Subject: Segway Control Term Project – Part 1
Student: Howard Chiang
Class: ME5659 – Control and Mechatronics
Professor: Rifat Sipahi
Date: 3/31/2015

Introduction

By evaluating the governing dynamics of the Segway, the transfer function of outputs theta [Θ(t)] and velocity [xVelocity(t)] were found. The written maple program to evaluate these functions is displayed in the appendix. The values of the variables that were used displayed below. **Note that the transfer function for theta based on voltage has a negative term in front of it so the PID gains can be positive.**

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Variable Initialization
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% theta;            % ~ % Platform deviation from vertical
% x;                % ~ % Horizontal displacement of Segway vehicle
% V;                % ~ % Motor voltage
% T;                % ~ % Motor torque
% d;                % ~ % Horizontal displacement of rider load/mass (m)

R = 0.5;            % C % Wheel radius (m)
L = 0.4;            % C % Distance from wheel centerline to center of mass (m)
l = 1;              % C % Distance from wheel centerline to rider load/mass (m)
M = 27;             % C % System (vehicle) mass (~120 lbs)
m = 33.72;          % C % Rider (load) mass(N) (~150 lbs)
k_t = 0.75;         % C % Torque constant (N*m/A)
k_bemf = 0.5;       % C % Back emf constant (V*sec)
r_a = 1.4;          % C % Armature resistance
c1 = 0.01;          % C % Small rotational damping with appropriate units
c2 = 0.01;          % C % Small linear damping with appropriate units
g = 9.81;           % C % Gravitational acceleration (m*s^2)
J = M*L^2+m*l^2;            % C % Moment of inertia related to all the mass rotating
around the wheels (kg*m^2)(calculated)
alpha = k_t/r_a;            % C % Motor constant (calculated)
beta = k_t*k_bemf/(R*r_a);  % C % Motor constant (calculated)
```

Figure 1 – Variables, constants, and Equations

Task 1

$invlaplace(G_{xVelocity}, s, t);$

$$0.008710801394\, e^{-0.008792102207\, t}$$

$simplify(invlaplace(G_{\theta V}, s, t));$

$$-4.647029483\, 10^{-8}\, e^{-0.008792102207\, t} + 4.647029483\, 10^{-8}\, e^{-0.00006510416667\, t} \cosh(3.390588998\, t) + 0.002039375548\, e^{-0.00006510416667\, t} \sinh(3.390588998\, t)$$

$simplify(invlaplace(G_{\theta d}, s, t));$

$$-2.561778353\, e^{-0.00006510416667\, t} \sinh(3.390588998\, t)$$

Figure 2 – Laplace Representations of Θ (t) and v (t)

Task 2

The Routh table analysis of the open looped system shows that velocity will be stable, but Θ will be unstable. However, the closed loop systems show that the velocity will be stable and the stability of Θ will be dependent of both the disturbance (d) and voltage (v).

> $RouthTable(denom(G_{xVelocity}), s);$

$$\begin{bmatrix} \frac{861}{20} & s \\ \frac{757}{2000} & 1 \end{bmatrix}$$

$RouthTable(denom(G_{xVelocity}), s, 'stablecondition' = true);$

$$true$$

$RouthTable(denom(G_{\theta}), s);$

$$\begin{bmatrix} \frac{82656}{25} & -\frac{7601768243}{200000} & s^3 \\ \frac{294993}{10000} & -\frac{6683553}{20000} & s^2 \\ -\frac{3636154692811}{6555400000} & 0 & s \\ -\frac{6683553}{20000} & 0 & 1 \end{bmatrix}$$

$RouthTable(denom(G_{\theta}), s, 'stablecondition' = true);$

$$false$$

Figure 3 - Open Loop Routh Table Analysis for Velocity

> $RouthTable(denom(tf_{\theta}), s);$

$$\begin{aligned}
&[[330624000000000000000, -380088412200000000000 \\
&\quad + 2286160714000000000\,V - 28717794000000000000\,d, s^3], \\
&[294993000000000000, -3341776500000000000 \\
&\quad + 20276785710000000\,V - 252489780000000000\,d, s^2], \\
&\left[-\frac{181807736279400000000000}{32777} + \frac{44468973852200000000}{32777}\,V \right. \\
&\quad \left. - \frac{137366781300000000000000}{32777}\,d, 0, s\right], \\
&[-3341776500000000000 + 20276785710000000\,V \\
&\quad - 252489780000000000\,d, 0, 1]]
\end{aligned}$$

$RouthTable(denom(tf_{\theta}), s, 'stablecondition' = true);$

$$0 < -909038681397000 + 222344869261\,V - 686833906500000\,d$$
$$\textbf{and}\ \ 0 < -371308500000 + 225297619\,V - 280544200000\,d$$

Figure 4 - Closed Loop Routh Table Analysis for Velocity and Θ

Task 3

The program that evaluated the closed loop transfer functions of the system with a proportional, intergral, and derivative (PID) controller was written in maple. The maple program that was created is displayed in the appendix. Arbitrary gain values were created, where the system will remain stable according to the Routh table analysis. The chosen PID gains were verified with the equations derived from the Routh table analysis.

```
> CE := sort(simplify(denom(tf)));
```

$$4628.736000\,s^4 + 32.00625000\,s^3\,k_D + 41.29902000\,s^3 + 0.2838750000\,s^2\,k_D + 32.00625000\,s^2\,k_P - 53212.37770\,s^2 + 0.2838750000\,s\,k_P + 32.00625000\,s\,k_i - 467.8487100\,s + 0.2838750000\,k_i$$

```
simplify(RouthTable(CE, s, 'stablecondition' = true));
```

$$0 < k_i \textbf{ and } 0 < 2753268 + 2133750\,k_D \textbf{ and } 0 < -2128495108 + 1280250\,k_P + 11355\,k_D - \frac{185149440\,(-31189914 + 18925\,k_P + 2133750\,k_i)}{2753268 + 2133750\,k_D} \textbf{ and } 0 < -31189914 + 18925\,k_P + 2133750\,k_i - (11355\,(2753268 + 2133750\,k_D)\,k_i) \Big/ \left(-2128495108 + 1280250\,k_P + 11355\,k_D - \frac{185149440\,(-31189914 + 18925\,k_P + 2133750\,k_i)}{2753268 + 2133750\,k_D}\right)$$

 # Arbitrary controller values were selected to satisfy the stable condition requirements as required by the Routh Table analysis.

$k_P := 1800;$
$k_i := 20;$
$k_D := 80;$

$$1800$$
$$20$$
$$80$$

```
simplify(RouthTable(CE, s, 'stablecondition' = true));
```

true

Figure 5 - Closed Loop Routh Table Analysis with a PID Controller

Task 4

By implementing the PID controller created in task 3 a stable voltage input for the velocity transfer function was created. The input voltage is the response to stabilize the Segway and counteract the disturbance.

Based on the Routh table analysis in task 2, the open loop velocity system is expected to be stable without additional controls. This is determined to be true by the simulation performed with an open loop velocity system.

