

Tensors Revisited

(10)

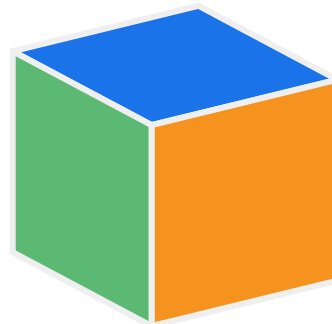
Scalar



Vector



Matrix



Tensor

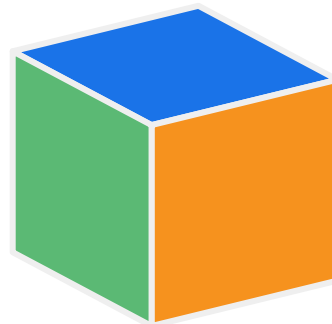
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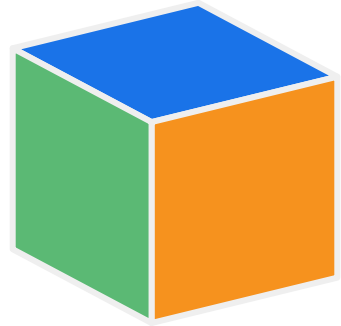
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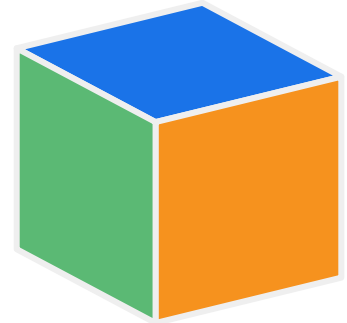
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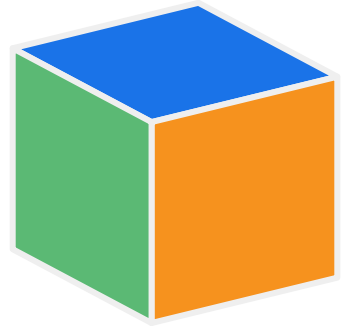
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Vector



Matrix



Tensor

Some types of tensors

Variables

tf.Variable

```
tf.Variable("Hello", tf.string)
```

Constants

tf.constant

```
tf.constant([1, 2, 3, 4, 5, 6])
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Characteristics of a tensor

Tensor

Shape

Data type

```
tf.Tensor([4 6], shape=(2,), dtype=int32)
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Inspect variables of a built-in Keras layer

```
model = tf.keras.Sequential([  
    tf.keras.layers.Dense(1, input_shape=(1,))  
])
```

```
>>> model.variables
```

```
[<tf.Variable 'dense_1/kernel:0' shape=(1, 1) dtype=float32,  
  numpy=array([[1.4402896]], dtype=float32)>,  
 <tf.Variable 'dense_1/bias:0' shape=(1,) dtype=float32,  
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Creating Tensors with `tf.Variable`

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vector = tf.Variable(initial_value = [1,2])
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vector = tf.Variable([1,2], tf.float32) # don't do please!
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```
vector = tf.Variable([1,2,3,4], shape=tf.TensorShape(None))
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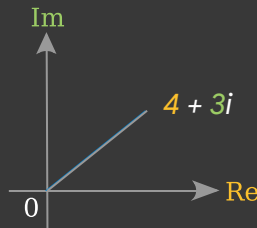
Creating Tensors with tf.Variable

```
mammal = tf.Variable("Elephant", dtype=tf.string)
```

```
its_complicated = tf.Variable(4 + 3j,  
                               dtype=tf.complex64)
```

```
first_primes = tf.Variable([2, 3, 5, 7, 11],  
                            dtype=tf.int32)
```

