1 NE101 Mid Term 2

NE101 Equation Sheet, Joshua Howland, 11/3/2014

Constants:
$\mu_0 = 4\pi \times 10^{-7} NA^{-2}$ or Hm^{-1}
$\epsilon = 8.854187 \times 10^{-12} \frac{F}{m}$
$eV = 1.6022 \times 10^{-16} J$
mass hydrogen $= 1.00794u$
$1u = 1.66053 \times 10^{-27} kg$
fine structure constant:
$\frac{e^2}{4\pi\epsilon_0} = 1.439976 Mev \cdot fm$

Nuclear radius ($\approx$): $R = 1.25A^{1/3} fm$
Shell model:
Intermediate Form: (unlikely on exam):
Magic Numbers: 2,8,20,40,58,92,112

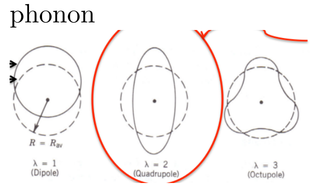
Intermediate form with spin orbit (Extreme independent shell model):
$1s^2_{1/2}1p^4_{3/2}1p^2_{1/2}1d^6_{5/2}2s^2_{1/2}1d^4_{3/2}1f^8_{7/2}2p^4_{3/2}$
$1f^6_{5/2}2p^2_{1/2}1g^{10}_{9/2}1g^8_{7/2}2d^6_{5/2}2d^4_{3/2}$
Magic Numbers: 2, 8, 20, 28, 50, 82, 126, 184
Notation: 1(s-0, p-1, d-2, f-3, g-4, h-5)

Even-Odd nuclei:
To calculate parity, find unpaired proton/neutron shell state. l is the shell (s p d f..) and I is the subscript. Find parity from l. For excited states, bump the unpaired neutron up the number of excited states. Example: Give the expected shell-model spin and parity assignments for the ground states of: (a) ^7Li: 3p, 4n: 1 unpaired proton. Proton shells are: $1s^2_{1/2}1p^1_{3/2}$. $\ell = 1 (p = 1), \pi = -, J_\pi = \frac{3}{2}^-$ (b) ^{11}B: 5p, 6n: 1 unpaired proton. Proton shells are: $1s^2_{1/2}1p^3_{3/2}$. $\ell = 1(p = 1)$, $\pi = -$, $J_\pi = \frac{3}{2}^-$ (c) ^{15}B: 6p, 9n: 1 unpaired neutron. Neutron shells are: $1s^2_{1/2}1p^4_{3/2}1p^2_{1/2}1d^1_{5/2}$. $\ell = 2(d = 2)$, $\pi = +$, $J_\pi = \frac{5}{2}^+$ Note: ^{15}C prediction wrong, observed value is $\frac{1}{2}^+$
ADD EXCITATION EX!

For odd-odd nuclei: $\vec{I} = \vec{j}_p + \vec{j}_n$
In ground state, $\vec{s}_p$ and $\vec{s}_n$ are parallel
$j_p = \ell_p + s_p$ and $j_n = \ell_n + s_n$
Note: ℓ and s not always parallel!
To calculate J_π, match ℓ to correct orbital for p and n, with spins parallel and multiply parities!
Ex: $^{40}_{19}K_{21}$. $J\pi$ found by coupling J from upaired p and n. P: $1d_{3/2}$, N $1f_{7/2}$. Put spins parallel, combine, mult parities. Answer: 2^-

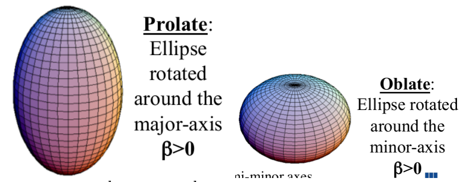
EVEN EVEN:
Shell model predicts that even-even nuclei will have a spin of 0^+ For excited states (even-even), the SHELL model predicts that the ground state is 0^+, the first excited state is 2^+, second is 4^+ and so on.
Example: Calculate I^π assignments of: (a) ^{26}Na: 11p, 15n. p: $1d^3_{5/2}$, $\ell = 2$, $2 + \frac{1}{2} = \frac{5}{2}$. n: $2s^1_{1/2}$, $\ell = 0$, $0 + \frac{1}{2} = \frac{1}{2}$. Total $J^\pi = 3^+$ (b) ^{28}Na: 11p, 17n. p: $1d^3_{5/2}$, $\ell = 2$, $2 + \frac{1}{2} = \frac{5}{2}$. n: $1d^1_{3/2}$, $\ell = 2$, $-2 + \frac{1}{2} = |-\frac{1}{2}|$. Total $J^\pi = 1^+$