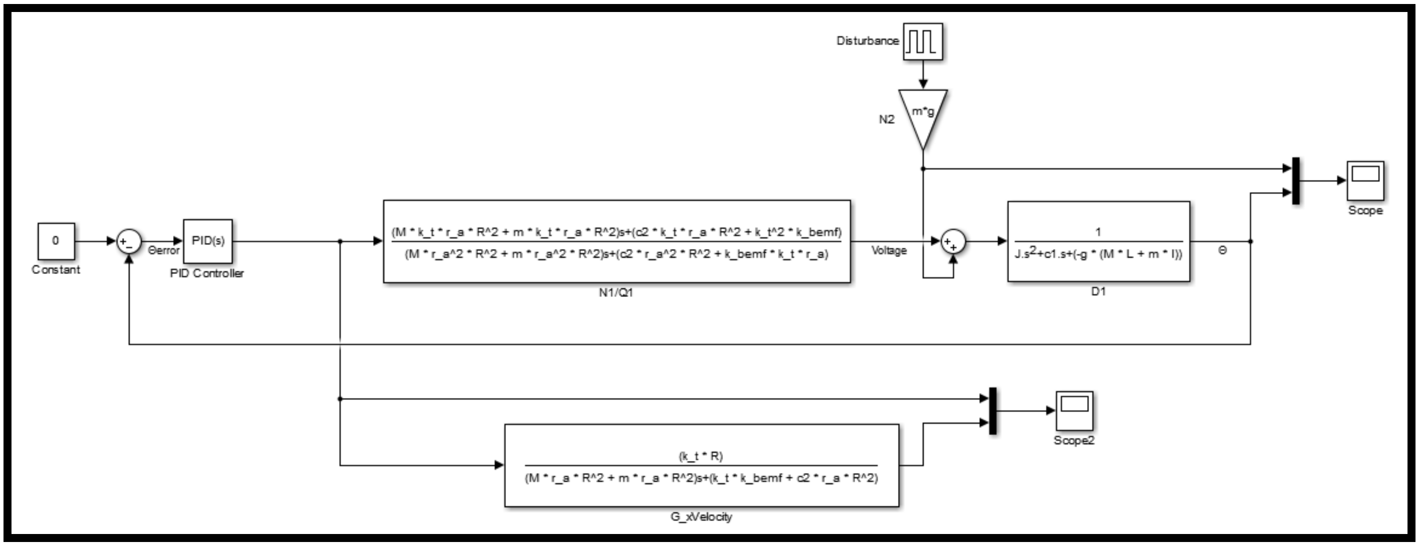

Figure 6 - Open Loop Velocity Stability Analysis

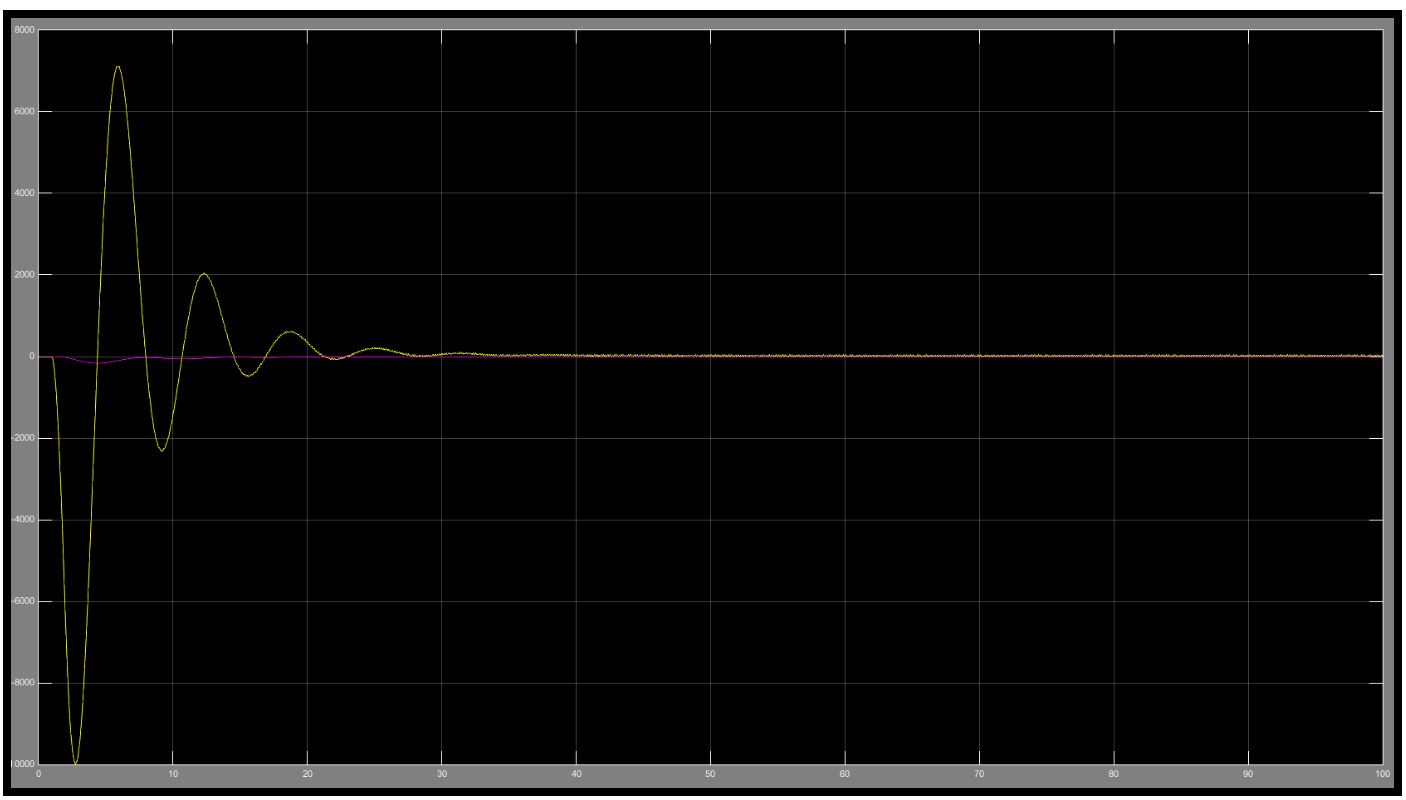

Figure 7 - Projected Open Loop Velocity Output

Task 5

The root locus analysis was performed through multiple series to refine the kp, ki, and kd gains. 2 out of the 3 gains are set to predetermined values, and the optimal value for the last gain will be determined using root locus analysis in MATLAB.

\> ***# Arbitrary controller values were selected to satisfy the stable condition requirements as required by the Routh Table analysis.***

k_P;

$k_i := 20$;

$k_D := 80$;

$$k_P$$

$$20$$

$$80$$

CE

$$4628.736000\,s^4 + 2601.799020\,s^3 - 53189.66770\,s^2 + 32.00625000\,s^2\,k_P + 0.2838750000\,s\,k_P + 172.2762900\,s + 5.677500000$$

Figure 10 – CE when ki=20 and kd=80

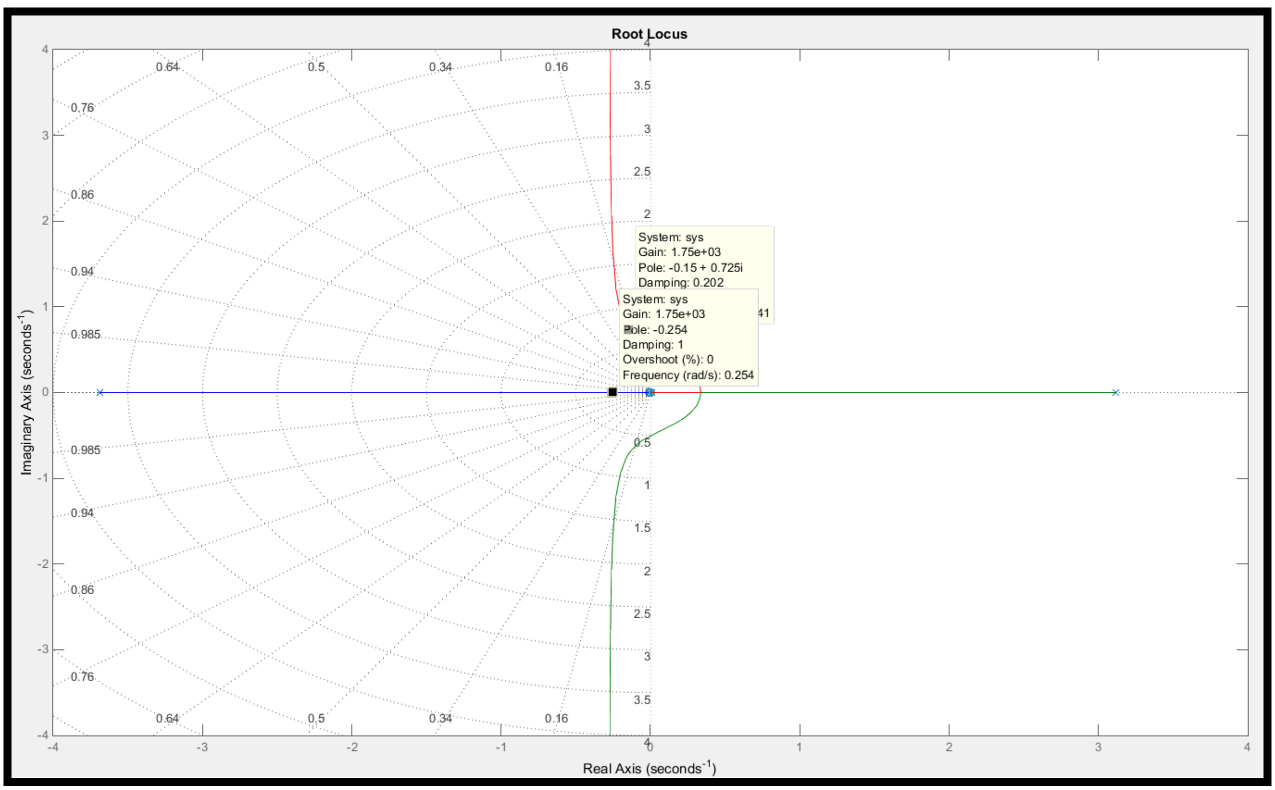

Figure 11 – Root Locus Analysis when ki=20 and kd=80

It seems that 1750 is the most appropriate kp gain when ki is set to 20 and kd is 80.

