Heuristics for Master Thesis

Håvard Notland

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1 Introduction

We assume that a trip s is an ordering of nodes where the first and last node is b, called the base. Each trip is a cycle starting and ending in b. Any trip is allowed to visit a node more than once, including b. The trip is not seen as terminated if b appears in the middle of s, only at the end. There are two possible metrics of optimizing the algorithm:

1.1 Limit number of trips

Given a graph G, a subset P of V(G), a start node b, a limit k and bound w. Is there a set of trips $s_1, s_2, ..., s_n$ which visits all of P, such that the size of $s_i \in S$ of each trip appearing in order is at most k, and the number of trips is at most w?

$$
\begin{aligned}
&Input : G, P \subseteq V(G), start\ b,\ limit\ k, constraint\ w\\
\\
&Problem : \exists\ set\ of\ trips\ S,\ s.t.\ \forall\ p\ \in P,\ p \in s_i \in S,\ and\\
&for\ each\ member\ of\ S : s_1, s_2, ..., s_n\ s_i\ starts\ and\ ends\ with\ b\ and\\
&|s_i|\ is\ at\ most\ k\ and\ n \leq w?\\
&The\ size\ of\ s_i\ is\ defined\ as\ the\ number\ of\ p_m \in P\ s.t.\ p_m \in s_i\ and\\
&p_m \notin s_j\ where\ 0 < j < i
\end{aligned} \tag{1}
$$

This heuristic is based on limiting the number of trips, where a trip may have some inherent cost function for doing. This would mean that we are encouraged to maximize the size of s_i subject to a limit k. Assuming the bound w is no less than $|P|/k$. Assuming that the members of P are all pairwise reachable, there should always be a solution which meets this bound. Hence, there is little point of optimizing on this metric as we always assume the stated to be true.

1.2 Limit sum of length of trips

Given a graph G, a subset P of V(G), a start node b, a limit k and bound w. Is there a set of trips $s_1, s_2, ..., s_n$ which visits all of P, such that the size of $s_i \in S$

of each trip appearing in order is at most k, and the sum of the length of each trip is at most w?

$$
\begin{aligned}
&Input : G, P \subseteq V(G), start\ b,\ limit\ k, constraint\ w\\
\\
&Problem : \exists\ set\ of\ trips\ S,\ s.t.\ \forall\ p\ \in P,\ p \in s_i \in S,\ and\\
&for\ each\ member\ of\ S : s_1, s_2, ..., s_n\ the\ following\ is\ true\\
&1.\ s_i\ starts\ and\ ends\ with\ b\\
&2.\ |s_i|\ is\ at\ most\ k\\
&3.\ \textstyle\sum_{i=1}^{n} |s_i| \leq w\\
&The\ size\ of\ s_i\ is\ defined\ as\ the\ number\ of\ p_m \in P\ s.t.\ p_m \in s_i\ and\\
&p_m \notin s_j\ where\ 0 < j < i
\end{aligned} \tag{2}
$$

This heuristic is a more complex and valuable limitation, as the grouping and ordering of which elements of P are to be visited in a trip is not arbitrary. A trip should minimize the total distance between each node including a base. It is trivial to observe that this is an NP-Hard formulation as a specialized instance of this problem is the Traveling Salesman Problem. The problem becomes TSP if we set $P = V(G)$

2 Conclusion

The latter heuristic can be solved using various techniques, though it is uncertain if any of these guarantee a minimum solution. A simple greedy solution is to simply visit the closest non-visited p at each step, and then returning to base once k is reached. This is an approximation algorithm, in which the approximation factor is currently unknown.

Another more sophisticated solution is as follows: First one would obtain a distance matrix between all members of P using a path-finding algorithm. Secondly we would use this distance matrix in a clustering algorithm to get a grouping where the size of each group is at most k, and the distance within the group is minimized. Finally we could use a traditional TSP solving technique for each group.

Although the final step is exponential, we can expect k to be of a relative small size ($k < 30$) and should therefore not be too resource intensive. There is a possibility that this is also an approximate solution and not an exact. This would depend on the clustering technique, as it is likely it will not be able to group members in an optimal manner considering the fastest solution is not necessarily guaranteed to have the minimal amount of trips. This statement is subject to a future proof.