1 First Section

NE180, Plasma Physics, Mid Term 2 Equation Sheet

Constants:
$\mu_0 = 4\pi \times 10^{-7} NA^{-2}$ or Hm^{-1}
$\epsilon = 8.854187 \times 10^{-12} \frac{F}{m}$
$eV = 1.6022 \times 10^{-16} J$
mass hydrogen $= 1.00794u$
$1u = 1.66053 \times 10^{-27} kg$
Charge e^-: 1.6022×10^{-19} C
1 u = 1.6605 $\times$ 10^{-27} kg
$10^6 (mega)$;$10^3 (kilo)$;$10^{-3} (mili)$;
$10^{-6} (micro)$;$10^{-9} (nano)$;$10^{-12} (pico)$
General Notes:
$E = \frac{hc}{\lambda}$
ν_i^* is usually around 10^5
Sanity check: $v_{sound} < \nu_i^* < c$
$\omega_{ci}\tau_i \approx 10^6$ $n\omega$ in plasma formulary in cgs ($cm^{-3}s$), and Temperatures in eV
$\frac{m_i}{m_p} = 2.5$ for DT Plasma

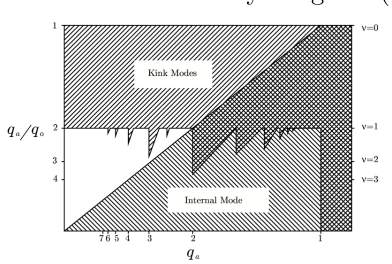
Hydrogen-Like Ions:
$E_{\text{inf}}^{Z-1} = 13.6 Z^2 [eV]$, near-stripped when $T_e >= \frac{1}{3} E_{\text{inf}}^{Z-1}$

Tokamak Safety Factor:
$q = \frac{r}{R}\frac{B_t}{B_p}$
R = major radius, r = minor radius
Note: $B_p = B_\theta$ and $B_t = B_\phi$
$A = R/r$ Note: r = a
$B_\theta(a) = \frac{\mu_0 I(a)}{2\pi a}$ and $B = \frac{\mu_0 I}{2\pi r}$
Adiabatic Invariant: $\mu = \frac{wperp}{B}$ FIX

Bennets Pinch Relation:
$< p > = \frac{B_\theta(a)^2}{2\mu_0}$

Plasma Drifts:
ExB Drift: $v_{drift} = \frac{\vec{E}\times\vec{B}}{B^2} = \frac{\vec{F}\times\vec{B}}{qB^2}$
Troyon Beta Limit: $\beta_{max}(\%) = \beta_N \frac{I}{aB}$
Fusion power density scales with n^2:
$p_f = \frac{n^2}{4} < \sigma v > E_f$
$c_s = (\frac{ZT_e + \gamma}{T}{}_i m_i)^{1/2}$
$v_A = \frac{B}{\sqrt{\mu_0 \rho}}$, $\frac{c_s}{v_A} = \sqrt{2\beta}$. In most plasmas, Alfven wave much faster than sound wave

Tokamak Stability Diagram (Wesson)

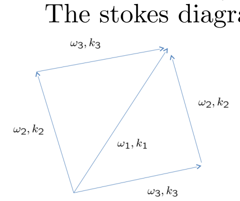


Alfven velocity: low frequency (compared to the ion cyclotron frequency) travelling oscillation of the ions and the magnetic field. $v_A = B/(4\pi n_i m_i)^{1/2}$. Note $n_i m_i = \rho$

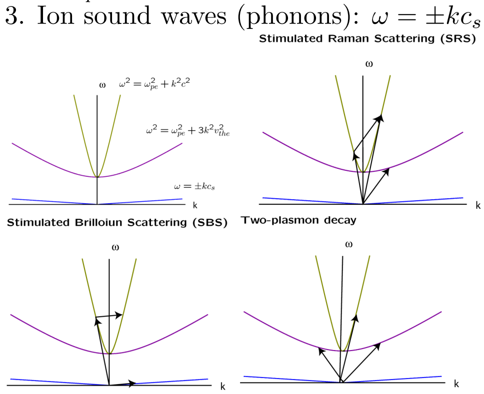
Lorentz Force Law: $\vec{F} = q(\vec{v}\times\vec{B} + \vec{E})$. Ex: derive e and ion cyclotron frequencies from LFL: $\vec{E} \to 0$, $qvB = m\frac{v^2}{r}$. $[\frac{v}{r} = \omega]$. $\frac{qB}{m} = \omega$. $[2\pi f = \omega]$. $f = \frac{qB}{2\pi m}$