> # *Arbitrary controller values were selected to satisfy the stable condition requirements as required by the Routh Table analysis.*

$k_P := 1750;$

$k_i;$

$k_D := 80;$

CE

$$4628.736000\, s^4 + 2601.799020\, s^3 + 2821.26980\, s^2 + 28.9325400\, s + 32.00625000\, s\, k_i + 0.2838750000\, k_i$$

Figure 12 – CE when kp=1750 and kd=80

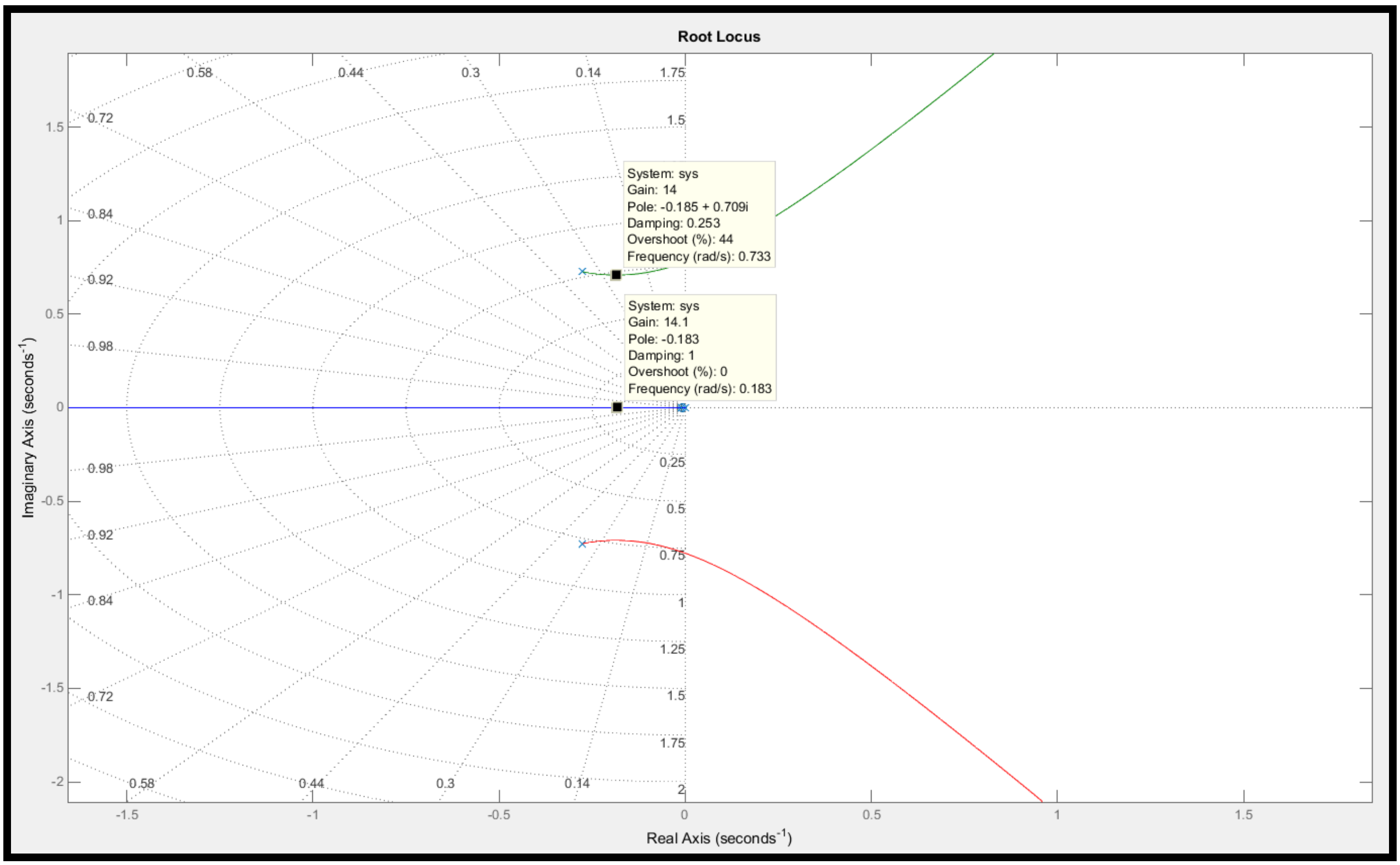

Figure 13 – Root Locus Analysis when kp=1750 and kd=80

It seems that 14 is the most appropriate ki gain when kp is set to 1750 and kd is 80.

