GDP Growth Forecast

Multivariate Time Series Analysis

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August 4, 2021

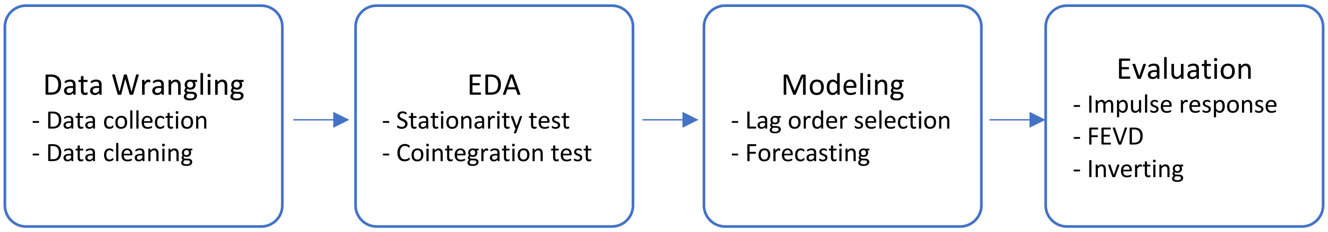
Content

Problem Statement

Macroeconomics affects our everyday life in all kinds of aspects. For examples, employment and unemployment, government welfare, inflation, daily goods and services, investment, housing, and more. We hear news such as “71K jobs lost in May”, “manufacturing sales declined further in November at -0.12%”, “housing reports edge Q2-2020 nowcast down to 2.70%”, or “retail sales leaves Q3-2020 roughly unchanged at 1.20%”, etc. All of these in turn influence our decision making, from grocery shopping (what to eat) to residence location (where to live). Hence, understanding GDP growth and other components in the equation is important. Prior to the notorious 2008 subprime mortgage crisis, people’s investment decisions might have changed had they realized the unpaid household debt was piling up before the bubble burst.

The main goals of this project are to inform us how the economic indicators affect each other, and to provide insights of predicting how they may play out in the next 2 years. We take care of GDP growth, unemployment rate, inflation, government-spending-to-GDP ratio, rent-to-income ratio, and producer price indices (PPI) to find their relationship, if any. And we target on U.S. because it is still the world’s financial centre to this date. We hope that money managers, economists or any individuals who rely on macroeconomics information will be able to extract the ideas delivered in this project to make better decisions on their work.

Workflow



In this project, we would predict GDP growth from other variables of the U.S. using multivariate time series analysis. We obtained data from the Organization for Economic Co-operation and Development (OECD). After cleaning it, we selected 5 variables to forecast GDP growth in a Vector Autoregressive (VAR) model.

We applied Augmented Dickey-Fuller and KPSS tests for stationarity, and Johansen test for cointegration. We checked their correlation relationship with heatmap. Then, we fit the data into a VAR model, selected the lag order and forecasted the results.

In the end, we evaluated the time series using impulse response analysis and forecast error variance decomposition (FEVD) before inverting them back to their original form to plot the prediction trend.

Data Wrangling

<u>Data collection:</u>
OECD is a reliable source to provide all the data below for free.

1. Household debt to income ratio. We are all aware of what caused the subprime mortgage crisis in 2008 – right, too much debt. So, this will be the main focus.
2. Unemployment rate. We always hear about the unemployment rate of a country, and how many workers lose their jobs in a month.
3. Housing price to income ratio and housing price to rent ratio. Housing problem is a global concern. Everyone complains how uncontrolled the housing market is. These ratios represent a measure of affordability
4. Government spending to GDP ratio. Does it push the GDP growth or is it correlated if the government spend more to stimulate the economy? We will find out.
5. Inflation rate (or CPI). As long as the country is not experiencing hyperinflation, I believe inflation rate is proportional to GDP growth.
6. Producer price indices (PPI). This is related to manufacturing price. It measures the average changes in prices received by domestic producers for their output.

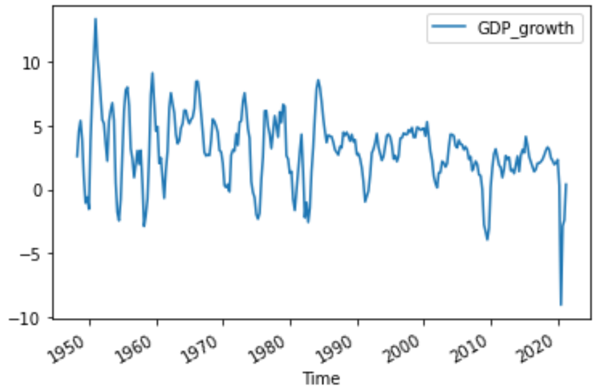
<u>Data cleaning:</u>
While the datasets we obtain from OECD are decent, missing values are expected. In general, more developed countries tend to have longer history of data available.

We need to take into consideration of the followings:
- Frequency. Do we want to work with monthly, quarterly or annual data?
- What are the values we are comparing against? For examples, over last period or over last year?
- What are the availabilities of our dataset? Some of them has a broad range of frequency available, e.g., monthly, quarterly and annual. While some only has annual data.

For practicality and standard metrics, I decided to go with quarterly data, comparing against the same period from last year, e.g., Q3-2021 compared to Q3-2020. If the most frequent time frame of any dataset is annual, we will have to up-sample the data to quarterly and linearly interpolate the missing values in between the periods. On the other hand, if monthly data of any dataset is available, we will down-sample and only take its quarterly data and ignore the months in between. Once every time series data is cleaned, they are visualized with basic plots to check for any suspicious entries before being merged into a single dataframe for further analysis.

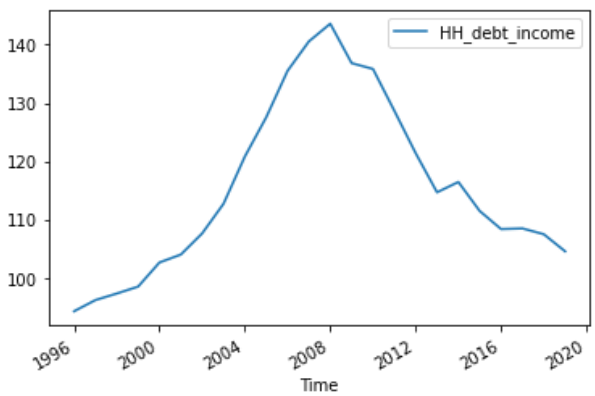
GDP growth

We can tell that the GDP growth is constant despite some irregular up and down movement. In fact, this should be stationary as it has already taken the percentage growth of the quarter from its previous year.



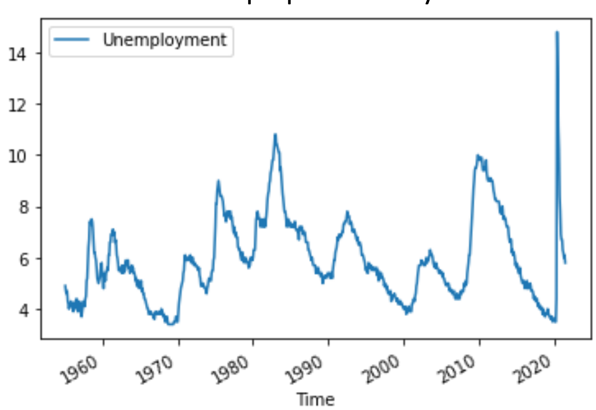
Household debt to income ratio

An obvious uptrend and downtrend in pre- and post-2008, respectively. We will see how first differencing looks like.



