

Farshod Mosh | Math I B

HONORS PROJECT

Howon Kim

01 *Computer + Math*

Introduction

Numerical Analytics
Discrete Mathematics

02

03 *Numerical Integration*

Simpson's rule

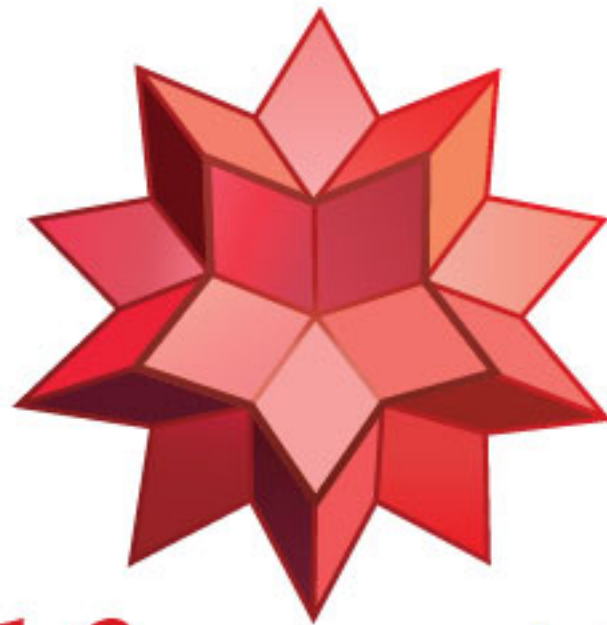
Simpson's rule
Implementation in C++

04

Computer + Math

01

Do you know Wolfram Alpha?



WolframAlpha®

integrate mosh



 Examples
  Random

Interpreting as: **integrate**



Assuming "integrate" refers to a computation | Use as [a general topic](#) or [referring to a mathematical definition](#) or [a word](#) instead

■ function to integrate:

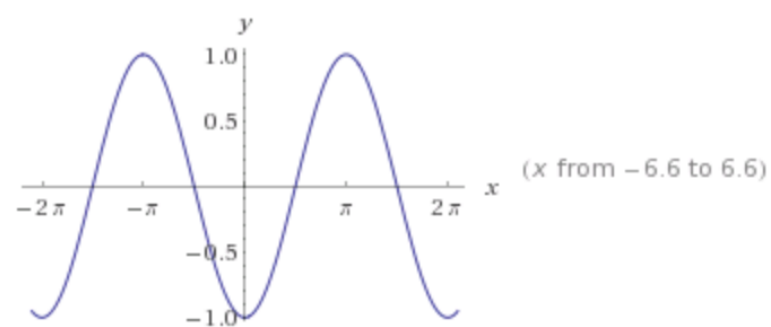
Also include: [domain of integration](#) | [variable](#)

Indefinite integral:

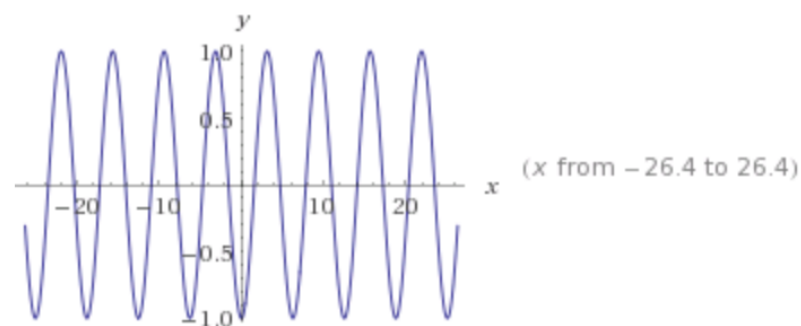
 Step-by-step solution

$$\int \sin(x) dx = -\cos(x) + \text{constant}$$

Plots of the integral:



 Enable interactivity



 Enable interactivity

Computer + Math

Definite integral from Wolfram Alpha

Definite integral over a half-period:

$$\int_0^{\pi} \sin(x) dx = 2$$

Definite integral mean square:

$$\int_0^{2\pi} \frac{\sin^2(x)}{2\pi} dx = \frac{1}{2}$$

How?