\> # *Arbitrary controller values were selected to satisfy the stable condition requirements as required by the Routh Table analysis.*

$k_P := 1750;$

$k_i := 14;$

$k_D;$

$$1750$$

$$14$$

$$k_D$$

CE

$$4628.736000\,s^4 + 32.00625000\,s^3\,k_D + 41.29902000\,s^3 + 0.2838750000\,s^2\,k_D + 2798.55980\,s^2 + 477.0200400\,s + 3.974250000$$

Figure 14 – CE when kp=1750 and ki=14

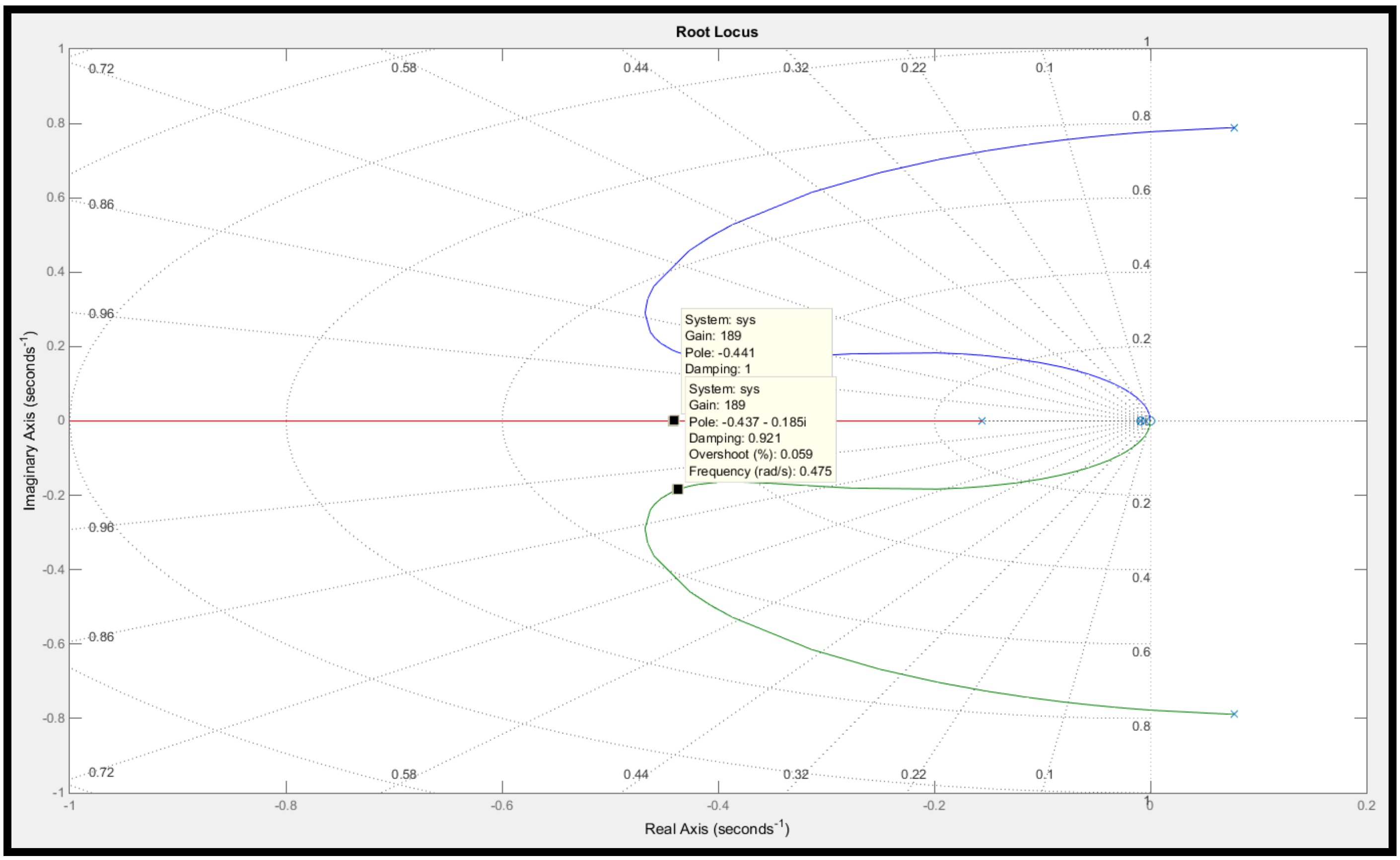

Figure 15 – Root Locus Analysis when kp=1750 and ki=14

It seems that 68.7 is the most appropriate kd gain when kp is set to 1750 and ki is 14.

Another series of root locus analysis was performed, refining the gains for kp, ki, and kd even further. The final values of the PID gains are:

$$\boxed{\begin{array}{c} kp = 1750; \\ ki = 14; \\ kd = 189 \end{array}}$$

Figure 16 – Root Locus Analysis Optimized PID Gains

Task 6

(i) Impulse Disturbance

The system settles and the value of theta returns to zero after a few seconds. There is only one visible oscillation in the response, and this is likely due to inertia. The voltage demanded by the system is similar to the inverse of Θ.

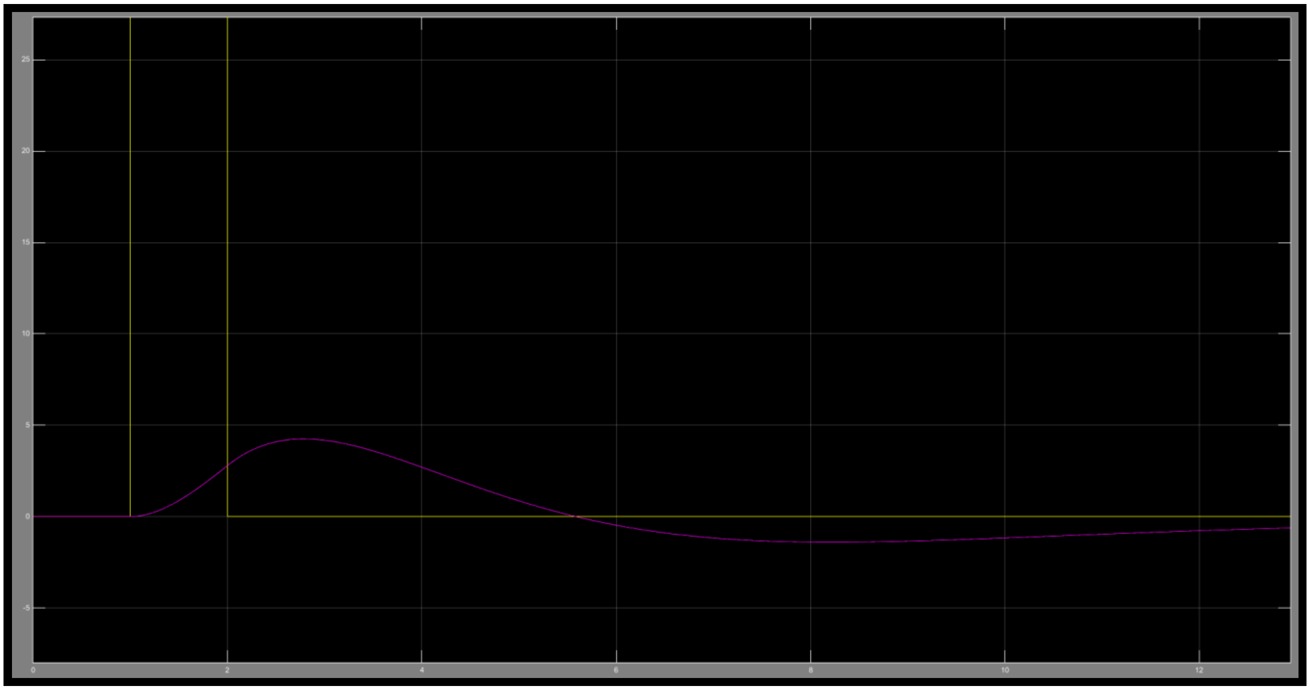

Figure 17 – Scope of Θ after Impulse Disturbance

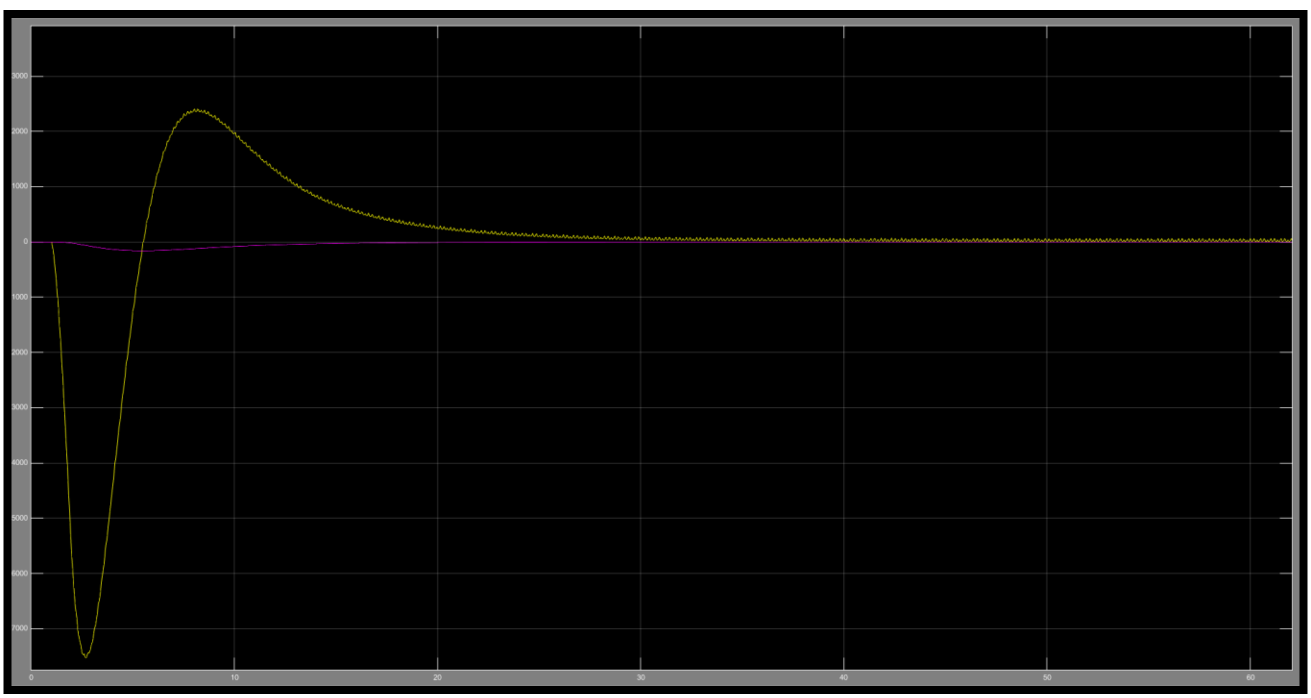

Figure 18 – Scope of velocity after Impulse Disturbance

(ii) Constant Step Disturbance

The system settles, but the value of Θ remains at a constant value that is not equal to zero. There seems to be a constant voltage demand by the system, and the velocity will eventually converge to a steady speed. The segway will be tilted at all time, and be moving either forward or backwards.