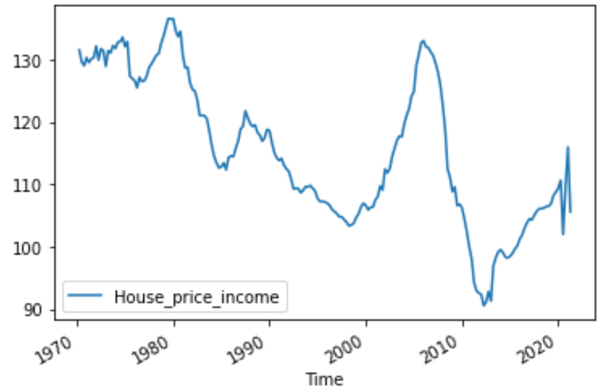
Unemployment rate

It has the same shape periodically. First differencing should make it stationary.



House price to income ratio

It is going down in general, but the few is unpredictable in terms of period and magnitude. We will difference it and see what it looks like.

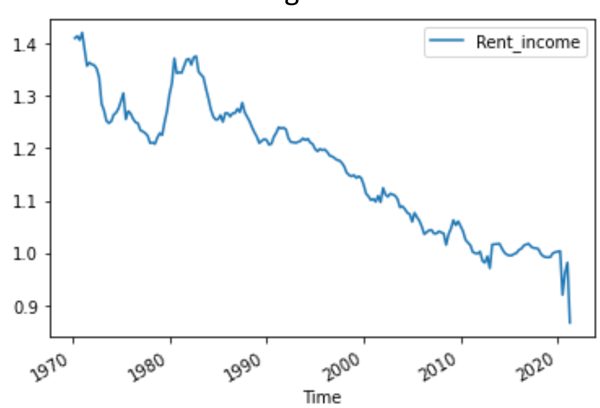


House price to rent ratio

Same as house price to income ratio above, the shape is unpredictable and will be transformed using differencing to see what it looks like.

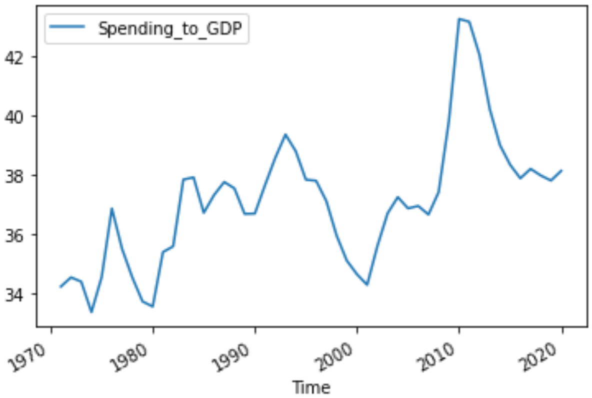
Rent to income ratio

Calculated from dividing the house price to income ratio by the house price to rent ratio, the rent to income ratio shows a general downtrend for the past 50 years.



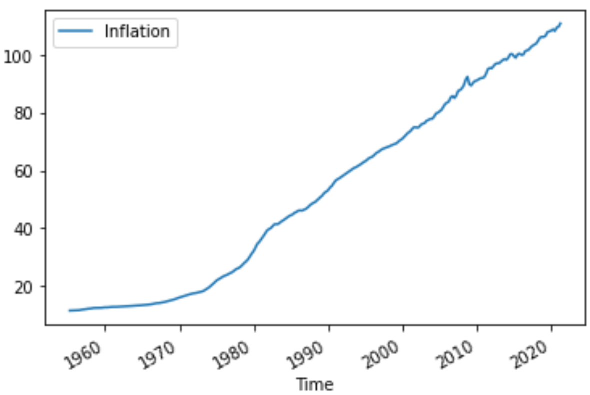
Government spending to GDP ratio

A subtle uptrend for the past 50 years. We will see if first differencing suffices to make the time series stationary.



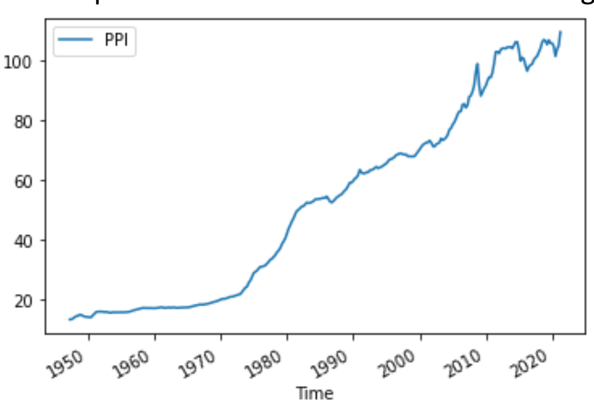
Inflation rate

First differencing should be able to transform the inflation rate time series to become stationary because of its constant uptrend.



Producer price index (PPI)

Trend is upward, but rate is not constant and there are some bumps along the along which makes it hard to predict. We will see if first differencing is enough to transform the time series to be stationary.



Exploratory Data Analysis

This is the step we need to perform the stationarity test and cointegration test, and transform any non-stationary time series through differencing, log, or square root methods.

Stationarity test – why is stationarity in time series analysis important?
When forecasting or predicting the future, time series models assume each point is independent of one another, and the statistical properties of a time series do not change over time. In other words, stationarity is required for sample statistics such as means, variances, and correlations to accurately describe the data at all time points of interest. Therefore, many analytical tools and statistical models rely on stationarity to make higher probabilistic prediction.

Stationarity tests are used to determine the presence of unit root in the series through different ways. Augmented Dickey-Fuller test is used as a difference stationarity test, while KPSS test is used as a trend stationarity test. Our goal is to make the trend to be strict stationary.

		Augmented Dickey-Fuller test (Test for difference)	
		Stationary	Non-stationary
KPSS test (Test for trend)	Stationary	Strict stationary	Trend stationary
	Non-stationary	Difference stationary	Non-stationary

Below is a snippet of the stationarity test on GDP growth, which passes both Dickey-Fuller and KPSS tests at a significant level of 95%. All the time series in this project will be passed through both tests to decide if it is needed for transformation. Note that GDP growth does not need to be differenced, as the "growth" is already differenced from the same period of last year. Hence, we see that it has satisfied the 95% confidence level without further adjustment.

```
=============================================
GDP_growth
Dickey-Fuller test (difference test):
Null hypothesis: series is not stationary
Test statistics         -3.394415
p-value                  0.011151
Lags used                0.000000
Observations used       97.000000
Critical value (1%)     -3.499637
Critical value (5%)     -2.891831
Critical value (10%)    -2.582928
dtype: float64
Fail to reject null hypothesis at critical value 1: series is not stationary
Reject null hypothesis at critical value 2: series is stationary
Reject null hypothesis at critical value 3: series is stationary

KPSS test (trend test):
Null hypothesis: series is stationary
Test statistics           0.401707
p-value                   0.076420
Lags used                12.000000
Critical value (10%)      0.347000
Critical value (5%)       0.463000
Critical value (2.5%)     0.574000
Critical value (1%)       0.739000
dtype: float64
Reject null hypothesis at critical value 1: series is not stationary
Fail to reject null hypothesis at critical value 2: series is stationary
Fail to reject null hypothesis at critical value 3: series is stationary
Fail to reject null hypothesis at critical value 4: series is stationary
```

Inflation rate, on the other hand, will have to be transformed before making any prediction on the trend, same as other time series that does not pass the stationarity tests.

