

WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

SCUT Final Exam

Mathematical Analysis I Exam Paper A (2019-2020-1)

- Notice:**
1. Make sure that you have filled the form on the left side of seal line.
 2. Write your answers on **the exam paper**.
 3. This is a **close**-book exam.
 4. The exam with full score of 100 points lasts 120 minutes.

Question No.	I	II	III	IV	Sum
Score					

Score

I. Please fill the correct answers in the following blanks. (4' × 5 = 20')

1. $\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n =$ _____.

2. If $\sin(x + y) = y^2 \cos x$, then $dy =$ _____.

3. The inflection points of the curve $f(x) = \frac{1}{2}x^2 + \frac{9}{10}(x-1)^{\frac{5}{3}}$ are _____.

4. Suppose f is continuous with the property that $|f(x)| \leq x^2$ for all x , then $f'(0) =$ _____.

5. If $f(x)$ is continuous, and $f(x) = \cos^4 x + \frac{1}{\pi} \int_0^{\frac{\pi}{2}} f(x) dx$, then $f(x) =$ _____.

II. Finish the following calculations. (6-11: 6'×6 = 36')

Score

6. $\lim_{x \rightarrow +\infty} \left(x - x^2 \ln \left(1 + \frac{1}{x} \right) \right).$

7. $\lim_{x \rightarrow 3} \left(\frac{x}{x-3} \int_3^x \frac{\sin t}{t} dt \right)$

8. If $y = \arctan x$, find $y^{(n)}(0)$.

9. Evaluate the indefinite integral $\int \frac{1}{x^2 \sqrt{x^2 + 1}} dx$

10. (a) Find the tangent to the cycloid $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ ($a > 0$ is a constant)

at the point where $\theta = \frac{\pi}{3}$.

(b) Find the area under one arch of the cycloid.

III. Prove the following conclusions. (8'×3=24)

Score

11. Prove that $\lim_{x \rightarrow 3} x^2 = 9$ by using the ε, δ definition of limit.

12. Let
$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x=0 \end{cases} .$$

Find $f'(x)$. Is $f'(x)$ continuous at $x = 0$? Show your reasons.

13. Assume that $f(x)$ is continuous on the closed interval $[a, b]$, and f is differentiable in the open interval (a, b) , $0 < a < b$. Use Cauchy's mean value theorem to prove that there exists $\xi \in (a, b)$ such that

$$\frac{af(b) - bf(a)}{a - b} = f(\xi) - \xi f'(\xi).$$

Score

IV. Finish the following questions. (10'×2 = 20)

14. Find the volume of the solid obtained by rotating about $y = 1$ the region between $y = x$ and $y = x^2$.

15. A sequence $\{a_n\}$ is given by $a_1 = \frac{1}{2}\left(a + \frac{1}{a}\right)$, $a_{n+1} = \frac{1}{2}\left(a_n + \frac{1}{a_n}\right)$ ($n = 1, 2, 3, \dots$), ($a > 0$).

Show that $\lim_{n \rightarrow +\infty} a_n$ exists and find it.