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The Referential Dynamics of Cognition and Action

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Referential behavior theory (RBT), a general dynamical approach to psychological and related systems that operate through a control or referencing process, is introduced. A review of existing evidence shows that this approach can apply to a variety of human and animal systems and tasks, whether the framing language is that of homeostasis, error correction, coupled biological oscillators, motor control, adaptive change, cognitive goal-setting, evaluation and refinement, or neural network learning. Thus, RBT provides a path for reconciliation of dynamical and information-processing accounts of action and cognition. RBT generates a class of mathematical equations, one of which, the discrete control equation (DCE), forms the basis for more detailed investigation. The primary focus here is on the application of the DCE to the temporal structure of regular human movement. Given certain conditions, the equation produces various standard (and new, more general) forms of the circle map class that governs relative phase in motor coordination and, hence, generates well-documented nonlinear "dynamical" motor phenomena such as behavioral attractors, phase transitions, critical slowing, and so on. Under certain other conditions, the DCE produces the linear stochastic timing models often associated with motor program traditions, accommodating hierarchical effects, open- and closed-loop conditions, and unilimb and multilimb movement. A number of new predictions are identified from the approach, and a review of experimental evidence gives support for the claim that the current formulation is an integrated generalization and improvement of several aspects of existing motor program and dynamic approaches.

Some human behavior, such as the knee-jerk reflex, is automatic and reflexive. Once triggered, it unfolds according to a preset muscular scheme. Other human behavior, such as an arm released to swing back and forth as a passive pendulum from the shoulder, has no such underlying scheme; it proceeds on a simple deterministic physical basis, under the driving force of gravity. In both cases, once the action has begun, we can (at least in principle) predict with considerable accuracy how it will continue, without any further information.

A contrasting, less predictable type of human behavior may be called *referential*: It is organized by ongoing, often adaptive interaction with some specific process, criterion, set of events, or plan (in general, a *referent*) that is used to guide or assess production (Pressing, 1988). The interaction with the referent allows determination of the aptness of the current behavior, which may be used to shape future actions. Aptness may be determined by such factors as cost (energetic or cognitive), discrepancy between actual and target actions, or degree of consistency with a long-range goal, among others. The referent used to guide the referencing process may come from the environment in real time or from an automatic physiological source; it may be an internal structure pulled up from memory, or one freshly invented by a cognitive process; it may be used in open- or closed-loop fashion (Pressing, 1988).

Referential processes are very widespread and occur over a range of time scales. The time scale of reference is taken to be the inverse of the mean frequency of access to the referent. Examples of short time scales (referencing time scale $\tau < \sim 1$ s) include simple motor tasks such as tracking, synchronized rhythmical tapping, balancing a pole, and interception. For example, in tracking a randomly moving target, the target is the referent, and discrepancy between target and cursor position is the variable used to determine aptness.

Examples at medium time scales ($1 \text{ s} < \tau < \sim 1$ min) are found in fielding questions in an oral presentation, musical improvisation, and knowledge of results (KR) and knowledge of performance (KP) in many sport or decision-making tasks. KR and KP are feedback techniques known to improve aptness of control (Proctor & Van Zandt, 1994; R. A. Schmidt, 1988).

Referencing at long time scales ($\tau = \sim 1$ min–decades) typically centers around processes that feature adaptation, learning, goal setting, and planning, such as building a children's swing, interpreting a series of experimental results, or updating a financial investment strategy. In deliberate practice of the kind that forms an essential element in the long-term development of high levels of expertise, development of self-monitoring and self-regulation skills (forms of referencing) is an important constituent (Ericsson & Charness, 1994; Pressing, 1998d).

The point of this reference-based characterization of many human activities is to set the stage for a unified theory of referential control that can operate over these various time scales and classes of phenomena. In trying to strike a balance between general theory and application, I have produced an article that is composed of two

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parts. In the first part, I try to justify the plausibility of this unified dynamic conception of referencing, which yields an approach I have termed *referential behavior theory* (RBT; Pressing, 1998c). This unified conception can be linked to a general class of mathematical equations, and I focus here on a particular form, the discrete control equation (DCE). I elaborate on the characteristic properties of this equation, using their implications to examine the schism often considered to exist between dynamical and information-processing accounts of cognition and human behavior. In the second part of this article, I develop this central equation in detail by application to the particular case of timed human movement, which operates predominantly over short time scales. This provides a framework for the possible reconciliation of dynamic and cognitive/motor program accounts of human movement. In subsequent discussion, I address some potentials and predictions of the approach.

Part 1: Control Theory and Human Behavior

When humans act, they control their bodies, and often, external objects. In the mathematically barest depiction of such control, the human system's output at time t is a vector of values, $\mathbf{y}(t)$, which is some function of the system's input vector $\mathbf{u}(t)$:

$$\mathbf{y}(t) = \mathbf{S}[\mathbf{u}(t)], \tag{1}$$

where $\mathbf{S}$ is called the transfer function. A more detailed and less behaviorist formulation involves the possibility of an internal (possibly unobserved) state variable, $\mathbf{x}(t)$, and sources of noise:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{W}[\mathbf{x}(t), \mathbf{u}(t), \xi(t), t], \tag{2}$$

$$\mathbf{y}(t) = \mathbf{R}[\mathbf{x}(t), \mathbf{u}(t), \eta(t), t]. \tag{3}$$

In this *state-space* formulation, the first equation is the equation of state, and the second equation is the observation equation. $\mathbf{W}$ determines the internal state behavior, and $\mathbf{R}$ translates from state space to output. $\mathbf{u}(t)$ is variously identified as system input, the decision vector, or the control vector (Aoki, 1989); $\xi(t)$ and $\eta(t)$ are exogenous noise sources. In this article, I focus on the equation of state, Equation 2, assuming that its output can be directly observed. This equation simply proposes that there is some general lawful relation among state variable, control source, and noise. It is a standard theorem that continuous state-based systems can always be written in this form, subject to a few mathematical caveats (Ashby, 1960).

The state equation is often assumed to be autonomous (contain no explicit dependence on t) and to be linearly separable, as follows (Aoki, 1989; Fleming & Rishal, 1975):

$$\frac{d\mathbf{x}(t)}{dt} = \phi\mathbf{x}(t) + \kappa\mathbf{u}(t) + \xi(t), \tag{4}$$

where the noise source $\xi(t)$ is purely additive. The parameter matrices ϕ and κ are represented here as time-invariant; they also can be generalized to include the effects of long-term adaptation ($\kappa[t]$) or multiplicative noise ($\phi[t]$). Aside from the separability assumption, this equation form is still quite general, because (a) systems of higher order, ordinary, differential equations that might be proposed can always be reduced to first-order differential equations by a standard procedure of defining higher derivatives as new independent variables and (b) nonautonomous (explicit time) effects can always be made autonomous by adding appropriate new components to the state vector (e.g., Burghes & Graham, 1980).

This approach also can be formulated in discrete time and is frequently found in that form. In that case, Equation 4 is replaced by

$$\mathbf{x}_{n+1} = \mathbf{\Phi}\mathbf{x}_n + K\mathbf{u}_n + \mathbf{Q}_n. \tag{5}$$

Here, $\mathbf{Q}_n$ is the noise source. Analogous discrete and continuous forms (e.g., Equations 4 and 5) are not in general equivalent.

Equations 2–5 and their alternative formulations find wide usage in domains forming the traditional focus of control theory: industrial, engineering, and economic contexts. However, they occur much more widely. The continuous form occurs in synergetics as the slaving–driving principle and in the physics of random processes as one form of the Langevin equation (Haken, 1987). A related equation occurs as a proposal for a dynamic theory of control that is based on environmental information, memory, or intention (Schöner & Kelso, 1988a), with application predominantly to human movement. The same mathematical formalism, coupled with developing methods of nonlinear time-series analysis, can also be labeled *stochastic dynamical systems theory,* which is being increasingly used to attack psychological and clinical problems (e.g., Masterpasqua & Perna, 1996; Tschacher, Schiepek, & Brunner, 1992).

The discrete form of the state equation occurs frequently in time-series analysis, where it generally constitutes a vector autoregressive (moving average) model with exogenous variables, or VAR(MA)X model: The dependence of the current state value on earlier values provides the autoregression, the noise term provides the moving average constituent (the term is not very apt but is traditionally used in time-series analysis in this way), and the variable $\mathbf{u}_n$ represents exogenous forces. Such time-series modeling has been widely applied to psychological processes (Gottman, 1981; Gregson, 1983). Whether the continuous or discrete form is more apt will depend on the system's characteristics and the preferences of the modeler.

One further point needs to be made about these central equations. The definition of the *system–environment boundary* will determine what processes contribute to the first (autoregressive) term and which processes are part of the second (control) term. For example, imagine two persons interacting in some behavior such as coordinated rowing or conversation. Suppose that Person A adjusts his or her rowing speed and phase to that of Person B but not the reverse or, in the conversational case, that A listens and responds to B but B just gives orders. Then B's acts can be modeled as environmental control input to A's equation of state, whereas B can be modeled as a system unaffected by outside control: $\mathbf{u}_n^{A\rightarrow B} = 0$; $\mathbf{u}_n^{B\rightarrow A} \neq 0$. Each person constitutes a separate scalar system, and the coupling between the two person–systems is unidirectional. However, one can instead define the two persons together to constitute one system, with a two-component vector state variable. Now there is no external control. One simply has unidirectional self-referential processes within one equation of state.

This interpretation, which appears useful for many types of psychological and physiological aspects of control, allows one to rewrite the central control equations, without loss of generality and without the control variable **u**, because it is now part of **x**. In other words, the system can be defined so that all control aspects are encoded in relations between components of the state vector. To make the central equation as general as possible, I propose to simply use the discrete analog of Equation 2, with the single assumption of the linear separability of noise, which may be written as

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \mathbf{G}(\mathbf{x}_n, \mathbf{x}_{n-1}, \ldots \mathbf{x}_{n-l+1}; \mathbf{v}) + \mathbf{Q}_n. \tag{6}$$

Because **G** is so far unspecified (e.g., it might be linear or nonlinear), I might just as well have incorporated the $\mathbf{x}_n$ on the right-hand side into it. However, the written form is handy, because then the function **G** is interpretable as a control or forcing or behavior-modification function: If there is no control ($\mathbf{G} = 0$), changes in state are due solely to noise fluctuations. Here, **v** is a vector of control parameters, in which values affect the kinds of behavior the system can exhibit (e.g., they may affect the existence and location of equilibrium points, limit cycles, or chaos). l is the memory span or temporal depth of reference of the system, that is, the number of earlier time points needed to fully specify the controlled change in state (I assume $l \geq 1$, so that l is the *order* of the difference equation). For Markov processes, $l = 1$, and such modeling is widespread. The value of l in general is situation-dependent and must be determined empirically. Although additive noise has been made explicit, it is a simple matter to retain the possibility of other types (notably multiplicative noise) by suitable choice of the function **G**.

I refer to this equation as the self-referential form of the DCE. An analogous continuous form can be readily written,[1] which has its own properties, but from this point on in this article, the DCE is primarily used. In simple language, the equation proposes that change in a behavior can be predicted from a sum of the effect of previous behaviors (including environmental relations) and noise. Its form provides a foundation for causal inference, because **G** suggests an antecedent mechanism for change.

Feedback-Based Control

The application of Equation 6 to a particular problem then depends on the choice of state variables and a characterization of the control function **G** and noise source $\mathbf{Q}_n$. The most common action of **G** is to provide error correction or discrepancy minimization that is based on negative feedback.

Such a choice has a considerable history of application in industry, as seen in the 19th-century steam engine governor of James Watt (Arbib, 1981) and common control devices such as the thermostat or regulator. These machines exhibit homeostatic control, a goal that also applies to many human systems, operating at levels varying from the chemical to the cognitive (Freeman, 1948). Homeostatic processes operate at the physiological level to regulate fluid level, body temperature, glucose blood concentration, light intensity on the retina, and many other variables. Homeostatic forces have been invoked in models of social reconciliation (van Hooff & Aureli, 1994), emotion (Solomon, 1980), and, in Freudian psychotherapy, ego maintenance (Grunbaum, 1993).

Of course, error-correction/discrepancy minimization is used not only for homeostasis. Specific motor tasks like tracking, interception, aiming, and steering require real-time adaptation to possibly unpredictable environmental change. Synchronization effects that are based on reciprocal referencing between entities occur in the simultaneous flashing of swarms of fireflies, in pacemaker cells of the heart, in circadian pacemaker circuits, in unison cricket chirping, in coordination of breathing and gait in running, in cascade juggling, and in phase-coherent menstruation of groups of women, to mention only a few cases (Beek & Turvey, 1992; Bramble & Carrier, 1983; Mirollo & Strogatz, 1990; Winfree, 1980, 1987). Such relations between interacting entities produce a common palette of effects, including multifrequency mode locking and phase entrainment.

The familiar design of a negative-feedback control system (a servomechanism) is shown in Figure 1. Feedback about regulated variables is compared with target (reference) variables. The reference variables may be internally generated in the controller, or they may come externally from the environment. The comparison process generates an error signal, a difference vector, which, after amplification, produces a forcing function that controls the system. This control is only partial, because the system is also affected by noise (disturbances).

The difference vector is customarily just a subtraction of the actual signal from the *target, intended,* or *reference* signal, with the preferred label depending on the teleological interpretation of the process. Depending on the nature of the system, these signals may be produced in various ways. For example, in human movement and cognition, natural signals may be based on spatial variables (e.g., velocity, position, orientation), force variables (e.g., linear acceleration, torque, pressure, muscle tension), or timing variables (e.g., time, relative phase).

Control through feedback necessarily involves a delay component, so it can be accurate only if motion is not too fast. Even with slower motions, feedback control may not operate well unless the control parameters are in appropriate ranges. If control parameters are too weak, resulting in undercompensation, the system is dominated by noise and is not stable. If the parameters are too large, resulting in overcompensation, high-amplitude oscillatory behavior may result. Such effects are visible, for example, as poor steering technique in the novice's use of a ship's rudder, as intention tremor associated with certain types of cerebellar dysfunction in humans (Arbib, 1981), and as a broader range of deficits attributed to disordered servo-like mechanisms in the cerebellum in temporary lesion studies with monkeys (e.g., Hore & Flament, 1986). At certain values of control parameters, depending on the nature of the equation of State 6, chaotic behavior may result (Ott, 1993).

Feedforward

Another process that can have a significant role in referential control is feedforward, which may directly use information about

[1] This equation can also be presented in continuous form, using simple analogy, as $\dot{\mathbf{x}}(t) = -\mathbf{G}[\int_0^\infty \mathbf{s}(\tau)\mathbf{x}(t - \tau)d\tau; \mathbf{v}] + \xi(t)$, where $\mathbf{s}(\tau)$ is a kernal weighting the effect of past experience. If $\mathbf{s}(\tau)$ is the Dirac delta function $\delta(\tau - \tau_0)$, the equation reduces to a *differential single-delay equation*, which is a variant of Equation 4. If τ_0 is zero (no discrete delays), then the equation becomes $\dot{\mathbf{x}}(t) = -\mathbf{G}[\mathbf{x}(t); \mathbf{v}] + \xi(t)$.

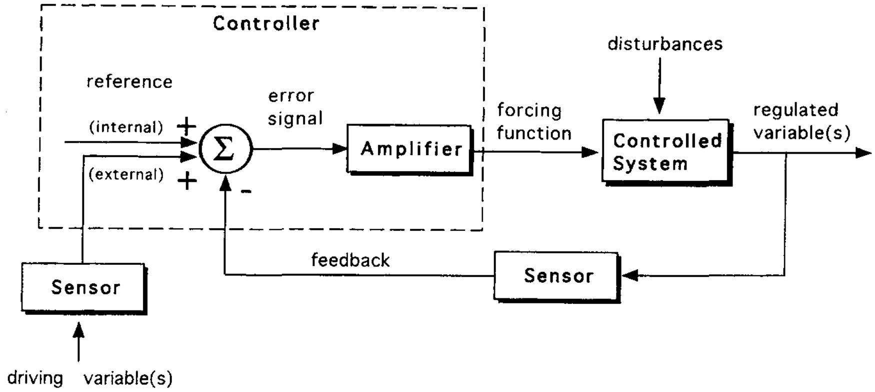

Figure 1. Control via feedback, showing internal and external reference.

the disturbances perturbing the system to make predictions. Feedforward avoids the delays found with sensory monitoring and, hence, is faster than feedback. In motor functions, it is essential for rapid, controlled movement and forms the basis for anticipatory postural adjustments, which occur widely (e.g., in the trunk and legs preparatory to impending arm action and in precision-grip maintenance tasks under varying inertial conditions; Flanagan & Wing, 1997). The use of feedforward information requires some sort of internal model of the system that can be used predictively to prepare for future input or intended behavior.

Feedback and feedforward frequently operate cooperatively and, in humans and animals, have close physiological associations (Haggard, Jenner, & Wing, 1994). For example, in the case of motor control of reaching or aiming, target position must be estimated using sensory afference data; current limb location may be determined by means of either sensory afference (feedback) or motor efference (feedforward). There is strong evidence for the use of either or both sources of information in varying circumstances (Bullock & Grossberg, 1988; Held, 1965; Winters, 1995).

Models specifying feedforward can be readily incorporated into the control function **G** above, although their precise form will vary with system type. In particular, feedforward is quite compatible with referential comparison. For example, feedforward can be used to effect compensation that is based on comparison of predicted system state (e.g., limb position) and predicted values of relevant environmental variables (e.g., target position). Another possibility is when feedforward multiplicatively modulates feedback, or vice versa (Winters, 1995).

Neural Networks as Dynamic Control Structures

Both biological neural networks (BNNs) and supervised artificial neural networks (ANNs) make much use of error-correction or discrepancy minimization, typically through feedback or feedforward mechanisms. BNNs often feature feedback loops designed to promote homeostasis, and the integration of different control sources (as in muscle agonist–antagonist pairs) is widely distributed throughout the bodies of animals. A well-known example is the cerebellum, which is widely acknowledged to function as a predictive controller (Buckingham, Houk, & Barto, 1994; Miall, Weir, Wolpert, & Stein, 1993; Paulin, 1989), integrating feedback and feedforward information.

Supervised ANNs are also based on what can be viewed as referential comparison: At each stage, an error (difference vector) is computed as actual output minus desired output in the output layer of the net. This vector is reduced by a systematic adaptation procedure, or learning rule, such as backpropagation, that increasingly connects input and output in the service of a particular goal (Anderson & Rosenfeld, 1990; Rumelhart, Hinton, & Williams, 1986). In fact, the general algebraic form of ANNs used for typical purposes such as identification or regulation is precisely the DCE, Equation 6, with **G** being the neural net function (Narendra, 1995). Autoregressive control focusing on timing patterns (as I emphasize later) typically implicates neural networks that are *recurrent,* that is, that store recent history in special input or context units (Hertz, Krogh, & Palmer, 1991).

Aspects of the Control Function

To fully elaborate the scope of the DCE, we need to consider the control function **G** more explicitly. To do this clearly, we narrow our focus for a time to cases in which system control parameters are relatively stationary, with referencing proceeding on a short to medium time scale. Given this, there are three important characterizing dimensions of **G**: linear–nonlinear, discrete–continuous, and immediate–delayed. These are discussed in turn, using primarily simple motor-control tasks as examples.

Linear versus nonlinear control. In the terminology here, linearity of the control function means, using the simplest possible form $\mathbf{G} = \mathbf{G}(\mathbf{x})$ for clarity of illustration, that $\mathbf{G}(a\mathbf{x} + b\mathbf{y}) = a\mathbf{G}(\mathbf{x}) + b\mathbf{G}(\mathbf{y})$; otherwise the control function is nonlinear. Many engineering control systems are based on the use of on-line feedback control of complex, often nonlinear systems (Panossian, 1987). In practice, most such situations can be converted (by design) into a situation of linear control that corresponds to differential or difference equations with one or more stochastic noise sources.