Wave Stuff!
Propagation: R and L cutoffs:
$\omega_R = \frac{1}{2}(\sqrt{\omega_{ce}^2 + 4\omega_{pe}^2} + \omega_{ce})$
$\omega_L = \frac{1}{2}(\sqrt{\omega_{ce}^2 + 4\omega_{pe}^2} - \omega_{ce})$
Upper hybrid resonance: Limit $\omega >> \omega_{ci}, \omega_{pi}$. In this case, condition $\epsilon_\perp = 0$ yields: $\omega^2 = \omega_{ce}^2 + \omega_{pe}^2$
Lower hybrid resonance: Limit $\omega << \omega_{ce}$. In this case, condition $\epsilon_\perp = 0$ yields: $\omega^2 \approx \omega_{ci}^2 + \frac{\omega_{pi}^2}{1+\frac{\omega_{pe}^2}{\omega_{ce}^2}}$. Say $\omega_{pe}/\omega_{ce} << 1$, underdense regime, dispersion relation becomes: $\omega^2 \approx \omega_{ci}^2 + \omega_{pi}^2$. In case that $\omega_{pe}/\omega_{ce} >> 1$, which is known as the overdense regime, dispersion relation becomes: $\omega^2 \approx \omega_{ce}^2 \frac{m_e}{m_i} \approx \omega_{ce}\omega_{ci}$

The stokes diagram:



Candidate waves for parametric instabilities:
1. Electromagnetic waves (photons): $\omega^2 = \omega_{pe}^2 + k^2c^2$
2. Electron Langmuir waves (plasmons): $\omega^2 = \omega_{pe}^2 + 3k^2 v_{th}^2$
3. Ion sound waves (phonons): $\omega = \pm k c_s$



Raman and two-plasmon cases have ω_2 and $\omega_3 > \omega_{pe}$, so $\omega_1 > 2\omega_{pe}$. Since $(\frac{\omega_{pe}}{\omega})^2 = \frac{n}{n_c}$, where n_c is critical density, locally $n < \frac{1}{4} n_c$ for these procees to occur.

Use transport equations out of plasma formulary.
Scaling laws for transport coefficients: Note that for equal temps and Z=1, $\frac{\tau_i}{\tau_e} \approx \sqrt{2}(\frac{m_i}{m_e})^{1/2}$ and so $\frac{k_{||}^e}{k_{||}^i} \approx (\frac{3.2}{3.9})(\frac{m_i}{m_e})^{1/2}\frac{1}{\sqrt{2}}$, however: $\frac{k_\perp^e}{k_\perp^i} \approx (\frac{4.7}{2})(\frac{m_e}{m_i})^{1/2}\frac{1}{\sqrt{2}}$ and the factors $\omega_{ci}\tau_i$ and $\omega_{ce}\tau_e$ are both very large. Conclusions: Parallel transport coefficients are much greater than cross-field transport coefficients. Cross-field ion heat conduction dominates over cross-field electron heat conduction. Parallel electron heat conduction dominates over parallel ion heat conduction.
Characteristic length $L = |\frac{T_e}{\nabla T_e}|$

