Signal structure

With $C \in \{0,1\}$ the context, $X \in \{0,1,2,3\}$ the cue, and $S \in \{0,1\}$ the signal, the chain structure is:

$$C \to X \to S. \tag{1}$$

The decision-maker (DM) uses the signal structure $P(S \mid x)$ to inform their belief about the true context.

Inference without history

Equipped with a prior $P(C)$ over contexts and assuming independence of contexts across trials, the DM wants to infer C from s. This is done according to:

$$P(C \mid s) \propto P(s \mid C)P(C) = (\sum_x P(s \mid x)P(x \mid C))P(C). \tag{2}$$

Then, in order to maximize their probability of being correct in the trial, the DM will use the following decision rule:

$$\hat{c}(s) = \operatorname*{argmax}_c P(C = c \mid s) = \mathrm{round}(P(C = 1 \mid s)). \tag{3}$$

Inference with history

On realizing signal s_{t-1} in trial $t-1$, the DM will update their prior over contexts using their posterior for their context in trial $t-1$ and their belief about the transition probabilities of the context between trials (Markov hypothesis):

$$P(C_t \mid s_{t-1}) = \sum_{c_{t-1}} P(C_t \mid c_{t-1})P(c_{t-1} \mid s_{t-1}). \tag{4}$$

Then, this prior will be used in inference on the signal in trial t. Recursively, exploiting the assumed conditional independence of s_t from s^{t-1} given c_t:

$$P(C_t \mid s^t := [s_1, ...s_t]) = P(C_t \mid s_t, s^{t-1}) \propto P(s_t \mid C_t)P(C_t \mid s^{t-1}). \tag{5}$$