In psychological circumstances, where one does not have the engineer's luxury of designing the system from scratch but only (at most) varying the experimental conditions, the control function can nevertheless often be successfully modeled as linear. Notably, linear correction that is based on positional or higher order error is found in human tracking tasks (e.g., Bösser, 1984; Licklider, 1960) and also in similar experiments with monkeys (e.g., Miall, Weir, &

Stein, 1986, 1988). Linearized control around system fixed points has been proposed by Schöner and Kelso (1988a, 1988b, 1988c) in addressing the effects of environmental information, memorized information, and intention on human performance. In stable synchronized tapping tasks, linearity of error correction provides excellent agreement with experimental results (Hary & Moore, 1987a, 1987b; Mates, 1994a, 1994b; Pressing, 1998a; Pressing & Jolley-Rogers, 1997; Vorberg & Wing, 1996).

However, nonlinear effects are also consistently reported, notably in tracking (Bock, 1987; Licklider, 1960; McCormick, 1970). Error-correction processes are often considered to have a small central dead zone of zero sensitivity, whether the task be tracking (Miall, Weir, & Stein, 1986), synchronized tapping (Mates, 1994a, 1994b), or maintenance of balance (Collins & De Luca, 1993). Error correction may incorporate threshold effects or higher algebraic power terms (Houk & Rymer, 1981; Mates, 1994b), particularly at higher frequencies (Bock, 1987) or with large errors (Bösser, 1984). Nonlinear effects are also widely reported in bimanual motor-coordination tasks, attributed to oscillatory coupling (e.g., Haken, Kelso, & Bunz, 1985; Kelso, Scholz, & Schöner, 1986; Turvey & Schmidt, 1994; Zanone & Kelso, 1992). Such tasks generate characteristic domains of phase-based oscillatory stability and instability that are due to the nonlinearity of the control function (Kelso, 1994a, 1994b), in terms of the language used here. In delayed visual feedback tracking experiments, recent findings show that first-order stochastic differential delay equations using a hyperbolic tangent control function and one or two delays (corresponding to separate visual and proprioceptive feedback loops) can successfully model both the actions of healthy participants and patients with Parkinson's disease (Beuter, Bélair, & Labrie, 1993; Vasilakos & Beuter, 1993). Finally, looking outside the motor domain, modern theories of behavior and personality have repeatedly suggested the fundamental importance of nonlinear relations between these variables and with the environment under certain conditions (Abraham, 1990; Heiby, 1995).

The most apt conclusion appears to be that referential processes are in general nonlinear but are often well approximated as linear in regimes of behavioral stability. This is understandable in the RBT framework because state-based systems, whether they are continuous or discrete, and provided the control function is continuous and suitably differentiable, can with relatively few exceptions be cast in linearized form in the neighborhood of an equilibrium point (Ashby, 1960; Khazin & Shnol, 1991).[2] Nonlinearities that produce solutions that are qualitatively similar to the linearized form of control can be called *perturbative.* Systems operating away from equilibrium points, where linearization breaks down, can exhibit *essential* nonlinearities (e.g., bifurcations to qualitatively new classes of behavior).

An *equilibrium point* (the terms *stationary point* or *fixed point* are also sometimes used) is a point where the control function goes to zero. Equilibrium points can be locally stable (so that perturbations are damped out, and the system tends back to the equilibrium point), unstable (perturbations are amplified, and the system leaves the equilibrium point), or neutral (the system exhibits no tendency to return or depart from equilibrium due to perturbations; Ashby, 1960; Khazin & Shnol, 1991). Which condition is obtained depends in the first instance on the properties of the first derivatives of the control function at the equilibrium point (positive, negative, or zero, respectively; cf. Equation 6); however, in the zero case the properties of higher derivatives at the control point must also be considered (see footnote 2). Stable fixed points are also called attractor points, and unstable fixed points are also called repeller points. In multivariate cases, an unstable fixed point may be a saddle point; that is, the partial derivative of the control function is positive in one direction and negative in another. In the presence of significant levels of additive noise, only stable equilibria have significant durations, as states at other types of equilibria diverge from the equilibrium point in response to fluctuations.

Stability in the neighborhood of an equilibrium point is neither a necessary nor a sufficient condition for linearity: As demonstrated in footnote 2, some equilibrium points cannot be linearized, and some linear systems are known to be unstable (Vorberg & Wing, 1996). Given this, the considerable incidence of linearity and (quasi-linear) perturbative nonlinearity in behavior appears to tell researchers something about general human control predilections.

Discrete versus continuous control. Referencing or control may be formulated as a discrete-time process (difference equation) or as a continuous-time process (differential equation). Clearly, the nature of the system will affect this choice but so will the nature of system measurement and the purposes of the modeler.

For example, automated engineering control systems typically have physical variables such as temperature or pressure that are continuously monitored to very high accuracy, so that the underlying processes are effectively continuous. On the other hand, there is recognition that industrial processes that include humans in the control loop may be better modeled by discrete-time rather than continuous-time equations, partly because of information-transmission latencies and partly because of intermittency in motor output or decision making (Panossian, 1987). Both continuous and discrete models of spatially and temporally continuous coordinated human movement are common (Haken et al., 1985; Kelso, 1994b; Turvey & Schmidt, 1994; Vorberg & Wing, 1996).

In deciding on the optimal representation of control, it is important to consider the potential relations between corresponding continuous and discrete forms, bearing in mind that corresponding forms will not typically yield equivalent solutions, even at the

[2] Linearization of control can be readily shown as follows. For simplicity of demonstration, let us use the simplest first-order form of the control function in Equation 6, $\mathbf{G} = \mathbf{G}(\mathbf{x}_n)$. (The story is readily generalized to other cases.) Then at an equilibrium (stationary) point, $\mathbf{X}$, $\mathbf{G}(\mathbf{X}) = 0$ by definition. Write $\mathbf{x}_n = \mathbf{X} + \xi_n$ so that $\mathbf{x}_n$ is measured as a deviation from its equilibrium value. From Taylor's theorem (if $\mathbf{G}$ is arbitrarily differentiable and these derivatives are continuous), $\mathbf{G}(\mathbf{x}_n) = \mathbf{G}(\mathbf{X}) + \xi_n\mathbf{G}'(\mathbf{X}) + \xi_n^2/2!\,\mathbf{G}''(\mathbf{X}) + \cdots$. (In the multivariate case, each right-hand-side term would be replaced by a sum of products of the components of $\mathbf{x}_n$ and appropriate partial derivatives—see Gillespie, 1960, for details). Because the first term in this expansion is zero, and provided the first derivative term $\mathbf{G}'(\mathbf{X})$ does not identically vanish, for small deviations from equilibrium ξ_n, the terms beyond first order will be negligible, yielding from Equation 6 a linear control equation, $\xi_{n+1} \approx \xi_n[1 - \mathbf{G}'(\mathbf{X})] + \mathbf{Q}_n$. Equilibrium is stable if $\mathbf{G}'(\mathbf{X})$ is positive, unstable if it is negative, and apparently neutral if it is zero—"apparently" neutral because investigation of the properties of higher derivatives at $\mathbf{X}$ is required to assess stability (see Khazin & Shnol, 1991) in this case. However, if these conditions do not obtain (e.g., $\mathbf{G}'[\mathbf{X}] = 0$), the equation is not linearizable; the first term will be a higher power of ξ_n (Khazin & Shnol, 1991).

discrete time points where both are defined. There are three primary ways that discrete control can arise in a system—ignoring the discretization commonly used to numerically solve differential equations (Palm, 1983). First, the nature of the system may be intrinsically discrete—for example, with a process that is based on sequentially presented, distinct, experimental judgment trials. Second, discrete control may arise from an underlying continuous process by sampling. Many of the properties of the variables in this sampled system (e.g., autocorrelation) will then vary with the sample rate. Third, a discrete state equation may arise from a continuous one by the technique of Poincaré section. A Poincaré section is a surface in the phase space of control that is chosen so that it is repeatedly pierced by system trajectories. The surface may correspond to some kind of process of conditional observation of the system's evolution. Sequential piercings by a trajectory define a sequence of points, which yields a sequence of discrete states reflecting discrete control. Because of the piercing process, the resulting discrete system has at least one less dimension than the parent continuous system (Jackson, 1989).

Likewise, there are three common pathways to continuous control models. First, continuous control may be intrinsic to the nature of the system. Second and third, difference equations may be converted to differential ones by one of two techniques. In the first of these, which is not always possible, one constructs a differential equation that is designed so that its solution matches that of the original discrete equation at every discrete time point. In the second, more common, technique, which is valid for sample-based discrete equations, one lets the sample size shrink asymptotically to zero, and, in this limit, differences become appropriate differentials. Such a conversion has been invoked in the area of motor control (e.g., Kelso, 1994a; Turvey & Schmidt, 1994).

However, three caveats about this last method must be mentioned. First, in general there is no unique or "canonical" (natural and favored) path of limit-based conversion between difference and differential equations. Particularly beyond first order, there may be several ways to group a given pattern of differences to formulate differentials, and vice versa.

Second, even if the formal conversion can be unambiguously made, corresponding discrete and continuous equations not only produce different numerical values but have under some conditions topologically distinct properties. For example, an autonomous set of first-order, nonlinear, differential equations cannot exhibit deterministic chaotic behavior (as defined by nonlinear systems theory), unless there are at least three independent variables (Ott, 1993). In contrast, a single-variable, first-order, nonlinear difference equation can exhibit chaos if its functional form is noninvertible (Ott, 1993).

Third, in the conversion between sample-based difference and differential equations, it is often found that the limiting process toward zero sample size can be meaningfully effected only if system parameters have particular relations or limiting behaviors, whose validity must be separately assessed. Given these points, it is essential that the choice of a particular discrete or continuous representation for human behavior be carefully justified, even in formulations that only aim to produce qualitative understanding (see also Kelso, 1994b).

It is worth emphasizing that the division between continuous and discrete control is by no means the same as the classification of tasks as continuous or discrete. To illustrate this, consider a task (e.g., driving a car) that is effected by discrete and temporally bounded control interventions—that is, intermittency of control. Such intermittency is well established in the movement of both humans and nonhuman primates (Miall et al., 1986; Neilson, Neilson, & O'Dwyer, 1992; Poulton, 1981), and it may apply whether the movement itself is intermittent or continuous. One way such control can be effected is if the scheduling of interventions is determined by monitoring some underlying criterion variable that is continuous, or at least changing at a much shorter time scale than the intervention rate. In this case, despite the intermittency of control, a continuous state model might be essential. Alternatively, intermittent control might be predictably regular or cued by external signals and require no explicit specification of an underlying continuous criterion variable, in which case, a discrete model would be appropriate.

Immediate versus delayed control. The fundamental issue here is how far back in time referencing recedes and, accordingly, the latency of all pertinent control loops. One approach to this is through the order of the control. In the case of continuous state-space modeling, the order is the number of derivatives of the state variable or control used in a task. Hence, zero-order control in a spatial task is based on position, first-order control on velocity, second-order control on acceleration, and so on. Higher order control is used in complex mechanical systems; for example, the rudder and flaps of aircraft are based on second-order control, and submarines normally have at least third-order control (McCormick, 1970). Typically, higher order control is more difficult for humans than lower order control (Jagacinski & Hah, 1988; McCormick, 1970). Indeed, *display quickening,* a technique that recodes higher order information as position information, generally improves performance in tasks that require use of higher derivative information (Jagacinski & Hah, 1988; Poulton, 1974).

In tracking tasks, one primary result of practice is a shift from the use of purely low-order information, such as position, to the inclusion of higher order control information, such as velocity and acceleration (Proctor & Dutta, 1995). Combined with work showing that performers regress to lower orders of information control when under stress or when competing attentional demands exist, there is good evidence for the progression–regression hypothesis (Fitts, Bahrick, Noble, & Briggs, 1961; Fuchs, 1962; Jagacinski & Hah, 1988), which states that a performer's control order reflects training, task demands, and the performer's state.

In many formulations of continuous motor control, first-order effects are found to be sufficient (Kelso, 1994a; Turvey & Schmidt, 1994), although second-order models are also found (e.g., Mussa-Ivaldi, 1995). In adaptive industrial control processes, so-called proportional, integral, and derivative (PID) controllers, where the control signal is a sum of terms based on the error, its integral, and time derivative, account for more than 90% of control loops (Åström, 1995). This is a second-order control process.

A parallel situation exists in the literature on discrete control. In this case, order is defined as the maximum difference in time indexes occurring in the control equation. Thus, if control at time $n + 1$ is determined from state values at times n and $n - 1$, the system is second order. Explicit accounts of compensatory discrete motor control have normally relied on first-order autoregressive processes (e.g., Hary & Moore, 1987a; Kelso, 1994a; Mates, 1994a; Pressing & Jolley-Rogers, 1997; Turvey & Schmidt, 1994;

Vorberg & Wing, 1996). However, second-order effects have been noted by some workers. Vos and Helsper (1992) proposed a second-order linear equation in asynchronies in an offbeat tapping task. Pressing and Jolley-Rogers (1997) found that second-order linear autoregression occurred in a synchronous tapping task with an expert performer at fast tapping speeds, whereas nonexpert performance was characterized by first-order regression, and Pressing (1998a) found that second-order error correction occurred in some complex musical tasks (e.g., polyrhythms). In these last three cases, the model was based on additivity of the first- and second-order autoregressive terms.

Explicit effects of time delays may also appear in continuous equations, yielding differential delay equations. Such effects are empirically well documented (Kelley, 1968) and may represent latencies in information processing or transmission. Examples of differential difference control in biological systems include the Mackey–Glass equation (Glass & Mackey, 1988) and the work of Beuter and coworkers on delayed visual tracking (Beuter et al., 1993; Vasilakos & Beuter, 1993).

General Properties of Human System Noise

The final detail in characterizing the DCE is to consider the character of the noise term, $\mathbf{Q}_n$. Stochastic noise generally arises from the averaged effects of many variables that cannot be directly measured. In the general systems approach, the effects of noise can be divided into three categories: control-dependent noise, state-dependent noise, and purely additive noise (Panossian, 1987). Which types should apply to human behavior? This is an empirical question that has not yet been fully answered. Human sensory processing appears to entail both additive and multiplicative noise (McGill & Teich, 1991). For systems exhibiting large variations in system control parameters, the conclusion of control theorists tends to be that control- and state-dependent noise are most realistically modeled as multiplicative (Harris, 1978; Panossian, 1987). Such conclusions are understandable as the result of substantial amplification processes in the control chain.

However, in many cases, particularly where the system parameters are relatively stable over the analytical time frame, additive noise alone is normally successfully used (Aoki, 1989; Box & Jenkins, 1976), as given above. This is standard in time-series analysis (e.g., Gottman, 1981). The typical design of psychological experiments, which is based on the repeated examination of set conditions, would therefore be expected to be well modeled by additive noise.

Such additive noise is often successfully modeled as white, Gaussian, and temporally uncorrelated. However, this is an empirical issue, and in some human control systems, noise is colored and shows nonzero autocorrelation at Lag 1 and sometimes also at higher lags (Box & Jenkins, 1976; Panossian, 1987). Recent work (Gilden, 1997; Gilden, Thornton, & Mallon, 1995; Pressing & Jolley-Rogers, 1997) has also established the presence of long-range noise memory processes in certain types of psychological control. These processes have no direct effect on the local models developed here, and their treatment is deferred to a later time.

Adaptation, Learning, Goal Setting, and Planning

I now return to a broader focus and consider referencing over medium to long time scales, which inevitably involves such phenomena as adaptation, learning, goal setting, planning, and evaluation, which are subserved by information-processing entities like memory and attention. How does this show up in the DCE? In the state-based dynamic approach given here, adaptation and learning may be reflected as changes in initial or boundary conditions or (more typically) as changes in the control parameter vector over time ($\mathbf{v}$ in Equation 6 becomes $\mathbf{v}[t]$). This can lead to altered and perhaps qualitatively very different solutions to the governing equations. Yet if the time scale of change in $\mathbf{v}(t)$ is slow relative to the time scale of behavior, as is often the case, then one can solve the DCE locally in any chosen time neighborhood using an average $\mathbf{v}$. Alternatively, the control parameter vector may itself be modeled over time and used functionally in the now nonautonomous DCE. Browne and Du Toit (1991) successfully applied this technique to model learning of an air traffic controller task, resulting, after Gompertz learning curve fitting to global means, in a linear form of the DCE that produced better fits to data than latent variable models. Transformation of the state vector, for example by differencing or detrending, may also promote stationarity or other valuable statistical attributes, yielding effective modeling for learning or adaptation.

With more complex behaviors, these approaches will not tell the whole story. Adaptation must also include the possibility that over time certain new or formerly negligible terms in the state equation become significant or that certain initial ones fall to zero. The relevant components of the state vector may also change from task to task, with other components becoming unimportant "dummy" variables. Formally, such major changes can be handled in various ways. One approach is through a master equation of control for the system in question, operating over a large number of dimensions, and a set of mechanisms by which the control that is appropriate for a certain task is projected out to a lower dimensional control space, as in the theory of projection operators, used in statistical and quantum physics (Jackson, 1989).[3] Projection operators reformulate a problem by transforming a full state equation to a simpler one based on fewer variables, usually by appropriate choice of coordinate systems or variable transformations.

Another approach is to suggest that goals and planning enable a higher order selection–control process that pulls up and continually modifies different contextually appropriate local $\mathbf{G}$ functions from memory. Long-term memory provides some kind of semantic indexing of such functions, and the homunculus problem is kept at bay by an insistence on heterarchical organization, under which control can shift as needed between competing brain regions in response to varying afferent data sets and efferent expectancies. In the global sense, researchers can speak of memories as dynamically stable attractors (Hertz, Krogh, & Palmer, 1991). Nevertheless, the particularities of how goals, plans, and schemas arise from dynamic systems have not yet been successfully articulated.

In any case, the setting and attempted fulfillment of goals that are chosen to satisfy an organism's needs are clearly referential. These needs are dynamic, and include those inculcated by evolution, as well as those that come about because of the complex web

[3] For example, this view of projection is one approach to solving Bernstein's classic problem in motor control, which asks how humans can so readily solve the problem of having too many degrees of freedom in the control of multijoint movements (cf. Turvey, 1990).

of social, cultural, developmental, personal, and physical circumstances affecting human life. Actions and plans of action to satisfy these needs are repeatedly evaluated and repeatedly modified to improve need satisfaction, often amidst the distracting effects of noise. Evaluation typically entails a referencing process that operates along germane physical and psychological dimensions: Outcomes or predicted future consequences are compared with desired outcomes, and new behaviors tried out or corrective modifications to existing behaviors made on this basis. Goals may be broken down into subgoals, each of which may be autonomously referenced, a process related to means-end analysis (Newell, Shaw, & Simon, 1959). Evaluation processes are affected by such considerations as the veridicality of internal models used and by choices between primary control (in which the organism attempts to change the environment to accommodate its preferences) and secondary control (where the organism adapts itself to environmental dictates; Heckhausen & Schulz, 1995). Emotional evaluation acts as an amplification circuit. If evaluation is not possible, then behavior may become dominated by randomness, or by acquiescing to a specific external control source (i.e., *vicarious control*).

Thus, the incorporation of a wide span of time scales into the dynamical RBT approach leads inevitably into relations between dynamical, cognitive, information-processing, and even emotional perspectives. Whichever of these languages of reference is most apt, they each may vary across a range of causal styles—from automatic, unconscious, environmentally driven, or hardwired to conscious, volitional, goal-oriented, preplanned, or designed. At all levels, outcomes are somehow referenced to goals or control sources to produce reliably apt or adaptive behavior.

One advantage of this wide perspective is that a forced choice between philosophical orientations that are sometimes considered dichotomous (e.g., top-down vs. bottom-up, case-based vs. rule-based learning, or self-organization vs. computation) is not a prerequisite for consideration of the general properties of the central control equation, and discussion of these interpretive issues can be profitably deferred until the context prescribes their relative use, as discussed more fully below.

Different Languages, Philosophies, and Contexts of Control

To summarize, the perspective of referential control (RBT) presented here is quite general and can apply whether the controlling process is classed as muscular, chemical, diffusive, electromechanical, magnetic, emotional, neurophysiological, cognitive, informational, energetic, and so on. Of course each such process is likely to have its own unique characteristics (i.e., parameters, variables, initial–boundary conditions, noise and control functions), but when these processes occur in living creatures, they will display links and commonalities by virtue of their integrated contributions to the functioning of the organism. The capacity of the current formulation to span distinct structural levels, types, and time scales of analysis seems therefore attractive. Indeed, parallels between phase transitions in motor behavior and in spatiotemporal organization of brain activation point to advantages of just this kind (Fuchs, Kelso, & Haken, 1992; Kelso et al., 1992).