Unit of EM energy called photon. "Quantum" of vibrational energy called phonon



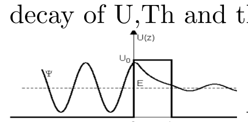
Vibrational model best for even-even nuclei with A¡150?(methinks)
Energy of vibration: $E = hv$, v = frequency in $\frac{vibrations}{sec}$ Magnetic moments for first 2^+ states predicted to be $2(\frac{Z}{A})$ and $\frac{E(2^+)}{E(4^+)}$ is predicted to $= 2$
Example: Compute vibrational frequency associated with typical quadrupole vibrations, then compare typical lifetimes of 2^+ excited states in vibrational nuclei to their vibrational frequency: ^{120}Te is a good choice (even-even, A¡150), low lying excited state at 2^+. $E = hv$, $v = \frac{E}{h} = \frac{0.5604Mev}{4.135\times10^{-21}\frac{Mev}{s}} = 1.355 \times 10^{20} \frac{1}{sec}$. T is much shorter than half life, so vibrates many times before decaying. $< \alpha >$ goes to 0, so we cant observe anything based on it, however $< \alpha^2 >$ does not!

Nuclear Rotations: Can only be observed in nuclei with non-spherical equilibrium shapes
$R(\theta, \phi) = R_{av}[1 + \beta Y_{20}(\theta, \phi)]$
$\beta = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{\Delta R}{R_{av}}$, $R_{av} = R_0 A^{\frac{1}{3}}$
$E = \frac{\hbar^2}{2\Im}J(J + 1)$, $\Im$= moment of inertia



$P = \frac{N_p^{valence}N_n^{valence}}{N_p^{valence}+N_n^{valence}}$, $P > 4$ deformed
If the nucleus was rigid: $\Im_{rigid} = \frac{2}{5}MR^2_{av}(1 + 0.31\beta) \approx 6kev$ for A = 170, but the observed value is 15Kev, so rotation is non rigid. $\omega_{rotation} \approx 10^{20}\frac{rad}{sec} \approx 0.002c$ on surface. Nuclear rotation is much slower than motion of nucleons, thus it is a type of collective motion. Deformation lowers energies of states moving inside the nucleus, raises energies of states spending more time outside nucleus
??Add backbending/nisson model??
Collective model magnetic moment: $\mu(I) = I\frac{Z}{A}\mu_N$

Alpha Decay: First type of decay discovered, least penetrating, emitted almost entirely by large nuclei, p conserved
$(Z, A) \to (Z - 2, A - 4) + ^4_2He_2$
$Q_\alpha = M(Z, A)c^2 - M_{product}c^2 - M(^4_2He_2)$
$Q_\alpha = K.E.(Z - 2, A - 4) + K.E.(^4_2He_2)$
$K.E. = T = \frac{p^2}{2m}$
$Q = T_\alpha + T_{Z-2,A-4} = T_\alpha + \frac{p^2}{2M_{Z-2,A-4}}$
$Q = T_\alpha + T_\alpha(\frac{M_\alpha}{M_{Z-2,A-4}})$
$T_\alpha = \frac{Q}{1+\frac{M_\alpha}{M_{Z-2,A-4}}} = \frac{Q}{1+\frac{4}{A}}$
$T_{x'} = \frac{Q}{1+\frac{m_{x'}}{m_\alpha}}$
Example: $Q_\alpha = [M(^{235}U) - M(^{231}Th) - M(\alpha)]c^2 = 4.678Mev$. $T_\alpha = 4.678Mev(1 - \frac{4}{235}) = 4.598Mev$, $T_{^{231}Th} = Q_\alpha - T_\alpha = (4.678 - 4.598)Mev = 0.080Mev$
Majority of ^4He on earth comes from α decay of U,Th and their daughters



Barrier Penetration

$T = \frac{1}{1+\frac{1}{4}\frac{U_0^2}{E(U_0-E)}sinh^2(\alpha L)}$
$\alpha = \sqrt{\frac{2m_\alpha E_\alpha}{\hbar^2}} \approx 17.5fm$, $T \approx 10^{-29}$
BAD. Instead, break up barrier into smaller chunks
$P \approx e^{-2\alpha a} = exp(-2\sqrt{\frac{2m_\alpha\frac{1}{2}(B-Q)}{\hbar^2}}x\frac{1}{2}(b - a))$