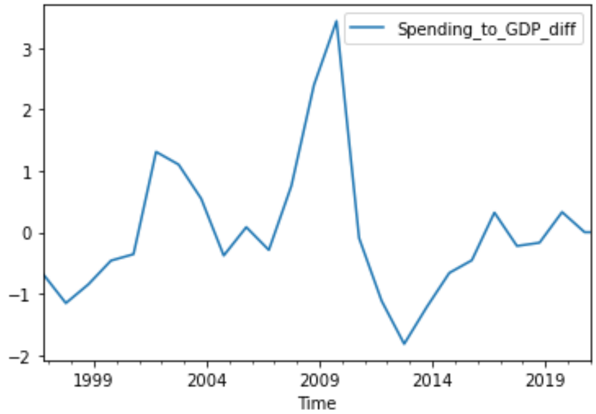
```
=============================================
Inflation
Dickey-Fuller test (difference test):
Null hypothesis: series is not stationary
Test statistics          -0.764169
p-value                   0.829421
Lags used                11.000000
Observations used        86.000000
Critical value (1%)      -3.508783
Critical value (5%)      -2.895784
Critical value (10%)     -2.585038
dtype: float64
Fail to reject null hypothesis at critical value 1: series is not stationary
Fail to reject null hypothesis at critical value 2: series is not stationary
Fail to reject null hypothesis at critical value 3: series is not stationary

KPSS test (trend test):
Null hypothesis: series is stationary
Test statistics            0.863227
p-value                    0.010000
Lags used                 12.000000
Critical value (10%)       0.347000
Critical value (5%)        0.463000
Critical value (2.5%)      0.574000
Critical value (1%)        0.739000
dtype: float64
Reject null hypothesis at critical value 1: series is not stationary
Reject null hypothesis at critical value 2: series is not stationary
Reject null hypothesis at critical value 3: series is not stationary
Reject null hypothesis at critical value 4: series is not stationary
```

We will iterate the process on each time series through differencing. Second differencing will be performed if such time series does not pass the stationarity test after the first differencing. If it is still non-stationary after second differencing, I would drop it and only work with the remaining ones for forecasting.

The resulting graphs below show the remaining time series I will use to predict GDP growth in this project.

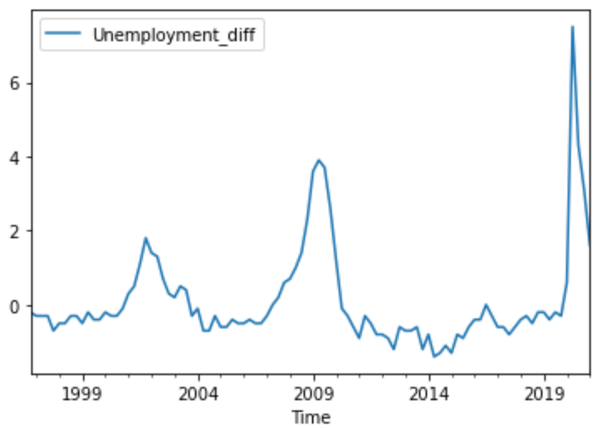
<u>First differenced government spending to GDP ratio</u>

Government spending to GDP ratio is a lot more stationary after differencing.



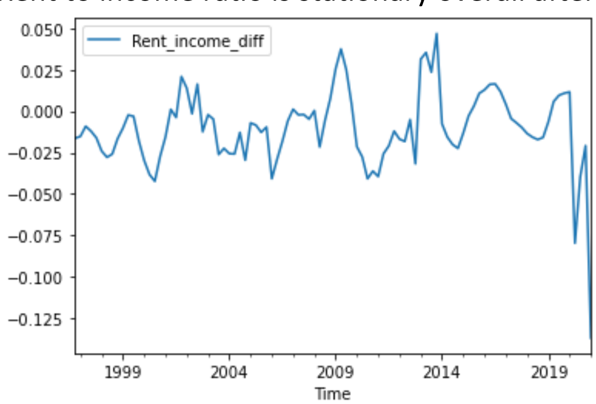
<u>First differenced unemployment rate</u>

Despite few humps, we see a much cleaner stationary trend in unemployment rate after differencing.



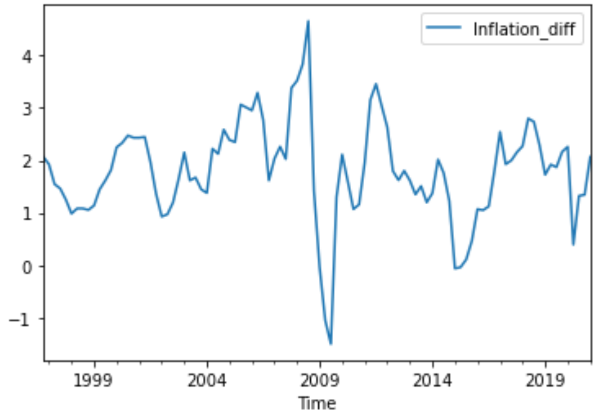
<u>First differenced rent to income ratio</u>

Rent to income ratio is stationary overall after differencing.



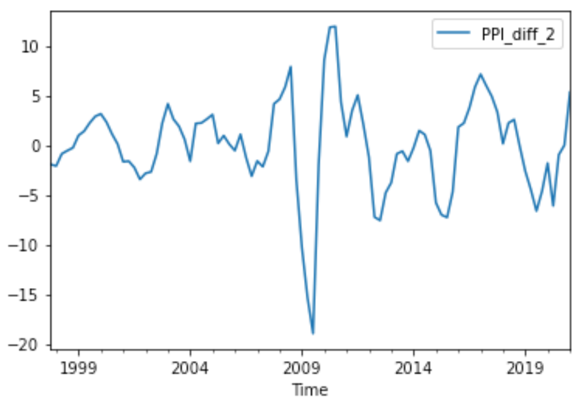
First differenced inflation rate

Inflation rate is stationary after differencing.