Because the scope of referencing in RBT is so broad, the approach here is more general than the dynamical viewpoint that posits a unique informational foundation to dynamics in living organisms (Kelso, 1994b). Likewise, RBT does not demand a fundamental schism or irreducibility between symbolic and dynamic modes of operation of living systems, a view held by some (Kugler & Turvey, 1987; Pattee, 1977).[4] Rather, depending on the state variables and the interpretation assigned to the components of the fundamental equation, the focal process may be used to characterize behaviors as diverse as the following: real-time control that is based on error correction or discrepancy minimization, referential compensation, iterative behavioral modification, informational signaling between entities, coupling between two or more dynamic processes (e.g., oscillators), interlimb coordination, attentional dynamics, tracking of an environmental signal, on-line search in a problem space with constraints, adaptation and learning, optimal prediction, and goal setting and tuning.

These descriptions point in a variety of directions and are certainly far from equivalent, but the formalism here is general enough to accommodate all of them, and I believe that this can be usefully exploited in a unified approach to modeling psychological processes. It may have particular use in the continuing debate about relations between dynamical and information-processing accounts of human behavior and the brain, accounts whose potentials are often set in marked contrast (Abernethy & Sparrow, 1992; Schöner & Kelso, 1988a; Skarda & Freeman, 1987; van Gelder, 1998). The perspective here is that a credible, general dynamical approach must show how it can be reconciled not only with memory and attentional effects, as in recent neural network approaches, but also with traditional high-level informational phenomena such as case-dependent learning, expertise, planning, beliefs, and representation.

This is the conclusion of the first section. In the following section, I narrow the focus and apply the general principles formulated above to the production of human temporal patterns.

Part 2: Production and Control of Temporal Patterns

Temporal structure in behavior can seemingly come about in two ways (cf. Traub & Miles, 1991, p. 120). In the first way, temporal structure has an autonomous (nonreferential) and fundamentally energetic, noninformational basis. Simple physical examples include a resonating circuit, a mass oscillating on the end of a spring, and a swinging pendulum. These examples were not chosen idly; they all have been used in motor modeling. Resonating circuits can function as clocks to coordinate body activity (Winfree, 1987), and both energy-driven mass-spring (e.g., Bizzi & Mussa-Ivaldi, 1989; Buckingham et al., 1994; Cooke, 1980) and pendular (e.g., Kugler & Turvey, 1987) models have been used successfully in understanding certain aspects of limb movement, including the organizing effect of external constraints, such as gravity.

The second way in which temporal order can arise is by referencing or control, and this is the major focus here. The nature of the variable of reference is not restricted, but commonly it will

[4] One possible specific link between such modes is that of "symbolic dynamics" (e.g., Hao, 1989), which considers how dynamical systems can yield preferential patterns of occupation of distinct regions of phase space, which can be directly mapped to symbols. See also the later discussion on cognitive criteria for temporal pattern phase transitions.

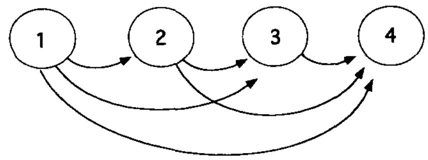

Figure 2. A chain of elements that can create a timing sequence. Each connection represents unidirectional inhibition, so that an increasing activation applied to all units, each with a threshold to fire, yields the firing sequence 1–2–3–4.

have an energetic or informational basis. Here, it is important to note that biological information, unlike the information on this page, is typically stored in dynamic, self-sustaining energetic systems, so that the boundary between energetic processes and informational processes is not always crisp. Accordingly, models of information processes often implicitly use energetic language. Consider the example shown in Figure 2.

This figure shows a familiar chain of inhibitory elements as occurs in models of serial behavior, or as might be found in particular regions of connectionist models that have an asynchronous update rule (Estes, 1972; Rumelhart, McClelland, & the PDP Research Group, 1986; Rumelhart & Norman, 1982). In the language used here, this example depicts a simple, nonlinear referencing process, because the outputs of units are based on threshold functions of the inputs from other units.

Because of the pattern of inhibitory connections, all that is necessary for the sequential behavior 1–2–3–4 to emerge is that a steady input be applied to all elements simultaneously and that each element accumulates activation, firing when it reaches a certain threshold. Activation is clearly an energetic concept, yet this arrangement dependably produces a sequence of actions, because of the pattern of inhibitory connections, and hence can act as an information-storage device. Indeed, the output can be considered symbolic. The limitations of models of this simplicity in solving the problem of sequential behaviors are well-known (e.g., Rosenbaum, 1991) but are not important here. What is noteworthy is that one can readily couch the timing structure in terms of information and symbols, even though its genesis is purely energetic. This perspective dovetails with the work of Frieden (1998), who derived the central energy-based laws of physics from an observer-based definition of the measurement process and use of the principle of extreme physical information.

Temporal referencing provides a natural forum for application of the DCE, and the referencing process need not be exclusively energetic or informational. The reference stream may exist inside the system, outside the system, or both. As shown earlier, we can convert the last two cases to the first case by appropriate choice of the system–environment boundary.

The temporal reference stream may be continuous, or it may be a discrete sequence of reference events. Typically, it is a clock, an oscillator, or some other organized temporal process, which enables the generation of the target pattern. The process is shown in Figure 3 in schematic discrete form: A stream of reference events (R_n) is somehow used to produce a stream of behaviors (S_n), aimed to be located at certain target positions (defined by the intervals T_n). Familiar laboratory examples of this process include tracking and synchronous tapping experiments.

Although clocks and oscillators are not the only kinds of temporal reference processes, they are the most common, and they are closely related concepts. Essentially, a clock is a combination of an oscillator and a readout mechanism, typically with a stabilizing mechanism (e.g., the escapement in mechanical clocks). The readout mechanism either counts numbers of recurrences of a specified oscillator position (for a "fast" oscillator, as in quartz crystal watches) or measures calibrated distances along a continuum (for a "slow" oscillator, as with a sundial). It typically does, but need not, produce equally spaced temporal output.

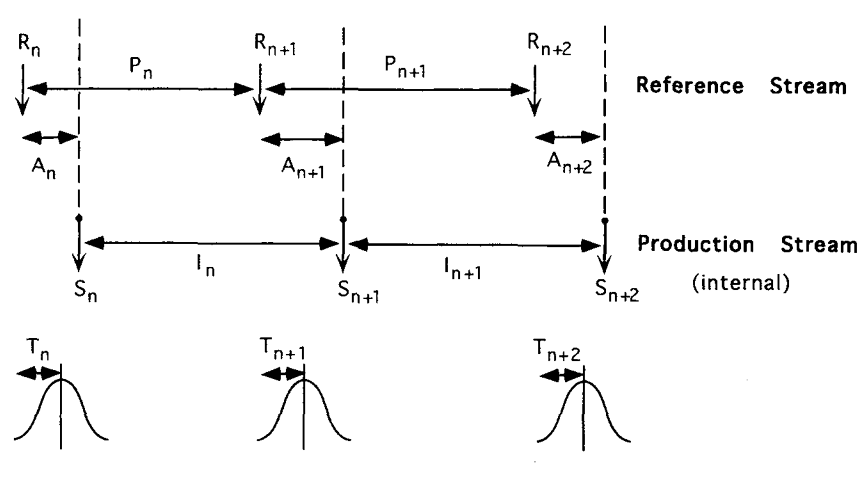

Figure 3. The basic referencing or error-correction process. A stream of reference events (R stream) is used as a basis for the timing of a produced stream of motor events (S stream). The target intervals (T_n) for the asynchronies (A_n) are shifted by certain intervals from exact synchrony. The distributions drawn illustrate the range of actual asynchronies. P_n = *n*th reference stream interval; I_n = *n*th interval between events.

The Motor Theory "Paradigm Crisis"

The general split between dynamical and information-processing accounts of behavior mentioned above has percolated into the area of motor control (Meijer & Roth, 1988). This has reached the dimensions of a Kuhnian paradigm crisis, according to Abernethy and Sparrow (1992), with a number of coexisting competing theoretical positions, variously labeled *cognitive, motor program, dynamical,* and *connectionist.* This view may be rather sensationalist, but the motor program and dynamical approaches do remain far apart in the eyes of many, despite some recent attempts toward limited accommodation. (Thus, Heuer [1993] has used a motor program approach to broach instabilities in bimanual performance, normally the mainstay of dynamic system theory, and Kelso [1994b] has viewed behavioral information as central to dynamics.)

One distinction that has remained has been a tendency to associate distinct mathematical models of timed recurrent behavior with the two perspectives (Pressing, 1998c): Nonlinearly coupled oscillator equations form the essential foundation for the instabilities focused on by the dynamical approach (e.g., Haken et al., 1985; Kelso, 1994a; Turvey & Schmidt, 1994), whereas linear stochastic equations used in modeling stable behavior have often been explicitly linked to the motor program approach (Beek, Peper, & Stegeman, 1995; Heuer, 1988, 1993; Vorberg & Wing, 1996).

In what follows, I show that both types of equations follow under certain conditions from the central control equation given here. I then review research that provides sound experimental support for this new approach, including excursions into new terrain. In general, both steady-state and transitional–unstable domains have essential and complementary roles to play in behavioral description. Humans predominantly operate in the stable regions of the behavioral landscape, and so an emphasis here is ecologically sound, yet a full characterization of dynamic repertoire requires system information on breakdown.

Diversity of Temporal Pattern Control

Temporal patterns occur widely in human motor behavior, and their sources appear to be manifold. Anatomically, the human motor system is built on extensive parallel and hierarchical neural connections (Kandell, Schwartz, & Jessell, 1991). Developmentally, there is a shift from subcortical control of movement to overarching cortical control, as early reflex actions of infancy fade, are inhibited, or become subject to conscious modification or override (Kandell et al., 1991). Evolutionarily, this duplication and dispersal of function are often considered to represent an encoding of successive solutions to particular adaptive problems over the course of phylogenesis (G. C. Williams, 1992). Functionally, in mature organisms, control typically involves the coordinated activity of many centers and appears able to be initiated and directed from a variety of structural levels acting singly or in tandem (Carlson, 1994; Neilson et al., 1992), by using open-loop or closed-loop control (Summers, 1981) and integrating central and peripheral influences (Neilson et al., 1992). For example, where feedforward is lost because of cerebellar dysfunction, less efficient conscious volitional control can often be substituted (Arbib, 1981).

The presence of low-level automatic control is indicated by such things as oscillatory central pattern generators in lower animals, many postpartum automatic reflexes, and preprogramming for the walking gait in infants. Adaptive intermediate-level control can show rapid modification of reflex actions (e.g., changes in stereotyped muscular responses to perturbations of stance; Nashner, 1976) and often involves the cerebellum as a learning device that is capable of error correction, tracking, and prediction (Paulin, 1993). High-level control uses these lower system components and in addition exploits potentials for consciously planned and extensively rehearsed motor skills constructed in the service of intentions and goals that are configured in relation to specific environmental conditions.

This diversity of control sources provides a challenge to a unified account of timed motor behavior and skill. The contemporary split between different paradigms rests in no small part on this diversity. Thus, one may wonder, is control top-down, bottom-up, bidirectional, hierarchical, heterarchical, emergent, or context-driven? Do these simple choices suppose false dichotomies? One advantage of the current formulation is that, in its general form, it is neutral with regard to the answers to these questions, possibly providing a unified context within which to interpret particular systems and conditions.

The General Control Model Applied to Movement Patterns

To apply the central equation to the case of movement timing, three straightforward steps are needed. The first of these is to identify the main variable as the scalar asynchrony A_n, which is defined as the timing difference between the *n*th behavior (e.g., action, timing point) and the nearest previous event (e.g., reference point) in the reference stream (recall Figure 3). Second, it is convenient to consider that the referencing process operates on the *relative asynchronies,* which are defined as $A_n^* = A_n - Z_n$, where Z_n is the zero point interval for the *n*th asynchrony (i.e., the zero point value for referencing). Z_n will normally be quite close to, but not identical to, the intended target interval for the *n*th asynchrony, T_n.

Finally, in line with the general review and specific evidence about timing tasks given above, I assume that correction–control is well modeled in nearly all situations by an, at most, second-order autoregressive process. There is certainly evidence that expert musical performers can use information over greater ranges, as in the rubato control of musical phrases or execution of certain complex instrumental patterns (Shaffer, 1981; Sloboda, 1985; Sundberg, Friberg, & Frydén, 1991; Todd, 1985). The formalism here can certainly accommodate these possibilities; however, such cases appear to be specific to certain conditions of deep expertise and have not been linked clearly to error correction. Hence, they are omitted from discussion in this article.

In what follows, I limit timing correction to referencing based on phase, although it can also be framed in terms of period adjustment (Vorberg & Wing, 1996). Large and Jones's (1999) model of pattern-based attentional dynamics uses DCE-type equations for both phase and period variables.

Given these assumptions, the general form for the $(n + 1)$th asynchrony is as follows (from Equation 6):

$$A_{n+1}^* = A_n^* - G(A_n, A_{n-1}) + Q_n. \tag{7}$$

Here, G is the control function (with the parameter vector $\mathbf{v}$ not explicitly displayed), and in this experimental context it may represent error correction or discrepancy minimization. In addition, because of the second convention above, G goes to zero if there is no control/error correction; in that case, only noise moves each successive asynchrony away from its previous value.

This equation is still quite general: It can apply to humans and nonhumans, and it can include cognitive and noncognitive components, because target behaviors may be the automatic result of low-level control predilections in spinal cord, brain stem, and so on, or consciously intended actions that are governed by a complex combination of sensory feedback, feedforward, and cortical decision making. For terminological convenience, I describe the central process as either control or error correction, but no large-scale, top-down connotation is necessarily intended: Control may be reciprocal or only locally hierarchical, and no central or intelligent agent is necessarily meant to be invoked, in accord with alternative descriptions given earlier, such as discrepancy minimization or referencing.

Elaboration to Include Complex Patterns

The aim here is to make the most general elaboration of Equation 7 with respect to discretely controlled, timed motor behavior, to allow the inclusion of the full range of possible behaviors, ranging from simple isochrony to cognitively complex repeating patterns, perhaps articulated by multiple limbs. Consider, then, that the sequence of events is based on a repeating pattern or cycle of L events, irrespective of length and cognitive complexity. The time series of observations can then be written using two indexes as

$$\{A_{j,n}\} = A_{11}, A_{21}, A_{31}, \cdots A_{L1}, | A_{12}, A_{22}, \cdots A_{L2}, | A_{13}, A_{23}, \cdots A_{L3}, | \cdots A_{LN}, \quad (8)$$

where the first index, $j = 1, 2, \cdots L$, indicates position within the cycle and the second index, $n = 1, 2, \cdots N$, indicates cycle number. If the noise source has a mean of zero, then one would typically assume that the best experimental estimator for the zero point value of the jth position asynchrony is the mean value of the asynchrony there; that is, $Z_j = \mu_j$, so that $A^*_{j,n} = A_{j,n} - \mu_j$, where μ_j is the mean asynchrony for position j in the cycle. Hence, the bias in production, which is the difference between mean actual and targeted positions, is $\mu_j - T_j$.

Under these conditions Equation 7 becomes:

$$A_{j+1,n} = A_{j,n} + (\mu_{j+1} - \mu_j) - \alpha g(A_{j,n}) - \beta g(A_{j-1,n}) + Q_{j,n}, \; 1 \leq j \leq L, \quad (9)$$

where $A_{j,n}$ is the asynchrony of the jth tap in the nth cycle measured in relation to the nth reference signal. Here, I have further assumed that the function G is expressible as a sum of terms from times j and $j - 1$ and that the error-correction function applying to these two times is the same. These assumptions can be relaxed at the cost of a more cumbersome formalism, but there is currently no information to suggest that such complications are necessary. Here, the parameters α and β are, respectively, first- and second-order error-correction (or control) parameters. They can be interpreted as measuring plasticity of short-term response. These should depend primarily on the individual participant, training, the sizes of the $(j, j + 1)$ and $(j - 1, j + 1)$ time intervals, and the nature of the task. This equation is the practical form of the DCE for timed human movement. Where this form needs to be specifically referred to, I call it the discrete temporal control equation, or DTCE.

For notational simplicity, the convention is used here that $A_{L+1,n} = A_{1,n+1}$ and $A_{0,n} = A_{L,n-1}$; in other words, the equation "wraps around" to the first point of the next cycle. To use this equation practically it is necessary to collate some empirical information about the form of the error-correction function g and the noise function $Q_{j,n}$, with respect to this class of tasks.

The noise function. In view of various experimental investigations of both discretely timed (Wing, 1980; Wing & Kristofferson, 1973a, 1973b) and continuous (Turvey, Schmidt, & Rosenblum, 1989) movement tasks, $Q_{j,n}$ contains noise that has zero (or nearly zero) autocorrelation for lags greater than one. That is, to a good approximation, one may write

$$\mathrm{Cov}(Q_{j,n}, Q_{k,n}) = \begin{cases} Var(Q_{j,n}) & j - k = 0 \\ \gamma_j(1) & |j - k| = 1 \\ 0 & |j - k| > 1. \end{cases} \quad (10)$$

In some cases, $\gamma_j(1)$ may also prove to be zero (Gentner, 1987; Vorberg & Wing, 1996), but for tapping experiments, it is nonzero—the noise is colored.

Such a distribution can be generally described by an at least two-level random noise structure. In the simplest cases, when taps and targets are synchronous, or when tapping occurs without an external reference, the standard Wing–Kristofferson noise formulation can be applied, as shown in Figure 4 (Vorberg & Wing, 1996; Wing & Kristofferson, 1973a, 1973b). In this approach, an internal model that is based on two processes is supposed: a recurring clock or timekeeper process, C_n, that triggers a motor process, M_n. Asynchronies, where tapping is referential, are measured as before. The two noise sources combine to produce a noise term with the proper covariance structure as follows:

$$Q_{j,n} = C_n - P + (M_{n+1} - M_n). \quad (11)$$

Here, P is the reference stream period, which is assumed to have negligible variance in the experimental design (this assumption can be relaxed; see below). Because in this case $L = 1$, j does not vary, and it has been dropped from the right-hand side.

This characterization of the two noise sources as clock and motor may not be fully apposite, and Ivry and Hazeltine (1995) have proposed the terminology *central* and *implementational* to highlight this fact. The essential point is that there are two processes: one, essentially white noise, with variance empirically found to be period dependent (C_n), and the other producing nonzero Lag 1 autocorrelation, with variance empirically found to be period independent (M_n; Ivry & Hazeltine, 1995; Pressing & Jolley-Rogers, 1997; Wing, 1980).

Heuer and coworkers (Heuer, 1988; Heuer, Schmidt, & Ghodsian, 1995; Spijkers & Heuer, 1995) developed a related two-level process formulation. In their models, the clock or central noise source for the jth interval C_n is replaced by $R_n\omega$, with ω being a prototypical duration and R_n being a stochastic rate parameter that governs all intervals of a given cycle. This has the effect of allowing ready examination of claims of invariant relative timing at the central level, a possibility associated with the generalized

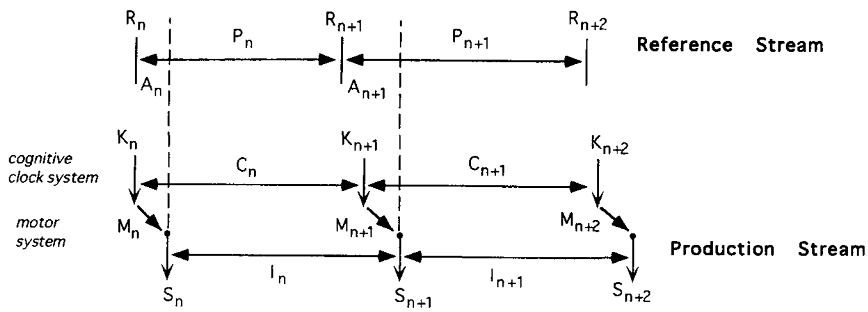

Figure 4. Referencing with an elaborated internal process (Vorberg & Wing, 1996; Wing & Kristofferson, 1973b). A stream of reference events (R stream) is used as a basis for the timing of a produced stream of motor events (S stream) by a two-tiered internal process comprising a Gaussian white-noise "clock" system, C_n, and a Gaussian white-noise "motor" system, M_n. The effect of the motor system noise is to create nonzero Lag 1 autocorrelation between the I_n. P_n = reference stream periods; A_n = *n*th asynchrony; K_n = cognitive clock events; M_n = motor delays; I_n = *n*th interval between events.

motor program approach (Heuer, 1988). It also allows a mechanism for nonzero covariances at lags that are greater than one within a cycle. Although I do not pursue this approach further here, it is perfectly consistent in principle with RBT, and its viability is a matter for empirical investigation.