```
linear_squares = tf.Variable([[4, 9], [16, 25]],  
                              dtype=tf.int32)
```


$$\begin{bmatrix} 2 & 3 & 5 & 7 & 11 \end{bmatrix}$$
$$\begin{bmatrix} 4 & 9 \\ 16 & 25 \end{bmatrix}$$

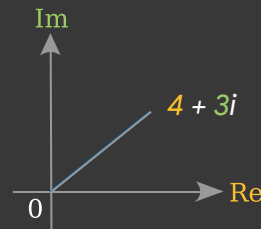
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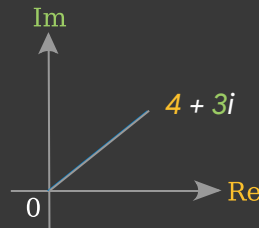
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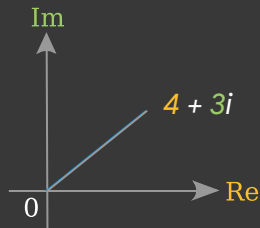
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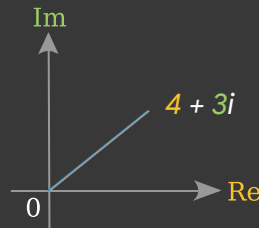
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Use `tf.constant` to create various kinds of tensors

Constant 1-D Tensor populated with value list.

```
tensor = tf.constant([1, 2, 3])
```

```
>>> tensor
```

```
[1 2 3]
```

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Constant 2-D Tensor populated with value list.

```
tensor = tf.constant([1, 2, 3, 4, 5, 6], shape=(2, 3))
```

```
>>> tensor
```

```
[[1 2 3], [4 5 6]]
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Constant 2-D tensor populated with scalar value -1.

```
tensor = tf.constant(-1.0, shape=[2, 3])
```

```
>>> tensor
```

```
[[ -1.  -1.  -1.]
```

```
 [ -1.  -1.  -1.]]
```

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

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Operations

`tf.add`



`tf.subtract`



`tf.multiply`



...

Applying operations

```
>>> tf.add([1, 2], [3, 4])  
tf.Tensor([4 6], shape=(2,), dtype=int32)
```

```
>>> tf.square(5)  
tf.Tensor(25, shape=(), dtype=int32)
```

```
>>> tf.reduce_sum([1, 2, 3])  
tf.Tensor(6, shape=(), dtype=int32)
```

Operator overloading is also supported

```
>>> tf.square(2) + tf.square(3)  
tf.Tensor(13, shape=(), dtype=int32)
```


Applying operations

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Eager execution in TensorFlow

- Evaluate values immediately
- Broadcasting support
- Operator overloading
- NumPy compatibility

Evaluate tensors

```
x = 2
```

```
x_squared = tf.square(x)
```

```
>>> print("hello, {}".format(x_squared))
```

```
hello, 4
```

Evaluate tensors

```
x = 2
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x_squared = tf.square(x)
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Evaluate tensors

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x_squared = tf.square(x)
```

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>>> print("hello, {}".format(x_squared))
```

```
hello, 4
```


Broadcast values

```
a = tf.constant([[1, 2],  
                 [3, 4]])
```

```
>>> tf.add(a, 1)
```

```
tf.Tensor(  
  [[2 3]  
   [4 5]], shape=(2, 2), dtype=int32)
```

Broadcast values

```
a = tf.constant([[1, 2],  
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```
tf.Tensor(  
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```

Overload operators

```
a = tf.constant([[1, 2],  
                 [3, 4]])
```

```
>>> a ** 2
```

```
tf.Tensor(  
[[ 1  4]  
 [ 9 16]], shape=(2, 2), dtype=int32)
```

Overload operators

```
a = tf.constant([[1, 2],  
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```
>>> a ** 2
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tf.Tensor(  
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Overload operators

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a = tf.constant([[1, 2],  
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```
>>> a ** 2  
tf.Tensor(  
[[ 1  4]  
 [ 9 16]], shape=(2, 2), dtype=int32)
```