Second differenced PPI

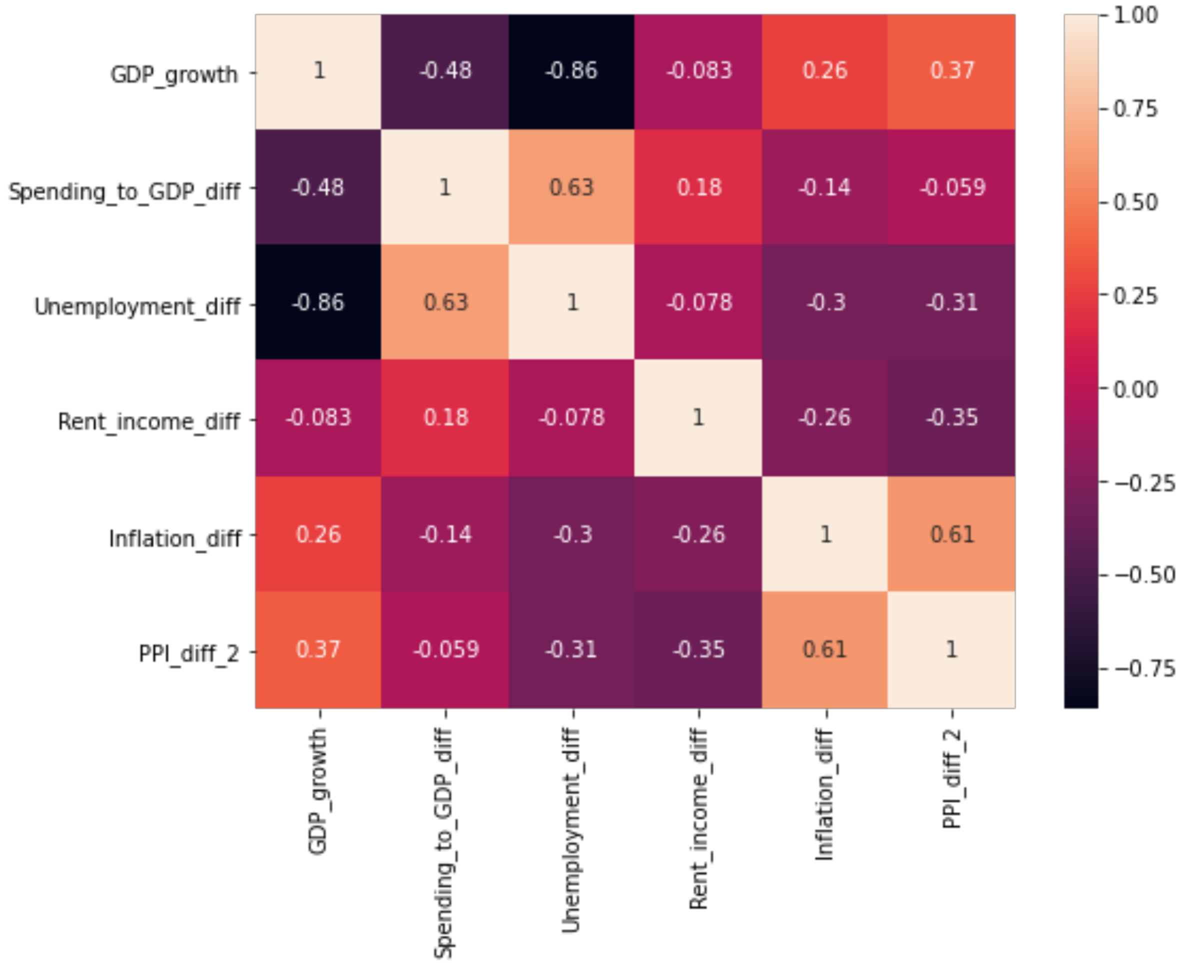
PPI is stationary after differencing twice.



Correlation heatmap

We also looked at correlation heatmap to have a preliminary idea of how correlated each variable is against each other. And we found that unemployment rate is highly correlated with GDP growth, followed by government spending to GDP, producer price index and inflation rate. Rent to income ratio does not seem to have a significant correlation with GDP growth.

I can think of a reason that contribute to the low correlation between the rent to income ratio and the GDP growth. It may be due to the extra calculation of dividing the house-price-to-rent ratio by the house-price-to-income ratio to get the rent-to-income ratio, which may have "dampen" the effect of how house price is correlated to the GDP growth. Regardless of the reason, we will move on to the next steps to check cointegration between the time series.



Cointegration test – why is it important?
Time series that are cointegrated with each other suggests that they will follow the same long-run path since they presumably have some common factor, such as they are in the same sector, or they have the same price movement. Therefore, cointegration can be applied for estimating long-run equilibrium between variables.

Johansen test is used to check the cointegration up to a maximum of 12 time series. Compared to Augmented Dickey-Fuller test that is based on autoregression, Johansen test is based on time series analysis, and it generates results regardless of the order of the time series.

Trace statistics
Null hypothesis: time series are not cointegrated
The trace statistics below tells us whether the sum of the eigenvalues is 0. At r ≤ 5, we can reject the null hypothesis at 95% confidence level and accept the alternate hypothesis, suggesting the series are cointegrated.

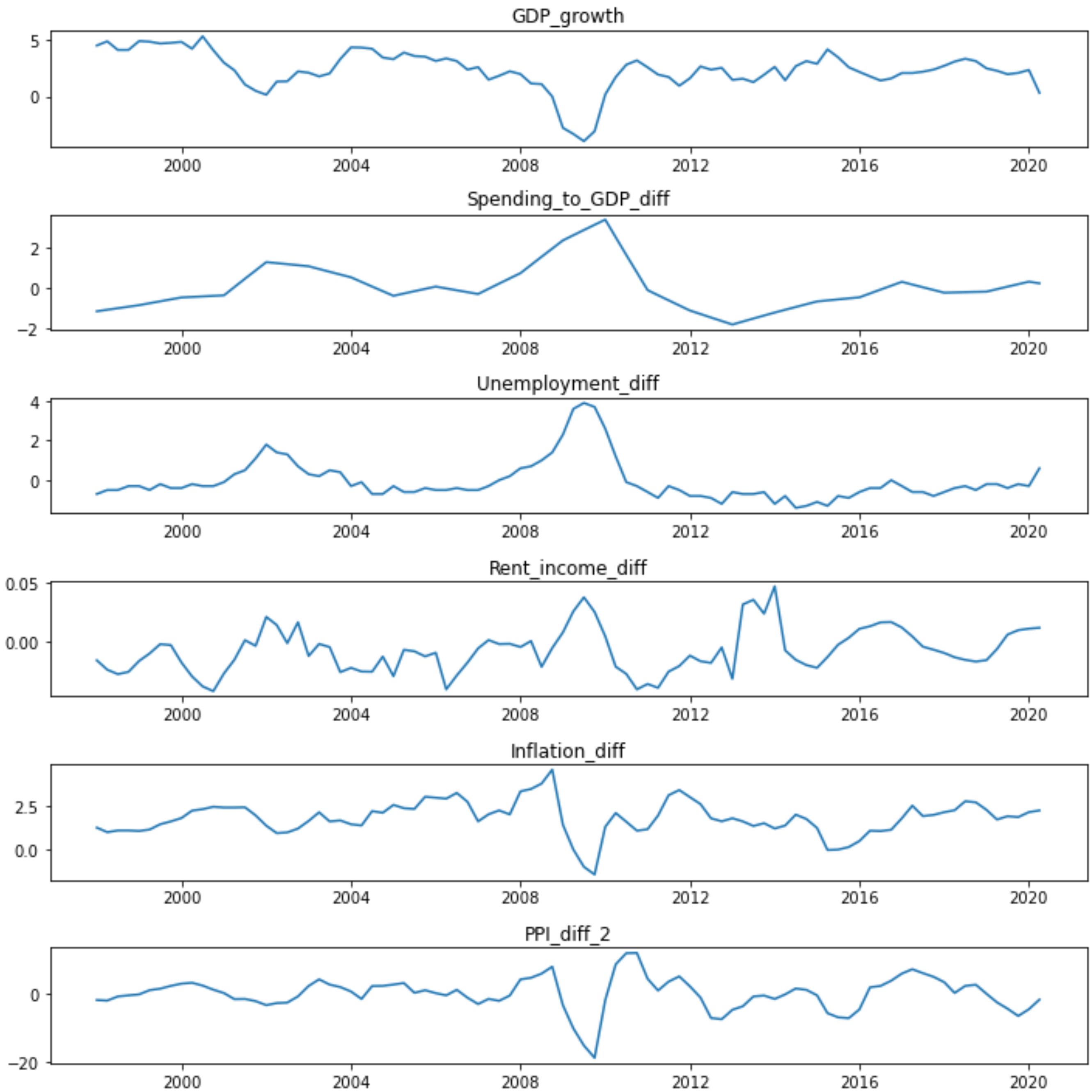
	cv=90%	cv=95%	cv=99%	trace stat	r
0	91.1090	95.7542	104.9637	132.680942	r = 0
1	65.8202	69.8189	77.8202	83.916761	r <= 1
2	44.4929	47.8545	54.6815	47.276366	r <= 2
3	27.0669	29.7961	35.4628	30.106856	r <= 3
4	13.4294	15.4943	19.9349	18.158044	r <= 4
5	2.7055	3.8415	6.6349	8.801103	r <= 5