Coulomb barrier height (usually about 34 MeV for typical heavy nucleus):
$B = \frac{1}{4\pi\epsilon_0}\frac{zZ'e^2}{a}$
Probability to penetrate the complete barrier is:
$P = e^{-2G}$
$G = \sqrt{\frac{2m}{\hbar^2 Q}}\frac{zZ'e^2}{4\pi\epsilon_0}[arccos(\sqrt{x}) - \sqrt{x(1-x)}]$
Example: $^{210}Po \to {}^{206}Pb + \alpha$. $E_\alpha = 5.4Mev$. Estimate height of coulomb barrier that α must tunnel through: $r = \frac{1.25(210)^(1/3)}{2} = 3.714fm$,
$B = \frac{e^2}{4\pi\epsilon_0 \frac{(2)(82)}{3.714}} = 2(31.78)MeV$?
Estimate the probability that a 5.4MeV alpha particle in a head on collision wiht a ^{206}Pb nucleus will penetrate the Coulomb barrier: Use approx that $[arccos\sqrt{x} - \sqrt{x(x+1)}] \approx \frac{\pi}{2} - 2\sqrt{x}$ with $x = \frac{Q}{B}$. Calc G then P.
Angular Momentum in Alpha Decay: Any angular momentum carried away by the α-particle is purely orbital
Parity change $= (-1)^{\ell_\alpha}$. If the initial and final parities are the same, ℓ is even. If they are different, ℓ is odd. For a given α energy Q, the probability of barrier penetration decreases with increasing ℓ
Example: ^{235}U decay:
$\frac{\ell(\ell+1)}{2mr^2} = \frac{\ell(\ell+1)(197.3Mevf`m)^2}{2(4)(931.5Mev)(7.4fm)^2}$,
$\ell(\ell+1)x0.087Mev \to 0.174MeV$ for $\ell = 1$
Alpha particle spectroscopy: 1. Make source of α-decaying nuclei. Must be thin for α's to escape without losing much energy. 2. Measure the number of α's as a function of their energies. You can also measure g-rays emitted from excited states populated by α decay.

Copy hw 5 problems 8 and 10!!!

Beta Decay:
Neutrinos: No electric charge, interacts 'weakly' with matter (small xs), tiny mass, carries energy and linear momentum, has intrinsic spin of $\hbar/2$
$(Z, A) \to (Z-2, A-4) + {}^4He_2$
$(Z, A) \to (Z \pm 1, A) + e^{\mp} + \nu_e (or) \bar{\nu_e}$
β^- decay:
$^{210}_{83}Bi_{127} \to {}^{210}_{84}Ar_{126} + e^- + \bar{\nu_e}$
$(n \to p + e^- + \bar{\nu_e})$
β^+ decay:
$^{22}_{11}Na_{11} \to {}^{22}_{10}Ne_{12} + e^+ + \nu_e$
B^+ decay can occur if $Q_{ec} > 2m_e c^2$ $(p \to n + e^+ + \nu_e)$
Electron Capture:
$^{22}_{11}Na_{11} + e^+ \to {}^{22}_{10}Ne_{12} + \nu_e$
$(p + e^- \to n + \nu_e)$
$Q_{\beta^-} = [M_{atomic}({}^A_Z X_N) - M_{atomic}({}_{Z+1}^{A}Y_{N-1})]c^2$
$Q_{\beta^-} = T_e + T_{\bar{\nu}}$, since $T_{\bar{\nu}} \approx E_{\hat{\nu}}$
$T_e^{max} = E_{\bar{\nu}}^{max} = Q_{\beta^-}$
$Q_{\beta^+} = [M_{atomic}({}^A_Z X_N)$
$-M_{atomic}({}_{Z-1}^{A}Y_{N+1}) + 2M_e]c^2$
EC-Decay: ${}^A_Z X_N + e^- \to {}_{Z-1}^{A}Y_{N+1} + \nu_e$
$Q_{ec} = [M_{atomic}\,{}^A_Z X_N$
$-M_{atomic}\,{}_{Z-1}^{A}Y_{N+1}]c^2 - B_n$
B_n = binding energy of captured e^-
Minimum Q value for β^+ decay: $Q_{\beta^+} > 2m_e c^2 = 1.022MeV$
Observed Q_β values range from $\approx$ 2keV to $\approx$ 20 MeV. (Typical is around 1 MeV).