Example: Parallel heat conduction: plasma at $n_e = 10^{20} m^{-3}$ has $1eV/m$ temp gradient at a temp of $15KeV$. What is electron heat conduction along $\vec{B}$? $k_{||}^e = 3.2\frac{nT_e\tau_e}{m_e}$. $\tau_e = \frac{3.44\times10^5 T_e[eV]^{3/2}}{ln(\triangle) n_e [cm^{-3}]} = \frac{2.0\times10^4 T_e[eV]^{3/2}}{n_e[cm^{-3}]} = \frac{2\times10^4(15000)^{3/2}}{10^{14}} = 3.67\times10^{-4}s$. Now, $k_{||}^e = 3.2\frac{nT_e\tau_e}{m_e} = 3.2\frac{10^{20}(15000\times1.6\times10^{-19})3.67\times10^{-4}}{9.11\times10^{-31}} = 3.10\times10^{32} m^{-1}s^{-1}$. $\nabla T_e = 1.6\times10^{-19} Jm^{-1}$ so that $k_{||}^e \nabla T_e = 3.10\times10^{32} 1.6\times10^{-19} = 4.96\times10^{13} Wm^{-2}$.
Example: Interspecies heating: D+ plasma at $n_e = 10^{20}m^{-3}$ with e temp 15Kev and i temp 20Kev. What is volumetric rate of electron heating? Note that $\tau_e = 3.67\times10^{-4}s$ as before. Then $Q_e = 3\frac{m_e}{m_i}\frac{n(T_i - T_e)}{\tau_e} = 3(\frac{9.11\times10^{-31}}{2(1.67\times10^{-27})})10^{20}(\frac{((20{,}000-15{,}000)1.6\times10^{-19})}{3.67\times10^{-4}}) = 178kWm^{-3}$
Example: Cross field ion heat conduction: Ion temp near edge in D+ plasma is 1.0Kev and temp grad there is -5 kev per m. Mag field is 4T. What is ion heat conduction there? $k_\perp^i = 2.0\frac{nT_i\tau_i}{m_i(\omega_{ci}\tau_i)^2}$. $\tau_i = \frac{2.09\times10^7\sqrt{m/m_p}T_i[eV]^{3/2}}{ln\triangle n_i[cm^{-3}]} =$

$\frac{2.09\times10^7\sqrt{2}(1000)^{3/2}}{16\cdot10^{14}} = 5.84\times10^{-4}s$. $\omega_{ci} = \frac{1.6\times10^{-19}\cdot4}{2\cdot1.67\times10^{-27}} = 1.916\times10^8 s^{-1}$. $\omega_{ci}\tau_i = 1.916\times10^8\cdot5.84\times10^{-4} = 1.12\times10^5$. $k_\perp^i = 2.0\frac{10^{20}\cdot1000\times1.6\times10^{-19}\cdot5.84\times10^{-4}}{2\times1.67\times10^{-27}\cdot(1.12\times10^5)^2} = 4.47\times10^{17}m^{-1}s^{-1}$. $\nabla T_i = 5\cdot1.6\times10^{-16} = 8.0\times10^{-16}Jm^{-1}$. $(q_i)_\perp = -k_\perp^i\nabla T_i = 4.47\times10^{17}\cdot8.0\times10^{-16} = 357Wm^{-2}$

Collisionality:
$\nu_* = \frac{qR}{v_{th}\tau_i} = \frac{\nu/\epsilon}{\omega_b} = \frac{\nu/\epsilon^{3/2}}{\nu_T/qR} >> 1$
$\nu_i^* = \frac{qR}{\nu_{th}\tau_i}$ If $\nu_i^* > 1$, Pfirsch-Schutler
$1 > \nu_i^* > \epsilon^{3/2}$, Plateau
$\nu_i^* < \epsilon^{3/2}$. $\epsilon = \frac{r}{R_0}$

Ware pinch: Effect in the banana regime where trapped particles move into the plasma core under the influence of a parallel electric field. Ware pinch velocity: $\Delta v = (v_D)_{ware} \approx \frac{cE_\zeta}{B_p}$

Rutherford island growth rate: $\frac{\tau_R}{r^2}\frac{d\omega}{dt} = \Delta'$, τ_R is the local resistive time from Spitzer resistivity, r is the minor radius at q = m/n, Δ' is the classical stability index defined as the logarithmic jump of the radial magnetic field perturbation across the rational surface

$\Delta' > 0$ Unstable, $\Delta' < 0$ Stable

Neoclassical correction to Rutherford growth rate: $\frac{\tau_R}{r^2}\frac{d\omega}{dt} = \Delta' + \epsilon^{1/2}\frac{L_q}{L_p}\frac{\beta_{pol}}{\omega}$

Example: Tokamak, $B_\theta = 5.2$T, $A = 3$, $q(0.9a) = 3$, $50-50DT$, circular cross section, $q = 2$ surface is located at $r = 0.5a$ and Δ' is $-50.0m^{-1}$ there. Answer: At threshold value for growth, $\frac{d\omega}{dt} = 0$, $\rightarrow 0 = \epsilon^{\frac{1}{2}}\frac{L_q}{L_p}\frac{\beta_{pol}}{\omega}$. $\Delta' = \epsilon^{\frac{1}{2}}\beta_{pol}\omega$, $\rightarrow \epsilon =$ inverse aspect ratio, $\beta_{pol} = \frac{-\Delta'\omega}{\epsilon^{\frac{1}{2}}} = \frac{(50)(0.01)}{(1/3)^{1/2}} = 0.86$

