座位号________ 专业________ 学院________ 学号 姓名________

（密封线内不答题）

密 封 线

诚信应考, 考试作弊将带来严重后果!

[redacted]

《线性代数与解析几何 》(全英课)试卷(A)—2020年1月6日

注意事项：1. 考前请将密封线内填写清楚;

2. 所有答案请直接答在试卷上;

3. 考试形式：闭卷;

4. 本试卷共 8 大题, 满分 100 分, 考试时间 120 分钟.

题　号	一	二	三	四	五	六	七	八	总　分
得　分									
评卷人									

1. (14 points) Compute the following determinants:

$$(1)\ \begin{vmatrix} 3 & 5 & 7 & 9 \\ 11 & 13 & 15 & 17 \\ 19 & 21 & 23 & 25 \\ 27 & 29 & 31 & 33 \end{vmatrix}; \qquad (2)\ \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 8 & 16 \\ 3 & 9 & 27 & 81 \\ 1 & 4 & 16 & 64 \end{vmatrix}.$$

2. (8 points) Calculate the area of the parallelogram determined by the points $(-3,-5)$, $(-1,0)$, $(3,-4)$ and $(5,1)$.

3. (15 points) For two matrices $A = \begin{pmatrix} 7 & 2 \\ -4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 \\ 0 & y \end{pmatrix}$,

(1) (6 points) if matrix equation $A\vec{x} = B\vec{x}$ has nonzero solutions, what is the value of y?

(2) (3 points) If A and B are similar to each other, what is the value of y?

(3) (6 points) Find a formula for A^k.

4. (16 points) For the following quadratic form

$$\vec{x}^T A\vec{x} = 3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 + 2x_2x_3,$$

(1) (4 points) Give the matrix A of the quadratic form, and indicate which type this quadratic form is? (For example, negative definite, positive definite or indefinite?)

(2) (12 points) Find an orthogonal matrix P such that the change of variable $\vec{x} = P\vec{y}$ transforms $\vec{x}^T A\vec{x}$ into a new quadratic form with no cross-product term. Give the new quadratic form.

5. (20 points) In vector space $\mathbb{P}_2$, the vector sets $B = \{-1,\, 2-t,\, -3-t-t^2\}$, and $C = \{1+t^2,\, t+t^2,\, -t\}$ are two bases.

(1) (6 points) Find the coordinate vector $[p]_B$ of $p(t) = 1 - 2t + t^2$ relative to B.

(2) (6 points) Find the coordinate vector $[p]_C$ of $p(t) = 1 - 2t + t^2$ relative to C.

(3) (8 points) Find a matrix M, such that for any $p \in \mathbb{P}_2$, there is $[p]_B = M[p]_C$.

6. (11 points) For the vector space

$$H = \left\{ \begin{pmatrix} a - 3b + 6c \\ 5a + 4b + 4d \\ -2c - d \\ -a + 8b - 6c + 5d \end{pmatrix} \ : \ a, b, c, d \in \mathbb{R} \right\},$$

(1) (3 points) what is the dimension for H?

(2) (8 points) Find a set of basis for the orthogonal compliment $H^{\perp}$ of H.

7. (8 points) $M = \begin{pmatrix} 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 2 & -1 \\ 0 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}$ is a 6×6 matrix. Find M^{-1}

8. (8 points) For a matrix A, $det(A) = s$ and $s \neq 0$. If A has an eigenvalue λ, prove that the adjoint matrix A^* has an eigenvalue $\frac{s}{\lambda}$.