

PV of the portfolio is

$$(S_0 u \Delta - f_u) e^{-rT}$$

The initial cost of the portfolio is

$$S_0 \Delta - f$$

It follows that

$$S_0 \Delta - f = (S_0 u \Delta - f_u) e^{-rT}$$

or

$$f = S_0 \Delta (1 - u e^{-rT}) + f_u e^{-rT}$$

because we know that $\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$, we obtain

$$f = S_0 \left(\frac{f_u - f_d}{S_0 u - S_0 d} \right) (1 - u e^{-rT}) + f_u e^{-rT}$$

or

$$f = \frac{(f_u - f_d)(1 - u e^{-rT})}{u - d} + f_u e^{-rT}$$

or

$$f = \frac{f_u(1 - u e^{-rT}) - f_d(1 - u e^{-rT})}{u - d} + \frac{u f_u e^{-rT} - d f_u e^{-rT}}{u - d}$$

or

$$f = \frac{f_u(1 - d e^{-rT}) - f_d(1 - u e^{-rT})}{u - d}$$

or

$$f = e^{-rT} \left(f_u \left(\frac{e^{rT} - d}{u - d} \right) - f_d \left(\frac{e^{rT} - u}{u - d} \right) \right)$$

or

$$f = e^{-rT} \left(f_u \left(\frac{e^{rT} - d}{u - d} \right) + f_d \left(\frac{u - e^{rT}}{u - d} \right) \right)$$

since

$$p = \frac{e^{rT} - d}{u - d} \text{ and } (1 - p) = \frac{u - e^{rT}}{u - d}$$

we can convert the prior formula into

$$f = e^{-rT} (p f_u + (1 - p) f_d)$$

2.

	$\Delta t = 1/12$	$\Delta t = 1/52$	$\Delta t = 1/252$
u	1.0905	1.0425	1.0191
d	0.9170	0.9593	0.9813
p	0.5024	0.5012	0.5005
European Put	6.7874	6.7775	6.7570
American Put	7.5075	7.4856	7.4710

B-S solution for European Put Value = 6.76014

As the number of time steps rise, we can find that the European Put value calculated by MATLAB are closer to the B-S solution for European Put value which is 6.76014. However, it seems that we can not find out the value must be higher or lower than the B-S value.

[Bonus]

After plotting the calculation of value from 1 to 252 steps, we found out that the value was vibrating, decreasing the amplitude until the value approaches the B-S solution value.