More digits

02

Numerical Analytics Discrete Mathematics

*Numerical analysis is the study of **algorithms** that use numerical **approximation** for the problems of **mathematical analysis** (as distinguished from **discrete mathematics**)*

Symbolic Computation

$$x^2 - 5 = 0$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

Numerical Analytics

$$x^2 - 5 = 0$$

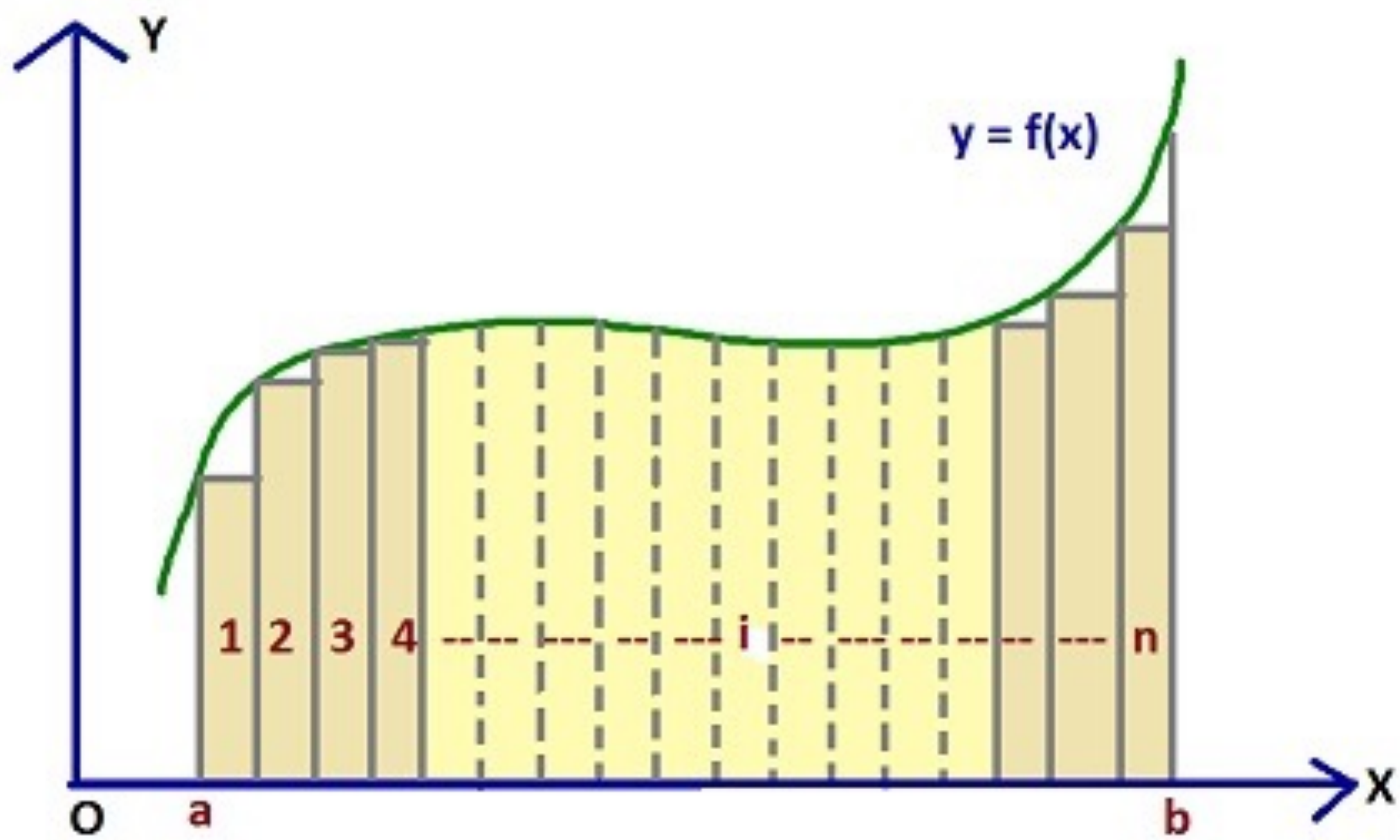
$$x = (0.00000, \text{infinite})$$

until it finds the answer

03

Numerical Integration

Approximate Integration



1

midpoint rule or rectangle rule.