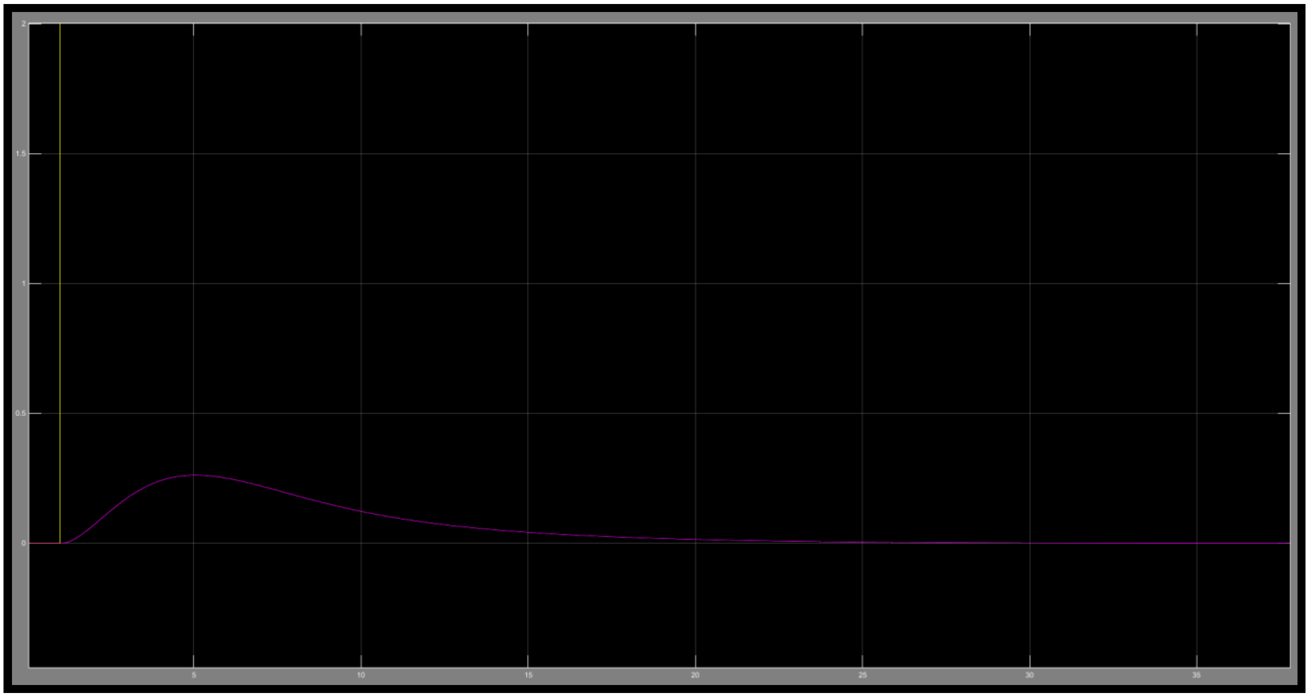

Figure 19 – Scope of Θ after Constant Disturbance

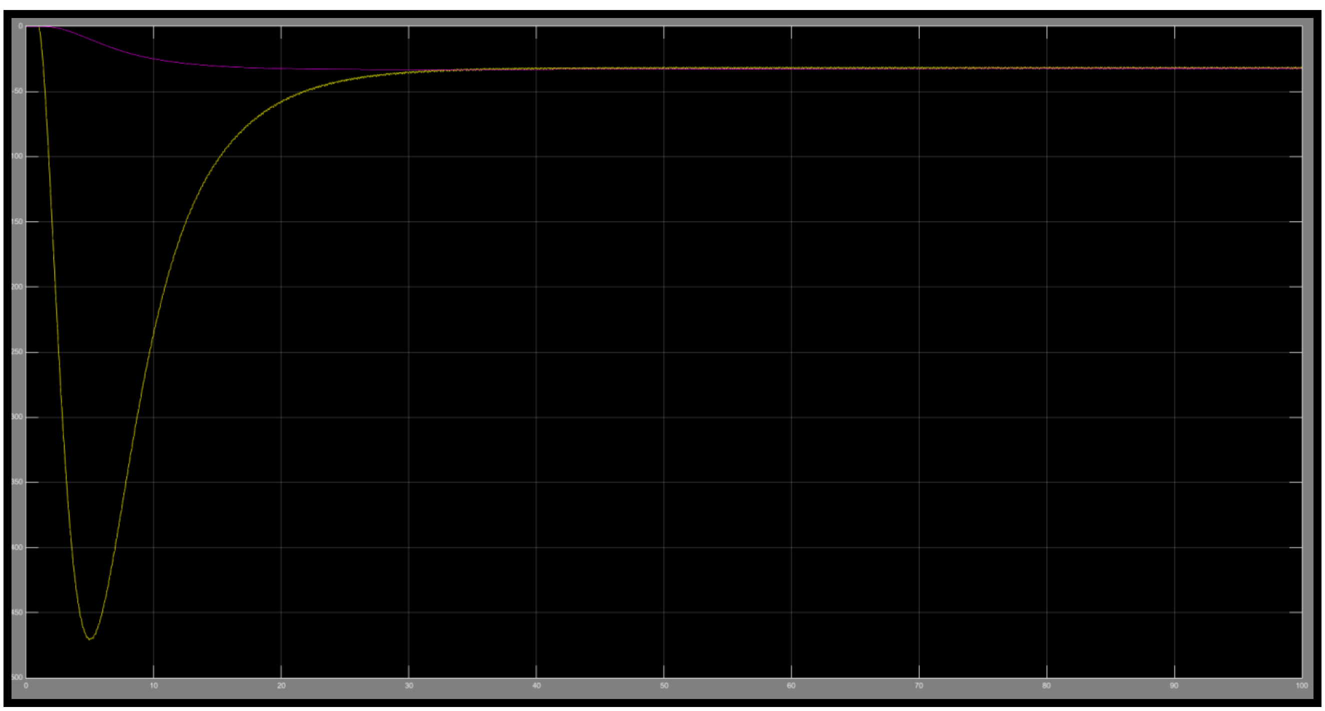

Figure 20 – Scope of velocity after Constant Disturbance

Task 7

It seems that with a saturated voltage, the system take slightly longer to settle. This is insignificant on the scope, but may alter performance and user experience greatly.