NumPy Compatibility

```
import numpy as np  
a = tf.constant(5)  
b = tf.constant(3)  
  
>>> np.multiply(a, b)  
15
```


Numpy interoperability

```
ndarray = np.ones([3, 3])  
>>> ndarray
```

```
[[1. 1. 1.]  
 [1. 1. 1.]  
 [1. 1. 1.]
```

```
tensor = tf.multiply(ndarray, 3)  
>>> tensor
```

```
tf.Tensor(  
[[3. 3. 3.]  
 [3. 3. 3.]  
 [3. 3. 3.]],  
shape=(3, 3),  
dtype=float64)
```

```
>>> tensor.numpy()
```

```
array([[3., 3., 3.],  
       [3., 3., 3.],  
       [3., 3., 3.]])
```

Numpy interoperability

```
ndarray = np.ones([3, 3])
```

```
>>> ndarray
```

```
[[1. 1. 1.]  
 [1. 1. 1.]  
 [1. 1. 1.]
```

```
tensor = tf.multiply(ndarray, 3)
```

```
>>> tensor
```

```
tf.Tensor(  
  [[3. 3. 3.]  
   [3. 3. 3.]  
   [3. 3. 3.]],  
  shape=(3, 3),  
  dtype=float64)
```

```
>>> tensor.numpy()
```

```
array([[3., 3., 3.],  
       [3., 3., 3.],  
       [3., 3., 3.]])
```

Numpy interoperability

```
ndarray = np.ones([3, 3])
```

```
>>> ndarray
```

```
[[1. 1. 1.]  
 [1. 1. 1.]  
 [1. 1. 1.]
```

```
tensor = tf.multiply(ndarray, 3)
```

```
>>> tensor
```

```
tf.Tensor(  
  [[3. 3. 3.]  
   [3. 3. 3.]  
   [3. 3. 3.]],  
  shape=(3, 3),  
  dtype=float64)
```

```
>>> tensor.numpy()
```

```
array([[3., 3., 3.],  
       [3., 3., 3.],  
       [3., 3., 3.]])
```

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```
>>> tensor.numpy()
```

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array([[3., 3., 3.],  
       [3., 3., 3.],  
       [3., 3., 3.]])
```

Evaluating variables

```
v = tf.Variable(0.0)
```

```
>>> v + 1
```

```
<tf.Tensor: id=47, shape=(), dtype=float32, numpy=1.0>
```

```
v = tf.Variable(0.0)
```

```
>>> v.assign_add(1)
```

```
<tf.Variable 'UnreadVariable' shape=() dtype=float32, numpy=1.0>
```

```
v = tf.Variable(0.0)
```

```
v.assign_add(1)
```

```
>>> v.read_value().numpy()
```

```
1.0
```


Evaluating variables

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v = tf.Variable(0.0)
>>> v + 1
<tf.Tensor: id=47, shape=(), dtype=float32, numpy=1.0>
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>>> v.read_value().numpy()
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Evaluating variables

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```

```
v.assign_add(1)
```

```
>>> v.read_value().numpy()
```

```
1.0
```

Examine custom layers

```
class MyLayer(tf.keras.layers.Layer):  
  
    def __init__(self):  
        super(MyLayer, self).__init__()  
        self.my_var = tf.Variable(100)  
        self.my_other_var_list = [tf.Variable(x) for x in range(2)]  
  
m = MyLayer()  
>>> [variable.numpy() for variable in m.variables]  
[100, 0, 1]
```

Examine custom layers

```
class MyLayer(tf.keras.layers.Layer):
```

```
    def __init__(self):  
        super(MyLayer, self).__init__()  
        self.my_var = tf.Variable(100)  
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```

```
m = MyLayer()
```

```
>>> [variable.numpy() for variable in m.variables]
```

```
[100, 0, 1]
```

Examine custom layers

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        self.my_var = tf.Variable(100)  
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```