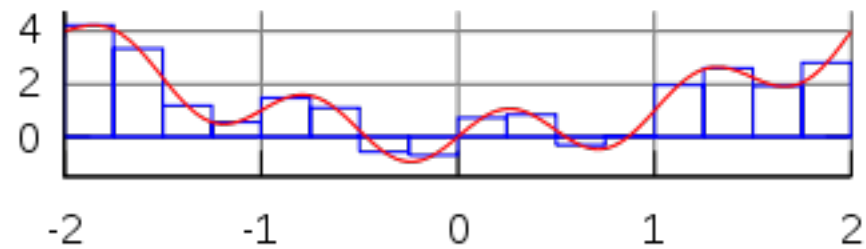
2

trapezoidal rule.

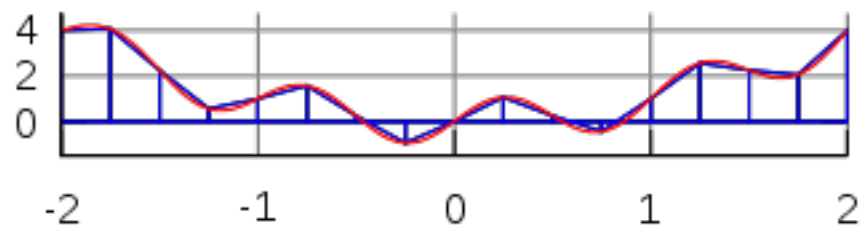
3

Newton–Cotes formulas (Simpson's rule)

