NBody Simulation

Understanding the Physics

Let's take a step back now and look at the physics behind our simulations. Our Body objects will obey the laws of Newtonian physics. In particular, they will be subject to:

- Pairwise Force: *Newton's law of universal gravitation* asserts that the strength of the gravitational force between two particles is given by the product of their masses divided by the square of the distance between them, scaled by the gravitational constant

$$G = 6.67 \cdot 10^{-11} \frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$$

The gravitational force exerted on a particle is along the straight line between them (we are ignoring here strange effects like the curvature of space). Since we are using Cartesian coordinates to represent the position of a particle, it is convenient to break up the force into its **x-** and **y-**components **(Fx, Fy)**. The relevant equations are shown below. We have not derived these equations, and you should just trust us.

- $F = \frac{G \cdot m_1 \cdot m_2}{r^2}$
- $r^2 = dx^2 + dy^2$
- $F_x = \frac{F \cdot dx}{r}$
- $F_y = \frac{F \cdot dy}{r}$

Look at the image below and make sure you understand what each variable represents!

Note that force is a vector (i.e., it has direction). In particular, be aware that dx and dy are signed (positive or negative).

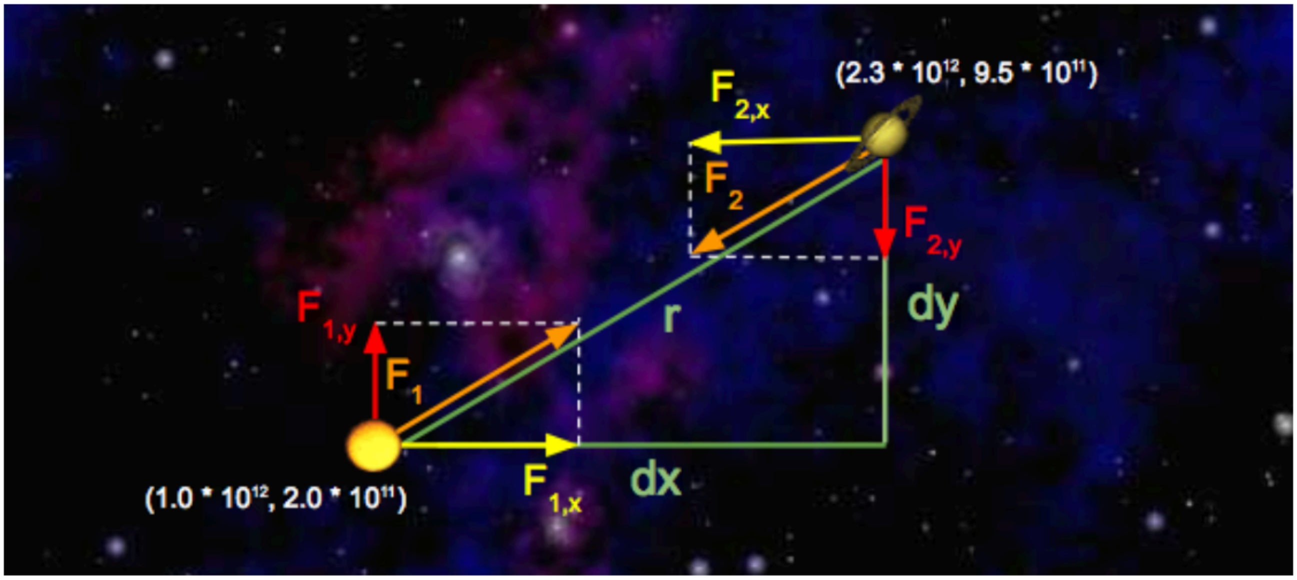
- Net Force: The *principle of superposition* says that the net force acting on a particle in the x- or y-direction is the sum of the pairwise forces acting on the particle in that direction.

In addition, all bodies have:

- Acceleration: Newton's *second law of motion* says that the accelerations in the x- and y-directions are given by:
    - $a_x = \dfrac{F_x}{m}$
    - $a_y = \dfrac{F_y}{m}$

Check your understanding!