In the general case, when one or more events per cycle are targeted to be nonsynchronous with the clock pulses, experimental work has shown that a separate subdivision process (called here B_n) is necessary for successful modeling (Jagacinski & Hah, 1988; Pressing, Summers, & Magill, 1996; Summers & Pressing, 1994; Vorberg & Hambuch, 1984). In the simplest case of offbeat tapping, the noise source becomes (Pressing, 1998c)

$$Q_{j,n} = C_n - P + (B_{n+1} - B_n) + (M_{n+1} - M_n). \tag{12}$$

The effect of this result is to decrease further the negative Lag 1 autocorrelation and, it is supposed, to undermine the period independence of the variance of this noise source. Figure 5 shows another example of this sort of internal event-generation process for a cycle with two events, using the notation of the DCE.

For this case the noise terms for the interonset intervals are, respectively,

$$Q_{1,n} = (B_n - \mu_{I_1}) + (M_{2,n} - M_{1,n}) \text{ and} \tag{13}$$

$$Q_{2,n} = (C_n - B_n - \mu_{I_2}) + (M_{1,n+1} - M_{2,n}). \tag{14}$$

Even with more complex patterns, there is no clear evidence of noise forms that require additional hypotheses, at least for tapping experiments (Pressing et al., 1996). This leads to the general hypothesis (Pressing, 1998a)

$$Q_{j,n} = \{C_n, P_n, B_n\}_j + (M_{j+1,n} - M_{j,n}), \tag{15}$$

where noise appears to be due to a cognitive clock process, C_n; a reference stream pulse, P_n; an optional clock subdivision process, B_n; and a motor-delay process, M_n (plus possible constant terms). In Equation 15, $\{\ \ \}_j$ is a linear combination (all coefficients either 1 or -1) of *C*s, *B*s, and *P*s that are structurally relevant to position *j*. Its specific form depends on the specific task and cognitive approach to it (Vorberg & Wing, 1996). The configuration of the motor-delay process variables always has the given form. In this formulation, the *M*s and *C*s are independent random variables (typically Gaussian white noise), and the *B*s are independent of all other variables except their enclosing *C*s, as described in Pressing et al. (1996).

The control/error-correction function. The control/error-correction function, *g*, is a priori unknown, but it must be congruent with several lines of evidence and theoretical considerations. Clearly, if the reference stream has negligible variance, as is the case in synchronized tapping to a standard electronic source ($P \equiv \bar{P}_n$), *g* must have periodicity *P*, and the equation must also operate modulo *P*; otherwise, it would not be consistent from cycle to cycle and could not represent a stationary time-series process.[5] Therefore, the function *g* can be expanded with complete generality in a Fourier series. The number of terms needed to describe it varies with the number of elements in the cycle, that is, *L*. As can be seen in Figure 6, which shows a two-harmonic case, each stable point (O) in the cycle must correspond to a point where the function *g* is zero and has positive slope. In other words, each positive-slope zero crossing point acts as an attractor; around each such point is a basin of attraction. Repeller points (X) occur at negative-slope zero crossing points.

Each such attractor point can also be represented as the bottom of a potential well, as in standard basic physics (e.g., Kelso, 1994b). The control/error-correction "force" derives from the potential energy well by the standard formula $F = -dV/dt$, although this is a formulation I do not pursue here. Each successive harmonic in *g* potentially introduces one additional independent stable point; evidently, one may need to consider a sum involving at least the *L*th harmonic for an *L*-element cycle.

Because there are strong empirical and theoretical grounds for

[5] In other words, if asynchronies become so great that the event they correspond to moves into the next *P* interval, then correction is made on the basis of the next reference pulse. It seems reasonable that an operator using error correction would use the most current value of information obtainable, and given the presence of feedforward, there should be a smooth transition across the cycle boundary. This assumption does not rule out the possibility of a central dead zone of no error compensation.

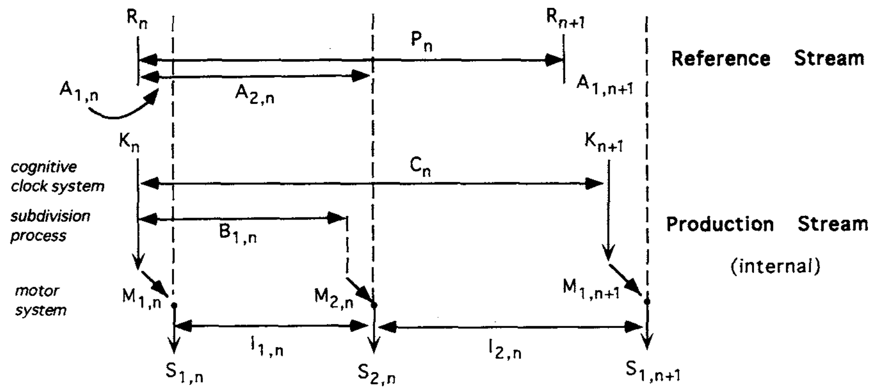

Figure 5. Referencing for a pattern with cyclic substructure, showing a more complex internal process. A stream of reference events (R stream) is used as a basis for the timing of a produced stream of motor events (S stream) by a three-component internal process comprising a Gaussian white-noise "clock" system, C_n; a Gaussian white-noise "motor" system, $M_{i,n}$ (creating nonzero Lag 1 autocorrelation as before); and a "subdivision" white-noise process, $B_{1,n}$ (in general there might be a number of these per cycle). The subdivision process may be correlated with its enclosing clock interval. P_n = *n*th reference stream interval; $A_{j,n}$ = asynchronies; K_n = cognitive clock events; $I_{j,n}$ = intervals between actions.

local linearity in error correction, as discussed above, for small deviations from target positions, g must satisfy

$$\alpha g(A_{j,n}) \approx \alpha_j(A_{j,n} - \mu_j), \tag{16}$$

for all j. This local linearity idea can be seen in Figure 7.

Given these considerations, there is a natural form for g:

$$\alpha g(A) = \frac{P}{2\pi} \sum_{m=1}^{m \geq L} a_m \sin[2\pi m(A + \phi_m)/P], \tag{17}$$

where a_ms are constants giving the contribution of each harmonic to the overall function and ϕ_ms are lags (phase shifts). I have chosen to use sine basis functions with distinct lags, rather than both sines and cosines, but the two are clearly equivalent. Sine functions not only are naturally suggested by the periodic boundary conditions but have extended zones of near-linearity around their zero crossing points.

The ϕ_ms are expected to be normally small lags that reflect two possible contributing source types. The first source type is based on relatively immutable effects corresponding to consistent delays (and compensations for them) in neuroanatomical transmission and processing, as have often been reported (Aschersleben & Prinz, 1995; Fraisse, 1982; Mates, 1994a). They can also reflect asymmetries in multilimb coordination (Stucchi & Viviani, 1993). These implementation delays are presumably the primary source of consistent bias in performance (Collier & Wright, 1995).

The second source type corresponds to adaptation to a particular set of task demands by learning. For example, particular external environmental forces may attempt to dictate optimal phase relations (cf. Schöner & Kelso, 1988b, 1988c), and in principle, it is possible that arbitrary lag relations that are not close to zero might be learned and lead to stable performance, as suggested by Zanone and Kelso (1994). However, available evidence seems to suggest that the arbitrary learning of phase relations that is not based on division of the time interval into a small number of roughly equal parts is very difficult (Summers, Todd, & Kim, 1993; Yamanishi, Kawato, & Suzuki, 1980). This is true even for trained musicians, which has led to the conclusion that there is a specialized rhythm production system for patterns of cognitive simplicity that is not available for complex patterns (Collier & Wright, 1995). In the framework of the current theory, attractor points cluster around positions that correspond to cognitive simplicity, which may be interpreted cognitively as the mental operation of subdividing a time interval into a small number of equal parts (Pressing et al., 1996) and, dynamically, as the truncation of the Fourier series of the control function with the first few terms (presuming quite constrained control of phase delays).

The possibility also exists that learning of phase delays could extend to voluntary on-line control, and this is suggested by small, controlled temporal shifts that sometimes appear in microanalysis of highly skilled musical performance (e.g., "swing" in jazz; Berliner, 1994; Shaffer, 1981). This has been researched very little, but the time scale of such shifting appears to be quite limited.

Is the given form for g unique? Certainly not. Another set of nonlinear basis functions that exhibits local linearity might do as well—for example, triangle wave functions. The two representations are computationally equivalent given sufficient terms in each. The question of which basis set forms a more efficient description, in the sense of requiring a minimum number of terms for accurate modeling, is an empirical one that has not really been addressed. Instead, Fourier forms have simply been assumed and shown to be compatible in a number of ways with experimental results (deGuzman & Kelso, 1991; Kelso, Scholz, & Schöner, 1986; Schöner & Kelso, 1988b).

Given this representation for g, what factors govern the values of the a_m? First, there are intrinsic dynamic effects that strongly suggest that a_1 and a_2 cannot be arbitrarily set on the basis of intention alone (e.g., Kelso, Holt, Rubin, & Kugler, 1981; Schöner & Kelso, 1988c), although the precise nature of this constraint is not clear. This suggests that some contribution to these coefficients comes from sources that are not subject to

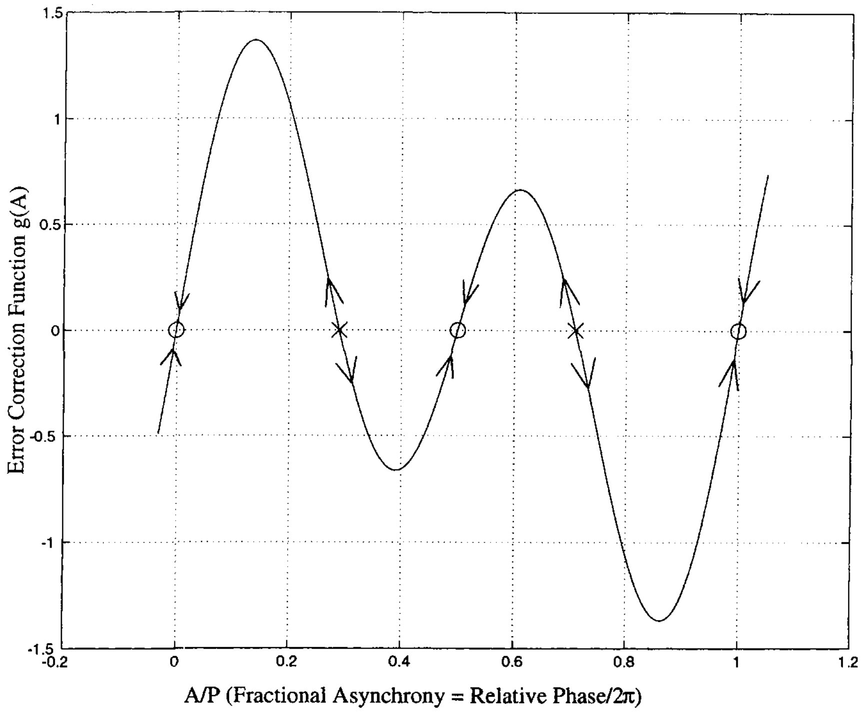

Figure 6. An example of the control/error-correction function $g(x)$. Attractor points occur at 0.000 and 0.500 (O) and repeller points at 0.290 and 0.710 (X). Here, $g(x) = \frac{1}{2}\sin(2\pi x) + \sin(4\pi x)$. A = asynchrony; P = period.

purely voluntary control by the participant, such as subcortical motor control systems like the spinal cord. Second, there are effects that are due to intention, attention, and previous learning. By adopting specific mental operations, practicing, invoking specific memories, and selectively focusing on certain motor subsystems and sets of sensory input, the performer can act to vary the a_m. Control of the a_m for $m > 2$ is, I propose, completely "volitional"; for $m \leq 2$, it is partly volitional and partly constrained by intrinsic system dynamics. This position is broadly consistent with previous work (e.g., Schöner & Kelso, 1988c; Turvey & Schmidt, 1994; Zanone & Kelso, 1994) but is distinguished by its emphasis on higher harmonics as providing natural potentials for learning. (The reason for the limitation of intrinsic system effects to $m \leq 2$, to indulge in speculation for a moment, may rest with evolutionary needs to coordinate two limbs in consistently forceful arrangements of unison or alternation.)

Therefore, the orientation here is toward a variable number of higher harmonics rather than truncation at $L = 2$, as in earlier approaches. This position can more readily accommodate the preferential use of simple rhythmic ratios in general temporal pattern production, as has been repeatedly found (Summers, Ford, & Todd, 1993; Yamanishi et al., 1980), allowing a natural connection with the history of musical notation and practice, although differences may exist here between continuous- and discrete-timed motor tasks. Thus, instead of having to suppose that a professional musician will have one control function that can characterize all possible rhythmic patterns he or she might ever be able to play, it seems more economical, and more in line with views of musical experts, to suggest that expertise consists of the ability to rapidly recall a range of control functions organized into generic classes, as may occur in musical sight reading. These control functions might correspond in informational terms to specialized cognitive modules or schemas, stored in memory. An ambitious attempt to take the opposite tack with polyrhythms is found in Haken, Peper, Beek, and Daffertshofer (1996), who used a continuous oscillator formulation with numerous nonlinear coupling terms to model

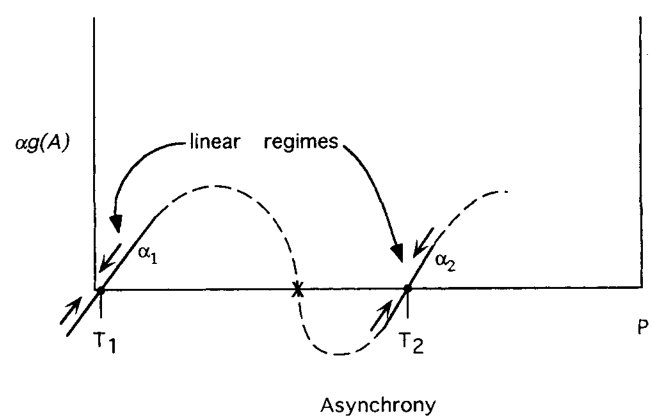

Figure 7. Locally linear regions of the control/error-correction function around attractor points T_1 and T_2. A = asynchrony; P = period.

polyrhythmic transitions. The approach has been successful in several cases and fully treats spatial variables but uses as many as 72(!) free parameters, which are given no cognitive interpretation. The tractability and generalizability of such complexity appear to be limited.

In general, the lags ϕ_m are related in a straightforward way to the means (μ_m). These relations, and relations between the alphas and *a*s, can be established by using the quasi-linearity of $g(A)$ around each of its attractor points, for each particular set of experimental conditions. Depending on the proposed number of harmonics in $g(A)$, the resultant equations may have a unique solution set, no solution, or multiple solutions. This is demonstrated by specific cases below.

In the linear regime of error correction, we can write the elaborated DTCE (using Equation 16 in Equation 9) as

$$A^*_{j+1,n} = (1 - \alpha_j)\, A^*_{j,n} - \beta_j A^*_{j-1,n} + Q_{j,n}, \qquad (18)$$

with wraparound applying to the subscripts, as before.

Selected Case Studies

I now examine a series of distinct examples and show how they are modeled from Equation 18 and why this formulation appears to offer advantages over alternative dynamic formulations and purely linear cognitive formulations. First, I demonstrate that the equation can readily apply to spontaneous (nonsynchronized) discrete motor behavior that is not based on error correction.

Spontaneous Isochronous Unimanual Tapping

In this case, there is no external reference pulse; tapping is self-driven. The reference stream reduces to a scale of measurement. Alpha and beta are zero, yielding from Equations 11 and 18,

$$A_{n+1} = A_n + (C_n - P) + (M_{n+1} - M_n), \qquad (19)$$

so that the interonset interval $I_n = A_{n+1} - A_n + P$ becomes

$$I_n = C_n + (M_{n+1} - M_n), \qquad (20)$$

which is the formulation of Wing and Kristofferson (1973a, 1973b).

Spontaneous Bimanual 4:3 Polyrhythm

For this bimanual polyrhythm, there are seven taps per cycle, four of which are produced with one hand and three with the other. This case has been investigated by Pressing et al. (1996) by using expert performers who were able to selectively apply different cognitive models in performance. The different cognitive models applied by the experts were based on distinct figure–ground relations that specified hand choice and performance intervals associated with the fundamental beat, corresponding to the use of different musical meters by the participants. In as much as error correction is not involved here, the different models are simply based on different structures for the noise components of the interonset intervals making up the musical pattern. For example, in accordance with Figure 8, a 3L4R* polyrhythmic model was found to be governed by the following equation set:

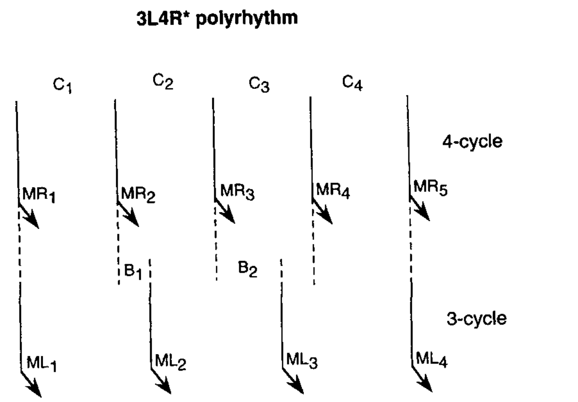

Figure 8. Cognitive model for a 3L4R* bimanual polyrhythm. C_j = clock intervals; MR_j = right-hand motor delays; B_j = subdivision intervals; ML_j = left-hand motor delays.

$$\begin{aligned}
I_0 &= \quad + (ML_1 - MR_1),\\
I_1 &= C_1 + (MR_2 - ML_1),\\
I_2 &= B_1 + (ML_2 - MR_2),\\
I_3 &= C_2 - B_1 + (MR_3 - ML_2),\\
I_4 &= B_2 + (ML_3 - MR_3),\\
I_5 &= C_3 - B_2 + (MR_4 - ML_3),\\
I_6 &= C_4 + (MR_5 - MR_4),
\end{aligned} \qquad (21)$$

where the Is are the successive intertap intervals in the polyrhythm. This equation set corresponds to a hierarchical bimanual model, with the cognitive ground linked to the right-hand taps, indicated by R*, and the left-hand taps (the figure) produced by a subdivision process (the Bs). Here the MLs are left-hand motor delays, the MRs are right-hand motor delays, and Cs and Bs are as described for Equation 15. The other polyrhythms examined were 3L*4R, 4L*3R, and 4L3R*.

Pressing et al. (1996) found that noise structures of this type could successfully discriminate between mental models used by the performers. Specifically, they showed, through comprehensive structural equation modeling, that models that are based on this structural design provide a very good fit of the full interonset covariance matrix for spontaneous (and synchronous) tapping, relative to a considerable variety of other linear models (i.e., uncorrelated, serial, parallel, and in the synchronous case, ground-reversed hierarchical). For further details, see Pressing et al. (1996).

Next, I examine cases that are based on error correction: synchronous tapping.