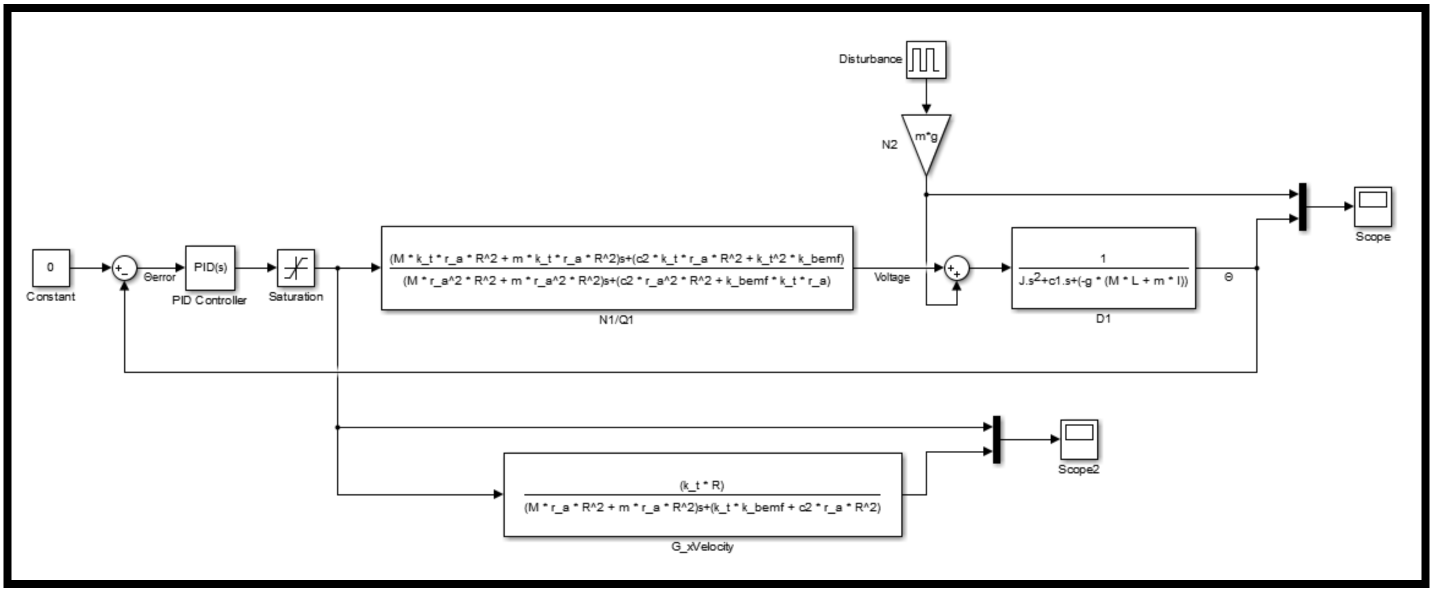

Figure 21 – Simulink Including a Saturation Block Limited Voltage

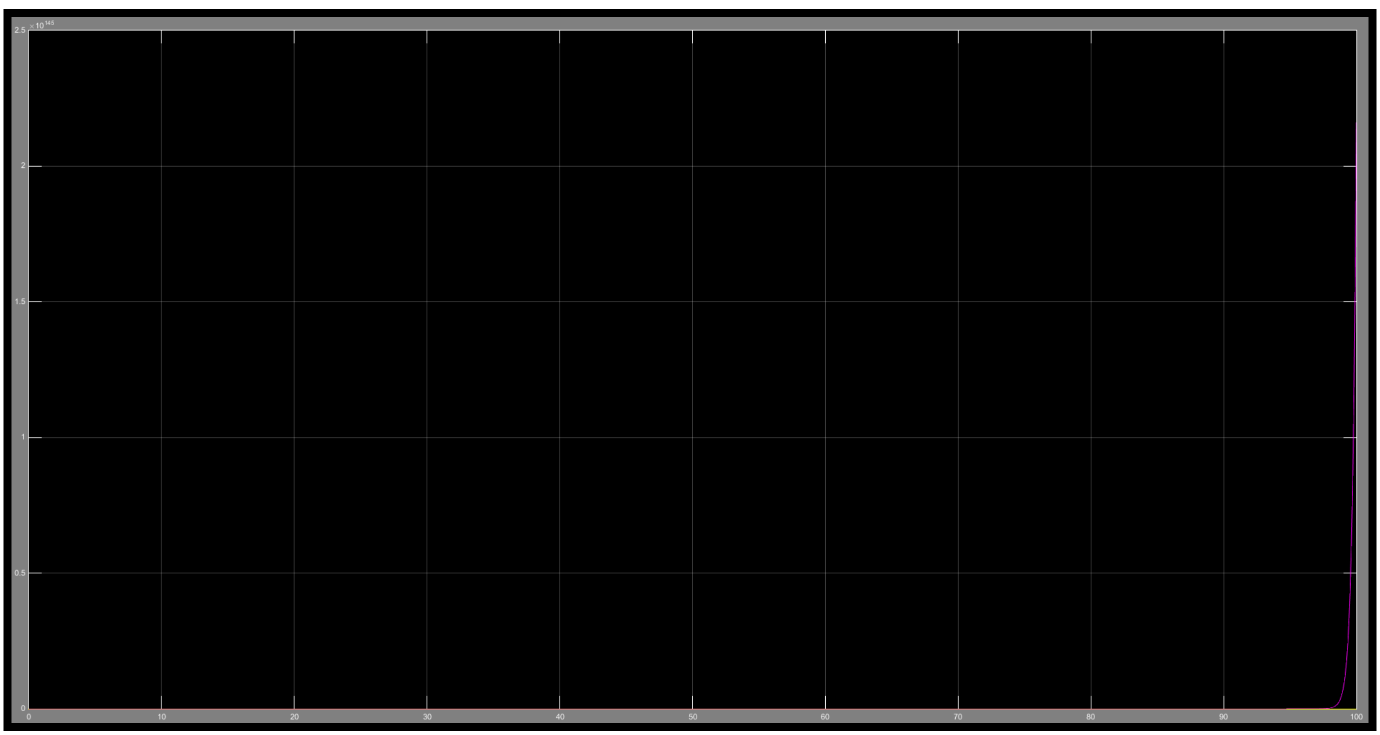

Figure 22 – Scope of Θ after Impulse Disturbance with Voltage Saturation

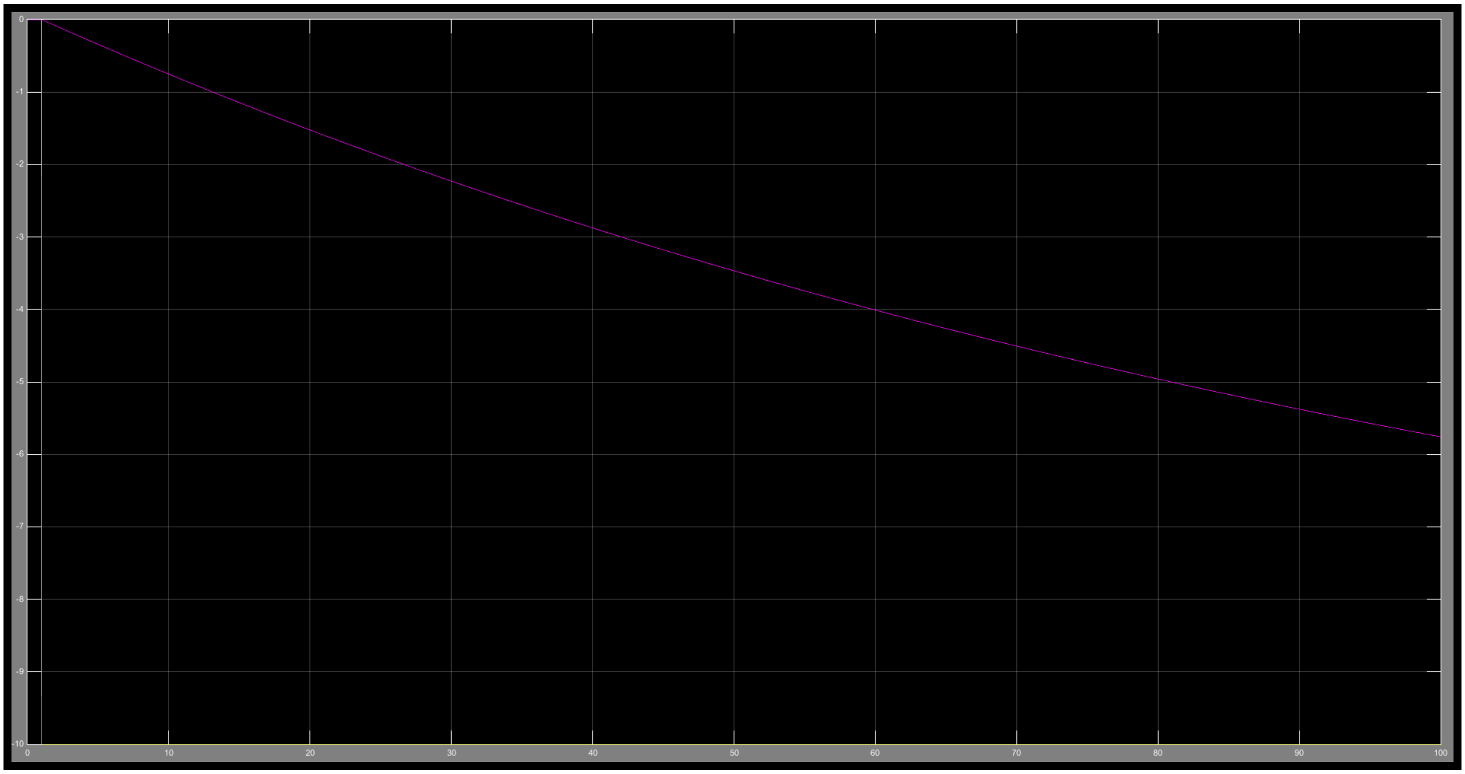

Figure 23 – Scope of Velocity after Impulse Disturbance with Voltage Saturation