Eigen statistics
Null hypothesis: time series are not cointegrated
The eigen statistics below tells us how strongly cointegrated the series are or how strong is the tendency to mean revert. At r ≤ 5, we can reject the null hypothesis at 95% confidence level and accept the alternate hypothesis, which means the series are cointegrated.

	cv=90%	cv=95%	cv=99%	eig stat	r
0	37.2786	40.0763	45.8662	48.764181	r = 0
1	31.2379	33.8777	39.3693	36.640395	r <= 1
2	25.1236	27.5858	32.7172	17.169509	r <= 2
3	18.8928	21.1314	25.8650	11.948812	r <= 3
4	12.2971	14.2639	18.5200	9.356941	r <= 4
5	2.7055	3.8415	6.6349	8.801103	r <= 5

Modeling

We used Vector Autoregression (VAR) for multivariate time series analysis because it is able to understand and use the relationship between several variables, which is particularly useful for describing the dynamic behavior of the data and also provides better forecasting results. Below is a quick comparison of all the time series used in the forecast.



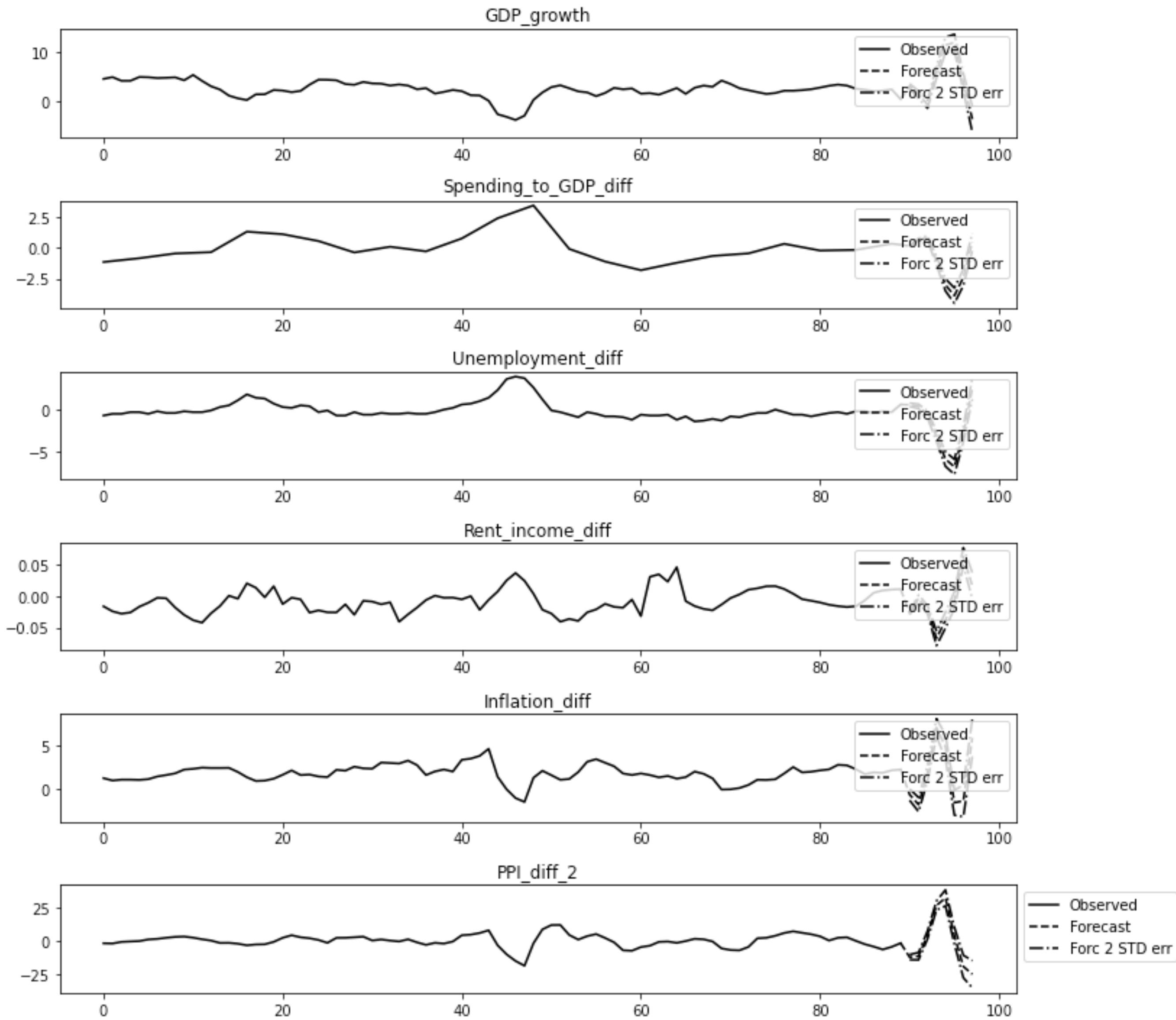
Lag order selection – why is it important?
We chose lag = 12 because it returns the lowest AIC between in the range of 1 to 12 without returning errors. As you can see in the graph below, the AIC for 12 lags is -79.2804.

Choosing AIC over BIC
As a rule of thumb, if you look for a higher predictive power, choose AIC. But if you look for a higher explanatory power, go for BIC. Both indicate the optimal model at their lowest values.