```
m = MyLayer()  
>>> [variable.numpy() for variable in m.variables]  
[100, 0, 1]
```

Change data types

```
tensor = tf.constant([1, 2, 3])
```

```
>>> tensor
```

```
tf.Tensor([1 2 3], shape=(3,), dtype=int32)
```

```
# Cast a constant integer tensor into floating point
```

```
tensor = tf.cast(tensor, dtype=tf.float32)
```

```
>>> tensor.dtype
```

```
tf.float32
```

Change data types

```
tensor = tf.constant([1, 2, 3])  
>>> tensor  
tf.Tensor([1 2 3], shape=(3,), dtype=int32)
```

```
# Cast a constant integer tensor into floating point  
tensor = tf.cast(tensor, dtype=tf.float32)  
>>> tensor.dtype  
tf.float32
```


Change data types

```
tensor = tf.constant([1, 2, 3])
```

```
>>> tensor
```

```
tf.Tensor([1 2 3], shape=(3,), dtype=int32)
```

```
# Cast a constant integer tensor into floating point
```

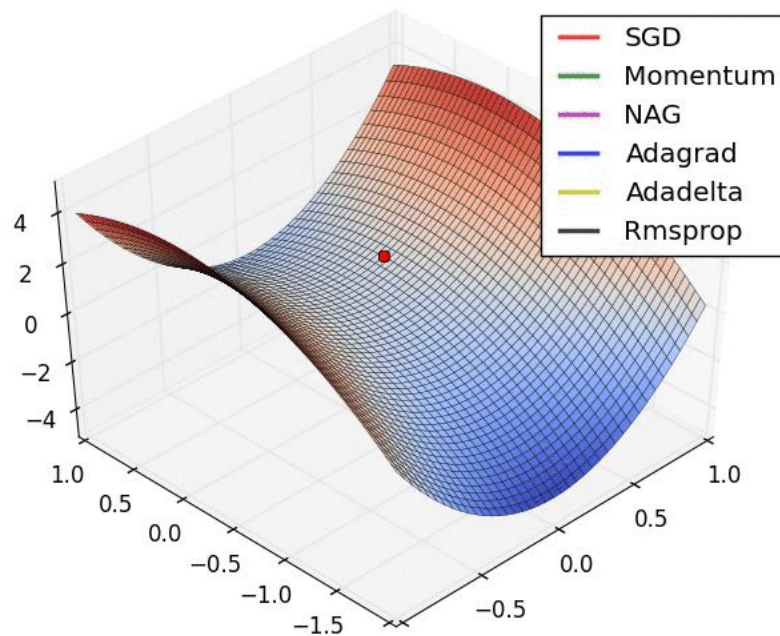
```
tensor = tf.cast(tensor, dtype=tf.float32)
```

```
>>> tensor.dtype
```

```
tf.float32
```

Eager execution

- Intuitive to use
- Easy to debug
- Works with Python's control flows



<http://cs231n.github.io/neural-networks-3/>

```
# Training data
x_train = np.array([-1.0, 0.0, 1.0, 2.0, 3.0, 4.0], dtype=float)
y_train = np.array([-3.0, -1.0, 1.0, 3.0, 5.0, 7.0], dtype=float)

# Trainable variables
w = tf.Variable(random.random(), trainable=True)
b = tf.Variable(random.random(), trainable=True)
```

```
# Training data
x_train = np.array([-1.0, 0.0, 1.0, 2.0, 3.0, 4.0], dtype=float)
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# Trainable variables
w = tf.Variable(random.random(), trainable=True)
b = tf.Variable(random.random(), trainable=True)
```

```
# Loss function
```

```
def simple_loss(real_y, pred_y):  
    return tf.abs(real_y - pred_y)
```

```
# Learning Rate
```

```
LEARNING_RATE = 0.001
```

```
for _ in range(500):  
    fit_data(x_train, y_train)  
  
print(f'y ≈ {w.numpy()}x + {b.numpy()}')
```

```
def fit_data(real_x, real_y):  
    with tf.GradientTape(persistent=True) as tape:  
        # Make prediction  
        pred_y = w * real_x + b  
        # Calculate loss  
        reg_loss = simple_loss(real_y, pred_y)  
  