Synchronous Isochronous Unimanual Tapping

Because there is only one stable point in the cycle, near zero, the simplest assumption is that only the first harmonic is needed for *g*. The linearization condition then requires that

$$g(A) = \frac{P}{2\pi\alpha} a_1 \sin[2\pi(A + \phi)/P] \approx A - \mu \equiv A^*, \quad (22)$$

for $A - \phi$ small, yielding $a_1 = \alpha$ and $\phi = -\mu$; the phase lag is just the negative mean asynchrony. Hence,

$$g(A) = \frac{P}{2\pi} \sin(2\pi A^*/P). \quad (23)$$

The fundamental Equation 9 becomes in this case

$$A^*_{n+1} = A^*_n - \frac{\alpha P}{2\pi} \sin(2\pi A^*_n/P) - \frac{\beta P}{2\pi} \sin(2\pi A^*_{n-1}/P) + C_n - P + M_{n+1} - M_n, \quad (24)$$

where $A^*_n = A_n - \mu_A$. Provided the asynchronies are not too large, $\sin(x) \approx x$, yielding

$$A^*_{n+1} = (1 - \alpha)A^*_n - \beta A^*_{n-1} + (C_n - P) + (M_{n+1} - M_n). \quad (25)$$

(Note that the inclusion of higher harmonics in g will not change the form of this linearized equation.) This equation has been found to yield an excellent fit for both expert and nonexpert participants for a variety of values of P (Pressing & Jolley-Rogers, 1997), in both temporal and spectral domains. The authors there found that the first-order AR1 form (i.e., $\beta = 0$) was indicated for both nonexpert and expert performers for periods in the range $P \geq 175$ ms, which included nearly all cases. Only for the fastest speeds with $P \leq 150$ ms for the expert performer was a second-order AR2 equation preferentially indicated ($\beta \neq 0$). (The nonexpert could not play this fast.) This transition between the AR1 and AR2 forms of control is shown in Figure 9, which presents some results from Pressing and Jolley-Rogers (1997) for the expert performer operating at his four fastest speeds. The figure shows the power spectra of the tapping process and reveals an increasing peak at high frequencies as the period is reduced, indicative of an AR2 process.

Pressing and Jolley-Rogers (1997) rationalized the change to a second-order autoregressive process at shorter periods on the basis of known values for auditory reaction time of about 140 ms, a value that is consistent with recent estimates of minimum visual error-correction time (Glencross & Barrett, 1992) as little as 130 ms. Specifically, they found that as period drops within this range ($\leq$150 ms), information from the first asynchrony becomes progressively less reliable and is given progressively less weight ($1 - \alpha$ falls), while the coefficient of the second-to-last asynchrony is given progressively more weight ($-\beta$ rises).

Because Pressing and Jolley-Rogers (1997) collected a large amount of data, their results can also be used to attempt to compare the relative descriptive accuracy of the nonlinear (sine) and linear forms of the equation. This is done here using only the first-order autoregressive cases for both participants. When actual data are considered, the difference between the equations is found to be very slight in all cases, indicating that the production of stable patterns tends to use values of error-correction (control) parameters that set up close approximations to a linear process.

This can be illustrated by simulations. The left panel in Figure 10 shows the covariogram for a typical linear simulation, and the right panel in Figure 10 shows a typical nonlinear (single-sine) simulation, which are both generated using the same value of alpha. In each case, the best fit of the linear theory is also shown,

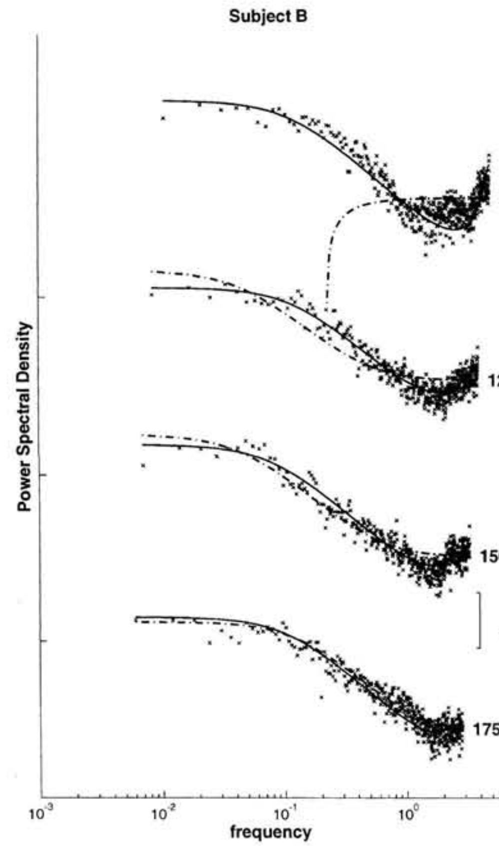

Figure 9. Asynchrony power spectral density plots for an expert musician in his fastest speed range of synchronous tapping. The fits at 100, 125, and 150 ms reveal a second-order autoregressive process (AR2); at 175 ms (and at slower speeds, which are not shown here), best fit is by a first-order autoregressive process (AR1). The distinguishing feature in the spectral domain between the two processes is the peak at high frequencies. AR2 fit = __; AR1 fit = _._. (data from Pressing & Jolley-Rogers, 1997).

using alpha calculated from the data from the method of Pressing and Jolley-Rogers (1997). The noise parameters here are those that correspond to the nonexpert's maximal asynchrony deviations: $P = 500$ ms, $\mu_C = 500$ ms, $\mu_M = 30$ ms, $\sigma_C = 21$ ms, and $\sigma_M = 4$ ms (800 trials). Such conditions should correspond to the maximum difference between sine and linear formulations in the experimental data, and indeed different values of alpha are estimated from the two data sets. However, the difference in fit between the two models is very slight; it is not even clear from inspection whether the linear theory fits the linear simulation better than it fits the nonlinear simulation. Such results are typical.

This finding can be partly understood from the nature of the sine function, which has a wide linear zone around zero. Using the standard series expansion $\sin x = x - x^3/3! + x^5/5! - \ldots$, the difference between the linear and sine functions for small x is based on neglect of the cubic term relative to the linear (first) term: $x^2/3! < 1$. For the left-hand side of this expression to reach even .05, one must have $x \geq 0.548$. Hence, to obtain a difference of even 5% between the linear and sine models, a majority of asynchronies of over $0.548/2\pi \rightarrow 8.7\%$ error from

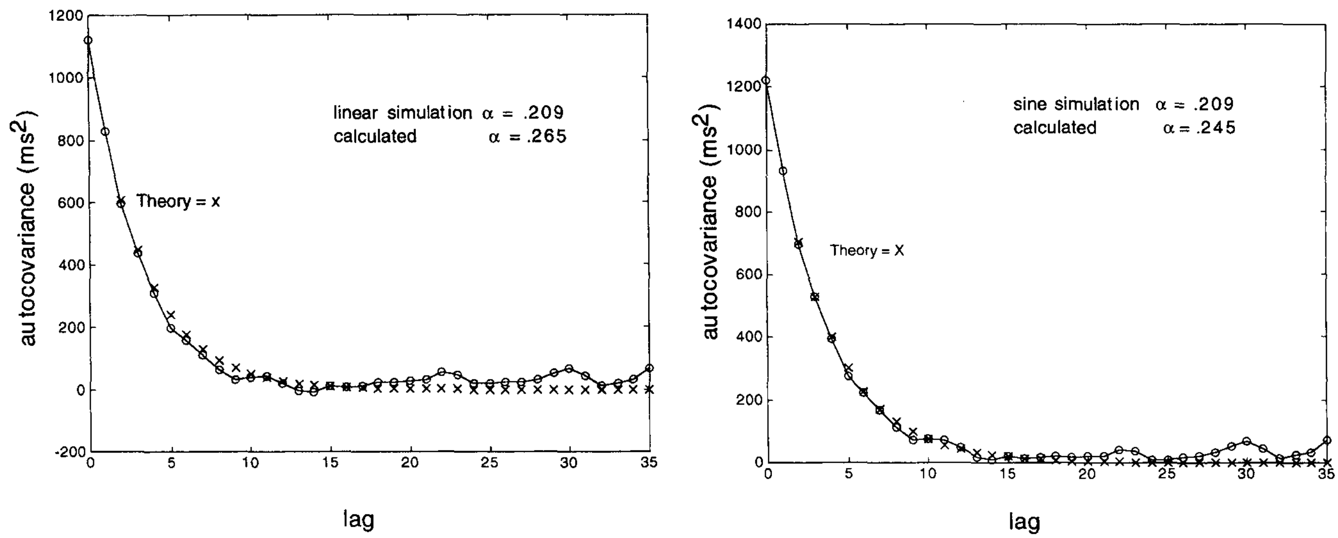

Figure 10. A comparison of covariograms of linear and nonlinear simulations. Left: Covariogram of a linear simulation ("data") of 800 trials (Os) compared with that of the best fitting linear model ("theory," Xs), using alpha calculated directly from the simulation data. Right: Covariogram of a nonlinear (sine) simulation (data) of 800 trials (Os) compared with that of the best fitting linear model with estimated parameters (Xs), with alpha calculated directly from the simulation data.

the target will be needed, in the complete absence of noise.[6] Errors for expert performers are typically less than this, and so the difference between models will be readily masked by noise. Nonexperts show larger deviations of asynchronies from their means, which should allow better discrimination between models, although this may be offset in part by greater masking that is due to greater noise.

A model discrimination test was constructed as follows, using the extensive data of Pressing and Jolley-Rogers (1997). Confining our attention to the majority of cases, where $\beta = 0$, the two alternative forms are linear (Equation 26) and nonlinear (sine; Equation 27):

$$A^*_{n+1} = (1 - \alpha)A^*_n + (C_n - P) + (M_{n+1} - M_n). \qquad (26)$$

$$A^*_{n+1} = A^*_n - \frac{\alpha P}{2\pi} \sin(2\pi A^*_n/P) + (C_n - P) + (M_{n+1} - M_n). \qquad (27)$$

Pressing and Jolley-Rogers (1997) estimated alpha in the linear equation using the analytically modeled covariogram. Such a procedure is not available for the nonlinear case, and so a different "local" procedure was developed for the two cases here, as follows (see Pressing, 1998a, for additional detail).

The procedure begins by placing all the A^*_ns, for $1 \leq n \leq N$, into bins arranged in increasing values. If the number of bins is B and the total range of the A^*_ns is $R = \max_n (A^*_n) - \min_n (A^*_n)$, then the width of each bin is $W = R/B$. Suppose that Bin b contains k_b A^*_ns: $A^*_{n_1}, A^*_{n_2}, \cdots, A^*_{n_{k_b}}$. Then, compute the average relative asynchrony in the bin as

$$[A^*]^{(b)} = \frac{1}{k_b} \sum_{i=1}^{k_b} A^*_{n_i}, \qquad (28)$$

where $[\;\;]^{(b)}$ denotes binned average. Next, compute the mean of the asynchronies that immediately follow the respective asynchronies in Bin b:

$$[A^*]^{(b,1)} = \frac{1}{k_b} \sum_{i=1}^{k_b} A^*_{n_i+1}. \qquad (29)$$

Then, consider a sum over all members of this same bin, applied to both sides of Equation 26. The noise terms will average to near zero, provided the number of bin members is large enough.[7] Hence, for each Bin b, there exists a linear estimate of alpha:

$$\alpha^{(b)}_{\text{lin}} \cong 1 - [A^*]^{(b,1)}/[A^*]^{(b)}. \qquad (30)$$

By averaging over all bins after discarding outliers (due primarily to cases where $[A^*]^{(b)}$ is near zero), an accurate estimate and a standard error can be attained. This was verified by extensive simulations in Pressing (1998b). It is similarly possible to apply the average over bins to the nonlinear Equation 27, from which a nonlinear (sine function) estimate of alpha can be obtained:

[6] It is not difficult to show that the region of quasi-linearity shrinks with increasing numbers of higher harmonics. For example, for the case of a pure nth harmonic g function, the 5% threshold differentiating linear and nonlinear forms can be shown to require an asynchrony fractional error of at least $0.548/n$. This can be interpreted as evidence of the reduced stability of more complex temporal patterns.

[7] In fact, the noise terms do not average precisely to zero, but a quite small residual bias effect remains in the estimation of alpha because of the nonzero Lag 1 autocorrelation effects of the motor delays, a bias that can be eliminated by systematic compensation (see Pressing, 1998a). However, this bias effect and its compensation are the same for both nonlinear and linear cases in zones of stability, and so, it does not significantly affect the investigation of relative fit of the two models here (Pressing, 1998a).

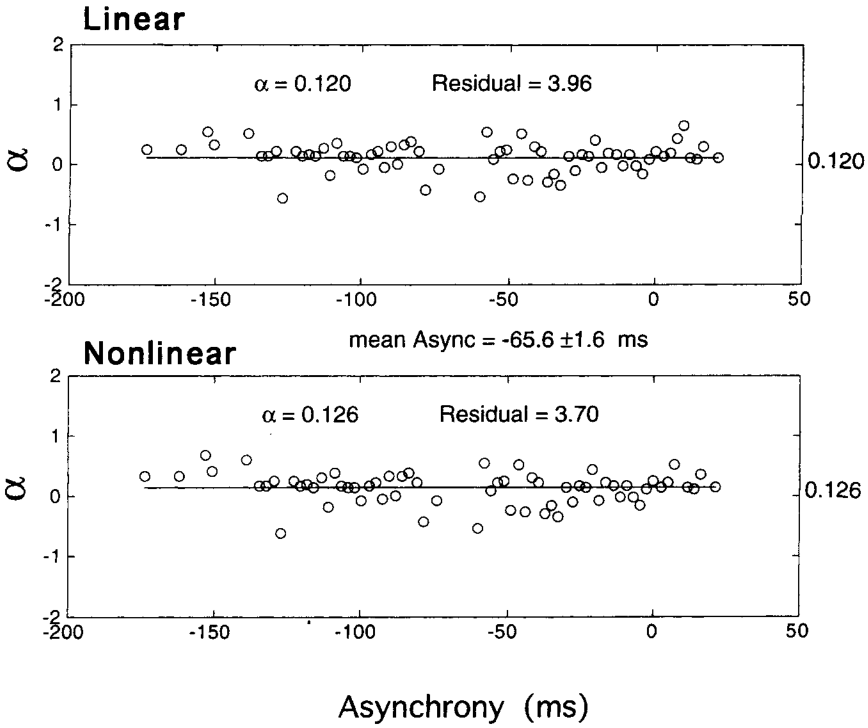

Figure 11. An example of estimates of alpha for synchronous tapping, using the method of bins. Top: based on the linear form of the fundamental equation (Equation 26). Bottom: based on the nonlinear (sine) form of the fundamental equation (Equation 27). Async = asynchrony.

$$\alpha_{\sin}^{(b)} \cong \frac{2\pi([A^*]^{(b)} - [A^*]^{(b,1)})}{\mathrm{P}[\sin(2\pi A^*/P)]^{(b)}}. \qquad (31)$$

This method of error-parameter estimation also has the advantage that it can be readily extended to patterns of arbitrary complexity (Pressing, 1998a). Typical results are as shown in Figure 11.

As with the covariograms, it is very difficult to tell the results of the two models apart by eye. Points that are more than four standard errors from the mean were excluded in computing residuals; this consistently excluded most points near $A_n^* = 0$, because the effects of small errors are amplified. This does not present a problem, as this region a priori provides very little model discrimination capacity. The results of binomial comparison for all first-order cases for both participants, using 80 bins, are shown in Table 1.

In terms of individual P groups, results did not show a significant difference between the two equations. However, overall, there was a significant difference in favor of the nonlinear equation (binomial $z = 2.41$, $p < .008$, one-tailed) for the nonexpert but only a tendency (in the same direction) toward significance for the expert (binomial $z = 1.26$, $p < .1038$, *ns*, one-tailed). A clearer effect was expected for the nonexpert participant, because his asynchronies were in general much larger than those of the expert.

Hence, there is evidence for the underlying nonlinearity of the control equation even in the stable regimes of simple isochronous tapping, although in practice the difference between the two forms is very small compared with system noise and only quantifiable with an extensive series of trials, as here, that entails over 20,000 taps per participant.

The effect of a dead-zone nonlinearity was also examined by simulations. When dead zones were 20 ms or less, using experimentally found parameters, there was little apparent difference between the normal and dead-zone covariograms. Because participants can sense auditory asynchronies as small as 1–2 ms under conditions paralleling those here (rapid tone onsets, as in Small & Campbell, 1962), dead zones may have a very limited effect on time-based control tasks.

Table 1
Assessment of Nonlinearity in Synchronous Tapping

Participant	Period (ms)	Fraction of runs with residual (sine) < residual (linear)
Nonexpert	250	5/8
	375	5/7
	500	7/8
	750	4/7
	1,000	6/9
Expert	250	3/8
	375	5/8
	500	5/8
	750	5/8
	1,000	6/8

These same data were also examined using techniques of nonlinear time-series analysis, primarily the correlation dimension, in an attempt to assess deterministic nonlinearity. Computation of the correlation dimension is one standard technique for assessing system dimensionality (Grassberger & Procaccia, 1983; Ott, 1993), which bears on the number of independent variables driving a system. With this method, a log–log plot of the so-called correlation integral versus length should have a straight-line region in which the slope is the correlation dimension. This slope is calculated for different embedding dimensions, and it should saturate to a stable value as the embedding dimension increases if the system is dominated by deterministic processes. When, in contrast, the system has significant contributions from random noise, there may be at best only weak linearity over a limited region, and there will be no saturation.

Given the model here, it would be predicted that high dimensional noise sources (the Q_n) should dominate calculations of dimensionality, because they are substantial in size and act on the same time scale as the error-correction process. This was borne out: Log–log plots of correlation integral versus length for asynchronies (using embedding lag of 1 or 3 steps) showed at best limited linear scaling regions, and up to embedding dimensions as high as 25 to 30, there was no trend toward saturation. This supports the idea that high dimensional random noise is significant in discrete human coordination systems, which is consistent with the assumptions of the motor program modeling approach and with previous dynamic dimensional analyses of rhythmic behavior in human movement (e.g., Kay, Saltzman, & Kelso, 1991). I also examined the Lyapunov exponent for these systems, an indicator that quantifies the extent of divergence over time of two initially very close points in phase space. These were weakly positive, with findings in the range of 0.02 and 0.10 in those cases examined, which suggests that the systems may be weakly chaotic. These findings should be considered tentative, as the data series, though long by the standards of psychology, are short by the standards of nonlinear time-series analysis (Gregson & Pressing, in press).

Dependence of Error Parameters and Noise on a Time Scale

The model presented here allows a natural interpretation of alpha as the effective fraction of information fed back for control—an index of short-term behavioral plasticity. Given this, the variation of alpha with time scale of tapping can be adumbrated from the following argument. In perception, information about the most recent event is registered at the sensory periphery and then must be transmitted to central nervous system areas for processing (possibly including decision making) and movement-production control. Both processing and transmission require time, and such latencies, despite feedforward processes, are known to decrease system stability. This, in conjunction with noise effects, suggests that a minimum value of error correction greater than zero may be necessary for stability. (A simple first-order treatment without delays predicts only that alpha must be in the range 0 to 2; cf. Vorberg & Wing, 1996.)

Above this possible threshold, a reasonable starting hypothesis appears to be that information transmission and processing proceed at a roughly constant rate until all information has been transmitted and processed. This assumption is made plausible by the roughly constant speed of transmission of action potentials in any given neural circuit (Carlson, 1994); the assumption is given more explicit support by the finding underlying the Hick–Hyman law (Hick, 1952; Hyman, 1953; Luce, 1986): that reaction time in a choice reaction task is generally a linear function of amount of information, some exceptions under special conditions notwithstanding. On this basis, a linear relation between alpha and period is expected, up to a certain point; thereafter, the curve should plateau because of saturation (all the information has been transferred and processed). (The decay of short-term memory suggests that this plateau level might subsequently slowly decay with further increases in period, with the rate depending on various conditions, but this effect has not so far been experimentally probed.)

In other words, above a minimum threshold and below a maximum threshold, there should be a linear zone where $\alpha(P) \approx kP + D$, with k and D being constants and P being the time period. It is to be expected that the lower threshold will be reached in the range of 130–200 ms, as this corresponds to the typical range from auditory reaction thresholds and Hick–Hyman law zero intercepts (Glencross & Barrett, 1992; Schmidt, 1988). No a priori value for the upper boundary is evident, although it would be expected to be clearly less than the auditory sensory store duration, which is commonly considered to be at least 2 s (e.g., Proctor & Van Zandt, 1994), and it may depend on experience and individual differences. The clearest precedent for this boundary value may be in Rosenbaum and Patashnik (1980), who found a plateau in reaction times in a related task that required production of temporal durations defined by two successive taps, with feedback given on interval accuracy after each pair of taps. Their plateau set in for time intervals above about 750 ms. It is further to be suspected that this proposed zone of linearity for alpha may not necessarily occur for sufficiently complex patterns, especially bimanual ones, because such cases might involve interleaved transmission and processing of different sets of information, and this might yield nonlinear and case-specific effects.

