---

1. (1 point) The total revenue function is $r = 80q - 0.5q^2$, where q is the number of units sold.
The marginal revenue function is ____________
When $q = 70$, the marginal revenue is ____________
Answer(s) submitted:
- `80-q`
- `10`

(correct)

---

2. (1 point) The average cost per unit is $\bar{c} = 0.05q + 6 + \frac{600}{q}$, where q is the number of units produced.
The total cost function is ____________
The marginal cost function is ____________
When $q = 50$, the marginal cost is ____________
Answer(s) submitted:
- `0.05q^2+6q+600`
- `0.1q+6`
- `11`

(correct)

---

3. (1 point) The total cost (in dollars) of producing x coffee machines is

$$C(x) = 2100 + 60x - 0.4x^2.$$

(A) Find the exact cost of producing the 21st machine. (Hint: this involves the difference of two values of C.)
Exact cost of 21st machine = ____________

(B) Use marginal cost to approximate the cost of producing the 21st machine. (Hint: marginal cost is $C'(x)$. It says approximately how much you will spend to make one more unit in addition to x already made.)
Approximate cost of 21st machine = ____________

Answer(s) submitted:
- `43.6`
- `44`

(correct)

---

4. (1 point) For the given total cost function

$$C(x) = 12100 + 800x + x^2$$

find:
a) The total cost at the production level 1150 ____________
b) The average cost at the production level 1150 ____________
c) The marginal cost at the production level 1150 ____________

Answer(s) submitted:
- `2254600`
- `1960.52173913`
- `3100`

(correct)

1. Total revenue function $r = 80q - 0.5q^2$.

- Marginal revenue function

$$\frac{dr}{dq} = \frac{d}{dq}(80q - 0.5q^2)$$

$$= 80 - q$$

- when $q = 10$, the marginal revenue is

$$r'(10) = 80 - 10$$

$$= 10$$

2. Average cost per unit is $\bar{c} = 0.05q + 6 + \frac{600}{q}$

- The total cost function

The marginal-cost function is the derivative of the total cost function c.
Thus, we find c by multiplying $\bar{c}$ by q.

$$c = q\bar{c}$$
$$= q\left(0.05q + 6 + \frac{600}{q}\right)$$
$$= 0.05q^2 + 6q + 600$$

- Marginal cost function

$$\frac{dc}{dq} = \frac{d}{dq}\left(0.05q^2 + 6q + 600\right)$$
$$= 0.1q + 6$$

- When $q = 50$, the marginal cost is

$$c'(50) = 0.1(50) + 6$$
$$= 5 + 6$$
$$= 11$$

3. The total cost of Producing x coffee machines.

$C(x) = 2100 + 60x - 0.4x^2$

A) Find exact cost for Producing 21st machine.

$C(21) = 2100 + 60(21) - 0.4(21)^2$

$= 3183.6$

$C(20) = 2100 + 60(20) - 0.4(20)^2$

$= 3140$

$\Rightarrow \dfrac{C(21) - C(20)}{1} = 3183.6 - 3140 = 43.6$

B) Use marginal cost to estimate the cost of Producing 21st machine

$\dfrac{dc}{dx} = \dfrac{d}{dx}(2100 + 60x - 0.4x^2)$

$= 60 - 0.8x$

$\Rightarrow C'(20) = 60 - 0.8(20)$

$= 44$

Applications of Rate of Change to Economics

A manufacturer's **total-cost function,** $c = f(q)$, gives the total cost c of producing and marketing q units of a product. The rate of change of c with respect to q is called the **marginal cost.** Thus,

$$\text{marginal cost} = \frac{dc}{dq}$$

For example, suppose $c = f(q) = 0.1q^2 + 3$ is a cost function, where c is in dollars and q is in pounds. Then

$$\frac{dc}{dq} = 0.2q$$

The marginal cost when 4 lb are produced is dc/dq, evaluated when $q = 4$:

