Social Computing - Homework 2

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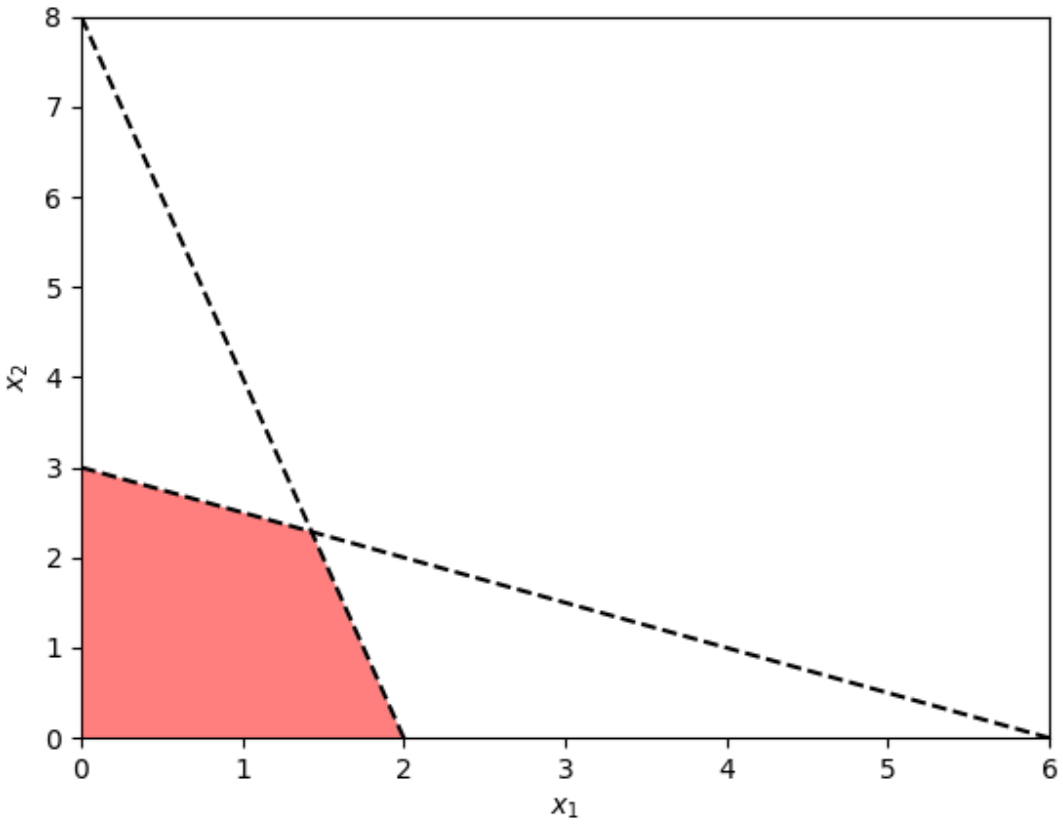
Problem 1.

(a)