Available data for on-beat tapping show promising agreement with these ideas (more complex patterns are discussed further in a later section). First, a minimum error-correction threshold in the range of 0.15–0.20 (achieved at short periods) has been observed by Pressing and Jolley-Rogers (1997) in relation to the single first-order error-correction parameter α and also as the sum of the two error-correction parameters ($\alpha_2 + \beta$) for the second-order autoregressive case. Second, as seen in Figure 12, a plot of alpha versus period is close to linear for both participants used by Pressing and Jolley-Rogers for data below 800 ms. These plots use data from Pressing and Jolley-Rogers and also additional runs collected at 625 ms and 875 ms for the same expert participant. Alpha–period size correlations (r^2) for the expert and nonexpert participants are, respectively, .972 and .997 for the range below 800 ms. Above 800 ms linearity fails, in line with the discussion above. Only four points are available for the nonexpert in this range because although further trials were run with him at 625 ms and 875 ms, they occurred after many intervening runs of tapping practice in other experiments, and a clear learning effect was found: His clock variances were consistently reduced, and his error parameters clearly increased over those of the previous sets of runs. In short, the nonexpert was developing expertise. The expert was not affected in this way, presumably because he had automatized and stabilized his production system well before the start of the experiments. For the expert, the alpha intercept (D) was very close to zero. For the non-

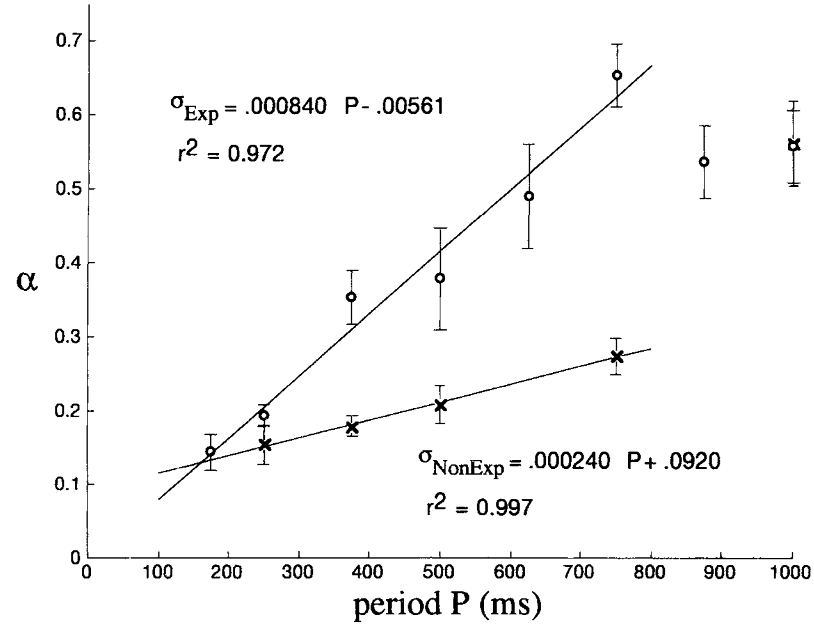

Figure 12. Error-correction parameter alpha (α) versus period (P) for the expert and nonexpert participants of Pressing and Jolley-Rogers (1997), with additional data for the expert participant. Straight lines are fit to points below 800 ms. Vertical lines depict $\pm$ 1 *SE*. NonExp = nonexpert; Exp = expert.

expert, the straight line fit was good and the intercept *D* was .092, distinctly nonzero. Further measurements would be useful here to confirm linearity and evaluate *D* for other participants.

Another testable linearity is that between the standard deviation of interval production and period. Such linearity is a form of Weber's law and allows comparison with time-perception studies, where such linearity has been previously observed in time estimation over the range 100–2,000 ms for humans and has also been shown to hold for rats and pigeons (for a review, see Ivry & Hazeltine, 1995). Because the natural variable for averaging across runs is variance, it is easier, and closely equivalent, to examine production variance by examining the linearity of a plot of clock variance versus (period)^2. Empirical support for this is strong, as seen in Figure 13, where the correlations (r^2) for expert and nonexpert between-clock variance and P^2 are, respectively, .998 and .933, again using the data of Pressing and Jolley-Rogers (1997).

This is in accord with other recent results (Grondin, 1992; Ivry & Hazeltine, 1995). A recent reanalysis of the best known excep-

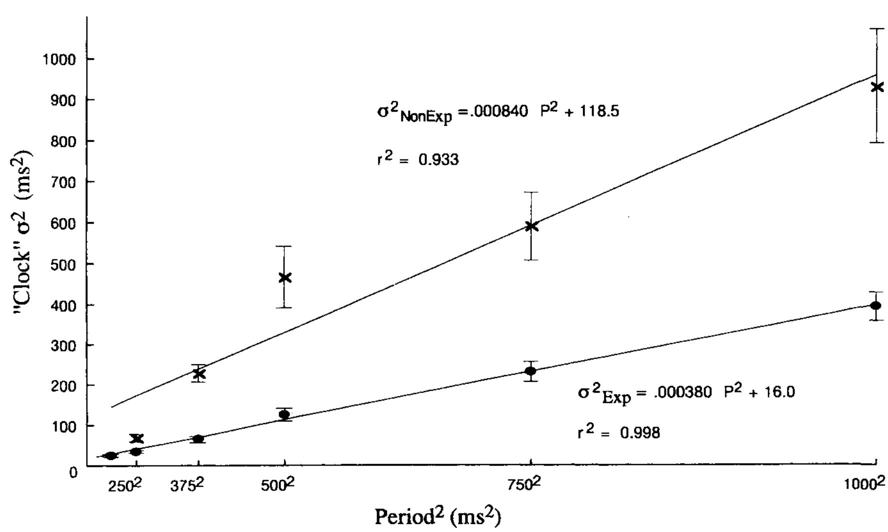

Figure 13. Clock motor variance versus period^2 (P^2) for the participants of Pressing and Jolley-Rogers (1997). Vertical lines depict $\pm$ 1 *SE*. NonExp = nonexpert; Exp = expert.

tion to this case, the data of Wing (1980), which found a linear relation between noise variance and period, showed that it too can be interpreted as supporting a Weber's law–type relation (Ivry & Hazeltine, 1995). In the experimental work here, comparative linear fits of variance to period yielded credible values of r^2 (expert) = .974, intercept = -78 ms^2; r^2 (nonexpert) = .971, intercept = -180 ms^2. However, in both variance–period plots, the $P = 0$ intercept for the linear period–variance form is clearly negative, which is an unphysical result; this problem was not found in the Weber's law-based fit. The preferential conclusion is that Weber's law, supporting a linear relation between "clock" standard deviation and period, has strong experimental support for the case of error-corrective temporal production over the range 200–1,000 ms.

Derivation of Circle-Map Behavior

The next logical example in this series of examples might be the case of bimanual synchronous and asynchronous coordination, where considerable elegant experimental work has shown the presence of phase transitions between antiphase and inphase motion, under a variety of experimental conditions (e.g., Kelso et al., 1981, 1986; Turvey & Schmidt, 1994).

Such phase transitions (and those of other cases) have been modeled in several ways, all featuring essential nonlinearities: by the use of second-order continuous equations of nonlinearly coupled oscillators (e.g., Haken et al., 1985, 1996), by the use of discrete circle maps (e.g., deGuzman & Kelso, 1991; Glass & Mackey, 1988; R. C. Schmidt, Beek, Treffner, & Turvey, 1991), and by first-order continuous equations based on relative phase that have a control structure analogous to that of the circle maps (e.g., Haken et al., 1985; Turvey & Schmidt, 1994).

Because the formulation here is discrete, I address this issue initially by showing the connection between the RBT and circle-map approaches. It turns out that the fundamental equation here (DCE) contains, as special cases, various forms of the circle map widely used in traditional dynamical modeling. After a demonstration of this and a discussion of implications, I compare the circle-map approach with the continuous approaches cited above and further case studies, which includes a discussion of stability.

Consider then a situation where the asynchronies need not be small, so that the general nonlinear form of the equation applies. Let us also assume that for the process under consideration, error correction is confined to first-order effects, as is frequently found. In this section, I also return to the more general case where the pulse stream may have nonzero variance. Under these conditions, the DTCE becomes, from Equation 9,

$$A_{n+1} = A_n - \alpha g(A_n) + (C_n - P_n) + (M_{n+1} - M_n). \quad (32)$$

I have used the form of the noise term appropriate for simple isochronous tapping, but the same derivation proceeds with minor modifications for other forms. There are several cases in which this equation can be shown to reduce to the circle map with an added noise term.

Case 1: Negligible Reference Stream Variance

Consider first the case in which the reference stream has negligible variance, so that $P_n = P$, as earlier. From the previous properties of g,

$$g(A_n + P) = g(A_n) \pmod{P}. \quad (33)$$

Then set $\theta_n = A_n/P$. This variable is just the discrepancy between actual behavior and synchronizing pulse, normalized by the period; in the language of oscillatory dynamics, this is a relative phase. Then, if we define $h(\theta_n) \equiv g(A_n)/P$, it follows that $h(\theta_n) = h(\theta_n + 1) \pmod 1$, because $h(\theta_n + 1) = g[P(\theta_n + 1)]/P = g(A_n + P)/P = g(A_n)/P \pmod P = h(\theta_n) \pmod 1$. Then, dividing Equation 32 by P, and substituting, yields

$$\theta_{n+1} = \theta_n + \Omega - \alpha h(\theta_n) + \xi_n \pmod 1, \quad (34)$$

$$\text{where } \Omega = (\mu_C - P)/P \text{ and} \quad (35a)$$

$$\xi_n = \frac{1}{P}[(C_n - \mu_C) + (M_{n+1} - M_n)], \quad (35b)$$

as may be verified by direct substitution. If Equation 34 is interpreted mod 1, Ω may be replaced with $\Omega + 1$ (or omega plus any other integer) without affecting the character of the solution, that is, $\Omega \rightarrow \mu_C/P$. (This ambiguity of definition is a well-documented aspect of this type of equation; Jackson, 1989). Thus, Ω acts as a ratio between eigenperiods of the two primary timing streams, clock (μ_C) and reference (P), in the absence of coupling (error correction) and noise. The noise term, ξ_n, has expectation zero and a nonzero Lag 1 autocorrelation structure, as previously.

Equation 34 is thus just the familiar circle map (Treffner & Turvey, 1993) with an added mean zero noise term of ξ_n, with otherwise familiar (if not exactly identical) interpretations for all variables.[8] The noise term often has been introduced in continuous differential analogues of the circle map, with considerable success. In the work here, I am able to identify the noise source in terms of constituent clock and motor random variables and show that it may have a nonzero Lag 1 correlation, which represents a generalization of the more common assumption that ξ_n is uncorrelated white noise. (As stated before, the specific form of the noise term may change with task conditions, and I have used one specific form here for illustration.) The effect of the noise term is to appreciably reduce stability of limit cycle solutions of the equation. For low values of noise, the regime diagram for subcritical parameter values for this equation will be very similar to that of the ordinary circle map but with reduced basins of attraction.

Case 2: Nonnegligible Reference Stream Variance

If the reference stream variance is not negligible, an additional consideration emerges. How should the asynchrony error-correction process change with reference interval size? If the variation in P_n is modest, as is typically the case—because rhythmic behaviors are often predicated on a steady pulse—it might be supposed that error correction occurs relative to the mean reference interval $\bar{P}$. In this case, Equation 34 is attained again, with the substitution of $\bar{P}$ for P. That is, with $\Omega = (\mu_C - \bar{P})/\bar{P}$, and

$$\xi_n = \frac{1}{\bar{P}}[(C_n - \mu_C) + (\bar{P} - P_n) + (M_{n+1} - M_n)]. \quad (36)$$

[8] Note though that the genesis of the motor-domain circle map has been typically through the Poincaré section of a continuous differential equation (Treffner & Turvey, 1993), whereas the DCE-based form comes directly from discrete-control processes.

Because the reference stream now displays its own timing structure, this equation can also be considered to apply to bimanual coordination, inasmuch as the asynchrony now corresponds to a relative phase between the reference (sometimes called *driver*) and follower behaviors. In this case, the experimental protocols include spontaneous bimanual coordination, without external synchronization.

In the most general case, where P_n varies widely or coupling between the two streams is asymmetrical, the DTCE does not in general scale to produce the noisy circle map. This suggests that the experimental condition of a noisy reference stream would bring useful insights into the issues here. So far, work on the noisy metronome problem (Schulze, 1992) has not addressed these particular concerns.

Case 3: Relative Error Correction

There is another condition under which a circle-map form might emerge. This arises from a suggestion that when P_n varies markedly, error-correction effects might be scaled by the length of the last reference interval. Such relational perceptual sensitivity is a form of the familiar Weber fraction law, and as noted earlier, this is experimentally supported for temporal estimation and production. A similar Weber effect for ballistic movement occurs in the spatial domain, usually labeled Craik's ratio rule (Poulton, 1981).

If one assumes that under such conditions it is relative error correction that is the appropriate process, then the main equation might become

$$A_{n+1}/P_n = A_n/P_{n-1} - \alpha h(A_n/P_{n-1}) + (C_n - P_n)/P_n + (M_{n+1} - M_n)/P_n, \quad (37)$$

where noise terms have been scaled by local period size to obtain correct dimensions, and so the equation reduces to Equation 32 when P_n does not vary. This again yields

$$\theta_{n+1} = \theta_n + \Omega - \alpha h(\theta_n) + \xi_n \pmod 1, \quad (38)$$

provided the defining variables are

$$\theta_n = A_n/P_{n-1},\ \Omega = (\mu_C - \bar{P})/\bar{P}, \text{ and}$$

$$\xi_n = \frac{1}{P_n}[C_n + (M_{n+1} - M_n)] - \frac{\mu_C}{\bar{P}}, \quad (39)$$

as may be verified by direct substitution.

The Sine Circle-Map Class

The circle map is a class of maps, and to produce specific members of the class that pertain to certain conditions, one needs the precise map (control) function. In all of the above cases, if g has a single sine term ($L = 1$, Equation 23), the sine circle map is obtained (with added noise term):

$$\theta_{n+1} = \theta_n + \Omega - \frac{\alpha}{2\pi}\sin(2\pi\theta_n) + \xi_n. \quad (40)$$

For $L = 2$, with zero phase lags, from Equations 17 and 38,

$$\theta_{n+1} = \theta_n + \Omega - \frac{\alpha}{2\pi}\sin(2\pi\theta_n) - \frac{\alpha^*}{2\pi}\sin(4\pi\theta_n) + \xi_n, \quad (41)$$

which is equivalent to the so-called phase-attractive circle map (with added noise; deGuzman & Kelso, 1991). These are two standard map forms used to model dynamic coordination, and hence I have shown that the DCE contains the motor circle-map models as special cases, as claimed above, including what are termed symmetrical and asymmetrical coupling terms. Because Equations 40 and 41 have been well documented as producing experimentally substantiated nonlinear phenomena like bistability, subcritical phase locking, entrainment, critical slowing, phase transitions, and certain appropriate general forms of regime diagrams, as described earlier, I have verified that the DCE can also readily model such effects.

Multilimb applications of these derivations can proceed in several ways. If two limbs coordinate relative to each other, then, as discussed in Case 2 above, Equation 40 or 41 can be interpreted with $\theta_n = \theta_n^{(1)} - \theta_n^{(2)}$, a relative phase variable. There is experimental evidence supporting the fundamental similarity of bimanual coordination between limbs and unimanual coordination to a driving signal (Treffner & Turvey, 1993). If two limbs coordinate relative to a separate reference stream, then the equations for relative phase will have a different form. The subtraction of sine terms for the two limbs can be rearranged, in the case of symmetrical coupling, to yield frequency terms beginning with $\sin(\pi[\theta_n^{(1)} - \theta_n^{(2)}])$, a first subharmonic.

Because Equation 40 is identical in form with traditional sine circle-map accounts of motor coordination (provided the noise term is ignored), these derivations suggest that the coupling constant K found in such formulations and the error-correction parameter alpha used here are one and the same, provided additive noise levels are low enough (and the other assumptions of the derivation hold experimentally). If this is so, then coupling constants of such dynamic systems (and hence their zones of instability) might be assessed by simple tapping experiments carried out in regions of behavioral stability. Furthermore, the interpretation of K as an error-correction parameter allows process-based derivations of its value, as discussed above. These conclusions appear to be broadly consistent with the suggestions of dynamical modelers, who have found that K (here alpha) should vary inversely with the fundamental frequency of oscillation, ω_c, in an oscillatory system (Sternad, Turvey, & Schmidt, 1992; Turvey & Schmidt, 1994). From the current perspective, because the inverse of the fundamental frequency is just the period, the previous discussion about the linearity of alpha with period yields the following:

$$K \equiv \alpha(P) \approx kP + D = k\omega_c^{-1} + D. \quad (42)$$

The constant D may not always be zero, as assumed by Turvey and Schmidt (1994). As seen above for 1:1 tapping, for the expert participant for whom the data are clearly the most accurate, D was close to zero, but for the nonexpert, it was nonzero. Further work with other participants is required to assess the universality of the claim that $D = 0$. In any case, the above findings on alpha suggest that linearity of K may hold only for a limited frequency domain of 1.2–6.5 Hz.

If the assumptions required to yield the traditional circle-map formulations are relaxed, the general DCE approach to nonlinear discrete control displays additional complexities: Additional second-order terms (beta nonzero) may appear, various phase delays may be nonzero, second and higher harmonics can occur, and the effects of task-specific structured noise arise.[9] A period-based variable such as phase is not necessarily fundamental. A full treatment of the quantitative differences between various nonlinear forms of the DCE and the simple sine circle maps would take this discussion too far afield. However, the issue of stability is further treated below in the sections on 2:1 and 3:1 tapping. Furthermore, the use of phase delays and second-order control has been experimentally demonstrated in the linear domain by several recent publications, and these results form a foundation for further evaluation, as discussed below.

Limitations of the Sine Circle Map

Although the formulation here can yield the sine circle map used in standard dynamical motor modeling in a principled way, and I have taken this as evidence that the current approach can model phase transitions and generally act in support of previous experimental findings about essential behavioral instabilities, I believe the current DCE approach is both more general and more powerful than the simple map approach. Why? Because, in my view, there are at least five reasons why the application of the simple circle map (or the phase-attractive circle map; deGuzman & Kelso, 1991) to general movement patterns can be inadequate, all of which appear circumventable with the integrative stance used here.

First, the circle map does not incorporate the effects of task-specific, high-dimensional noise sources (as here). Second, it does not accommodate general phase lags of the kind modeled here, which go beyond simple symmetry breaking and for which there is good experimental support (see below). Third, the circle map does not accommodate second-order error-correction processes, which have been experimentally found in some cases (Pressing, 1998a; Pressing & Jolley-Rogers, 1997). Fourth, the circle map basically generates patterns that are based on pulse trains of the form of $n:m$ and appears unable to accommodate patterns of arbitrary temporal design (e.g., most musical rhythms). In contrast, the current approach can, in a principled way, accommodate any temporal pattern (Pressing, 1998a, 1998c; also see Equation 9), at the same time finding a natural emphasis on those patterns that are consistent with cognitive traditions in music timing (subdivision and concatenation of time intervals to produce hierarchical temporal relations). There is also a fifth issue, which is based on pattern stability.

Pattern Stability

In the map-based dynamical formulations of movement control, stability has often been assessed in relation to the properties of the regime diagrams of the sine circle map. This has involved the invocation of the concept of the Farey tree, which can be interpreted to indicate relative stabilities of different polyrhythmic patterns, for which there is some experimental support (Treffner & Turvey, 1993).

However, consistent deviations from the Farey-based transition patterns have also been experimentally reported (Peper, 1995; Zanone & Kelso, 1992). These have sometimes been interpreted as stemming from learning (Zanone & Kelso, 1992); yet, other experiments have not found significant effects of practice on polyrhythm transition routes (Peper, 1995). One explanation of the prevalence of non-Farey transition routes is that the circle-map equation is operating in the supercritical regime: $K > 1$, when $L = 1$ (Peper, 1995).

