座位号________ 专业________ 学院________ 学号 姓名________

（密封线内不答题）

密 封 线

诚信应考, 考试作弊将带来严重后果!

[redacted]

《线性代数与解析几何 》(全英课)试卷(A)—2018年12月26日

注意事项：1. 考前请将密封线内填写清楚;

2. 所有答案请直接答在试卷上;

3. 考试形式：闭卷;

4. 本试卷共 8 大题, 满分 100 分, 考试时间 120 分钟.

题　号	一	二	三	四	五	六	七	八	总　分
得　分									
评卷人									

1. (10 points) For a spanning space

$$H = \left\{ \begin{pmatrix} a-3b+3c \\ 5a+4c+4d \\ b-2c-d \\ 5c+5d \end{pmatrix} \ : \ a,b,c,d \in \mathbb{R} \right\}.$$

(1) Find the dimension of H.

(2) If H is a subspace of $\mathbb{R}^k$, then $k = ?$

2. (12 points) Justify whether or not the following matrix is diagonalizable. If the matrix is diagonalizable, find a matrix P and diagonal matrix D such that $P^{-1}AP = D$.

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

3. (18 points) Given vectors $\vec{v}_1 = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} 7 \\ a \\ b \end{pmatrix}$.

(1) (6 points) What are the numbers a and b such that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is an orthogonal set?

(2) (3 points) Calculate the distance between $\vec{v}_1$ and $\vec{v}_2$, that is, $\text{dist}(\vec{v}_1,\ \vec{v}_2)$.

(3) (3 points) Normalize $\vec{v}_1$ to produce a unit vector.

(4) (6 points) In $\mathbb{R}^3$, for a subspace $W = span\{\vec{v}_1, \vec{v}_2\}$, and a vector $\vec{y} = \begin{pmatrix} 18 \\ 4 \\ 8 \end{pmatrix}$, write $\vec{y} = \vec{y_1} + \vec{y_2}$, where $\vec{y_1} \in W$, and $\vec{y_2} \in W^{\perp}$.

4. (14 points) In vector space $\mathbb{P}_2$, solve the following two questions.

(1) (7 points) The set $B = \{1+t,\ 1+t^2,\ 1+t+t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector $[p]_B$ of $p(t) = 4 - t + 2t^2$ relative to B.

(2) (7 points) Determine whether the polynomial set $\{10t^2 - t - 9,\ 4t^2 + t - 5,\ 2t^2 - 3t + 1\}$ is linearly independent or linearly dependent in $\mathbb{P}_2$.

5. (14 points) For the following quadratic form

$$\vec{x}^T A\vec{x} = 7x_1^2 + 5x_2^2 + 9x_3^2 - 8x_1x_2 + 8x_1x_3,$$

(1) (4 points) Give the matrix A of the quadratic form, and indicate which type this quadratic form is? (For example, negative definite, positive definite or indefinite?)

(2) (10 points) Find an orthogonal matrix P such that the change of variable $\vec{x} = P\vec{y}$ transforms $\vec{x}^T A\vec{x}$ into a new quadratic form with no cross-product term. Give the new quadratic form.

6. (14 points) Given two bases of $\mathbb{R}^3$ as follows

$$B=\{\vec{b}_1,\vec{b}_2,\vec{b}_3\}=\left\{\begin{pmatrix}1\\-1\\0\end{pmatrix},\begin{pmatrix}-2\\3\\-1\end{pmatrix},\begin{pmatrix}3\\-4\\2\end{pmatrix}\right\}$$

$$C=\{\vec{c}_1,\vec{c}_2,\vec{c}_3\}=\left\{\begin{pmatrix}-2\\2\\1\end{pmatrix},\begin{pmatrix}-7\\5\\3\end{pmatrix},\begin{pmatrix}-9\\6\\4\end{pmatrix}\right\}$$

(1) (7 points) Find the transformation matrix $P_{B\to C}$, such that for any vector $\vec{x}\in\mathbb{R}^3$, it holds $[\vec{x}]_C=P_{B\to C}[\vec{x}]_B$.

(2) (7 points) Find the transformation matrix $P_{C\to B}$, such that for any vector $\vec{x}\in\mathbb{R}^3$, it holds $[\vec{x}]_B=P_{C\to B}[\vec{x}]_C$.

7. (8 points) Verify that

$$||\vec{u}+\vec{v}||^2 + ||\vec{u}-\vec{v}||^2 = 2||\vec{u}||^2 + 2||\vec{v}||^2.$$

8. (10 points) Suppose V is a 3-dimensional vector space. $B = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ and $C = \{\vec{c}_1, \vec{c}_2, \vec{c}_3\}$ are two basis of V, and

$$\vec{c}_1 = \vec{b}_1 + \vec{b}_2 + \vec{b}_3, \qquad \vec{c}_2 = \frac{1}{5}\vec{b}_1 + \frac{1}{5}\vec{b}_2, \qquad \vec{c}_3 = \vec{b}_2 + \vec{b}_3.$$

$T : V \to V$ is a linear transformation. If the matrix for T relative to B is

$$[T]_B = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 2 & -3 \\ 2 & 3 & -4 \end{pmatrix}, \text{ calculate } [T]_C = ?$$