Figure 24 – Scope of Θ after Step Disturbance with Voltage Saturation

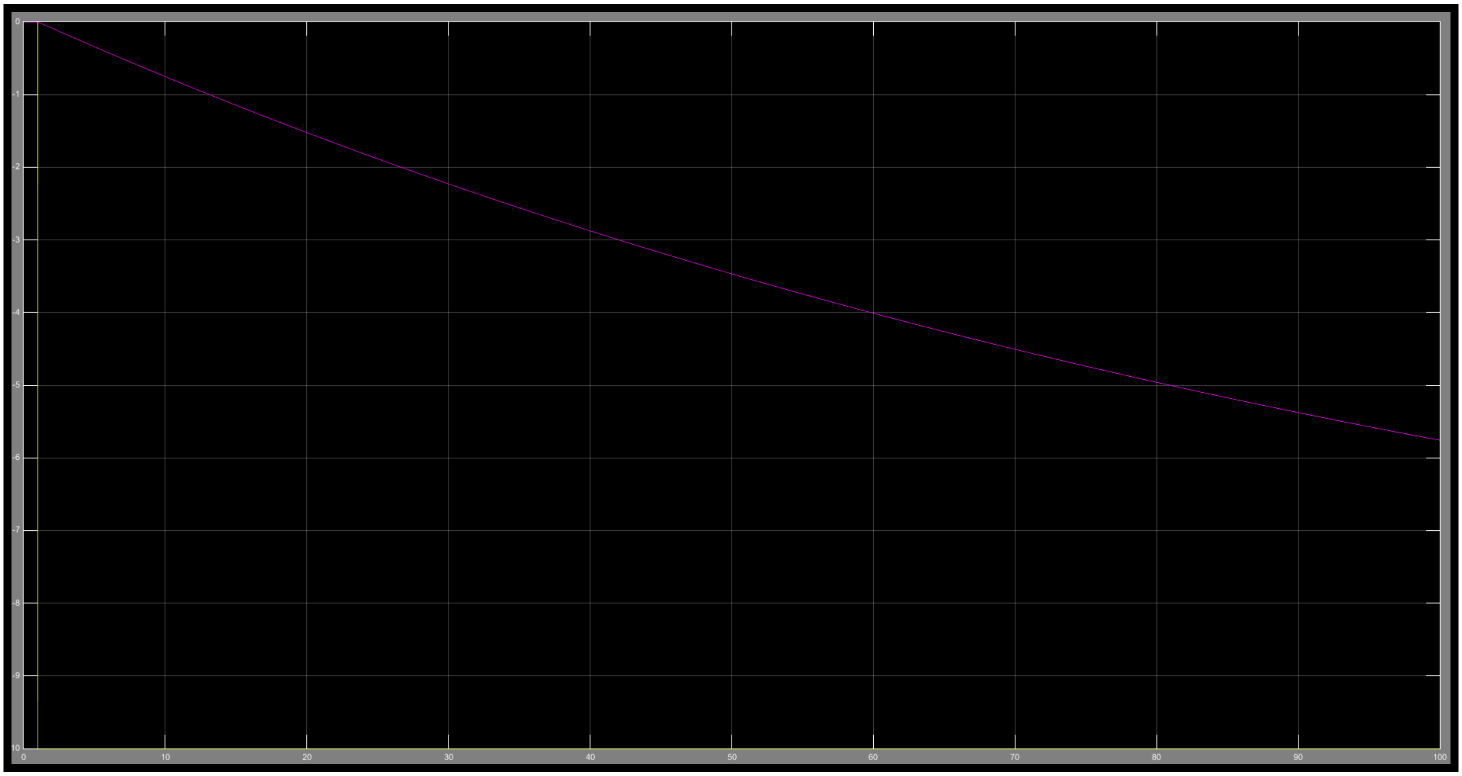

Figure 25– Scope of Velocity after Step Disturbance with Voltage Saturation

It seems that the segway cannot handle large disturbances due to the fact the voltage is saturated and can no longer deliver the power to restabilize the system, since the voltage is capped out at ±10V.