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Tue, 03, Aug, 2021
Time:                     14:40:04
--------------------------------------------------------------------
No. of Equations:         6.00000    BIC:                   -66.0466
Nobs:                     78.0000    HQIC:                  -73.9827
Log likelihood:           2865.87    FPE:                3.72855e-31
AIC:                     -79.2804    Det(Omega_mle):     1.73808e-33
--------------------------------------------------------------------
Results for equation GDP_growth
==========================================================================================
                            coefficient       std. error           t-stat            prob
------------------------------------------------------------------------------------------
const                          1.358866         1.507464            0.901           0.367
L1.GDP_growth                 -0.859822         0.410261           -2.096           0.036
L1.Spending_to_GDP_diff       -2.733166         1.520391           -1.798           0.072
L1.Unemployment_diff          -0.618433         1.037732           -0.596           0.551
L1.Rent_income_diff           26.931163        17.666132            1.524           0.127
L1.Inflation_diff             -2.021421         1.224211           -1.651           0.099
L1.PPI_diff_2                  0.575526         0.372005            1.547           0.122
L2.GDP_growth                 -0.402642         0.412703           -0.976           0.329
L2.Spending_to_GDP_diff        0.125916         2.693030            0.047           0.963
L2.Unemployment_diff          -0.592058         1.481904           -0.400           0.690
L2.Rent_income_diff           18.333273        13.118627            1.397           0.162
L2.Inflation_diff             -1.712199         1.181714           -1.449           0.147
L2.PPI_diff_2                  0.298473         0.388722            0.768           0.443
L3.GDP_growth                  0.036086         0.367740            0.098           0.922
L3.Spending_to_GDP_diff        4.107359         3.196769            1.285           0.199
L3.Unemployment_diff          -1.555735         1.467040           -1.060           0.289
L3.Rent_income_diff           -8.078692        25.395536           -0.318           0.750
L3.Inflation_diff             -2.154883         1.290066           -1.670           0.095
L3.PPI_diff_2                  0.173975         0.347061            0.501           0.616
L4.GDP_growth                 -0.270262         0.315999           -0.855           0.392
L4.Spending_to_GDP_diff        0.256761         1.977541            0.130           0.897
L4.Unemployment_diff          -3.460657         1.681094           -2.059           0.040
L4.Rent_income_diff           33.957997        17.001685            1.997           0.046
L4.Inflation_diff             -1.796284         1.563137           -1.149           0.250
L4.PPI_diff_2                  0.062464         0.416148            0.150           0.881
```

<u>Forecasting</u>

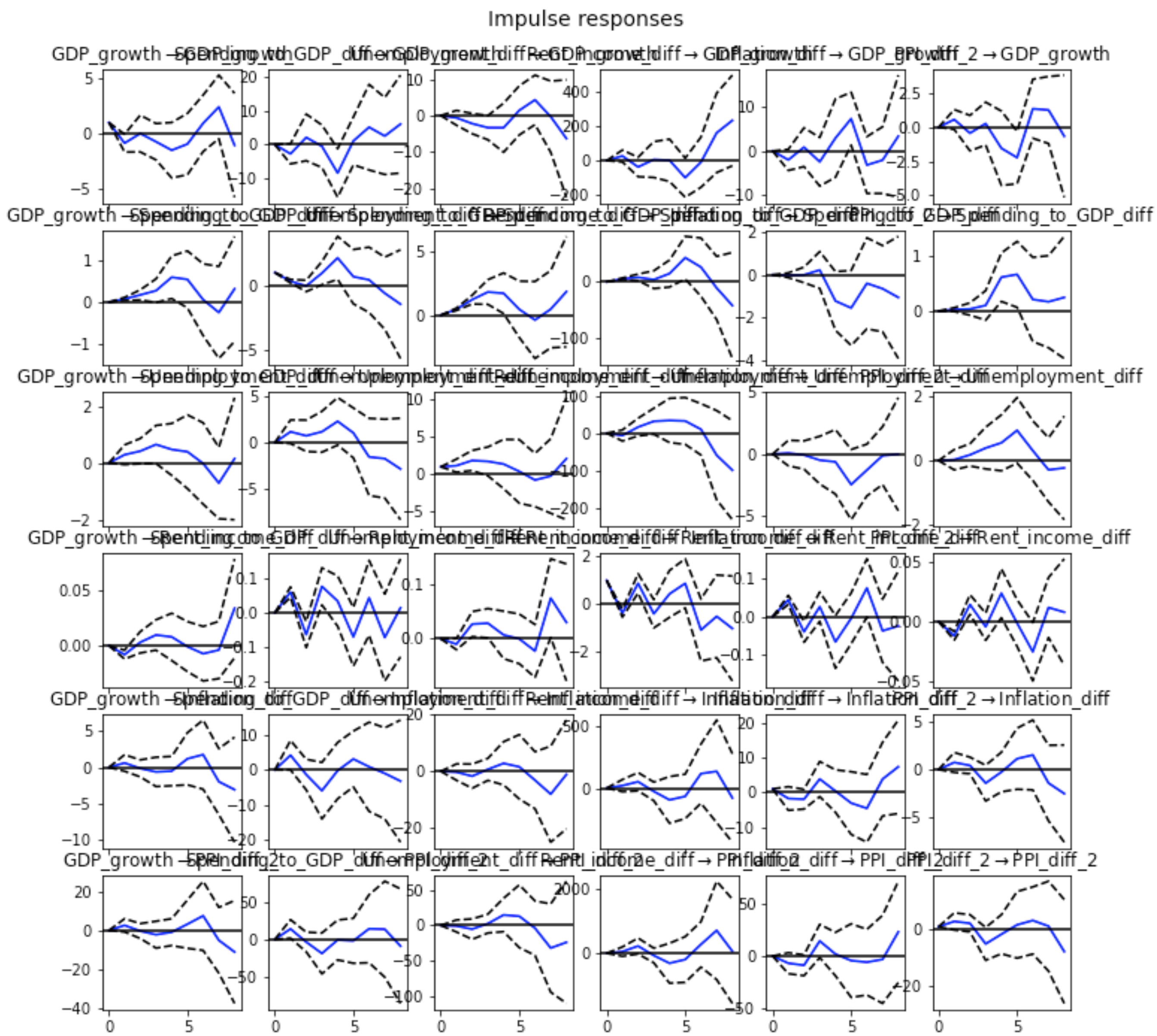
Below is the 8-step forecast of each variable with 12 lags length, along with 2 asymptotic standard errors. Each of the time series forecast shows a wider range compared to their observed values.



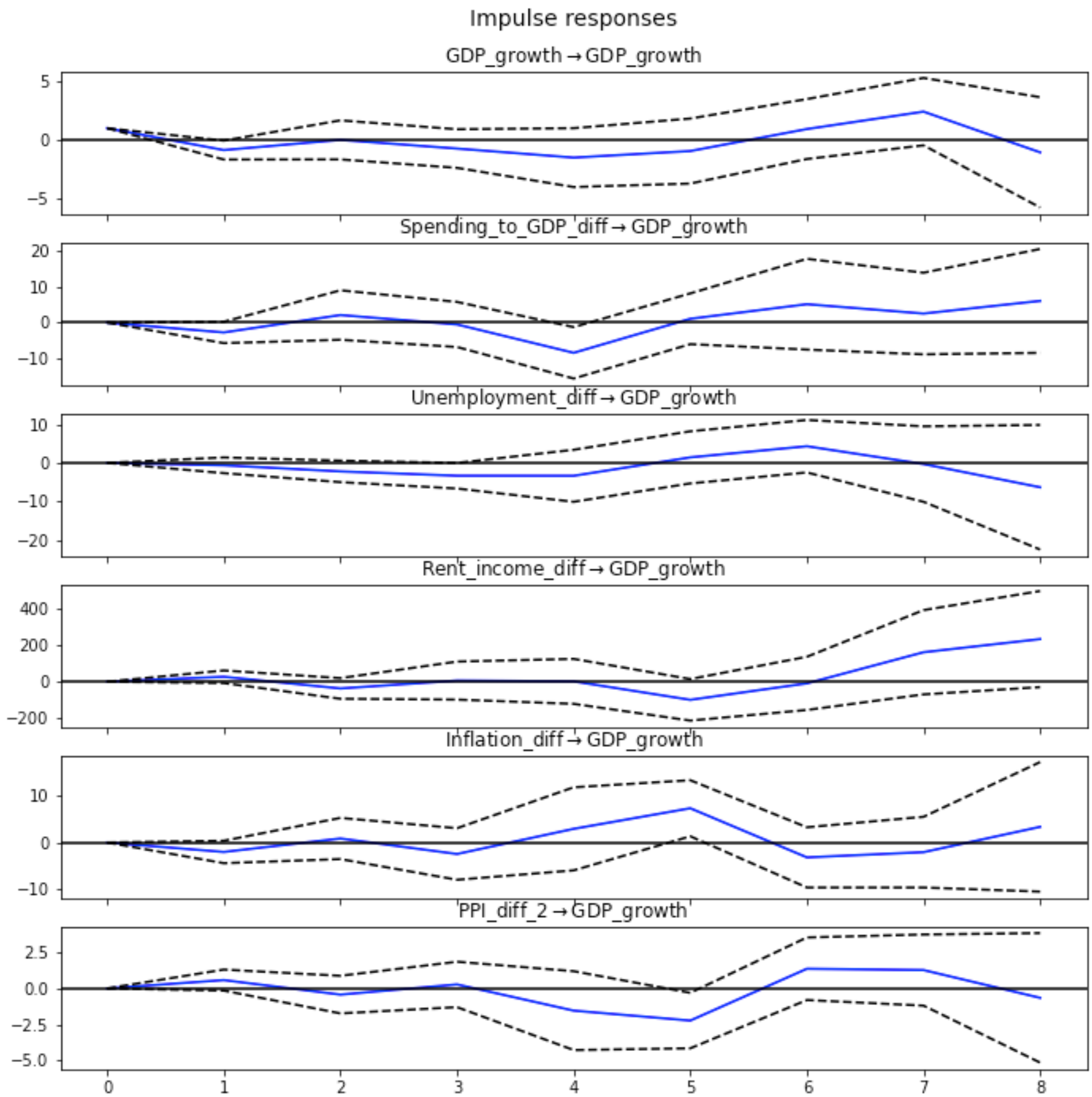
Evaluation/Structural Analysis

Impulse response analysis – why is it of interest?

We want to know the response of one variable to an impulse in another variable in this analysis. It is more of a causality test by checking how the inflation rate or other variables change will cause effects on the GDP growth. Let's say there is a reaction of unemployment rate to an impulse in GDP growth, then we may call GDP growth causal for unemployment rate. The first row below illustrates how GDP growth responds to the impulse of each variable.



Below is a closer look at the response of GDP to the impulse of each variable for forecasting the next 8 periods. Notice that the rent-to-income ratio is seen to induce the GDP growth to a high extent of level from period 6 onwards. We also know that the shocks can be considered as forecast errors and the impulse responses are sometimes referred to as forecast error impulse responses.



