Homework 3
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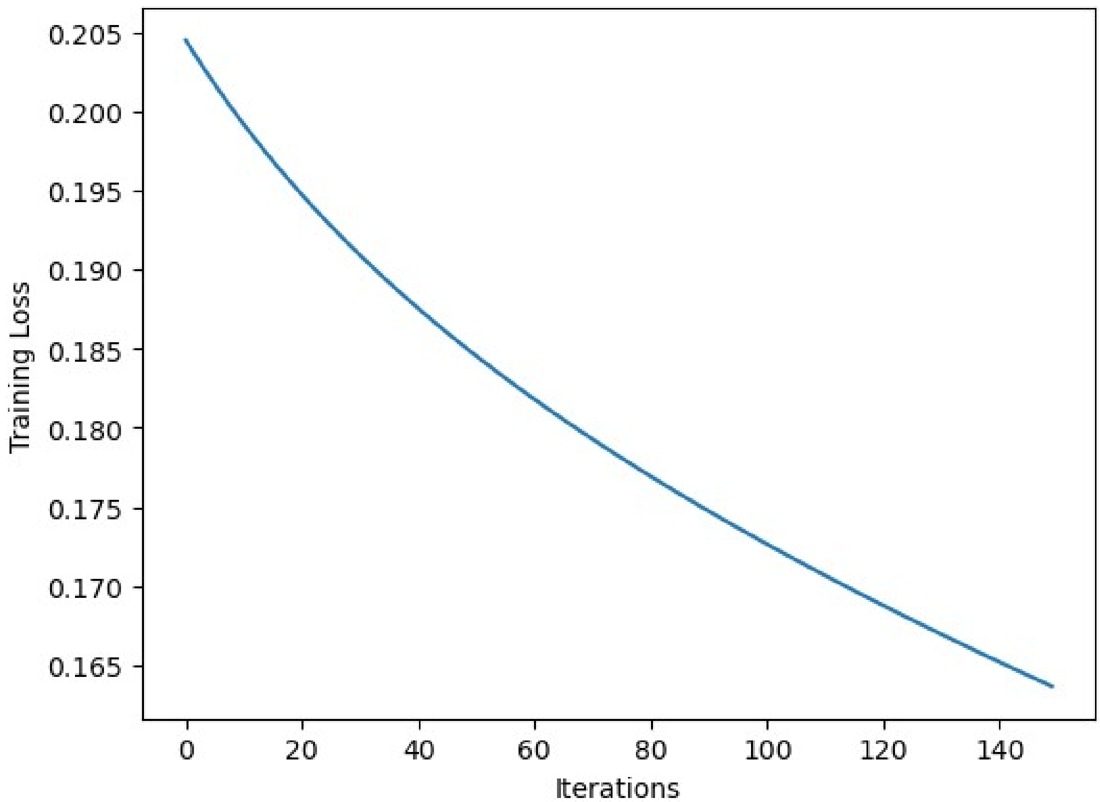
Answers rounded to 5 s.f. where required.

Q1



The final decision boundary is:

$y = 1$ if $0.58645 - 1.3883\,x_1 + 1.2986\,x_2 \geq 0$, and

$y = 0$ otherwise

Q2

(1)

$$\frac{\partial \hat{y}}{\partial \boldsymbol{w}_2}=\frac{\partial \hat{y}}{\partial \boldsymbol{z}_2}\frac{\partial \boldsymbol{z}_2}{\partial \boldsymbol{w}_2}$$

$$=\frac{\partial \hat{y}}{\partial \boldsymbol{z}_2}\boldsymbol{a}_1$$

$$\frac{\partial \hat{y}}{\partial b_2}=\frac{\partial \hat{y}}{\partial \boldsymbol{z}_2}\frac{\partial \boldsymbol{z}_2}{\partial b_2}$$

$$=\frac{\partial \hat{y}}{\partial \boldsymbol{z}_2}*1$$

$$=\frac{\partial \hat{y}}{\partial \boldsymbol{z}_2}$$

(2)

$$\sigma(x)=\log(1+\exp(x))$$

$$\frac{\partial \hat{y}}{\partial \boldsymbol{z}_2}=\frac{\partial}{\partial \boldsymbol{z}_2}\log(1+\exp(\boldsymbol{z}_2))$$

$$=\frac{\exp(\boldsymbol{z}_2)}{1+\exp(\boldsymbol{z}_2)}$$

$$=\frac{1}{1+\exp(-\boldsymbol{z}_2)}$$

Hence,

$$\frac{\partial \hat{y}}{\partial \boldsymbol{w}_2}=\frac{\boldsymbol{a}_1}{1+\exp(-\boldsymbol{z}_2)}$$

$$\frac{\partial \hat{y}}{\partial b_2}=\frac{1}{1+\exp(-\boldsymbol{z}_2)}$$

(3)

$$\frac{\partial \hat{y}}{\partial \boldsymbol{x}}=\frac{\partial \hat{y}}{\partial \boldsymbol{z}_2}\frac{\partial \boldsymbol{z}_2}{\partial \boldsymbol{x}}$$

$$=\frac{\partial \hat{y}}{\partial \boldsymbol{z}_2}\frac{\partial \boldsymbol{z}_2}{\partial \boldsymbol{a}_1}\frac{\partial \boldsymbol{a}_1}{\partial \boldsymbol{x}}$$

