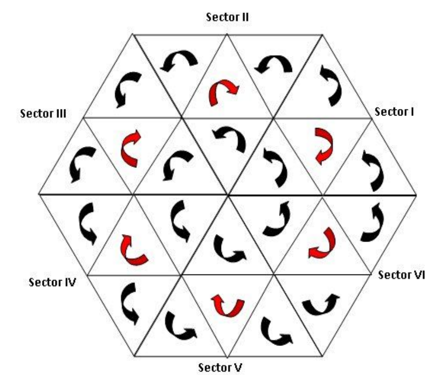
Chapter 5

Advanced Field Oriented Control



Professor Min-Fu Hsieh

Fall Semester – 2022

https://www.researchgate.net/publication/318930471_SIMULATION_AND_IMPLEMENTATION_OF_TWO-LEVEL_AND_THREE-LEVEL_INVERTERS_BY_MATLAB_AND_RT-LAB/figures?lo=1

Field Weakening (FW) Control

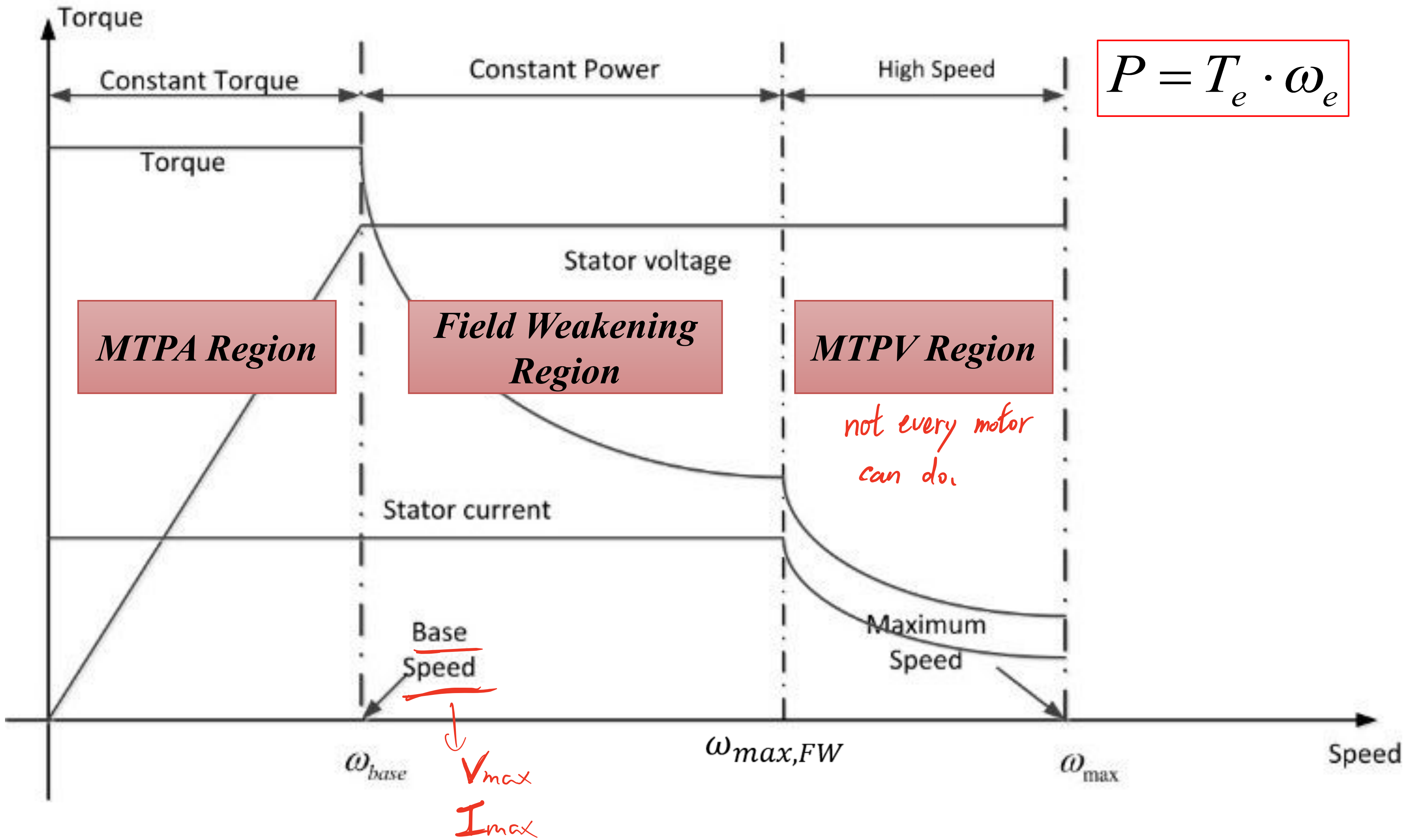
- Permanent magnet synchronous machines (PMSMs) have high efficiency, high power density, high torque-to-inertia ratio, and fast dynamic response. These features make this kind of machines very attractive for electric vehicle (EV) applications. However, because of their nature, i.e., constant magnet flux provided by magnets, these machines have a narrow constant power speed range (CPSR). This limitation is a strong drawback for application of PMSMs in electric vehicles, where high speed is the top requirement.
- In order to improve this disadvantage, it is necessary to give a thorough analysis about the control principles of PMSMs for different operation conditions.