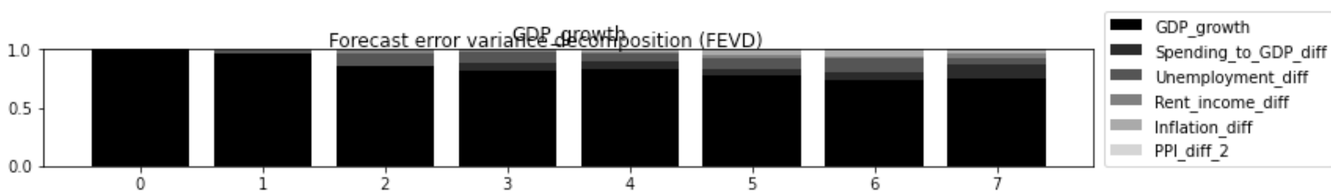
Forecast error variance decomposition (FEVD) – what does it imply?

It indicates how much of the forecast error variance of each variable can be explained by the exogenous shocks to the other variables in each of the steps in the forecast, 8 steps in this case. The decomposition below clearly shows that most of the error variance of GDP growth in the forecasting periods can be explained by itself, then around 6-12% of the error variance can be explained by each of government spending to GDP ratio and unemployment rate from the 3rd or 4th periods onward. A small 2-6% of error variance can be explained by each of rent-to-income ratio and inflation rate. Lastly, no error variance is related to PPI.

FEVD for GDP_growth

	GDP_growth	Spending_to_GDP_diff	Unemployment_diff	Rent_income_diff	Inflation_diff	PPI_diff_2
0	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1	0.968181	0.002702	0.016125	0.003038	0.009954	0.000000
2	0.858026	0.002989	0.102566	0.021371	0.015048	0.000000
3	0.817643	0.063195	0.099412	0.013462	0.006288	0.000000
4	0.830714	0.070067	0.063785	0.011868	0.023566	0.000000
5	0.769835	0.066011	0.089410	0.022449	0.052296	0.000000
6	0.732179	0.064685	0.121298	0.021144	0.060694	0.000000
7	0.743930	0.124775	0.061089	0.039123	0.031083	0.000000

Below is the graphical representation of the error contribution from other variables on GDP growth in each of the 8 forecasting periods.



Inverting

Finally, the graphs below are the prediction of each time series after being inverted back to their original form, except GDP growth, which is in its original form and what we wanted to predict from the beginning.

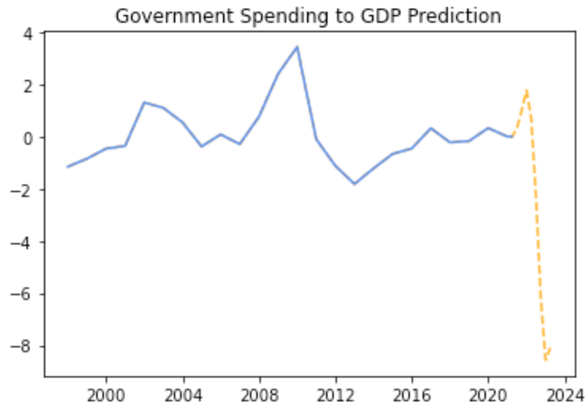
Time	GDP_growth_fc	Spending_to_GDP_diff_forecast	Unemployment_diff_forecast	Rent_income_diff_forecast	Inflation_diff_forecast	PPI_diff_2_forecast
2021-06-30	3.044324	0.304610	8.065243	-0.106711	-0.253518	-20.160563
2021-09-30	1.236768	0.995783	8.439629	-0.109911	-2.062504	-45.921510
2021-12-31	-0.709559	1.776458	7.852496	-0.131215	-0.224370	-67.260160
2022-03-31	5.891302	0.736458	4.690814	-0.198767	6.885260	-61.938310
2022-06-30	11.388645	-2.276143	-1.148166	-0.237064	11.793005	-24.221601
2022-09-30	11.936260	-6.163818	-7.907934	-0.243362	10.207746	17.332052
2022-12-31	3.813357	-8.570030	-10.818397	-0.179616	8.828006	39.398590
2023-03-31	-3.751944	-8.105402	-8.334871	-0.162680	14.764085	36.094787

GDP growth

The two highest predicted growth at around 11% in Q2-2022 and Q3-2022 looks reasonable to me as a bounce-back after plummeting to its lowest at -9% in Q2-2020.

Government spending to GDP ratio

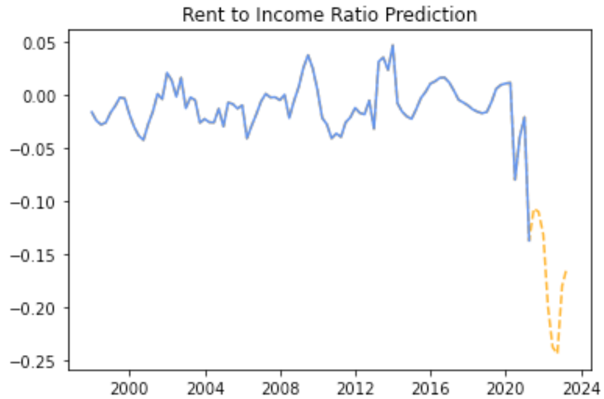
Instead of going -8% in 2022 and 2023, I believe that the spending would go the opposite direction especially after the pandemic hit that the government will have to spend more on rebuilding the economy.



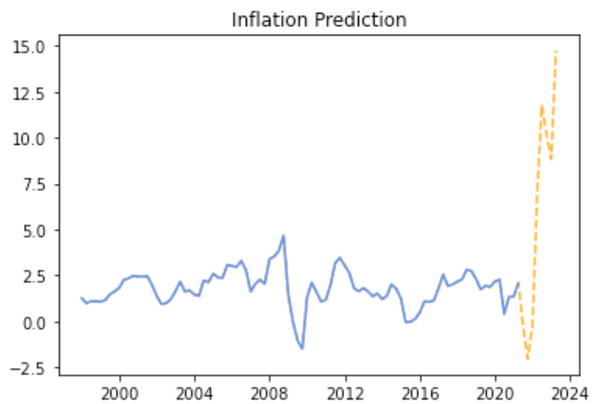
Unemployment rate

It is doubtful to me that the unemployment rate will drop to -10% in one quarter based on its track record. As we have seen in the sections above, unemployment rate may be the most important variable to forecast GDP growth as they are highly correlated and 6-12% of the error variance of GDP growth can be explained by the differenced unemployment rate.

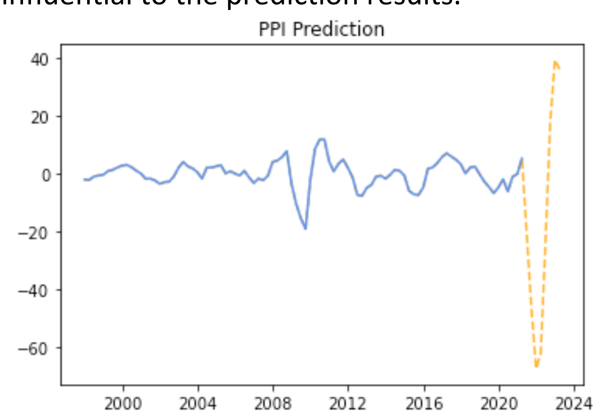
Rent to income ratio
The ratio continues to break down in the forecast. As we have seen in the impulse response analysis, it will induce GDP growth to a high extent of level, having significant in the GDP growth forecast.



Inflation growth
The forecast of inflation growth having an average of higher than 10% in the last 4 forecasting periods looks doubtful to me albeit not highly unlikely due to the economy recovery after the pandemic.



PPI
Forecast of PPI dropping to -67% in Q4-2021 seems surreal to me. However, the previous sections show that it is not likely to be an important metric to GDP growth forecast. So, its value may not be influential to the prediction results.