    # Calculate gradients  
    w_gradient = tape.gradient(reg_loss, w)  
    b_gradient = tape.gradient(reg_loss, b)  
  
    # Update variables  
    w.assign_sub(w_gradient * LEARNING_RATE)  
    b.assign_sub(b_gradient * LEARNING_RATE)
```



```
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    with tf.GradientTape(persistent=True) as tape:
```

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```

```
        pred_y = w * real_x + b
```

```
        # Calculate loss
```

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        reg_loss = simple_loss(real_y, pred_y)
```

```
    # Calculate gradients
```

```
    w_gradient = tape.gradient(reg_loss, w)
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```
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```

$$y \approx 1.9902112483978271x + -0.995111882686615$$

Gradient Descent with `tf.GradientTape`

```
def train_step(images, labels):  
    with tf.GradientTape() as tape:  
        logits = model(images, training=True)  
        loss_value = loss_object(labels, logits)  
  
    loss_history.append(loss_value.numpy().mean())  
    grads = tape.gradient(loss_value, model.trainable_variables)  
    optimizer.apply_gradients(zip(grads, model.trainable_variables))
```

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```


Gradient computation in TensorFlow

```
w = tf.Variable([[1.0]])  
with tf.GradientTape() as tape:  
    loss = w * w
```

```
>>> tape.gradient(loss, w)  
tf.Tensor([[ 2.]], shape=(1, 1), dtype=float32)
```

$$\frac{d}{dw} w^2 = 2w$$

Gradient computation in TensorFlow

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w = tf.Variable([[1.0]])  
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>>> tape.gradient(loss, w)  
tf.Tensor([[ 2.]], shape=(1, 1), dtype=float32)
```

Compute gradients of higher ranked tensors

```
x = tf.ones((2, 2))  
with tf.GradientTape() as t:  
    t.watch(x)
```

```
y = tf.reduce_sum(x)
```

```
z = tf.square(y)
```

```
# Derivative of z wrt the original input tensor x  
dz_dx = t.gradient(z, x)
```

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$1 + 1 + 1 + 1$$

$$4^2$$

$$\begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$$

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$$1 + 1 + 1 + 1$$

$$4^2$$

$$\begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$$

$$x = \begin{pmatrix} x_{1,1} & x_{1,2} \\ x_{2,1} & x_{2,2} \end{pmatrix}$$

$$y = x_{1,1} + x_{1,2} + x_{2,1} + x_{2,2} \quad \text{“reduce sum”}$$

$$z = y^2$$

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$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial x}$$

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$$\frac{\partial z}{\partial y} = 2 \times y$$

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$$y = x_{1,1} + x_{1,2} + x_{2,1} + x_{2,2} \quad \text{“reduce sum”}$$

$$z = y^2$$

$$\frac{\partial z}{\partial y} = 2 \times y$$

$$\frac{\partial y}{\partial x_{1,1}} = 1$$

$$\frac{\partial y}{\partial x_{2,1}} = 1$$

$$\frac{\partial y}{\partial x_{1,2}} = 1$$

$$\frac{\partial y}{\partial x_{2,2}} = 1$$

$$\frac{\partial z}{\partial x} = \begin{pmatrix} \frac{\partial z}{\partial x_{1,1}} & \frac{\partial z}{\partial x_{1,2}} \\ \frac{\partial z}{\partial x_{2,1}} & \frac{\partial z}{\partial x_{2,2}} \end{pmatrix}$$

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$$\frac{\partial z}{\partial x_{1,1}} = \frac{\partial z}{\partial y} \times \frac{\partial dy}{\partial x_{1,1}}$$