$$=\frac{\partial \hat{y}}{\partial \boldsymbol{z}_2}\frac{\partial \boldsymbol{z}_2}{\partial \boldsymbol{a}_1}\frac{\partial \boldsymbol{a}_1}{\partial \boldsymbol{z}_1}\frac{\partial \boldsymbol{z}_1}{\partial \boldsymbol{x}}$$

$$=\frac{1}{1+\exp(-\boldsymbol{z}_2)}\boldsymbol{w}_2\frac{1}{1+\exp(-\boldsymbol{z}_1)}\boldsymbol{w}_1$$

$$=\frac{\boldsymbol{w}_2\boldsymbol{w}_1}{(1+\exp(-\boldsymbol{z}_2))(1+\exp(-\boldsymbol{z}_1))}$$

$$=\frac{\boldsymbol{w}_2\boldsymbol{w}_1}{(1+\exp(-(\boldsymbol{w}_2\boldsymbol{a}_1+b_2)))(1+\exp(-\boldsymbol{z}_1))}$$

Hence, changing b_2 will affect $\frac{\partial \hat{y}}{\partial \boldsymbol{x}}$.

(4)

$$
\begin{aligned}
\frac{\partial \hat{y}}{\partial \boldsymbol{w}_1} &= \frac{\partial \hat{y}}{\partial \boldsymbol{z}_2} \frac{\partial \boldsymbol{z}_2}{\partial \boldsymbol{w}_1} \\
&= \frac{\partial \hat{y}}{\partial \boldsymbol{z}_2} \frac{\partial \boldsymbol{z}_2}{\partial \boldsymbol{a}_1} \frac{\partial \boldsymbol{a}_1}{\partial \boldsymbol{w}_1} \\
&= \frac{\partial \hat{y}}{\partial \boldsymbol{z}_2} \frac{\partial \boldsymbol{z}_2}{\partial \boldsymbol{a}_1} \frac{\partial \boldsymbol{a}_1}{\partial \boldsymbol{z}_1} \frac{\partial \boldsymbol{z}_1}{\partial \boldsymbol{w}_1} \\
&= \frac{\exp(\boldsymbol{z}_2)}{(1+\exp(\boldsymbol{z}_2))^2} \boldsymbol{w}_2 \frac{\exp(\boldsymbol{z}_1)}{(1+\exp(\boldsymbol{z}_1))^2} \boldsymbol{x} \\
&= \frac{\boldsymbol{w}_2 \boldsymbol{x} \exp(\boldsymbol{z}_2+\boldsymbol{z}_1)}{(1+\exp(\boldsymbol{z}_2))^2 (1+\exp(\boldsymbol{z}_1))^2}
\end{aligned}
$$

Q3

Frequency		Evade Tax	
		Yes	No
Refund	Yes	0	3
	No	3	4

Likelihood		Evade Tax	
		Yes	No
Refund	Yes	0/3	3/7
	No	3/3	4/7

Frequency		Evade Tax	
		Yes	No
Marital Status	Single	2	2
	Married	0	4
	Divorce	1	1

Frequency		Evade Tax	
		Yes	No
Marital Status	Single	2/3	2/7
	Married	0/3	4/7
	Divorce	1/3	1/7

$$\mu_{c=Yes}=\frac{95\text{e}3+85\text{e}3+90\text{e}3}{3}$$
$$=90\text{e}3$$

$$\sigma^2_{c=Yes}=\frac{(95\text{e}3-90\text{e}3)^2+(85\text{e}3-90\text{e}3)^2+(90\text{e}3-90\text{e}3)^2}{3-1}$$
$$=25\text{e}6$$

$$P(X_3=79000|Y=Yes)=\frac{1}{\sqrt{2\pi(25\text{e}6)}}\exp\left(-\frac{(79\text{e}3-90\text{e}3)^2}{2(25\text{e}6)}\right)$$
$$\approx 7.0949\text{e-}6$$

$$\mu_{c=No}=\frac{125\text{e}3+100\text{e}3+70\text{e}3+120\text{e}3+60\text{e}3+220\text{e}3+75\text{e}3}{7}$$
$$=110\text{e}3$$

$$\sigma^2_{c=No}=\frac{1}{7-1}((125\text{e}3-110\text{e}3)^2+(100\text{e}3-110\text{e}3)^2+(70\text{e}3-110\text{e}3)^2+(120\text{e}3-110\text{e}3)^2+(60\text{e}3-110\text{e}3)^2+(220\text{e}3-110\text{e}3)^2+(75\text{e}3-110\text{e}3)^2)$$
$$=2975\text{e}6$$

$$P(X_3=79000|Y=No)=\frac{1}{\sqrt{2\pi(2975\text{e}6)}}\exp\left(-\frac{(79\text{e}3-110\text{e}3)^2}{2(2975\text{e}6)}\right)$$
$$\approx 6.2234\text{e-}6$$

$\hat{Y}=\arg\max_Y P(Y|X_1=Yes, X_2=Married, X_3=79000) \propto \arg\max_Y P(Y)\Pi_{j=1}^{n} P(X_j|Y)$, where
Y = Evade Tax
X_1 = Refund
X_2 = Marital Status
X_3 = Taxable Income

$$\hat{Y}\propto \arg\max_Y \left\{\frac{3}{10}*0*0*7.0949\text{e-}6, \frac{7}{10}\frac{3}{7}\frac{4}{7}6.2234\text{e-}6\right\}=\text{No}$$