Lab 4 Kalyn Howes & Chloe Vancem

3)

a) if A and B are independent, show:

- $\bar{A}$ and B are independent:

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

& if A and B are independent then, $P(A \cap B) = P(A) * P(B)$

so, $P(\bar{A} \cap B) = P(B) - P(A) * P(B) = P(B) * (1 - P(A)) = \underbrace{P(\bar{A}) * P(B)}_{\text{independent}}$

- A and $\bar{B}$ are independent:

if A & B are independent, $P(A \cap B) = P(A) * P(B)$

& since $P(B) = 1 - P(\bar{B})$, $P(A \cap B) = P(A) - P(A)P(\bar{B})$

so $P(A)P(\bar{B}) = P(A) - P(A \cap B)$ & we know $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

therefore, $P(A \cap \bar{B}) = P(A)P(\bar{B})$ so they're independent

- $\bar{A}$ and $\bar{B}$ are independent:

if we already proved $\bar{A}$ & B and B & $\bar{A}$ are each independent, we know $\bar{A}$ and $\bar{B}$ are as well since if 2 events are independent, then each event is independent of the complement of the other

b) A: 30% sent, 5% defective B: 70% sent, 4% defective

i) probability that a product is sent to A & is defective

$(30\%) 5\% = (0.3) 0.05 = 0.015 = 1.5\%$

ii) probability that a product is sent to A & is not defective

$(30\%) 95\% = (.3) 0.95 = 0.285 = 28.5\%$

iii) sent to B & is defective

$(70\%) 4\% = (.7) .04 = 2.8\%$

iv) sent to B & not defective

$(70\%) 96\% = (.7) .96 = 67.2\%$

c) show that for A & B, $P(A|B) > P(A)$ implies $P(B|A) > P(B)$

$P(A|B) > P(A)$

$\frac{P(A \cap B)}{P(B)} > P(A)$

$\frac{P(B|A)P(A)}{P(B)} > P(A)$

mult & divide

$\frac{P(B|A)P(A)}{P(A)} > P(B)$

$P(B|A) > P(B)$