Extension 1: Water Source can only Change flow rate every 4 hours.

In this extension, the frequency of water source change flow rate is limited. In order to establish a particular formulation to add this constraint for the SAS architecture, we set a function 'bs' which represents 'binary source' to examine the cycle of source's activity at least 4 hours. The mathematical formulation subject to the constraint is above. The variable 'ns' is not considered in this extension because it would not change and affect the constraint.

$$sources\_schedule(h,j) - sources\_schedule(h-(h-1)\%4+j) \leq M \cdot bs(j);$$

$$sources\_schedule(h-(h-1)\%4+j) - sources\_schedule(h,j) \leq M \cdot bs(j);$$

Among them,

$$sources\_schedule(h,j) = \begin{cases} sources\_schedule(h+j), & h+j \leq 24 \\ sources\_schedule(h+j-24), & h+j > 24 \end{cases};$$

$$\sum_{j=0}^{3} bs(j) = 3;$$

$$j = 0, \ldots, 3.$$

The subtraction of two sources' function is used to ensure the schedule can be divided by numeric 4 integrally when it implements the operation of flow rate transform. The right-hand formula determined the feasibility of flow rate change, the situation when the output value of function 'bs' equal zero would be rejected because the difference of two integers which greater than zero on the left hand cannot meet the requirement of inequality. In contrast, both of two reciprocal inequalities are true when 'bs' is one. In another word, only once in four hours for the design of sources schedule can change the flow rate, the sources would keep the same or similar flow rate in four hours which originate from the moment of the latest flow rate change till the next cycle. With the establishment of this constraint, the optimal solution still stayed in 1910.25 dollars. However, the distribution of source schedule has been changed refer to Figure 1. The flow rates produced by Cornwall WTP were normalized at the same rate in four hours per unit.

sources_schedule:

[1]	sources_schedule
1	3500
2	800
3	800
4	3500
5	800
6	800
7	3500
8	800
9	800
10	3500
11	3500
12	800
13	3500
14	3500
15	3500
16	3500
17	2128
18	2084
19	1904
20	3500
21	800
22	3500
23	3500
24	3500

sources_schedule:

[1]	sources_schedule
1	800
2	800
3	800
4	3500
5	3500
6	3500
7	3500
8	1165
9	1165
10	1165
11	1165
12	3500
13	3500
14	3500
15	3500
16	2039
17	2039
18	2039
19	2039
20	3500
21	3500
22	3500
23	3500
24	800

Figure 1. The sources schedule without constraint (left) and the sources schedule with constraint (right).

Extension 2: A pump has to run for at least 2 hours.

The modified problem formulation is built consist of 3 functions which represent three various moments in the pumps' working process in this case. The function 'pumps_schedule' follows the Bernoulli distribution. It directs that '1' for open and '0' for close. The principle of this formulation is to verify if it is opened in all three moments, or closed in the second hour while the switch of pumps is opened either at the first hour of any period within 24 hours or the third hour. The variable 'np' does not vary in the whole process of iteration and will be not included. The specific formulations are as follows.

$$pumps\_schedule_1(h)+pumps\_schedule_2(h)\geq pumps\_schedule_3(h);$$

Among them,

$$pumps\_schedule_2(h)=\begin{cases} pumps\_schedule_2(h+2), & h+2\leq 24 \\ pumps\_schedule_2(h+2-24), & else \end{cases};$$

$$pumps\_schedule_3(h)=\begin{cases} pumps\_schedule_3(h+1), & h+1\leq 24 \\ pumps\_schedule_3(h+1-24), & else \end{cases}.$$

According to these formulations, the function 'pumps_schedule1' (P1) refers to the status of pumps at the first hour, the same reason can be proved that 'pumps_schedule2' (P2), 'pumps_schedule3' (P3) represents the second and the third hour respectively. The sum of P1 and P2 (max 2) determines that the value of P2 would not be 1 when the left-hand formula is 0. The pumps have to run for not least than two hours based on this formulation and the performance of it illustrates in Figure 2 and 3. All the pumps were running more than two hours after modified and stayed in the same flow rate during its working period.

pumps_flow_schedule						
	Cornwall_P1	Cornwall_P2	Cornwall_P3	Kingsland_P1	Kingsland_P2	Kingsland_P3
1	600	600	800	800	800	0
2	0	600	800	800	800	400
3	0	600	800	800	800	400
4	600	600	800	800	800	400
5	0	0	0	800	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	800	0	0
10	0	0	0	0	800	400
11	600	0	0	0	0	0
12	600	600	0	800	0	400
13	0	0	0	0	0	0
14	0	600	0	800	800	400
15	0	600	0	800	800	400
16	600	0	0	800	800	400
17	600	600	800	800	0	400
18	600	600	800	800	0	400
19	600	600	800	800	800	0
20	600	0	800	0	800	400
21	600	600	800	0	0	0
22	600	600	800	0	0	400
23	0	0	0	0	800	400
24	0	600	0	0	0	400