Allowed Decays:
If electron and neutrino created at r = 0, they cannot carry any orbital angular momentum. Change in angluar momentum of nucleus can only result from their spins. If spins are antiparallel, it is a Fermi Decay (total S = 0, and $I_i = |I_i - I_f| = 0$). If spins are parallel, called Gamow-Teller Decay, (total S = 1, and $I_i = I_f + 1$).
Selection rules for allowed β decay:
$\Delta I = 0, 1$; $\Delta\pi$(parity change) = no
Ex: Allowed β decay: $^{14}O \to {}^{14}N^*$ $(0^+ \to 0^+)$. Fermi Type.

Forbidden Decays (less probable):
First forbidden decays:$(l = 1)$
$\Delta I = 0, 1, 2$, $\Delta\pi = yes$
Ex: $^{17}N \to {}^{17}O$, $(\frac{1}{2}^- \to \frac{5}{2}^+)$
Second forbidden decays: $(l = 2)$

$\Delta I = 2, 3$, $\Delta\pi = no$
Ex: $^{22}Na \to {}^{22}Ne$, $(3^+ \to 0^+)$
Third forbidden decays: $(l = 3)$ $\Delta I = 3, 4$, $\Delta\pi = yes$
Ex: $^{87}Rb \to {}^{87}Sr$, $(\frac{3}{2}^- \to \frac{9}{2}^+)$
Fourth forbidden decays: $(l = 4)$
$\Delta I = 4, 5$, $\Delta\pi = no$
Ex: $^{115}In \to {}^{115}Sn$, $(\frac{9}{2}^+ \to \frac{1}{2}^+)$

Cross section for reaction $\bar{\nu} + p \to n + e^+$ is σ = probability per target atom for reaction / incident flux of $\bar{\nu}$

Helicity: All $\bar{\nu}$ have their spin vectors parallel to their momentum vectors, while all ν have spin opposite to momentum. This property is called the helicity and is defined to be:
$h = \frac{s \cdot p}{|s \cdot p|}$. h = 1 for $\bar{\nu}$ and -1 for ν

Example: Using helicity of emitted e^- and ν^-, deduce whether the e^- and ν^- tend to be emitted parallel or anti to one another: $\Delta L = 0$, so spin of emitted nuclei must be anti-parallel. Helicity tells us that the momentum is aligned with this for the n and anti-aligned with this for the e^- thus the particles tend to be emitted parallel to one another. For $1^+ \to 0^+$ β^-, using same logic but with $\Delta L = 1$, they tend to be emitted anti-parallel.

Gamma Decay: E - electric, M - magnetic, dipole radiation. Parity is:
$\pi(ML) = (-1)^{L+1}$
$\pi(EL) = (-1)^L$
Weisskopf Estimates:
$\lambda(E1) = 1.0 \times 10^{14} A^{2/3} E^3$
$\lambda(E2) = 7.3 \times 10^7 A^{4/3} E^5$
$\lambda(E3) = 34 A^2 E^7$
$\lambda(E4) = 1.1 \times 10^{-5} A^{8/3} E^9$
Note λ is in s^{-1} and E is in MeV
$\lambda(M1) = 5.6 \times 10^{13} E^3$
$\lambda(M2) = 3.5 \times 10^7 A^{2/3} E^5$
$\lambda(M3) = 16 A^{4/3} E^7$
$\lambda(M4) = 4.5 \times 10^{-6} A^2 E^9$

Angular momentum and parity selection rules:
$\Delta\pi = no$: even electric, odd magnetic
$\Delta\pi = yes$: odd electric, even magnetic

Example: List all permitted multipole modes and which is most likely (most intense) for $\frac{9}{2}^- \to \frac{7}{2}^+$. $\Delta\pi = yes$, so $L = 1, 2, 3, 4, 5, 6, 7$, or 8. Δpi = yes, so Odd electric, even magnetic $\to$ $E1, M2, E3, M4, E5, M6, E7, M8$. Most intense is E1.
λ_t = total decay probability
$\lambda_t = \lambda_\gamma + \lambda_e$
$\alpha = \frac{\lambda_e}{\lambda_\gamma}$
$\lambda_t = \lambda_\gamma(1 + \alpha)$
$\lambda_t = \lambda_\gamma + \lambda_{e,K} + \lambda_{e,L} + \lambda_{e,M} + ..$
$= \lambda_\gamma(1 + \alpha_K + \alpha_L + \alpha_M + ..)$
Nucleus Recoil: $T_r = \frac{E_\gamma^2}{2m_r c^2}$
Reaction Rate: $R = (\rho R)_{targ} \cdot I_{beam} \cdot \sigma_{rxn}$

Copy quarks and lepton stuff?
Copy forbidden decay problem from hw!!!!

Schrodingers Equation:
$-\frac{\hbar}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$
Parity: $|L| = \sqrt{l(l+1)}$