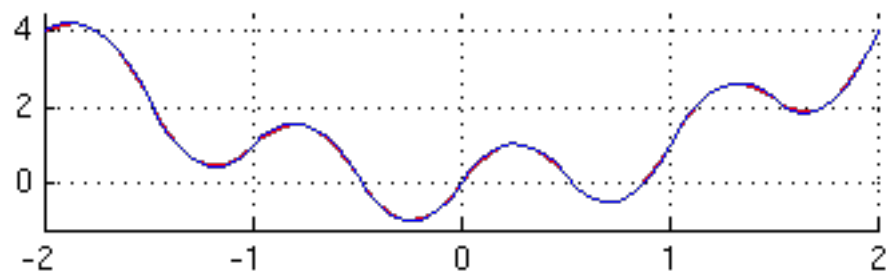
midpoint rule or rectangle rule.



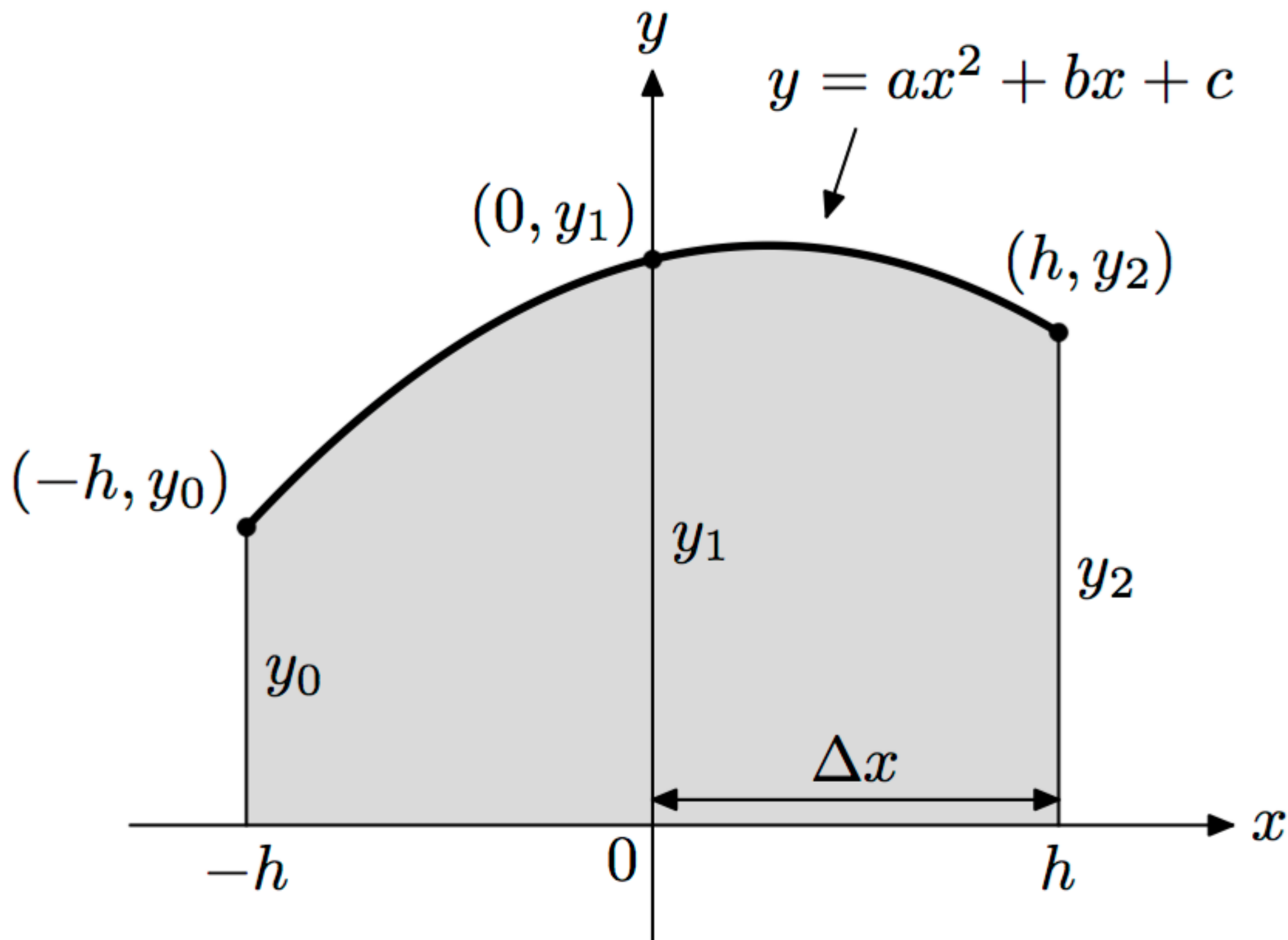
trapezoidal rule.