Consider a small example consisting of two celestial objects: Saturn and the Sun. Suppose the Sun is at coordinates $(1.0 \cdot 10^{12}, 2.0 \cdot 10^{11})$ and Saturn is at coordinates $(2.3 \cdot 10^{12}, 9.5 \cdot 10^{11})$. Assume that the Sun's mass is $2.0 \cdot 10^{30}$ kg and Saturn's mass is $6.0 \cdot 10^{26}$ kg. Here's a diagram of this simple solar system:



Let's run through some sample calculations. First let's compute F_1, the force that Saturn exerts on the Sun. We'll begin by calculating r, which we've already expressed above in terms of dx and dy. Since we're calculating the force exerted by Saturn, dx is Saturn's x-position minus Sun's x-position, which is 1.3×10^{12} m. Similarly, dy is $7.5 \cdot 10^{11}$ m.

So, $r^2 = dx^2 + dy^2 = (1.3 \cdot 10^{12})^2 + (7.5 \cdot 10^{11})^2$. Solving for r gives us $1.5 \cdot 10^{12}$ m. Now that we have r, computation of F is straightforward:

$$F = \frac{G \cdot (2.0 \cdot 10^{30}\ \text{kg}) \cdot (6.0 \cdot 10^{26}\text{kg})}{(1.5 \cdot 10^{12}\ \text{m})^2} = 3.6 \cdot 10^{22}\ \text{N}$$

Note that the magnitudes of the forces that Saturn and the Sun exert on one another are equal; that is, $|F| = |F_1| = |F_2|$. Now that we've computed the pairwise force on the Sun, let's compute the x- and y-components of this force, denoted with $F_{1,x}$ and $F_{1,y}$, respectively. Recall that dx is $1.3 \cdot 10^{12}$ meters and dy is $7.5 \cdot 10^{11}$ meters. So,

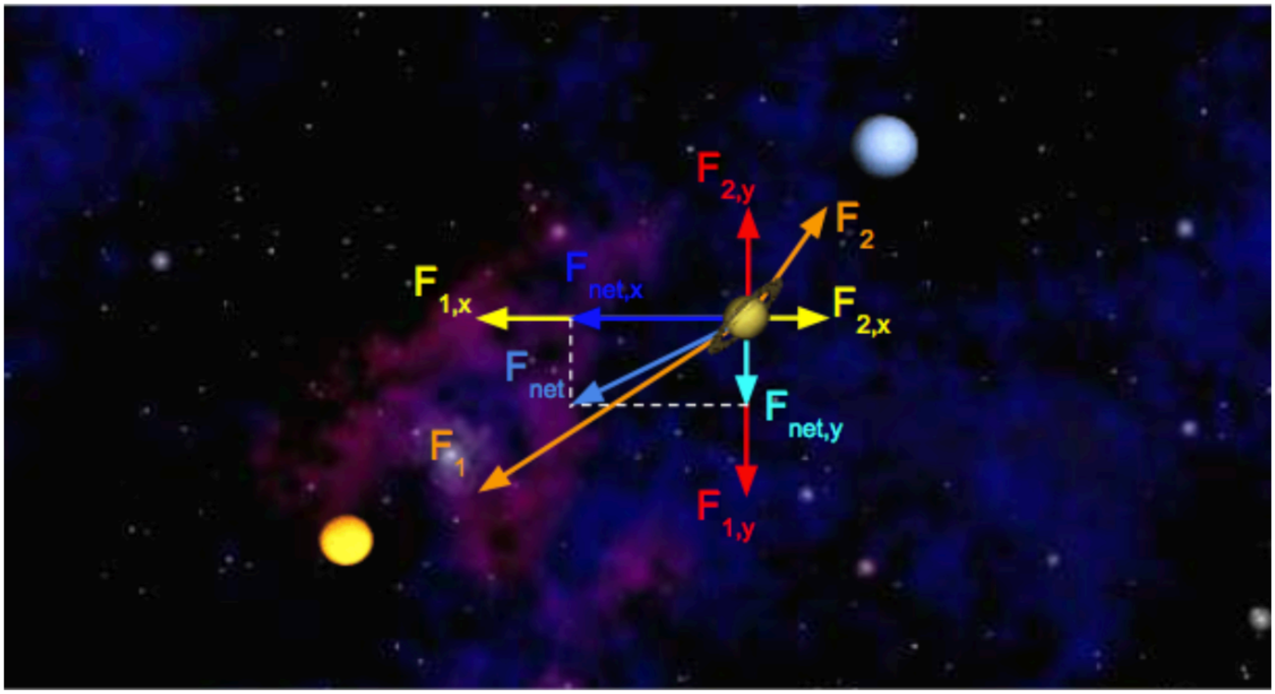
- $F_{1,x} = \dfrac{F_1 \cdot (1.3 \cdot 10^{12}\ \mathrm{m})}{1.5 \cdot 10^{12}\ \mathrm{m}} = 3.1 \cdot 10^{22}\ \mathrm{N}$
- $F_{1,y} = \dfrac{F_1 \cdot (7.5 \cdot 10^{11}\ \mathrm{m})}{1.5 \cdot 10^{12}\ \mathrm{m}} = 1.8 \cdot 10^{22}\ \mathrm{N}$