- Field Weakening (FW) control needs to be implemented efficiently to obtain wide constant power speed range (CPSR). However, a specific power inverter cannot drive PMSMs at high speeds because of the fact that the back-EMF is proportional to motor speed and air gap flux, thus, leading to higher back-EMF values. Once the back-EMF becomes larger than the maximum output voltage of the drive, the PMSM will be incapable of drawing current and hence incapable of developing torque, thus rotor speed of such a motor cannot be increased unless the air gap flux can be weakened.
- Considering that rotor magnetic field generated by the PMs can only be weakened indirectly through armature MMF demagnetization of the PMs. extended speed range can be achieved by means of FW control. While the resultant air-gap flux is indirectly reduced/weakened and correspondingly the motor speed is increased

Generally, the motor operation can be happened following modes:

- Mode 1 : **current-limited region (MTPA region)** This is the region from zero to rated speed where maximum torque is obtained by operating with rated current at the MPTA torque angle β. This corresponds to the point at which the torque hyperbola are tangent to the current-limit circle.
- Mode 2 : **current-and-voltage-limited region (FW region).** Here the drive is operated with rated current at the minimum current angle required to give rated terminal voltage, i.e. at the intersection of the voltage and current-limit loci.
- Mode 3: **voltage-limited region (MTPV).** Here the drive operates to give maximum torque with a limited voltage. This corresponds to the point where the torque hyperbolas are tangent to the voltage-limit ellipse.

T-N Characteristic of PMSM



$$P = T_e \cdot \omega_e$$

Supplied Current and Voltage

- In the real case, the current and voltage are subject to real constraints:

$Current\ limit\ equation$: ${i_d}^2 + {i_q}^2 = {i_s}^2 \le {i_{max}}^2 \rightarrow$ *a continuous circle form*

$Voltage\ limit\ equation$: ${v_d}^2 + {v_q}^2 = {v_s}^2 \le {v_{max}}^2 \rightarrow$ *difference of SPM and IPM*

- Assume that motor runs at high speed mode (and sufficiently high), so the phase resistance R is neglected because its voltage drop is much smaller than X_L , we have voltage on d-q axis as:

$$v_d = \omega_e L_q i_q$$

$$v_q = \omega_e L_d i_d + \omega_e \psi_f$$

Steady state and assume that the speed is fast enough to ignore the resistance voltage drop

$$\Rightarrow \quad \frac{\left(i_d + \frac{\psi_f}{L_d}\right)^2}{\left(\frac{V_S}{\omega_e L_d}\right)^2} + \frac{(i_q)^2}{\left(\frac{V_S}{\omega_e L_q}\right)^2} = 1 \qquad \text{(A)}$$