Figure 2. The performance of pumps before modified.

pumps_flow_schedule:

pumps_flow_schedule						
	Cornwall_P1	Cornwall_P2	Cornwall_P3	Kingsland_P1	Kingsland_P2	Kingsland_P3
1	600	600	800	800	800	0
2	0	0	800	800	800	400
3	600	600	800	800	800	400
4	600	600	800	800	800	400
5	0	0	0	0	800	-0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	800	0
10	0	0	0	0	800	0
11	600	0	0	0	800	0
12	600	0	0	0	800	0
13	0	600	0	0	0	0
14	0	600	0	800	0	0
15	0	600	0	800	800	0
16	600	0	0	800	800	0
17	600	600	800	800	0	0
18	0	600	800	0	0	0
19	-0	0	800	800	0	0
20	600	600	800	800	0	0
21	600	600	800	800	800	0
22	600	600	0	0	800	400
23	0	600	0	800	800	400
24	600	600	800	800	800	400

Figure 3. The performance of pumps after modified.

Extension 3: If a valve is open, it has to stay open at the same flow rate for at least 4 hours.

The connection established in this extension includes the decision of valves opening and the flow rate, which is controlled by valves. There are five binary variables set which are 'valves_open' and 'bv'. The function 'valves_open' has been splitted into three variables which represent the situation for valves opening or closing in the first hour, the forth hours after the first hour and the 'j'st hours after the first hour respectively. 'bv' is used to judge if the flow rate at the same level. 'M' is a huge integer number that it is available to expand the effect of function 'bv'. The variable 'nv' is the valve node in the water distribution network, however, it will not be discussed in this extension due to its immutability.

$$valves\_open_1(h) + valves\_open_2(h) \geq valves\_open_3(h,j);$$

$$valves\_schedule_1(h) \leq M \cdot \left[1 - bv_1(h)\right];$$

$$valves\_schedule_1(h) - valves\_schedule_2(h) \leq M \cdot bv_2(h);$$

$$valves\_schedule_2(h) - valves\_schedule_1(h) \leq M \cdot bv_1(h);$$

$$j = 1, \ldots, 3;$$

Among them,

$$valves\_open_2(h) = \begin{cases} valves\_open_2(h+4), & h+4 \leq 24 \\ valves\_open_2(h+4-24), & else \end{cases};$$

$$valves\_open_3(h) = \begin{cases} valves\_open_3(h+j), & h+j \leq 24 \\ valves\_open_3(h+j-24), & else \end{cases};$$

$$valves\_schedule_2(h) = \begin{cases} valves\_schedule_2(h+1), & h+1 \le 24 \\ valves\_schedule_2(h+1-24), & else \end{cases};$$

$$bv_2(h) = \begin{cases} bv_2(h+1), & h+1 \le 24 \\ bv_2(h+1-24), & else \end{cases}.$$

The first formulation indicates that the valves cannot be close between any period in four hours. Subsequently, the flow rate of the first hour is determined not more than 0 if it is closed. Furthermore, the inequality in reversed verification is used to ensure that the flow rate is maintained at the same water level from the first hour it is opened to the next hour. The effect of valves is optimized through this method which illustrates in Figure 4.

valves_schedule:

	valves_schedule	
	EricssonValve	GreenwoodValve
1	200	1500
2	0	1500
3	624	1500
4	1000	1500
5	1000	1500
6	1000	1500
7	1000	1500
8	200	1500
9	0	1500
10	0	1500
11	0	1500
12	0	1500
13	0	1500
14	0	1500
15	0	1500
16	0	1500
17	0	1500
18	0	1350
19	200	1500
20	0	1259
21	0	1500
22	0	1500
23	0	1500
24	1000	1500

valves_schedule:

	valves_schedule	
	EricssonValve	GreenwoodValve
1	0	1484
2	0	1484
3	850	1484
4	850	1484
5	850	1484
6	850	1484
7	850	1484
8	850	1484
9	0	1484
10	0	1484
11	0	1484
12	0	1484
13	0	1484
14	0	1484
15	0	1484
16	0	1484
17	0	1484
18	0	1484
19	0	1484
20	281	1484
21	281	1484
22	281	1484
23	281	1484
24	0	1484

Figure 4. The flow rate in valves before modified (left) and after modified (right).