However, in view of the RBT approach, it seems more likely that the Farey tree simply has limited general relevance for polyrhythmic transitions, because alternate forms of the circle-map class—for example, the phase-attractive circle map (deGuzman & Kelso, 1991)—or those based on control functions with extra harmonics or second-order terms (as found in the DCE) have different regime diagrams that do not lead to Farey-type predictions about pattern stabilities and transitions (deGuzman & Kelso, 1991; Pressing, 1995). Furthermore, the effects of stochastic noise must be addressed, in general, and even small amounts of noise can cause substantial changes to the noise-free regime diagram, as simulations readily show.

Moreover, the traditional Farey-based approach makes only simple disjunctive predictions (e.g., moving from Polyrhythm A to Polyrhythm B rather than to C as a result of accelerating tempo). Pressing (1995) carried the traditional approach a step further by looking at quantitative correlations between various novel dynamic indicators and empirical values for state stabilities and interstate transition probabilities, using bimanual pendular data from Treffner and Turvey (1993) and comparing these correlations with comparable ones made from cognitive indicators. The results were that a number of his dynamic and cognitive indicators were strongly correlated in predicted ways with the experimental findings of stability and interpattern transition rates. Empirically, cognitive and extended dynamical approaches were equally effective.

However, the situation changes markedly when one moves beyond polyrhythms to consider temporal patterns in general. Here, Farey or traditional dynamical logic does not obviously transfer, whereas a more explicitly cognitive approach does, as I show below. The explicit involvement of higher order cognition also allows a connection with more adaptive and extended phenomena than repetitive pattern production, such as real-time reading, speaking, musical sight-reading, and various forms of improvised behavior (Pressing, 1988), as discussed earlier.

Cognitive Principles Underlying Phase Transitions and Instabilities

In the cognitive formulation given here, there need be no universal nonlinear form for control for a given individual, from which universal considerations of pattern stability emerge. Although multiple stable states can certainly emerge from a single control function, the more general phenomenon is that different control strategies can be selectively recalled from memory and adapted on-line, in response to variations in task conditions, providing behavioral richness and flexibility.

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[9] For example, the full extension of Equation 41 to second order with two harmonics and arbitrary phase lags (φs) would yield

$$\theta_{n+1} = \theta_n + \Omega - \frac{\alpha}{2\pi}\sin[2\pi(\theta_n + \varphi_1)] - \frac{\alpha^*}{2\pi}\sin[4\pi(\theta_n + \varphi_2)] - \frac{\beta}{2\pi}\sin[2\pi(\theta_{n-1} + \varphi_3)] - \frac{\beta^*}{2\pi}\sin[4\pi(\theta_{n-1} + \varphi_4)] + \xi_n.$$

Under this view, the principles underlying pattern stability and interpattern transitions used by Pressing (1995) for polyrhythms can be generalized on the basis of subsequent theoretical (Pressing, 1998b) and experimental work (B. Williams & Pressing, 1999) as follows. Consider a situation where a performer is playing a recurring temporal pattern and experimental manipulations (e.g., acceleration of tempo) or internal changes of state (e.g., faltering attention) lead to different pattern outcomes. These changes can be viewed as bifurcations or phase transitions that arise because of essential nonlinearities in control. The suggested principles underlying pattern stability and interpattern transitions are:

1. Patterns (and their associated control functions) will be recalled on the basis of their accessibility in memory (+), fit to context (+), and cost of production (−).

2. Pattern stability will be inversely related to costs of production (−) and directly related to attention (+).

3. Transitions between patterns will be based on the "distance" between them (−), their relative production costs (−), predilections for transitions built into the experimental manipulations (+), and performer intentions (+).

(+) and (−) respectively indicate expected direct and inverse relations between the indicated variables.

Two essential considerations for the use of RBT with respect to these hypotheses are how to compute interpattern distance and how to evaluate production cost. In line with the nature of goal-directed action, I assume that interpattern distance should be computed from behaviorally meaningful dynamical variables that reflect task constraints and control strategies. For example, with a pattern made of two time intervals, one cognitively significant action variable would be interval ratio. As another example, Pressing (1995) showed that the incidence of transitions between polyrhythms showed highly significant negative correlation with interpattern distance, as measured by $|n/m - p/q|$, where transition is from an n:m to a p:q polyrhythm. This provides support for the first part of Hypothesis 3.

Cost of production is interpreted here to be directly related to the contextual complexity of a pattern. In identifying cost with complexity, I am using a central idea of algorithmic information theory (e.g., J. F. Traub, Wozniakowski, & Wasilowski, 1988). Now, there are many ways to define complexity operationally: One can count levels of hierarchical organization, compute numbers of free parameters in a dynamic model, calculate dimensional or entropic or predictability indicators, ask for the evaluations of domain-specific experts, or measure pattern learning difficulty, among others (Pressing, 1998c).

In the DCE approach, complexity of repeating patterns might be assessed on the basis of the structure of the control function: Patterns requiring more harmonic terms in the control function (in terms of subjective report by experts, this involves finer mental subdivision) require simultaneous maintenance of more system control parameters (weights and phase lags) in the correct ranges, with possibly narrower bounds of acceptability, and hence are more difficult to achieve and to stabilize.

This provides a rationale for the well-established human predilection to produce patterns that are based on simple harmonic ratios (Fraisse, 1982). It is further supported by Summers and Kennedy (1992) and Summers and Pressing (1994), who showed that performers who had difficulty with polyrhythmic performance tasks distorted the control structures by simplifying them, changing required mental subdivisions from three parts to two, and placing taps in the resulting nearest locations. Similar "decomplexifying" distortions were found by Summers, Ford, and Todd (1993), where patterns requiring mental subdivisions of three or four parts were simplified to two parts. These results provide support for Hypotheses 1 and 3 above.

Explicit support for Hypothesis 2 above was found by Pressing (1995) in analysis of Treffner and Turvey's (1993) polyrhythmic patterns. For example, one successful index of pattern complexity (hence cognitive cost) of an $n{:}m$ polyrhythm was $n + m$.

A procedure for computing cognitive cost of arbitrary temporal patterns was described by Pressing (1998b), who made the point that temporal pattern complexity is closely bound with the concept of syncopation, in musical terms. He showed how syncopation (degree of off-beat temporal emphasis) can be quantified by a systematic pattern-classification process embodied in the computer program Transcribe (Pressing, 1998b; Pressing & Lawrence, 1993), that is, based on five levels of syncopative complexity. Experimental work (B. Williams & Pressing, 1999) suggests that interpattern transitions for many kinds of patterns are well-handled by this syncopation-based model of temporal pattern complexity. These results are naturally presented in cognitive terms by use of symbolic dynamics, whereby symbols for different rhythmic patterns are defined by 1–1 correspondence between labels and specific regions of timing-phase space, yielding sequences of state symbols. For example, a transition from State *a* to *f* might proceed through the symbol string . . . *aaaacaaefaffafffff* . . . .

I now return to an examination of explicit mathematical modeling of two modestly complex patterns, to elaborate the ways in which the DCE approach examines stability and instability when a single control function applies.

2:1 Tapping

The central equation given in Equation 9 can be applied here by a characterization of the error and noise functions. For two taps per cycle, with $L = 2$, the approach suggests a control function of the following form (see Equation 17):

$$\begin{aligned}\alpha g(A_{j,n}) &= \frac{P}{2\pi}\{a_1 \sin[2\pi(A_{j,n} + \phi_1)/P] + a_2 \sin[4\pi(A_{j,n} + \phi_2)/P]\} \\ &\approx \alpha_j(A_{j,n} - \mu_j),\end{aligned} \tag{43}$$

for $j = 1, 2$, where the linear approximation relation holds for values of $A_{j,n}$ near the respective attractor points μ_1 and μ_2. Here, I have assumed that the process is first order. By expanding the sine functions in their linear regions, for stable points near zero and $P/2$, and matching the coefficients of the asynchronies and the constant terms at both points, it can be readily shown that

$$a_1 = \frac{\alpha_1 - \alpha_2}{2};$$

$$a_2 = \frac{\alpha_1 + \alpha_2}{4}. \tag{44}$$

The same linear expansion and matching also produces relations for the lags:

$$\phi_1 = -\frac{\alpha_1\mu_1 - \alpha_2(\mu_2 - P/2)}{\alpha_1 - \alpha_2};$$

$$\phi_2 = -\frac{\alpha_1\mu_1 + \alpha_2(\mu_2 - P/2)}{\alpha_1 + \alpha_2}. \quad (45)$$

An extensive treatment of the linear form of this case is given in Pressing (1998a). Experimental work is reported there for a number of 2:1 cases with variation across three independent variables: expert–nonexpert participant, unimanual–bimanual condition, and period. Results support a first-order linear process for virtually all cases (Pressing, 1998a). For all cases, $\mu_1 \cong \mu_2 - P/2$, and for a large fraction of cases, $\alpha_1 \cong \alpha_2$. When the second condition applies, error-correction processes are equally effective (stabilizing) at the two attractor points; hence $a_1 \cong 0$, and only the second harmonic is significantly present in g. Furthermore, the phase lags are approximately equal to each other and are just the mean relative asynchrony.

In general, the presence of attractor points depends on the locations of zero points with positive slope in g, and this can be shown to depend on the ratio

$$\frac{a_1}{2a_2} = (\alpha_1 - \alpha_2)/(\alpha_1 + \alpha_2).$$

In all measured cases, the alphas, and hence a_2, are positive. Given this, for $a_1/2a_2 < -1$, there is a single attractor point near $P/2$ (π radians). For $-1 < a_1/2a_2 < 1$, there are attractor points near 0 and π. For $a_1/2a_2 > 1$, there is a single attractor point near 0. (All these points would be exactly at the named locations if $\phi_1 = \phi_2 = 0$; as it is, they are found experimentally to differ slightly from these positions, indicating nonzero ϕ_1, ϕ_2.) Thus, this control function allows exclusive on-beat tapping, exclusive off-beat tapping, or cycles with both on- and off-beat tapping, and changes in this parameter ratio can lead to system bifurcations.

For such a 2:1 system to be stable at both 0 and π, and to be expressible in terms of two harmonics alone, it then follows that the relation $|(\alpha_1 - \alpha_2)/(\alpha_1 + \alpha_2)| < 1$ must be satisfied. This is equivalent to the property that either both alphas must be positive or both must be negative. This relation is not required in a purely linear system, and so it acts as a (weak) test of whether the system is consistent with the general nonlinear formulation. This can be examined for the data of Pressing (1998a), and the results are found in Table 2. In all cases, the relation holds. This ratio can also be considered an inverse indicator of the global stability of the process: The closer the ratio is to zero, the more stable is the process. *In all cases, the absolute value of the expert's stability parameter is less than that of the nonexpert, indicating greater stability.*

This interpretation of $a_1/2a_2$ as a stability parameter differs from that of some previous workers (deGuzman & Kelso, 1991; Treffner & Turvey, 1996), who have hypothesized that, in the notation of Equation 41, α^*/α should vary as $1/\omega_c$—hence, here, $P \propto$ 1/stability parameter. The data of Table 2 do not support this conjecture. However, this may be due to differences in task design between those experiments and the current work.

Table 2
Error-Correction Parameters α_1 and α_2 for 2:1 Tapping and the Resulting Stability Parameters

2:1 pattern type	Participant	Pulse size (ms)	α_1, α_2	Stability parameter: $\frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2}$
RR	Expert	1,000	0.259, 0.256	0.006
RR	Nonexpert	1,000	0.144, 0.119	0.095
RR	Expert	750	0.195, 0.233	−0.089
RR	Nonexpert	750	0.201, 0.084	0.411
RR	Expert	376	0.190, 0.149	0.121
RL	Expert	1,000	0.446, 0.296	0.202
RL	Nonexpert	1,000	0.048, 0.323	−0.741
RL	Expert	750	0.440, 0.483	−0.047
RL	Nonexpert	750	0.426, 0.196	0.370

Note. Data are from Pressing (1998a). R = right hand; L = left hand.

The linearized equations governing the tapping in this case follow directly from Equation 18:

$$A^*_{2,n} = (1 - \alpha_1)A^*_{1,n} - \beta_1 A^*_{2,n-1} + Q_{1,n} \quad (46)$$

$$A^*_{1,n+1} = (1 - \alpha_2)A^*_{2,n} - \beta_2 A^*_{1,n} + Q_{2,n}, \quad (47)$$

where $Q_{1,n}$ and $Q_{2,n}$ are given by Equations 13 and 14. These same equations apply whether the tapping is unimanual or bimanual.

3:1 Tapping

This pattern is readily made stable by expert participants and, with minimal practice, a large fraction of nonexpert participants. According to the formulation here, the error-correction function g may now contain three harmonics (see Equation 17):

$$\alpha g(A_{j,n}) = \frac{P}{2\pi}\{a_1 \sin[2\pi(A_{j,n} + \phi_1)/P]$$

$$+ a_2 \sin[4\pi(A_{j,n} + \phi_2)/P] + a_3 \sin[6\pi(A_{j,n} + \phi_3)/P]\}, \quad (48)$$

for $j = 1, 2, 3$. Attractors occur at points where $g(x) = 0$ and $g'(x) > 0$. As shown earlier, the linearity condition requires that $\alpha g(A_{j,n}) \approx \alpha_j(A_{j,n} - \mu_j)$ in the regions around μ_1, μ_2, and μ_3. These conditions produce equations that are algebraically messy but show that, in practice, there are one, two, or three attractor points. When all αs are roughly equal, and phase lags are small, mean asynchrony values are near 0, $P/3$, and $2P/3$, and this is the experimental finding, in accord with the task requirement. Precise values for the parameters a_1, a_2, a_3, ϕ_1, ϕ_2, and ϕ_3 can be estimated numerically on the basis of experimental results. This allows computation of the corresponding nonlinear error-correction function, and the result for one sample case is shown in Figure 14. The data are from Pressing (1998a), taken from a nonexpert participant 3:1 case with a period of 999 ms. The parameters required for the fit are $a_1 = .058$, $a_2 = .063$, $a_3 = .083$, $\phi_1/P = .110$, $\phi_2/P = .036$, and $\phi_3/P = .068$, which yields agreement to two decimal places with experimental values for the three target fractional asynchronies (A/P = relative phase/2π [experimental values of 0.929, 0.262, and 0.597]) and the three slopes (α_1, α_2, α_3 [experimental values of 0.423, 0.103, and 0.221];

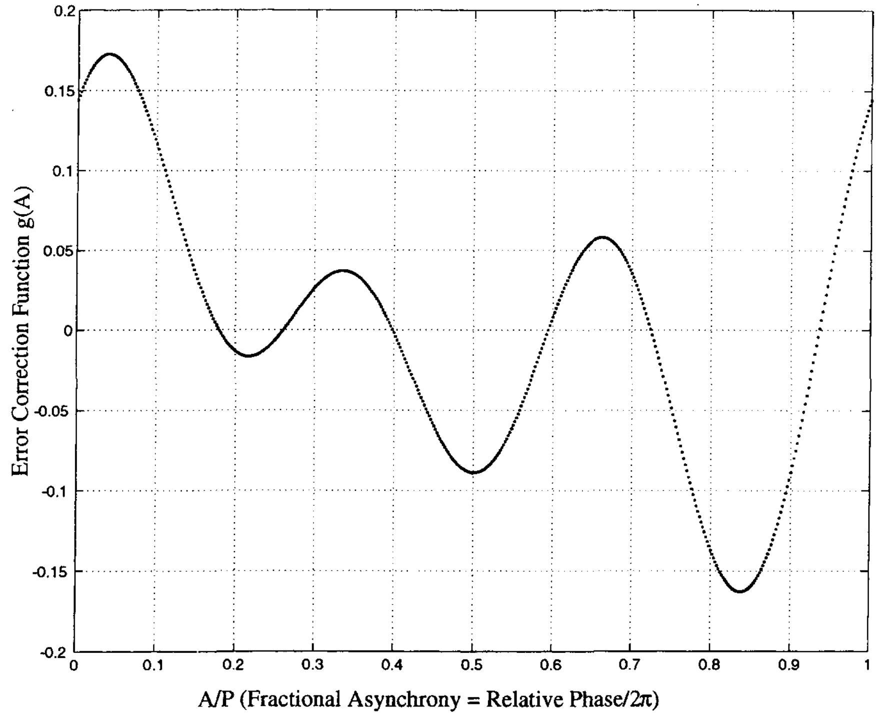

Figure 14. The calculated error-correction function for data from a nonexpert participant for 3:1 unimanual tapping with 999-ms period (data from Pressing, 1998a). The experimental attractor points are 0.262, 0.597, and 0.929, corresponding to the positive-slope zero crossing points in the diagram. The coefficients and lags of the three harmonics contributing to g(A) have the following values: a_1 = .058, a_2 = .063, a_3 = .083, ϕ_1/p = .110, ϕ_2/p = .036, and ϕ_3/p = .068; see Equation 48.

Pressing, 1998a). These experimental findings show the presence of anticipatory yet unequal phase lags in the harmonic components of the error-correction function.

With this information on the form of the error function, and using values of the Gaussian white-noise components to the fundamental equation (clock variance σ_C^2, and subdivision/motor variance $\sigma_{B/M}^2$) estimated from series analysis (Pressing, 1998a; Pressing & Jolley-Rogers, 1997), simulations of the data can be generated. A simulation and actual data for the above nonexpert 3:1 case with a period of 999 ms are compared in Figure 15. The two graphs are quite similar: Both show comparable levels of means and variances, and both show similar patterns of parallel local deviations from the mean for the three subseries. There is one slight difference: The variance of the participant data appears to decrease slightly near the end of the run, unlike the simulation. This is likely to be a learning effect of sustained practice over this long run and could easily be mimicked by a progressive reduction in clock variance.

In Figure 16, the effects of increasing noise are shown in a simulation; clock standard deviation is increased from 12 ms to 18 ms, with all other parameters held constant. The series is still mostly stationary, although with increased variance, but it also twice exhibits upward drift in asynchronies early in the run, which corresponds to the performer falling behind and then stabilizing in a new phase relation, having lost one beat (i.e., phase slippage). The first restabilization is brief (over the range 50–65 s), and then a second beat loss followed by semipermanent restabilization occurs. More precisely, this phenomenon (dragging leading to beat loss, or racing leading to beat gain, with subsequent cognitive misconstrual) was observed when the nonexpert was learning to perform this pattern. Still higher values of central noise or improper values of system parameters will, in general, produce continual phase drift or wandering between regions of temporary stability (intermittency), with frequent omissions or insertions of beats. These are hallmark effects of essential nonlinearity and have been frequently observed experimentally.

The effect of varying the Lag 1 negatively autocorrelated noise is somewhat different. Typically, within limits, increasing the negativity of autocorrelation (governed by $\sigma_{B/M}$), by providing more substantial autoinhibition, increased the higher frequency spectral components (fast jitter in the time series) and sometimes appeared to damp large white-noise excursions, at least on a short time scale. The inclusion of dead zones of up to 40 ms had only a very limited effect on stability of simulations.

Figure 17 shows a comparison of data for 3:1 tapping with a period of 999 ms and a simulation, using an expert performer as the participant. Data are from Pressing (1998a), with variance parameters estimated as in the previous case (the expert had lower values of variance than the nonexpert). The graphs are qualitatively very similar.