Newton–Cotes formulas (Simpson's rule)



Simpson's Rule | Derive Formula



Simpson's Rule | Integration

$$\begin{aligned} A &= \int_{-h}^h (ax^2 + bx + c) \, dx \\ &= \left(\frac{ax^3}{3} + \frac{bx^2}{2} + cx \right) \bigg|_{-h}^h \\ &= \frac{2ah^3}{3} + 2ch \\ &= \frac{h}{3} (2ah^2 + 6c) \end{aligned}$$

Simpson's Rule | $(-h, y_0), (0, y_1), (h, y_2)$

$$y_0 = ah^2 - bh + c$$

$$y_1 = c$$

$$y_2 = ah^2 + bh + c$$

Simpson's Rule | Add them up

$$y_0 + 4y_1 + y_2 = (ah^2 - bh + c) + 4c + (ah^2 + bh + c) = 2ah^2 + 6c.$$

$$A = \frac{h}{3} (y_0 + 4y_1 + y_2) = \frac{\Delta x}{3} (y_0 + 4y_1 + y_2) .$$

$$= \frac{h}{3} (2ah^2 + 6c)$$

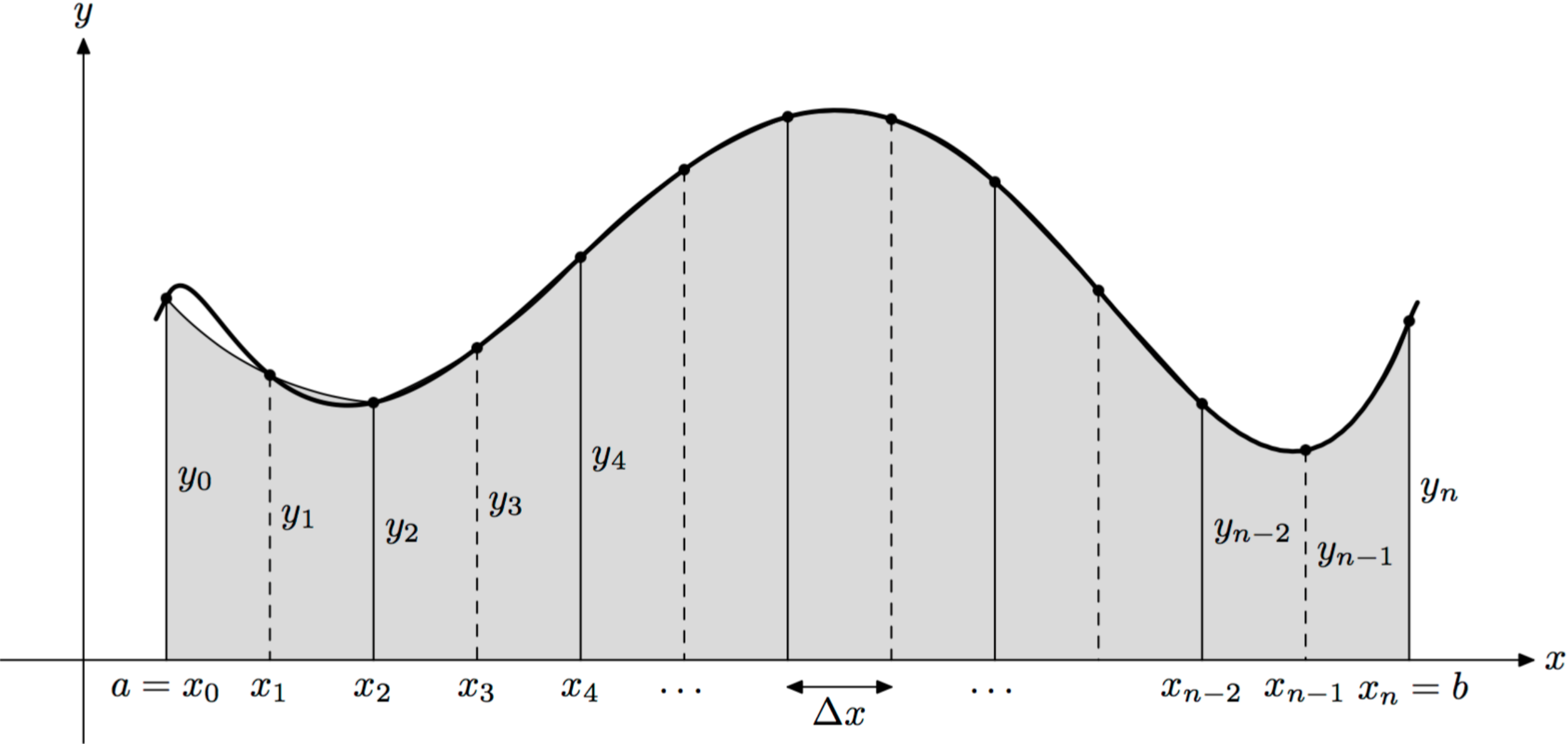
Simpson's Rule | Add them up

$$\int_a^b f(x) \, dx.$$

$$x_0 = a, \quad x_1 = a + \Delta x, \quad x_2 = a + 2\Delta x, \quad \dots, \quad x_n = a + n\Delta x = b.$$

$$y_0 = f(x_0), \quad y_1 = f(x_1), \quad y_2 = f(x_2), \quad \dots, \quad y_n = f(x_n).$$

Simpson's Rule | Graphical Explanation



Simpson's Rule | Derive Formula

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} (y_0 + 4y_1 + y_2) + \frac{\Delta x}{3} (y_2 + 4y_3 + y_4) + \cdots + \frac{\Delta x}{3} (y_{n-2} + 4y_{n-1} + y_n)$$