Appendix

(i) Calculate open loop tranfer functions

> restart; **# Include Libraries**with(inttrans); with(DynamicSystems);

[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable]

[AlgEquation, BodePlot, CharacteristicPolynomial, Chirp, Coefficients, ControllabilityMatrix, Controllable, DiffEquation, DiscretePlot, EquilibriumPoint, FrequencyResponse, GainMargin, Grammians, ImpulseResponse, ImpulseResponsePlot, IsSystem, Linearize, MagnitudePlot, NewSystem, NicholsPlot, NyquistPlot, ObservabilityMatrix, Observable, PhaseMargin, PhasePlot, PrintSystem, Ramp, ResponsePlot, RootContourPlot, RootLocusPlot, RouthTable, SSModelReduction, SSTransformation, Simulate, Sinc, Sine, Square, StateSpace, Step, StepProperties, System, SystemConnect, SystemOptions, ToDiscrete, TransferFunction, Triangle, Verify, ZeroPoleGain, ZeroPolePlot]

 # summation of all moments

$sumEqn_{\theta} := J\cdot\theta\cdot s^2 = M\cdot g\cdot L\cdot\theta + m\cdot g\cdot l\cdot\theta - T - c_1\cdot\theta\cdot s + m\cdot g\cdot d;$

$$J\theta s^2 = M g L \theta + m g l \theta - T - c_1 \theta s + m g d$$

 # summation of all forces in the x-direction

$sumEqn_{velocity} := (M+m)\cdot x\cdot s^2 = \frac{T}{R} - c_2\cdot x\cdot s;$

$$(M+m)\, x s^2 = \frac{T}{R} - c_2 x s$$

 # Predefined Equations

$\beta := \frac{k_t\cdot k_{bemf}}{R\cdot r_a};$

$$\frac{k_t k_{bemf}}{R r_a}$$

$\alpha := \frac{k_t}{r_a};$

$$\frac{k_t}{r_a}$$

$T := \alpha\cdot V - \beta\cdot x\cdot s;$

$$\frac{k_t V}{r_a} - \frac{k_t k_{bemf} x s}{R r_a}$$

 # This section solves for the transfer function of the velocity

$G_{xPosition} := solve\left(sumEqn_{velocity}, x\right);$

$$\frac{k_t V R}{s\left(k_t k_{bemf} + c_2 R^2 r_a + s R^2 r_a M + s R^2 r_a m\right)}$$

$G_{xVelocity} := \frac{G_{xPosition} \cdot s}{V};$

$$\frac{k_t R}{k_t k_{bemf} + c_2 R^2 r_a + s R^2 r_a M + s R^2 r_a m}$$

<u># This section solves for the transfer function of theta</u>

$x := G_{xVelocity} \cdot V;$

$$\frac{k_t R V}{k_t k_{bemf} + c_2 R^2 r_a + s R^2 r_a M + s R^2 r_a m}$$

$G_\theta := -solve\left(sumEqn_\theta, \theta\right);$

$$\left(-m g d r_a k_t k_{bemf} - m g d r_a^2 c_2 R^2 - m g d r_a^2 s R^2 M - m^2 g d r_a^2 s R^2 + k_t^2 V k_{bemf} + k_t V c_2 R^2 r_a + k_t V s R^2 r_a M + k_t V s R^2 r_a m - k_t^2 k_{bemf} V s\right) \Big/ \left(r_a \left(k_t k_{bemf} + c_2 R^2 r_a + s R^2 r_a M + s R^2 r_a m\right) \left(-M g L - m g l + c_1 s + J s^2\right)\right)$$

<u># This simplifies the TF of theeta into two components (V & d)</u>

$G_{\theta V} := coeff\left(G_\theta, V\right);$

$$\left(k_t^2 k_{bemf} + k_t c_2 R^2 r_a + k_t s R^2 r_a M + k_t s R^2 r_a m - k_t^2 k_{bemf} s\right) \Big/ \left(r_a \left(k_t k_{bemf} + c_2 R^2 r_a + s R^2 r_a M + s R^2 r_a m\right) \left(-M g L - m g l + c_1 s + J s^2\right)\right)$$

$G_{\theta d} := simplify\left(coeff\left(G_\theta, d\right)\right);$

$$-\frac{m g}{-M g L - m g l + c_1 s + J s^2}$$

(ii) Calculate closed loop transfer functions

> ***# This breaks down the transfer functions, seperating the disturbance from the model equation***

$N2 := numer(G_{\theta d});$

$$-m g$$

$D1 := denom(G_{\theta d});$

$$-M g L - m g l + c_1 s + J s^2$$

$N1 := numer(G_{\theta V});$

$$k_t \left(k_t k_{bemf} + c_2 R^2 r_a + s R^2 r_a M + s R^2 r_a m - k_t k_{bemf} s\right)$$

$Q1 := denom(G_{\theta V} \cdot D1);$

$$r_a \left(k_t k_{bemf} + c_2 R^2 r_a + s R^2 r_a M + s R^2 r_a m\right)$$

 # This creates a Closed Loop transfer function with PID Controller for theta

$C := k_P + \frac{k_i}{s} + k_D \cdot s;$

$$k_P + \frac{k_i}{s} + k_D s$$

$\theta = simplify\left(\frac{1}{1 + \frac{C \cdot N1}{Q1 \cdot D1}} \cdot d \cdot m \cdot g + \frac{1}{1 + \frac{C \cdot N1}{Q1 \cdot D1}} \theta_{ref}\right)$

$$\theta = \left(s r_a \left(k_t k_{bemf} + c_2 R^2 r_a + s R^2 r_a M + s R^2 r_a m\right) \left(-M g L - m g l + c_1 s + J s^2\right) \left(m g d + \theta_{ref}\right)\right) \Big/ \left(k_t k_P s^2 R^2 r_a M + k_t k_P s^2 R^2 r_a m + k_t k_i s R^2 r_a M + k_t k_i s R^2 r_a m + k_t k_D s^2 c_2 R^2 r_a + k_t k_D s^3 R^2 r_a M + k_t k_D s^3 R^2 r_a m - s^2 r_a^2 R^2 M^2 g L - s^2 r_a^2 R^2 m^2 g l + k_t k_P s c_2 R^2 r_a - s r_a k_t k_{bemf} M g L - s r_a k_t k_{bemf} m g l - s r_a^2 c_2 R^2 M g L - s r_a^2 c_2 R^2 m g l - s^2 r_a^2 R^2 M m g l - s^2 r_a^2 R^2 m M g L + k_P s k_t^2 k_{bemf} - k_P s^2 k_t^2 k_{bemf} - k_i k_t^2 k_{bemf} s + k_D s^2 k_t^2 k_{bemf} - k_D s^3 k_t^2 k_{bemf} + s^2 r_a^2 c_2 R^2 c_1 + s^3 r_a^2 c_2 R^2 J + s^3 r_a^2 R^2 M c_1 + s^4 r_a^2 R^2 M J + s^3 r_a^2 R^2 m c_1 + s^4 r_a^2 R^2 m J + k_t k_i c_2 R^2 r_a + s^2 r_a k_t k_{bemf} c_1 + s^3 r_a k_t k_{bemf} J + k_i k_t^2 k_{bemf}\right)$$

$tf := simplify\left(\frac{C \cdot N1 + d \cdot N2 \cdot Q1}{D1 \cdot Q1 + C \cdot N1}\right)$

$$-\left(-k_P s k_t^2 k_{bemf} - k_t k_P s c_2 R^2 r_a - k_t k_P s^2 R^2 r_a M - k_t k_P s^2 R^2 r_a m + k_P s^2 k_t^2 k_{bemf} - k_i k_t^2 k_{bemf} - k_t k_i c_2 R^2 r_a - k_t k_i s R^2 r_a M - k_t k_i s R^2 r_a m + k_i k_t^2 k_{bemf} s - k_D s^2 k_t^2 k_{bemf} - k_t k_D s^2 c_2 R^2 r_a - k_t k_D s^3 R^2 r_a M - k_t k_D s^3 R^2 r_a m + k_D s^3 k_t^2 k_{bemf} + d m g r_a s k_t k_{bemf} + d m g r_a^2 s c_2 R^2 + d m g r_a^2 s^2 R^2 M + d m^2 g r_a^2 s^2 R^2\right) \Big/ \left(k_t k_P s^2 R^2 r_a M + k_t k_P s^2 R^2 r_a m + k_t k_i s R^2 r_a M + k_t k_i s R^2 r_a m + k_t k_D s^2 c_2 R^2 r_a + k_t k_D s^3 R^2 r_a M + k_t k_D s^3 R^2 r_a m - s^2 r_a^2 R^2 M^2 g L - s^2 r_a^2 R^2 m^2 g l + k_t k_P s c_2 R^2 r_a - s r_a k_t k_{bemf} M g L - s r_a k_t k_{bemf} m g l - s r_a^2 c_2 R^2 M g L - s r_a^2 c_2 R^2 m g l - s^2 r_a^2 R^2 M m g l - s^2 r_a^2 R^2 m M g L + k_P s k_t^2 k_{bemf} - k_P s^2 k_t^2 k_{bemf} - k_i k_t^2 k_{bemf} s + k_D s^2 k_t^2 k_{bemf} - k_D s^3 k_t^2 k_{bemf} + s^2 r_a^2 c_2 R^2 c_1 + s^3 r_a^2 c_2 R^2 J + s^3 r_a^2 R^2 M c_1 + s^4 r_a^2 R^2 M J + s^3 r_a^2 R^2 m c_1 + s^4 r_a^2 R^2 m J + k_t k_i c_2 R^2 r_a + s^2 r_a k_t k_{bemf} c_1 + s^3 r_a k_t k_{bemf} J + k_i k_t^2 k_{bemf}\right)$$