Although an extensive comparison with circle-map-based formulations was not made, 3:1 behavior was studied by simulation, using $\Omega \cong ⅓$ and Equation 40, for this case. The results appeared to show that there were too few parameters to match all experi-

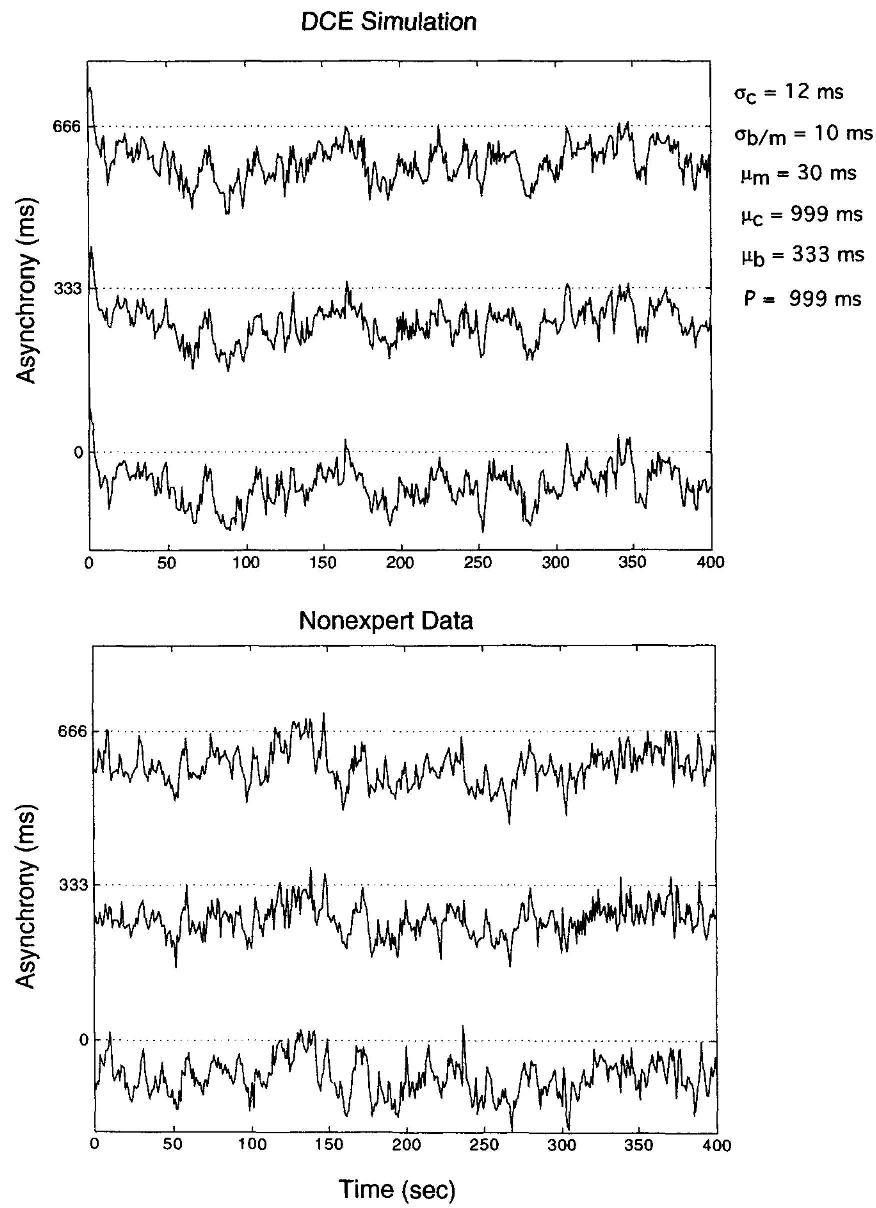

Figure 15. Comparison of a simulation of the asynchronies for 3:1 tapping using the discrete control equation (DCE) with the actual data found for the case of Figure 14 for the nonexpert participant. Phase lags and harmonic weights are as in Figure 14. In this and subsequent simulations, all noise constituents are Gaussian white noise.

mental observables and that stability there is considerably more sensitive to the effects of noise than in the DCE form. To even qualitatively match experimental results, considerably lower values of stochastic noise were required than with the current formulation.

Continuous Dynamical Models

The above equations are discrete. However, continuous forms that have structure that is analogous to the map formulations have been given a central role in traditional dynamical approaches to movement, with the relation between them traditionally considered to be that of Poincaré section (Treffner & Turvey, 1993). Thus, the standard continuous form of the elementary-coordination dynamic equation (e.g., Kelso, 1994a; Turvey & Schmidt, 1994) is a continuous-time analog of Equation 41 and can be written as

$$\dot{\Phi} = \Delta\omega - \alpha \sin(\Phi) - \alpha^* \sin(2\Phi) + \zeta_t, \qquad (49)$$

where $\Phi(t) \Leftrightarrow 2\pi\theta_n$ and ζ_t is a suitably defined Gaussian white-noise process (Turvey & Schmidt, 1994). A recent generalization of this to include variable phase delays is

$$\dot{\Phi} = \Delta\omega - \alpha \sin(\Phi + \phi_1) - \alpha^* \sin[2(\Phi + \phi_2)] + \zeta_t \qquad (50)$$

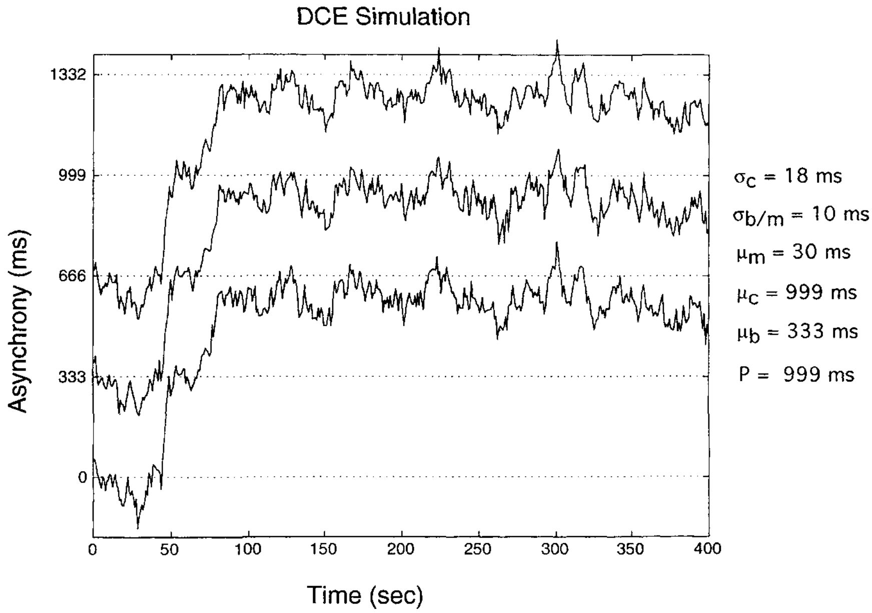

Figure 16. Simulation of the asynchronies for 3:1 tapping using the DCE, with the same parameters as those found in Figure 15 but with the one exception of σ_C = 18 ms. The greater noise source produces a series that is basically stable but twice shows a forward drift in asynchronies early in the run, which corresponds to the performer falling behind and ultimately losing one beat, a characteristic nonlinear phenomenon. DCE = discrete control equation.

(Treffner & Turvey, 1995), which shows similarities to the phase delays of the discrete model here. The aptness of these equations in particular circumstances has been well demonstrated (e.g., Kelso, 1994a; Treffner & Turvey, 1995). It is important to emphasize that these corresponding equations may yield quite different behavior from their discrete counterparts, and parallels between them need to be treated with caution; nevertheless, the potential for interpreting such equations as expressing continuous control seems clear. (To see this analytically, take the continuous form of the basic control equation given in footnote 1, without time delays, and substitute from Equation 17 for the control function with $L = 2$. Then, provided the noise term is appropriately defined—a standard issue in continuous stochastic differential equations—and interpreting the variable [asynchrony/period] as phase, as done above, the result is exactly Equation 50).

It is true that a more congruent relation exists between the current approach and these and other continuous dynamical formulations (e.g., Schöner & Kelso, 1988a, 1988b; Treffner & Turvey, 1995, 1996; Zanone & Kelso, 1994), inasmuch as the effects of intention, learning, and environmentally cued stability are addressed in general terms, a noise term is present, and additional free parameters of fit are included.

However, none of five objections given above for the circle map are fully eliminated. Specifically, these continuous-time approaches do not currently formulate case-specific principles of noise modeling (Objection 1) nor clarify the potential role of higher order harmonics or correction processes (Objections 2 and 3). With respect to Objection 4, the continuous-time approach just given does not provide a convincing rationale for why, independently of the presence of polyrhythm-type (n:m) patterns, approximations to simple temporal ratios are found so preferentially in human movement patterns, notably music performance.

Error Correction in General Temporal Configurations

The application of the approach proposed here to patterns of arbitrary complexity would proceed by an estimation of the nonlinear error-correction function, whose form is determined by cognitive constraints in the task design and whose detailed parametric values will also reflect the effects of structural and neurophysiological constraints (Carson, 1996). No such general evaluation has yet been carried out, but the validity of the current formulation for stable tapping in the linear regime has recently been experimentally examined by Pressing (1998a) for a wide variety of uni- and bimanual patterns, using both musically expert and nonexpert participants, and with a range of periods. The range of patterns examined is given in Table 3. Note that patterns such as 8th–16th–16th and paradiddle are included that have not been accounted for by earlier dynamical approaches (Pressing, 1998a).

I was able in that work to analytically solve for the error-correction parameters alpha and beta for all possible rhythmic patterns, using Equation 18. The resulting fits to data indicated that the processes are well modeled by linear error correction and that this is predominantly first order. Second-order (and possibly higher) effects occurred predominantly under conditions of high task demands and were associated with expertise. Specifically, second-order effects were found in expert performance of a complex polyrhythm and an off-beat triplet pattern, and for both expert and nonexpert performance of a slow off-beat pattern (Pressing, 1998a).

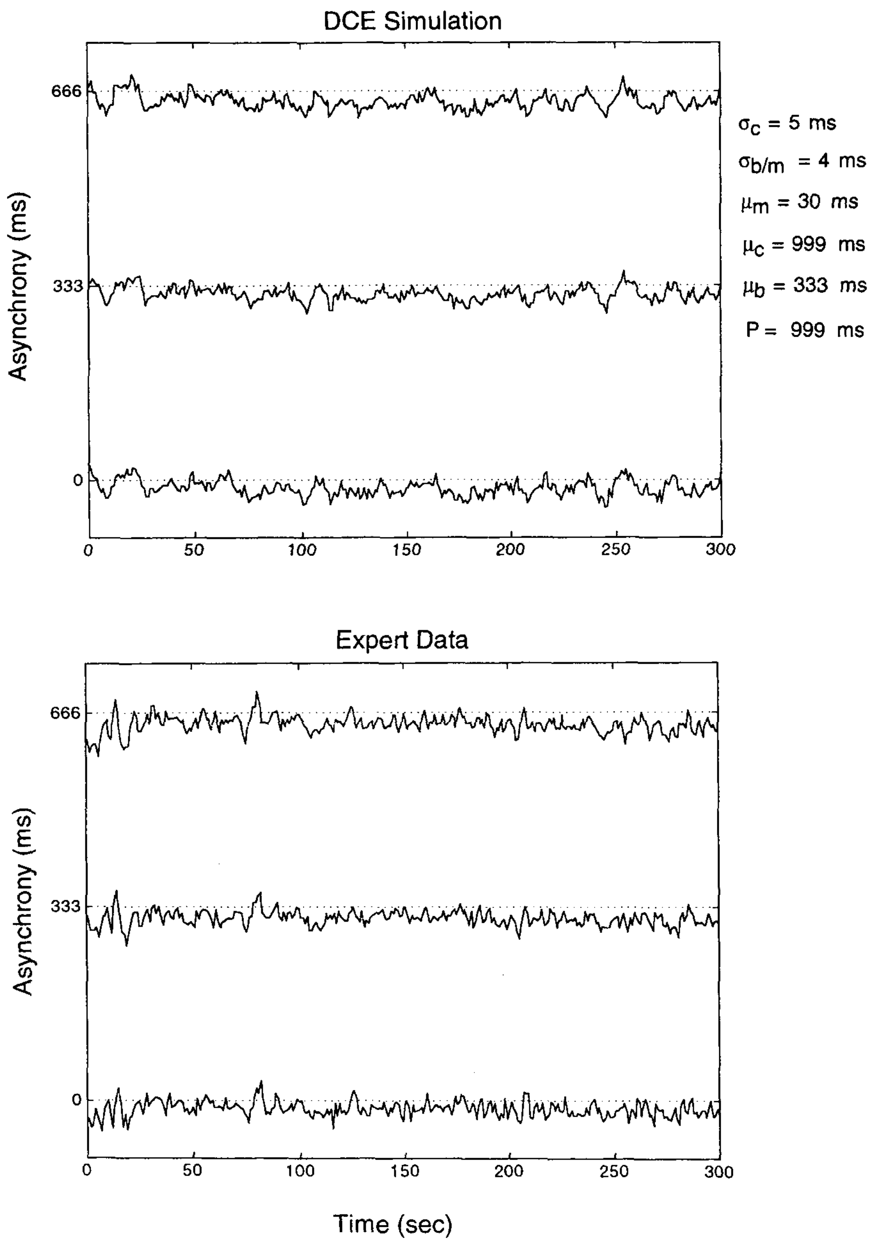

Figure 17. Comparison of a simulation of the asynchronies for 3:1 tapping using the discrete control equation (DCE) with the actual data found for the expert participant. Harmonic weights are as in Figure 14; phases shifted by −.05 from that case.

The standard errors of fit for the error-correction parameters of the different patterns for equivalent run lengths were comparable over all patterns and did not show any qualitative change in moving from simple 1:1 and 1:2 tapping to more complex patterns. This supports the unified approach used here, which considers that there is no hard and fast line between cognitive control and other types of control, despite the undeniable effects of intrinsic foundation constraints.

Pressing (1998a) also examined the issue of alpha variation with period for more complex patterns, as discussed earlier for the 1:1 and 2:1 cases. In most cases where it was examined, alpha was found to have a closely linear relationship with period, with correlations (r^2) falling in the range .893–.990. However, in a few complex cases, linearity dramatically failed—indeed, the relation between alpha and period was nonmonotonic in one case. This conditional failure of linearity accords with earlier arguments and suggests the need for a critical evaluation, which is based on experimental evidence, of the optimal interpretation of system control parameters for such cognitively complex cases.

In Pressing (1998a) and Pressing and Jolley-Rogers (1997), comparison of the results for the expert and the nonexpert participants showed that the expert generally has a more richly elabo-

Table 3
Experimental Designs Showing Good Fit to Linear Error-Correction Models

Pattern	Manuality	Cycle lengths (ms)	Participant type
On-beat (R)	Uni	100–1,000	E
On-beat (R)	Uni	250–1,000	N
Off-beat (−R)	Uni	375–1,000	E
Off-beat (−R)	Uni	750, 1,000	N
Off-beat triplet (− −R)	Uni	750	E
Isochronous 2:1 (RR)	Uni	376–1,000	E
Isochronous 2:1 (RR)	Uni	750, 1,000	N
Isochronous 3:1 (RRR)	Uni	564, 1,000, 1,500	E
Isochronous 3:1 (RRR)	Uni	1,000, 1,500	N
8th–16th–16th (R–RR)	Uni	752	E
Alternating 2:1 (RL)	Bi	750, 1,000	E, N
8th–16th–16th (R–RL)	Bi	752	E
Paradiddle[a]	Bi	1,600	E
3L4R* polyrhythm	Bi	1,800	E

Note. R = right hand; E = expert; N = nonexpert; L = left hand.
[a] Isochronous RLRRLRLL.

rated and refined error-correction process, which resulted in consistently larger values of error-correction parameters and lower values of central system (i.e., clock) noise. Also, he has a wider repertoire of pattern classes. Thus, learning is reflected in both qualitative system changes and evolution of control parameters (cf. Zanone & Kelso, 1994).

Conclusion

In this article, I have developed RBT, which is a state-based approach to human behavior that takes as its central process referencing, the use of a referent (e.g., process, criterion, set of events) to continuously modify behavior. The theory owes clear debts to control theory and also to synergetics, which itself draws on the older perspective of cybernetics. The referencing process may be discrete or continuous, but the focus here has been on the discrete form, generating a quite general DCE. The DCE has been characterized in terms of a number of psychological and physiological cases by identification of relevant state and control variables and modeling of the applicable fundamental noise sources and referencing processes. The DCE is cast in general dynamic terms but also allows for the natural inclusion of cognitive processes, because the control variables may act at any level of the system (be it hierarchical or heterarchical), varying from local to global in compass, and from automatic or reflexive to high-level cognitive or volitional in behavioral orientation. In particular, the setting, recursive assessment, and adaptive modification of goals and plans can be viewed as higher level referencing processes. Dynamical aspects of attention can also be readily framed as a first-order DCE (Large & Jones, 1999). Both stable and unstable behaviors can be addressed, the former by equilibrium solutions to the DCE and the latter by several paths, including state equation transitions, memory-assisted changes of class of control function, and complexity-based symbolic dynamics. This variety of operative variables and span of operational levels does make concrete the possibility of quantitative reconciliation of dynamic and information-processing accounts of cognition and action.

Turning from the general theory to a specific target domain to attempt to validate this claim, the DCE produced by the approach has been applied to the area of temporal control of human movement and shown to be able, under different but plausible conditions, to generate both characteristic linear stochastic approaches (with or without error correction), where the focus is on behavioral stability, and nonlinear dynamic approaches, where the focus is on behavioral instability. Essentially, this is because the DCE is a state-space-based method, which can subsume both nonlinear dynamical and linear autoregressive moving average (ARMA) approaches. This common genesis for both accounts achieves a degree of reconciliation of dynamics and motor programming and a partial resolution of the putative motor paradigm crisis, because the use of linear stochastic approaches has been long identified with the motor program–computational perspective on movement modeling (Beek et al., 1995; Haken et al., 1996; Heuer, 1993; Heuer et al., 1995).

Pursuing this reconciliation further, it appears that, like the current dynamical approach, the generalized motor program (Keele, Cohen, & Ivry, 1986; R. A. Schmidt, 1988), insofar as it is computationally defined, is a parametrically tunable control structure; insofar as it accommodates closed-loop processes, it involves error correction that is based on ongoing deviations from a reference stream, which may be based on sensory monitoring of output from the generalized motor program effectors, or motor efference. Open-loop processes (central to the traditional motor program concept) have already been shown to be compatible with the current theory, via the Wing–Kristofferson (1973a) approach, and may in other circumstances rely on internal referencing, which the theory can also accommodate. It is possible that the DCE can therefore be regarded, at least for the sake of argument, as one central control process driving the generalized motor program controlling action. Further applications of the RBT approach to motor control, including cases of interpersonal cooperation and force and spatial position-based referential control, are given in Pressing (1998c).

Thus, the DCE appears to present a coherent approach to movement that accommodates both energy-based and information-based control and also integrates perspectives that are based on cognition, error correction, human automaticity, the development of expertise (learning), and dynamical systems theory.

Experimental support for the current approach to movement organization is certainly strong, in no small part because of its ability to include earlier successful theories as subcases. However, this theory has also been demonstrated to successfully apply to a number of cases not previously investigated and to eliminate or alleviate some problems with existing formulations. In addition, the theory makes a number of novel predictions that should be subjected to further experimental examination, such as the equivalence of Stability Parameter K and Error-Correction Parameter α, new approaches to system stability and learning, nonlinear effects in regimes of stable behavior, the importance of second-order effects under certain conditions (yielding new mathematical formulations), the use of further examination of the noisy metronome problem, patterns of expert–nonexpert distinction, explicit cognitive hypotheses for pattern-based phase transitions, and the viability of DCE application to spatial and force error-correction processes.

The effects of learning and adaptation have not been fully developed here, partly for reasons of space. Nevertheless, the effects of training, as assessed by comparative examination of expert and nonexpert participants, strongly suggest that learning the control of timed patterns entails the development of richer and more effective referential control processes. This occurs by such means as increasing the error-correction parameters; building a control structure with greater temporal memory; developing new, alternative, or more effective state variables for control; or reducing the output variance by construction of an optimal set of parametrically tuned control processes, be they conscious, automatic, or an optimized mixture of the two. It seems perfectly possible that such changes could be modeled successfully by neural networks that encode the system variables as output-layer activations, which are compared with targets for supervised learning, whereas the control parameters could be represented by the weights connecting input and output (or possibly hidden) layer units. Intrinsic dynamic constraints might be codable as patterns of bias within the units of the net. As I have shown, neural nets are high-dimensional, nonlinear dynamic systems consistent with the DCE formulation.

In conclusion, the theory presented is quite general and, perhaps, ambitiously inclusive. It has seemed appropriate to delve deeply into a specific case, partly because of its domain-specific potential and partly to put flesh onto the bare bones of theory. Clearly though, the general referential approach will only be as good as the sum of the specific cases in which it is found to provide useful insights, predictions, and quantitative interpretations of experimental results.

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