Takeaways

The model created in this project forecasts the potential direction of all the time series affecting each other dynamically. We made the time series stationary to perform the analysis. Through cointegration test, we identified how many time series are integrated together. Subsequently, we selected the optimal lag order for the VAR model for forecasting. And we evaluated our findings with impulse response analysis and FEVD.

Similar to other machine learning models, the parameters here can be changed to yield different results. For examples, if we choose a different significance level in the Johansen test, we may add or reduce variables. In addition, we may choose a different lag order if we focus on BIC or HQIC instead of AIC. As British statistician George Box once said, "all models are wrong but some are useful." The prediction of this project may (or most likely) be inaccurate, it, however, sheds some light on what we should focus on when making dynamic prediction like this one.

In a nutshell, this project tells us few key takeaways:

- As shown in the heatmap, unemployment rate has the highest correlation with GDP growth, whereas rent to income ratio has the lowest correlation with GDP growth.
- From impulse response analysis, rent to income ratio has a huge effect on GDP growth.
- From FEVD, standard error of GDP growth forecast most likely won't be contributed by PPI, meaning it has little effect on the forecast.
- From Johansen test, we may only need 2 variables from our dataset to make the existing time series cointegrated to provide sufficient predictive power.

To summarize, unemployment rate is always in the same conversation with GDP growth. GDP growth may be explained by inflation, government spending and PPI, but not too relevant with rent-to-income ratio.

Future Work

2 quick fixes for better results:

- Based on the results we obtained from Johansen test for cointegration, we can eliminate all columns but 1-2 most significant ones along with the GDP growth, which is our target variable. This would in turn have less unnecessary effect or noise for our prediction.
- We can replace the most insignificant variables with other variables to see the new predictive power, as we have already found out some of them have little effect on GDP growth.

Going forward, we can also use the same approach to test the other countries of our choice. We may even find surprising results and get signals prior to a potential market crash.

Business Recommendation

As we have discussed above, the prediction of this project may be inaccurate. But every model has its own value that helps us answer the following questions.

<u>Why is the model valuable? How is it applied? Who will benefit from using it?</u>
The essence of this model is to show us the dynamic relationship between the variables, the number of time series used to be sufficient enough to make prediction, the ideal lag order to forecast, the portion of the error variance can be explained by which variable, etc. Knowing this will help us select more variables or remove existing variables, and fine-tune our parameters for better forecasting results.

This model was demonstrated to project GDP growth. However, we can use the same approach to project unemployment rate, inflation rate, or any other variables used in this project. We can also add other variables such as foreign investment, country current account balance, household spending, just to name a few, into our prediction. In trading and investing environment, we can change our variables into other types like stocks, interest rate, currencies or commodities to find their dynamic relationship using multivariate time series analysis.

As this project particularly focuses on GDP growth projection, any asset managers, economists or even politicians who take macroeconomics into consideration when performing their work will benefit from using it. But keep in mind that the end of this project is the beginning of a more advanced and customized ones. Let’s keep up the hard work.