Chapter 5

Integrability

5.3 The Fundamental Theorem of Calculus

Exercise 5.3.3

a) Answer is $-\frac{1}{2}$. Let $u = x^2 + 1$, $du = 2xdx$, and use Change of Variables Rule.

b) Answer is -4. Let $u = \sqrt{1-x^2}$, $u^2 = 1 - x^2$, $udu = -xdx$, and use Change of Variables Rule.

Exercise 5.3.4

a) Use Integration by Part.

b) Use Integration by Part.

c) Use Integration by Part.

Exercise 5.3.5

Set $g(x) = 1$, $\forall x \in [a, b]$

Exercise 5.3.6

Set

$$\begin{aligned} g(t) &= \alpha \int_a^t f(x)dx + \beta \int_t^b f(x)dx \\ \implies g'(t) &= \alpha f(t) - \beta f(t) \\ &= (\alpha - \beta) f(t) \end{aligned}$$

Exercise 5.3.9

$$\int_{f(a)}^{f(b)} f^{-1}(x)dx = \int_a^b f^{-1}(f(y))f'(y)dy$$

Then, use Integration by Part to get the equation.

Exercise 5.3.10

$$f \circ \phi = (f \circ \phi)|\phi'||\frac{1}{\phi'}|$$

Use Change of Variables Rule and the fact that $\phi' \neq 0$ and both ϕ' and $\frac{1}{\phi'}$ are continuous on $[a, b]$.

5.4 Improper Riemann Integration

Exercise 5.4.1

a) $\frac{3}{2}$

b) $\frac{1}{3}$

c) $\frac{3}{2}$

d) -1

Exercise 5.4.7

a) Assume $L > 0$ and use Comparison Theorem for Improper Integrals to lead a contradiction.

b) ???

Exercise 5.4.8

Let

$$\begin{aligned} & t = x^n \\ \implies & dt = nx^{n-1}dx \\ \implies & dx = \frac{dt}{nt^{\frac{n-1}{n}}} \end{aligned}$$

Use Change of Variables Rule and the fact that f is absolutely integrable on $[1, \infty)$. i.e.

$$\int_1^\infty |f(x)|dx < \infty$$