INSTITUTO POLITÉCNICO NACIONAL
ESCUELA SUPERIOR DE CÓMPUTO

Cryptography

Session 6: Operations in binary fields *March 20, 2019*

In this session we will work with some common operations in several encryption algorithms . Please do the following programming exercises in teams of two students. Please only consider the following programming languages to develop the exercises: C, C++, Python, or Java.

1. Theory

1. Consider the following operation used to calculate each entry of the AES S-box

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \mod 2.$$

where $b' = b'_7 b'_6 \dots b'_0$ is the bitwise vector representation of a^{-1}, where $a \in GF(2^8)$.

a) If $a = 25$ (in hexadecimal), use the operation above to obtain b, this value must be equal to the value stored in the AES S-box (column 2, row 5). The next table has the multiplicative inverse in $GF(2^8)$.

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	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	00	01	8D	F6	CB	52	7B	D1	E8	4F	29	C0	B0	E1	E5	C7
1	74	B4	AA	4B	99	2B	60	5F	58	3F	FD	CC	FF	40	EE	B2
2	3A	6E	5A	F1	55	4D	A8	C9	C1	0A	98	15	30	44	A2	C2
3	2C	45	92	6C	F3	39	66	42	F2	35	20	6F	77	BB	59	19
4	1D	FE	37	67	2D	31	F5	69	A7	64	AB	13	54	25	E9	09
5	ED	5C	05	CA	4C	24	87	BF	18	3E	22	F0	51	EC	61	17
6	16	5E	AF	D3	49	A6	36	43	F4	47	91	DF	33	93	21	3B
7	79	B7	97	85	10	B5	BA	3C	B6	70	D0	06	A1	FA	81	82
X 8	83	7E	7F	80	96	73	BE	56	9B	9E	95	D9	F7	02	B9	A4
9	DE	6A	32	6D	D8	8A	84	72	2A	14	9F	88	F9	DC	89	9A
A	FB	7C	2E	C3	8F	B8	65	48	26	C8	12	4A	CE	E7	D2	62
B	0C	E0	1F	EF	11	75	78	71	A5	8E	76	3D	BD	BC	86	57
C	0B	28	2F	A3	DA	D4	E4	0F	A9	27	53	04	1B	FC	AC	E6
D	7A	07	AE	63	C5	DB	E2	EA	94	8B	C4	D5	9D	F8	90	6B
E	B1	0D	D6	EB	C6	0E	CF	AD	08	4E	D7	E3	5D	50	1E	B3
F	5B	23	38	34	68	46	03	8C	DD	9C	7D	A0	CD	1A	41	1C

b) Explain why the operation above is equivalent to the operations that we studied in class to generate the S-box.

2. Programming exercises

1. Design a function to do multiplication in a binary field. This function must receive as a parameter an irreducible polynomial of degree n, m, and two a, b elements in $GF(2^n)$. Your function must return $a * b \bmod m$. Please use binary representation for a, b and m. The output must be in a binary representation also. Please **DO NOT USE** arrays to store a, b and m.
2. Design a function that take a binary string a and print the polynomial representation.
3. Design a function to implement the operation in the previous section and generate the S-box.

Products

- You must write a report, containing:
  1. Your personal information, date of the lab session and the topic that we are studying in this lab session.
  2. Your answers for section 1 (Theory).
  3. The most important parts of your source code, explaining what they do.

  You must submit a hard copy of your report next Wednesday (March 27). We will check your programs also the same day March 27.