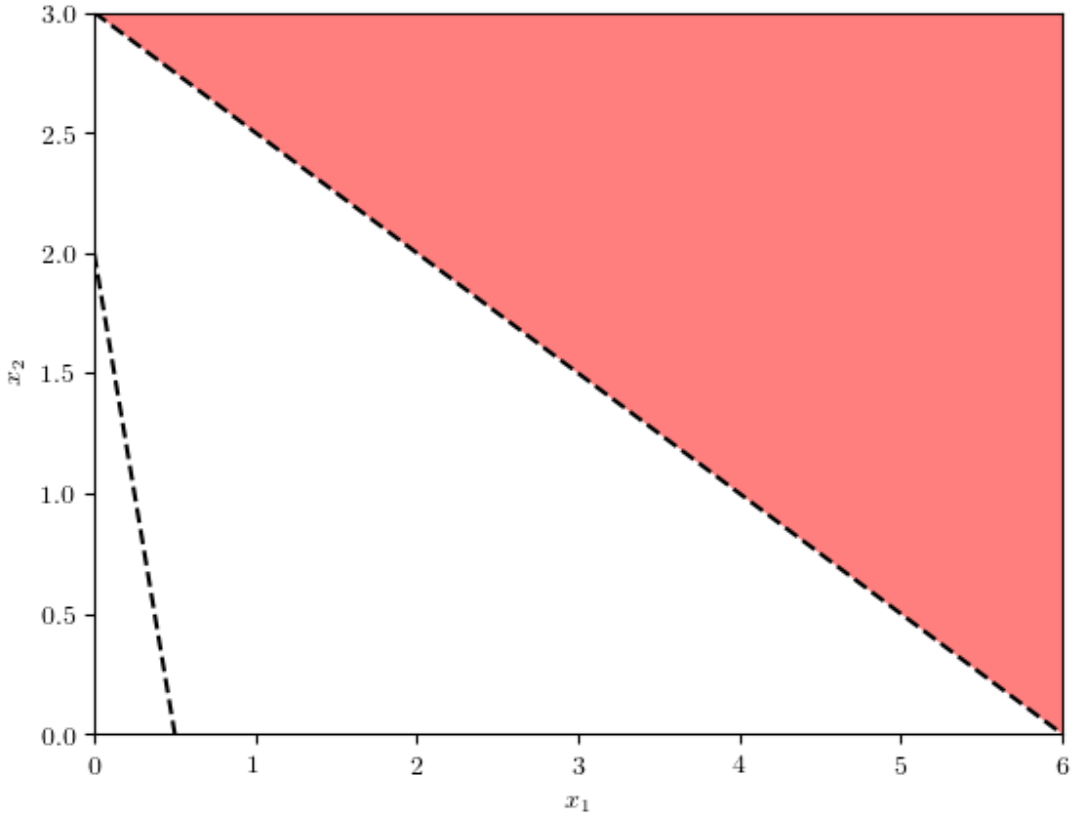


(b) The vertices are (x_1, x_2): $(0, 0), (0, 3), (2, 0), (10/7, 16/7)$.
(c) The optimal value is at $(0, 3)$: 18.

(d)

$$\begin{array}{ll} \text{minimize} & 8x_1 + 6x_2 \\ \text{subject to} & 4x_1 + x_2 \geq 2 \\ & x_1 + 2x_2 \geq 6 \\ & x_1, x_2 \geq 0 \end{array}$$

(e)



(f) The optimal value is at $(0, 3)$: 18.

Problem 2.

Let $P = \{(m, w), \dots\}$ be a male-optimal pairing output by the Gale-Shapley algorithm. We want to show that P is woman-pessimal by showing that any other pairing, $P^{'}$, that is more woman-pessimal that P is also unstable.

Let $P = \{(m^{'}, w), (m, w^{'}), \dots\}$ be a matching that is more woman-pessimal than P . This means that w must prefer m to her $P^{'}$ pairing $m^{'}$. Since our original pairing P is man-optimal, we know m must prefer w to $w^{'}$. Since w prefers m to her $P^{'}$ partner $m^{'}$ and m prefers w to her $P^{'}$ partner w , the pairing $P^{'}$ is unstable.

Since $P^{'}$ is unstable, we have show that P is the most woman-pessimal stable marriage.

Problem 3. (a) The proposed algorithm is based off the following fact. There must be a stable marriage which exists where the following are true: First, the marriage (m, w) is only stable if all the women which m prefers over w are paired with men who w prefers over m . And the inverse must also be true, all the men who w prefers over m must be paired with women who m prefers to w .

First, we remove m and w from each of the remaining preference lists. Then when running Gale-Shapley algorithm, if $rank([w][m^{'}]) > rank([w][m])$ where w is the woman in the stable pair to be tested, m is the man in the stable pair to be tested and $m^{'}$ is the currently choosing man, then we have $m^{'}$ choose a woman who m prefers to w who he has not previously proposed to. If at any point a man does not have a valid choice given that rule, then there is no stable matching. If the program terminates with a perfect matching, then there is a stable matching between (m, w).

(b) I would give the output of my algorithm for part **a**.

(c) I would give the stable matching where (m, w) had been removed and the facts from part **a** were true: The women which m prefers over w are paired with men who w prefers over m . And all the men who w prefers over m must be paired with women who m prefers to w .