GEOP591 Theoretical Seismology -- Final Exam

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1. Recall the Christoffel matrix $\Gamma_{ik}(\boldsymbol{x},\boldsymbol{p}) = a_{ijkl}p_j p_l$, where $p_i = \partial T/\partial x_i$ is the slowness vector. $G_m(\boldsymbol{x},\boldsymbol{p})$ denotes the eigenvalue of the matrix, where m = 1, 2, 3 for qP, qSV and qSH waves, respectively. $g_i(\boldsymbol{x},\boldsymbol{p})$ denotes the associated eigenvector.

a) Express $G_m(\boldsymbol{x},\boldsymbol{p})$ by $\Gamma_{ik}(\boldsymbol{x},\boldsymbol{p})$ and $g_i(\boldsymbol{x},\boldsymbol{p})$. [1 pts]

b) Write the Eikonal equation. [2 pts]

c) Write the Hamiltonian $H(\boldsymbol{x},\boldsymbol{p}) = ?$ [1 pts]

d) Using $H(\boldsymbol{x},\boldsymbol{p})$, find the ray-tracing equations $\frac{dx_i}{d\tau} = ?$, $\frac{dp_i}{d\tau} = ?$ [6 pts]

$\frac{dT}{dx_i} \cdot \frac{dx_i}{dx_i}$

uation,

$\omega^2 \rho =$

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2. Consider the wave equation for SH waves in 2D:

$$\ddot{v} = \frac{A}{\rho}\delta(x)\delta(z)\delta(t) + \beta^2\nabla^2 v.$$

a) Apply the Laplace transform to the t axis: $t \to s$. [3 pts]

b) Apply the Fourier transform to the x axis: $x \to k_x$. [3 pts]

c) Find the analytic solution in the (k_x, z, s) domain, i.e $v(k_x, z, s)$ = ?. [4 pts]

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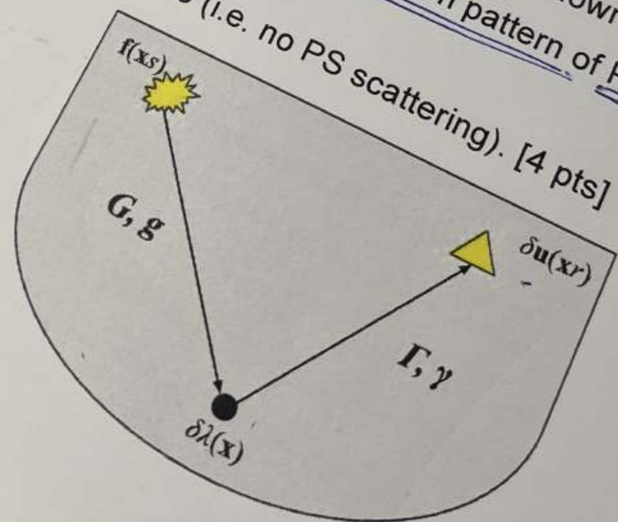
3. The graph below shows wave scattering due to a point $\delta\lambda$ scatter in a boundless homogeneous media. The source (f) is a vertical single force and the receiver is a vertical geophone, recording the scattered wave $\delta\mathbf{u}$.

a) Write the far-field Green's function for elastic P and S waves. Find the coefficients that describe the directional amplitude variations, i.e. the radiation patterns. [2 pts]

b) Let θ be the incident angle and ϕ be the scattering angle made from the downward z axis (both angles increase counter-clockwise). Find the diffraction pattern of PP waves as a function of θ and ϕ. [4 pts]

c) Show that the diffraction pattern of PS wave is zero (i.e. no PS scattering). [4 pts]



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4. Derive FWI gradients based on the variable-density acoustic wave equation,

$$\kappa^{-1}\ddot{p} = \nabla \cdot \rho^{-1}\nabla P + s.$$

a) Given a data misfit function $C(P)$, write down the Lagrangian. [2 pts]

b) Find the adjoint-state equation. [4 pts]

c) Find the FWI gradients with respect to κ and ρ. [4 pts]

$a i \omega^2 p \rightleftharpoons \nabla_u C$

$$\frac{1}{k}\frac{\partial^2 p}{\partial t^2} = \nabla\left(\frac{1}{\rho}\frac{\partial}{\partial}\right.$$

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Below is a graphic explanation of the Marchenko equation for 1D inversion,

a) Explain the physical meaning of these symbols: $\delta, F, R, A, \psi_f, \psi_a, \psi_a^R$. [4 pts]

b) Using these symbols and the graph, write down the Marchenko equation. [3 pts]

c) What operations do we perform on the data to solve the Marchenko equation? [3 pts]