Ara et al: (more accurate MHD gw)
$\gamma\tau_A = 0.55(\frac{\tau_A}{\tau_R})^{3/5}(\Delta' a)^{4/5}$
with: $\tau_A^{-1} = \frac{B_\theta}{\sqrt{\mu_0\rho}}\frac{1}{q}\frac{dq}{dr}$ and $\tau_R = \frac{\mu_0 a^2}{\eta}$

Mid Term 2 practice:
1. The ITER device ($R = 6.2m, a = 2.0m, B_\phi = 5.3T, q(0) = 1.0$) is assumed to operate with 40 MW of external heating and 80 MW of alpha-particle heating. Assume that at r = 0.9a the safety factor q is 3.0 and that the ion temperature is 3.0 keV. Assume that the density is 8×10^{19} at this point and that the plasma is Z = 1 with a 50-50 D-T mixture. Assume that ten percent of the heat leaves through the ion channel, neglecting ohmic heating. Assume that both heating sources mentioned above deposit all of the heat inside the r = 0.9a surface.

(a) At r = 0.9a, find τ_i, $\omega_{ci}\tau_i$ and $k_\perp^i$. Also find the neoclassical collisionality parameter, ν_i^*: $T_i(0.9a) = 3000eV$. $n_i(0.9a) = 8.0\times10^{19}m^{-3} = 8.0\times10^{13}cm^{-3}$. $n\tau_i = \frac{2.09\times10^7(\frac{m_i}{m_p})^{1/2}T_i^{3/2}}{ln(\Lambda)} = 3.61\times10^{11}cm^{-3}s$. $\tau_i = 0.0045s$. $\omega_{ci} = \frac{qB}{m_i} = 2.03\times10^8s^{-1}$. $\omega_{ci}\tau_i = 9.19\times10^5$. $k_\perp^i = 2.0\frac{nT_i\tau_i}{m_i(\omega_{ci}\tau_i)^2} = 9.80\times10^{16}m^{-1}s^{-1}$. $\nu_i^* = \frac{qR}{\nu_{th}\tau_i} = (3)(6.2)(\frac{T_i}{m_i})^{-\frac{1}{2}}/\tau_i = 0.0121$.

(b) For this value of ν_i^*, find the neoclassical collisionality regime (ie Pfirsch-Schluter, Plateau or Banana: $\nu = \frac{qR}{v_{th}\tau_i} = (3)(6.2)(T_i/m_i)^{-1/2}/\tau_i = 0.0121$ so ions are Banana

(c) If the ion heat transport is neoclassical, find the value of ∇T_i at $r = 0.9a$: $q^{''} = (40+80)\times10^6/612 = 19,607.8Wattm^{-2}$. Neoclassical factor $= Q_{neo} = 2q^2\epsilon^{-3/2} = 109.545$. Since $q^{''} = -Q_{neo}k_\perp\frac{dT(r)}{dr}$, we have $\frac{dT(r)}{dr} = 1.8\times10^{-15}Jm^{-1} = 11.35keVm^{-1}$.

(d) At the center of the device, the electron temperature is 25.0 keV and the ions are at 15.0 keV. The density there is $1.1\times10^{20}m^{-3}$. Find the electron-ion interspecies heating at $r = 0$, in megawatts per cubic meter: $Q_{ie} = \frac{3m_e}{m_i}\frac{n(T_i-T_e)}{\tau_e}$. $\tau_e = 2.0\times10^{10}T_e^{3/2}/n_e$ with T_e in eV and MKS density. Then $\tau_e = 718\mu s$ and $Q_{ie} = 160kWm^{-3}$