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{n-1} + y_n)$$

Simpson's Rule | Prove

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{n-1} + y_n)$$

$$y_0 + 4y_1 + y_2$$

$$y_2 + 4y_3 + y_4$$

$$y_4 + 4y_5 + y_6$$

$$y_6 + 4y_7 + y_8$$

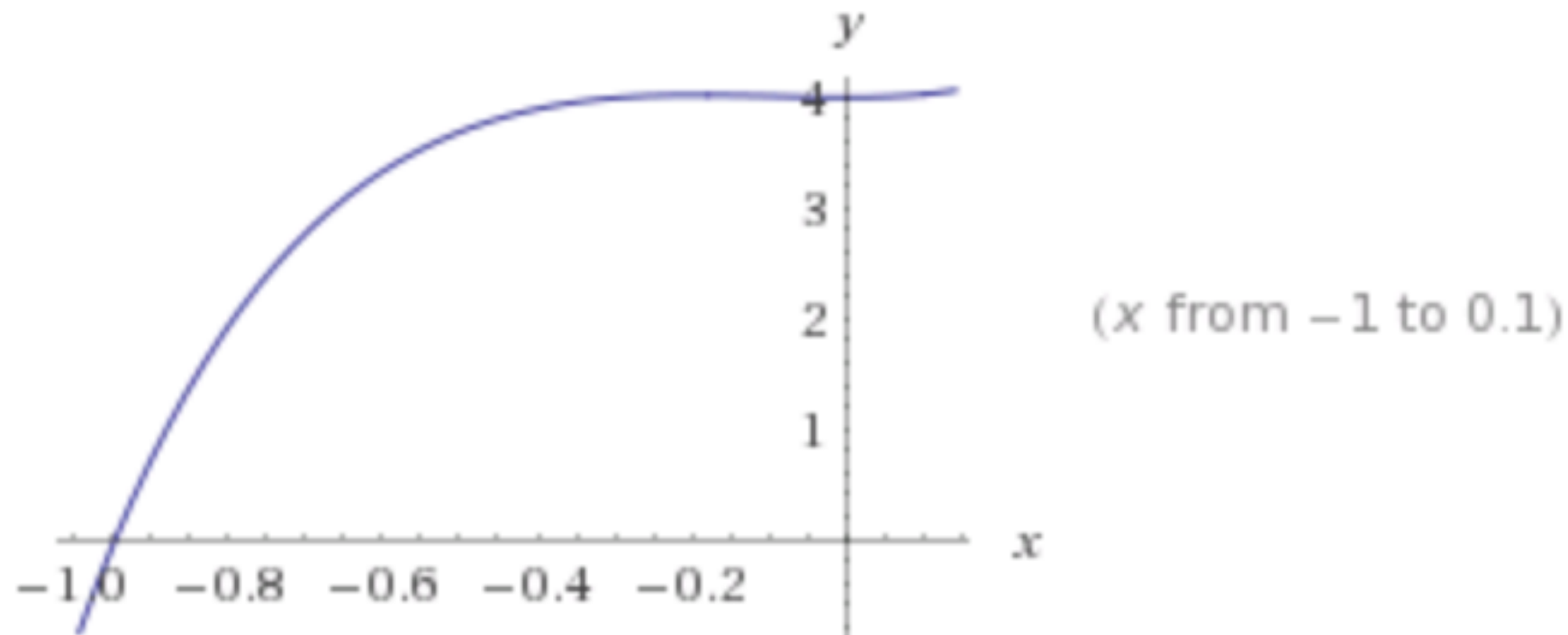
$$y_8 + 4y_9 + y_{10}$$

$$y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_6 + 4y_7 + 2y_8 + 4y_9 + y_{10}$$

Simpson's Rule | Example

$$4x^5 + 5x^4 + 8x^3 + 2x^2 + 4$$

Plots:



Lower limit : 1

$$a = 1$$

$$dx = (5-1) / 10$$

Upper limit : 5

$$b = 5$$

(upper limit - lower limit) / size

Dividing size : 10

$$n = 10$$

===== ENTER THE FUNCTION =====

Enter the highest exponent : 5
Enter the coefficient of X^5 : 4
Enter the coefficient of X^4 : 5
Enter the coefficient of X^3 : 8
Enter the coefficient of X^2 : 2
Enter the coefficient of X^1 : 0
Enter the coefficient of X^0 : 4

===== SIMPSON'S METHOD CALCULATION START =====

Enter the 'n' (even number) : 10
Enter the upper limit : 5
Enter the lower limit : 1

===== SIMPSON'S METHOD CALCULATION PROCESSING =====

dx = 0.4

x = 1		y = 23		
x = 1.4		y = 4 * 70.593	=	282.372
x = 1.8		y = 2 * 185.207	=	370.413
x = 2.2		y = 4 * 422.137	=	1688.55
x = 2.6		y = 2 * 861.871	=	1723.74
x = 3		y = 4 * 1615	=	6460
x = 3.4		y = 2 * 2827.14	=	5654.27
x = 3.8		y = 4 * 4683.83	=	18735.3
x = 4.2		y = 2 * 7415.48	=	14831
x = 4.6		y = 4 * 11302.3	=	45209
x = 5		y = 16679		

===== SIMPSON'S METHOD CALCULATION RESULT =====

The result of this integration is 14887.6

integrate $4x^5+5x^4+8x^3+2x^2+4$ from 1 to 5|



 **Examples**  **Random**

Definite integral:

[More digits](#)

 [Step-by-step solution](#)

$$\int_1^5 (4x^5 + 5x^4 + 8x^3 + 2x^2 + 4) dx = \frac{44660}{3} \approx 14887.$$

Demo