$$\left.\frac{dc}{dq}\right|_{q=4} = 0.2(4) = 0.80$$

This means that if production is increased by 1 lb, from 4 lb to 5 lb, then the change in cost is approximately \$0.80. That is, the additional pound costs about \$0.80. In general, *we interpret marginal cost as the approximate cost of one additional unit of output.* After all, the difference $f(q+1) - f(q)$ can be seen as a difference quotient

$$\frac{f(q+1) - f(q)}{1}$$

(the case where $h = 1$). Any difference quotient can be regarded as an approximation of the corresponding derivative and, conversely, any derivative can be regarded as an approximation of any of its corresponding difference quotients. Thus, for any function f of q we can always regard $f'(q)$ and $f(q+1) - f(q)$ as approximations of each other. In economics, the latter can usually be regarded as the exact value of the cost, or profit depending upon the function, of the $(q+1)$th item when q are produced. The derivative is often easier to compute than the exact value. [In the case at hand, the actual cost of producing one more pound beyond 4 lb is $f(5) - f(4) = 5.5 - 4.6 = \$0.90$.]

If c is the total cost of producing q units of a product, then the **average cost per unit,** $\bar{c}$, is

$$\bar{c} = \frac{c}{q} \tag{4}$$

For example, if the total cost of 20 units is \$100, then the average cost per unit is $\bar{c} = 100/20 = \$5$. By multiplying both sides of Equation (4) by q, we have

$$c = q\bar{c}$$

That is, total cost is the product of the number of units produced and the average cost per unit.

EXAMPLE 7 Marginal Cost

If a manufacturer's average-cost equation is

$$\bar{c} = 0.0001q^2 - 0.02q + 5 + \frac{5000}{q}$$

find the marginal-cost function. What is the marginal cost when 50 units are produced?

Solution:

Strategy The marginal-cost function is the derivative of the total-cost function c. Thus, we first find c by multiplying $\bar{c}$ by q. We have

$$\begin{aligned} c &= q\bar{c} \\ &= q\left(0.0001q^2 - 0.02q + 5 + \frac{5000}{q}\right) \\ c &= 0.0001q^3 - 0.02q^2 + 5q + 5000 \end{aligned}$$

Differentiating c, we have the marginal-cost function:

$$\begin{aligned}\frac{dc}{dq} &= 0.0001(3q^2) - 0.02(2q) + 5(1) + 0 \\ &= 0.0003q^2 - 0.04q + 5\end{aligned}$$

The marginal cost when 50 units are produced is

$$\left.\frac{dc}{dq}\right|_{q=50} = 0.0003(50)^2 - 0.04(50) + 5 = 3.75$$

If c is in dollars and production is increased by one unit, from $q = 50$ to $q = 51$, then the cost of the additional unit is approximately \$3.75. If production is increased by $\frac{1}{3}$ unit, from $q = 50$, then the cost of the additional output is approximately $\left(\frac{1}{3}\right)(3.75) = \$1.25$.

Now Work Problem 21 ◁

Suppose $r = f(q)$ is the **total-revenue function** for a manufacturer. The equation $r = f(q)$ states that the total dollar value received for selling q units of a product is r. The **marginal revenue** is defined as the rate of change of the total dollar value received with respect to the total number of units sold. Hence, marginal revenue is merely the derivative of r with respect to q:

$$\text{marginal revenue} = \frac{dr}{dq}$$

Marginal revenue indicates the rate at which revenue changes with respect to units sold. We interpret it as *the approximate revenue received from selling one additional unit of output.*

4. Total Cost Function $C(x) = 12100 + 800x + x^2$

(A) Total cost at the Production level 1150

$12100 + 800(1150) + (1150)^2$

$= 2254600$

(B) Average cost at the Production level 1150

* Average cost per unit => $\bar{C} = \frac{C}{q}$

$\bar{C}(1150) = \frac{C(1150)}{1150} = \frac{2254600}{1150}$

$= 1960.5217391 3$

(C) Marginal cost at the Production level 1150

$C'(x) = 800 + 2x$

$C'(1150) = 800 + 2300$

$= 3100$