Chinese Name ______ Student ID ______ School ______ Major ______ Sit NO. ______

（DONOY WRITE YOUR ANSWER IN THIS AREA）

Seal Line ...... Seal Line ...... Seal Line

WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

SCUT Exam

2018-2019 《Calculus I》 Exam Paper A

Notice:
1. **Make sure that you have filled the form on the left side of seal line.**
2. **Write your answers on the exam paper .**
3. **This is a close-book exam.**
4. **The exam with full score of 100 points lasts 120 minutes.**

Question No.	I	II	III	Sum
Score				

I. **Please fill the correct answers in the following blanks.** ($4'\times 5=20'$)

1. $\cos(\arcsin x)=$ ________________.

2. If $y=xe^{x}$, then $dy|_{x=1}=$ ____________.

3. If $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-x+1}-ax-b\right)=0$. then $a=$ ________, $b=$ ________.

4. If $f'(a)=2$, then $\lim\limits_{x\to 0}\dfrac{f(a+3x)-f(a)}{5x}=$ ______________.

5. $\lim\limits_{n\to\infty}\left[\dfrac{1}{\sqrt{n^2+1}}+\dfrac{1}{\sqrt{n^2+2}}+\cdots+\dfrac{1}{\sqrt{n^2+n}}\right]=$ ______________.

II. **Finish the following calculations.** (6-10: $8'\times 6=48'$)

6. $\lim\limits_{x\to\infty}\left(x\sin\dfrac{1}{x}-\dfrac{1}{x}\cos x\right)$.

7. $\lim\limits_{x\to 2}\arctan\left(\dfrac{x^2-4}{3x^2-6x}\right)$

8. $\begin{cases} x=\ln\sqrt{t^2+1} \\ y=\arctan t \end{cases}$, $\dfrac{d^2y}{dx^2}$

9. Use implicit differentiation to find an equation of the tangent line to the curve $x^{2/3} + y^{2/3} = 4$ at the point $(-3\sqrt{3}, 1)$.

10. Let $f(x) = x^{\cos x}$ $(x > 0)$, find the derivative of the function $f(x)$.

11. If $f(x) = x^2 e^x$, find $f^{(100)}(x)$

III. Finish the following questions. ($8' \times 4 = 32$ **)**

11. Prove that $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ using the $\varepsilon,\ \delta$ definition of limit.

12. Show that $\lim_{x\to 0}\sin\frac{1}{x}$ does not exist.

13. Let

$$f(x)=\begin{cases}\dfrac{\sin ax}{\sqrt{1-\cos x}}, & x<0\\ b, & x=0\\ \dfrac{1}{x}\left[\ln x\text{-}\ln(x^2+x)\right], & x>0\end{cases}.$$

Find $a,\ b$ such that $f(x)$ is continuous at $x=0$.

14. A sequence $\{a_n\}$ is given by $a_1 = 1,\ a_{n+1} = \sqrt{2 + a_n}\ (n = 1, 2, 3 \cdots)$. Show that $\lim_{n \to +\infty} a_n$ exists and find it. (Hint: Apply the Monotonic Sequence Theorem)