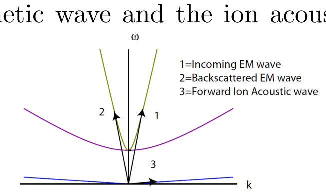
(e) Find the volumetric ohmic heating at $r = 0$. Give the answer in megawatts per cubic meter. Hint: note that the toroidal current can be derived from q: $J_\phi(0) = \frac{2B_\phi(0)}{\mu_0 q(0)R_0}$: $\eta = 2.8\times10^{-8}ZT_e^{-3/2} = 2.24\times10^{-10}\Omega m$. $J_\phi(0) = \frac{2B_\phi(0)}{\mu_0 q(0)R_0} = 1.36\times10^6Am^{-2}$. $P_\Omega = 414.6Wm^{-3}$

2. A hohlraum design for NIF is made with depleted uranium ($M_u = 238m_p$). The interior of the hohlraum is illuminated with frequency-tripled neodymium-glass laser light, $\lambda = 0.35\mu$. There are two laser entrance holes (LEHs), each with a 2.0mm diameter. The total laser power is $4.0\times10^{14}W$ with a duration of $4.0\times10^{-9}s$.

(a). If the fraction of the power loss due to the blackbody radiation through the LEHs is thirty percent, find the photon temperature in the hohlraun: Radiation loss: $q^{''} = 0.3P_{laser}/A_{hole} = \frac{0.3\times4\times10^{14}}{(2(\frac{\pi d^2}{4}))} = 1.91\times10^{15}Wcm^{-2}$. $q_{BB}^{''} = \sigma T_\gamma^4 = 1.03\times10^5T^4 = 1.9\times10^{15}Wcm^{-2}$. Then, $T_\gamma = (\frac{1.9\times10^{15}}{1.03\times10^5})^{1/4}$, or $T_\gamma = 369eV$

(b). Assume that the electron temperature at the hohlraum wall is 3.0 keV. Assume that the average charge $< Z >$ of the uranium is 30 and that the adiabatic index γ_e for the electrons is 1.0. Find the ion-acoustic wave speed, c_s in the uranium plasma. Ignore the uranium ion temperature. The ion-acoustic wave speed is given by: $c_s = (\frac{\gamma_e ZT_e+\gamma_i T_i}{m_i})^{1/2}$ Using $Z = 30$ and $m = 238mp = 2.38\times1.67\times10^{-27}$ with $T_e = 3000\times1.6\times10^{-19}$ gives $c_s = 1.90\times10^5ms^{-1}$

(c). Draw a Stokes diagram for stimulated Brillouin scattering (SBS), labeling the forward (1) and backscattered (2) electromagnetic wave and the ion acoustic wave (3).



(d). For SBS taking place where the plasma density is at 0.8x the critical density, find the value of the electron density there. The critical density n_c is defined as $\omega_{pe}(n_c) = \omega_L$, or $n_c = \frac{m_e\epsilon_0\omega_L^2}{e^2}$ Here $\omega_L = \frac{2\pi c}{\lambda_L} = \frac{2\pi3\times10^8}{0.35\times10^{-6}} = 5.39\times10^{15}s^{-1}$. Then $n_c = 9.13\times10^{27}m^{-3}$ and thus $n = 0.8n_c = 7.31\times10^{27}m^{-3}$

(e) Find the wavelength shift $\Delta\lambda$ for the backscattered electromagnetic wave by the following method: to first order, the wavenumber $k_3 = 2k_1$. Then find $\omega_3 = k_3c_s$ and use this to obtain $\omega_2 = \omega_1 - \omega_3$. Then solve for the (free-space) wavelength shift using $\Delta\lambda/\lambda \approx -\Delta\omega/\omega = -\omega_3/\omega_1$. Express your answer in angstroms. ($1\mathring{A} = 10^{-10}m$) For the incoming EM wave, $\omega_L^2 = \omega_{pe}^2 + k_1^2c^2$ Noting that $\omega_{pe}^2/\omega^2 = n/n_c$ and $k_0 = \omega/c$, this can be rewritten as $k_1 = k_0\sqrt{1-n/n_c}$ and thus $k = \sqrt{1-0.8}\frac{2\pi c}{\lambda_0} = 8.03\times10^6m^{-1}$. Then $\omega_3 \approx 2k_1c_s = 3.056\times s^{-1}$. Then $\Delta\omega/\omega_L = -5.675\times10^{-4}$ and $\Delta\lambda = (3500)(5.675\times10^{-4})\mathring{A} = 1.98\mathring{A}$