|f(x)| \le |x||\sin x|$ for all x , then $f'(0) =$ _______________.

5. If $f(x)$ is continuous, and $f(x) = \sin^4 x + \dfrac{1}{\pi}\int_0^{\frac{\pi}{2}} f(x)dx$, then $f(x) =$ ______________.

Score

II. **Finish the following calculations.** (6-11: $6' \times 6 = 36'$)

6. $\lim\limits_{x\to 0} \dfrac{e^{x^3} - 1 - x^3}{(\sin 2x)^6}$.

7. $\lim\limits_{x\to 3} \left(\dfrac{x}{x-3} \int_3^x e^{-t^2}\, dt \right)$

8. If $y = \arctan x$, find $y^{(n)}(0)$.

9. Evaluate the indefinite integral $\int \frac{1}{\sqrt{x^2 - a^2}} dx \ (a > 0)$

10. (a) Find the tangent to the cardioid $r = 1 + \sin\theta$ at the point where $\theta = \frac{\pi}{3}$.

(b) Find the length of cardioid.

III. Prove the following conclusions. $(8' \times 3 = 24)$

Score

11. Prove that $\lim\limits_{x \to a} \frac{1}{x} = \frac{1}{a}$ $(a > 0)$ by using the $\varepsilon, \ \delta$ definition of limit.

12. Let $f(x) = \begin{cases} x^2 \cos\frac{1}{x}, & x \neq 0 \\ 0, & x=0 \end{cases}$.

Find $f'(x)$. Is $f'(x)$ continuous at $x = 0$? Show your reasons.

13. If $ab > 0$. Use Cauchy's mean value theorem to prove that there exists $\xi \in (a,b)$ such that

$$ae^b - be^a = (1-\xi)e^{\xi}(a-b)$$

Score

IV. Finish the following questions. ($10' \times 2 = 20$ **)**

14. Find the volume of the solid obtained by rotating the region bounded $y = x - x^2$ and $y = 0$ about $x = 2$.

15. A sequence $\{x_n\}$ is given by $a > 0, 0 < x_1 < \frac{1}{a}, x_{n+1} = x_n(2 - ax_n), (n = 1, 2, \cdots)$. Show that $\lim\limits_{n \to +\infty} x_n$ exists and find it.