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$$\frac{\partial z}{\partial x_{2,1}} = \frac{\partial z}{\partial y} \times \frac{\partial dy}{\partial x_{2,1}}$$

$$\frac{\partial z}{\partial x} = \begin{pmatrix} \frac{\partial z}{\partial x_{1,1}} & \frac{\partial z}{\partial x_{1,2}} \\ \frac{\partial z}{\partial x_{2,1}} & \frac{\partial z}{\partial x_{2,2}} \end{pmatrix}$$

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$$\frac{\partial z}{\partial x_{2,2}} = \frac{\partial z}{\partial y} \times \frac{\partial dy}{\partial x_{2,2}}$$

$$\boldsymbol{x} = \begin{pmatrix} x_{1,1} & x_{1,2} \\ x_{2,1} & x_{2,2} \end{pmatrix}$$



$$\boldsymbol{x} = \begin{pmatrix} x_{1,1} & x_{1,2} \\ x_{2,1} & x_{2,2} \end{pmatrix}$$

$$\boldsymbol{x} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\boldsymbol{x} = \begin{pmatrix} x_{1,1} & x_{1,2} \\ x_{2,1} & x_{2,2} \end{pmatrix}$$

$$\boldsymbol{x} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$y = x_{1,1} + x_{1,2} + x_{2,1} + x_{2,2}$$

$$\boldsymbol{x} = \begin{pmatrix} x_{1,1} & x_{1,2} \\ x_{2,1} & x_{2,2} \end{pmatrix}$$

$$\boldsymbol{x} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$y = x_{1,1} + x_{1,2} + x_{2,1} + x_{2,2}$$

$$y = 1 + 1 + 1 + 1 = 4$$

$$x = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$y = 4$$



$$x = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\frac{\partial z}{\partial y} = 2 \times y = 2 \times 4$$

$$y = 4$$



$$x = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\frac{\partial z}{\partial y} = 2 \times y = 2 \times 4$$

$$\frac{\partial y}{\partial x_{1,1}} = 1$$

$$\frac{\partial y}{\partial x_{1,2}} = 1$$

$$y = 4$$

$$\frac{\partial y}{\partial x_{2,1}} = 1$$

$$\frac{\partial y}{\partial x_{2,2}} = 1$$



$$\begin{array}{llll}
 x = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} & \frac{\partial z}{\partial y} = 2 \times y = 2 \times 4 & \frac{\partial y}{\partial x_{1,1}} = 1 & \frac{\partial y}{\partial x_{1,2}} = 1 \\
 y = 4 & & \frac{\partial y}{\partial x_{2,1}} = 1 & \frac{\partial y}{\partial x_{2,2}} = 1
 \end{array}$$

$$\frac{\partial z}{\partial x_{1,1}} = 2 \times 4 \times 1 = 8$$

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$$\frac{\partial z}{\partial x_{1,1}} = 2 \times 4 \times 1 = 8$$

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$$x = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\frac{\partial z}{\partial x} = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$$



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Same as:

```
dz_dx = t.gradient(z, x)
```

Using persistent=True

```
x = tf.constant(3.0)
with tf.GradientTape(persistent=True) as t:
    t.watch(x)
    y = x * x
    z = y * y
dz_dx = t.gradient(z, x)  # 108.0 (4 * x^3 at x = 3)
dy_dx = t.gradient(y, x)  # 6.0
del t  # Drop the reference to the tape
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Higher-order gradients

```
x = tf.Variable(1.0)

with tf.GradientTape() as tape_2:
    with tf.GradientTape() as tape_1:
        y = x * x * x
        dy_dx = tape_1.gradient(y, x)
    d2y_dx2 = tape_2.gradient(dy_dx, x)

assert dy_dx.numpy() == 3.0
assert d2y_dx2.numpy() == 6.0
```

$$y = x^3$$

$$\frac{\partial y}{\partial x} = 3x^2$$

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