Note that the sign of dx and dy is important! Here, dx and dy were both positive, resulting in positive values for $F_{1,x}$ and $F_{1,y}$. This makes sense if you look at the diagram: Saturn will exert a force that pulls the Sun to the right (positive $F_{1,x}$) and up (positive $F_{1,y}$).

Next, let's compute the x and y-components of the force that the Sun exerts on Saturn. The values of dx and dy are negated here, because we're now measuring the displacement of the Sun relative to Saturn. Again, you can verify that the signs should be negative by looking at the diagram: the Sun will pull Saturn to the left (negative dx) and down (negative dy).

- $F_{2,x} = \dfrac{F_2 \cdot (-1.3 \cdot 10^{12}\ \mathrm{m})}{1.5 \cdot 10^{12}\ \mathrm{m}} = -3.1 \cdot 10^{22}\ \mathrm{N}$
- $F_{2,y} = \dfrac{F_2 \cdot (-7.5 \cdot 10^{11}\ \mathrm{m})}{1.5 \cdot 10^{12}\ \mathrm{m}} = -1.8 \cdot 10^{22}\ \mathrm{N}$

Let's add Neptune to the mix and calculate the net force on Saturn. Here's a diagram illustrating the forces being exerted on Saturn in this new system:



We can calculate the x-component of the net force on Saturn by summing the x-components of all pairwise forces. Likewise, $F_{\text{net},y}$ can be calculated by summing the y-components of all pairwise forces. Assume the forces exerted on Saturn by the Sun are the same as above, and that $F_{2,x} = 1.1 \cdot 10^{22}$ N and $F_{2,y} = 9.0 \cdot 10^{21}$N.

- $F_{\text{net},x} = F_{1,x} + F_{2,x} = -3.1 \cdot 10^{22}\text{ N} + 1.1 \cdot 10^{22}\text{ N} = -2.0 \cdot 10^{22}\text{ N}$
- $F_{\text{net},y} = F_{1,y} + F_{2,y} = -1.8 \cdot 10^{22}\text{ N} + 9.0 \cdot 10^{21}\text{ N} = -9.0 \cdot 10^{21}\text{N}$

Double check your understanding!

Suppose there are three bodies in space as follows:

- Samh: $x = 1, y = 0, \text{mass} = 10$
- Aegir: $x = 3, y = 3, \text{mass} = 5$
- Rocinante: $x = 5, y = -3, \text{mass} = 50$

Calculate $F_{\text{net},x}$ and $F_{\text{net},y}$ exerted on Samh. To check your answer, click here for $F_{\text{net},x}$ and here for $F_{\text{net},y}$.