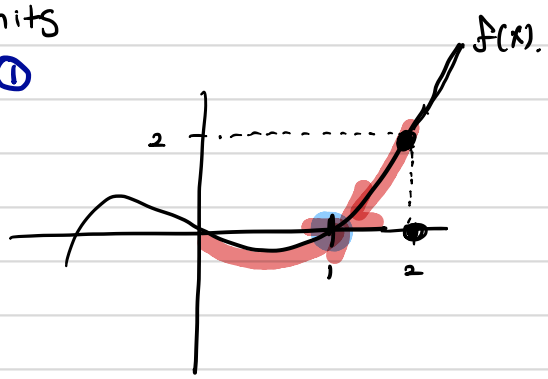
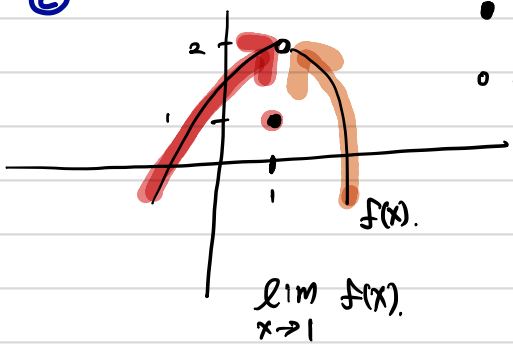
10.1 Limits

March 4th (Sunday)

① 

② 

● : Inclusive

o : Exclusive

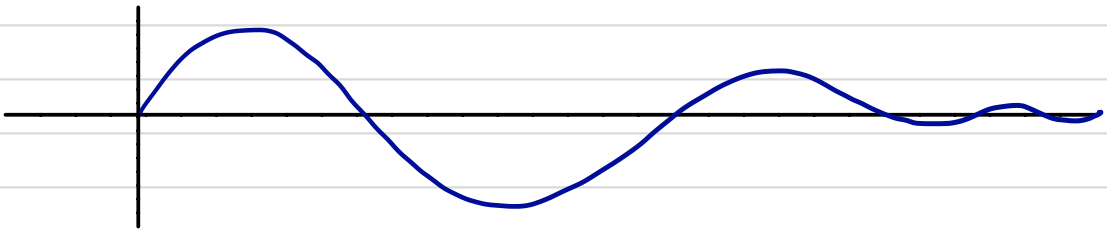
* LIMIT IS NOT EXACT VALUE

$f(1) = 1$

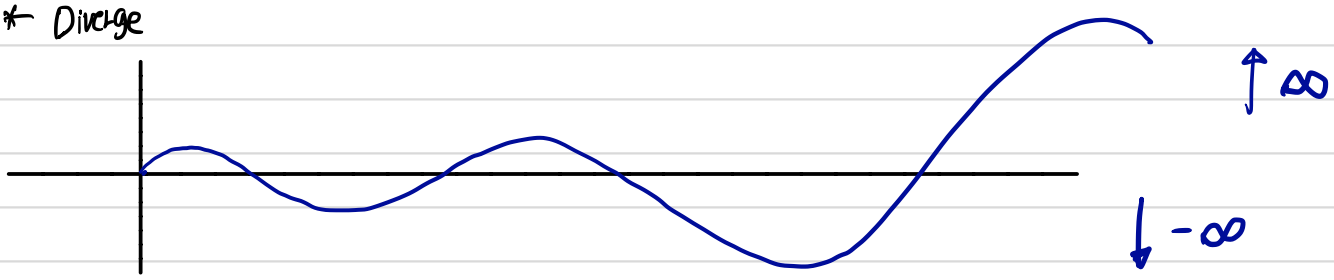
$\lim_{x \to 1} f(x) = 2$

* Converge and diverge

* converge



* Diverge

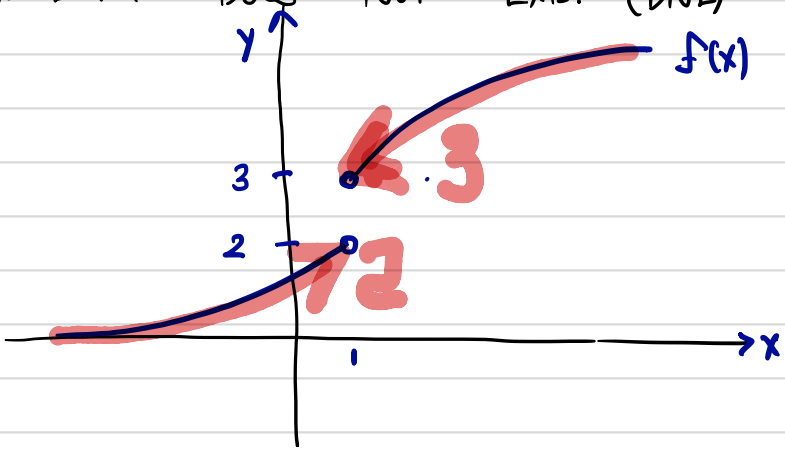


* Two ways to calculate Limit

① Graph

② limit properties.

* Limit DOES Not Exist (DNE)



$\lim_{x \to 1} f(x) = ?$ DNE

* $\lim_{x \to 1^-} f(x) = 2$

$\lim_{x \to 1^+} f(x) = 3$

2) Limit Properties

① Addition (+)

$$\lim_{x\to 0}[f(x)+g(x)] = \lim_{x\to 0} f(x) + \lim_{x\to 0} g(x)$$

② Subtraction (−)

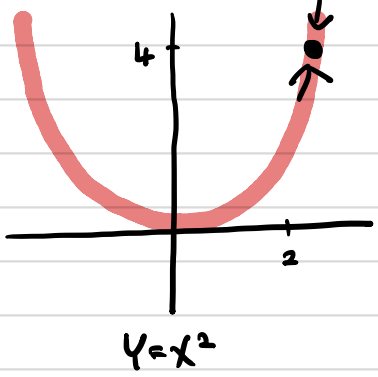
$$\lim_{x\to 0}[f(x)-g(x)] = \lim_{x\to 0} f(x) - \lim_{x\to 0} g(x)$$

$x(y+1) = xy + x$ (Distribution Method)

Ex) $\lim_{x\to 2}(x^2+x) = \lim_{x\to 2} x^2 + \lim_{x\to 2} x$

$= 4 + 2$

$= 6$



③ Division (÷)

$$\lim_{x\to 0}\left(\frac{f(x)}{g(x)}\right)=\frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} g(x)}$$

④ Multiplication (×)

$$\lim_{x\to 0}[f(x)\cdot g(x)]=\lim_{x\to 0} f(x)\cdot \lim_{x\to 0} g(x)$$

⑤ Constant Multiplication

$$\lim_{x\to 0}[cf(x)]=c\cdot \lim_{x\to 0} f(x) \quad (c \text{ is constant})$$

Ex)

$$\lim_{x\to 0}(2x^2)$$

$$=2\lim_{x\to 0} x^2$$

⑥ Root

$$\lim_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim_{x\to a} f(x)}$$

② ①

- When Limit is Polynomial Function

x^2
x^3
x^4
⋮

$$\lim_{x \to 0} (x^3 + x^2 + x + 1) = 1.$$

$$\lim_{x \to 0} f(x) = f(0).$$

- Form 0/0

$\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ → Numerator, Denominator

Ex)
$$\lim_{x \to 1} \frac{(x^3 - 1)}{x - 1}$$

- $x^3 - 1 = (x - 1)(x^2 + x + 1)$

$$= \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)} = \lim_{x \to 1} x^2 + x + 1 = 3$$