☐ Taking into account (A) in case of SPM and IPM

Supplied Current and Voltage

SPM: $L_d = L_q = L_s$

Circle equation

$$\frac{(i_d + \frac{\psi_f}{L_s})^2}{(\frac{V_S}{\omega_e L_s})^2} + \frac{(i_q)^2}{(\frac{V_S}{\omega_e L_s})^2} = 1$$

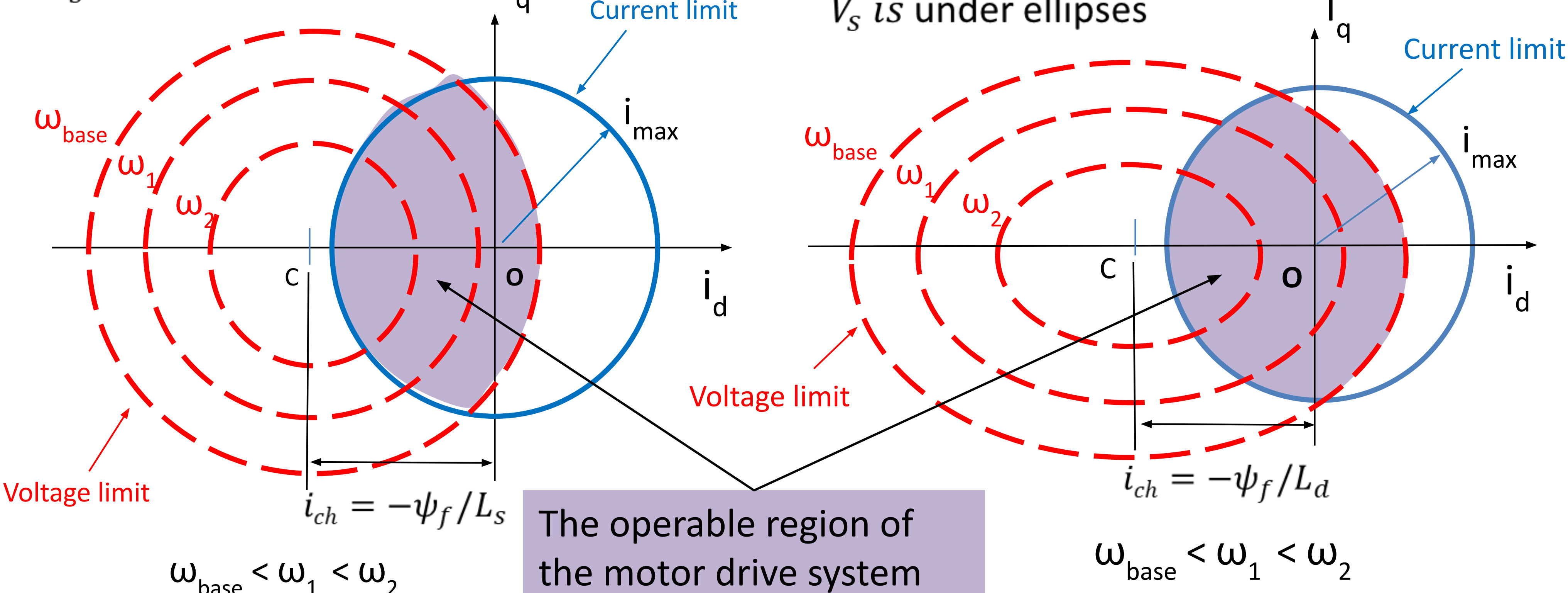
IPM: $L_d < L_q$

Ellipse equation

$$\frac{\left(i_d + \frac{\psi_f}{L_d}\right)^2}{\left(\frac{V_S}{\omega_e L_d}\right)^2} + \frac{(i_q)^2}{\left(\frac{V_S}{\omega_e L_q}\right)^2} = 1$$

V_s *is* under circles

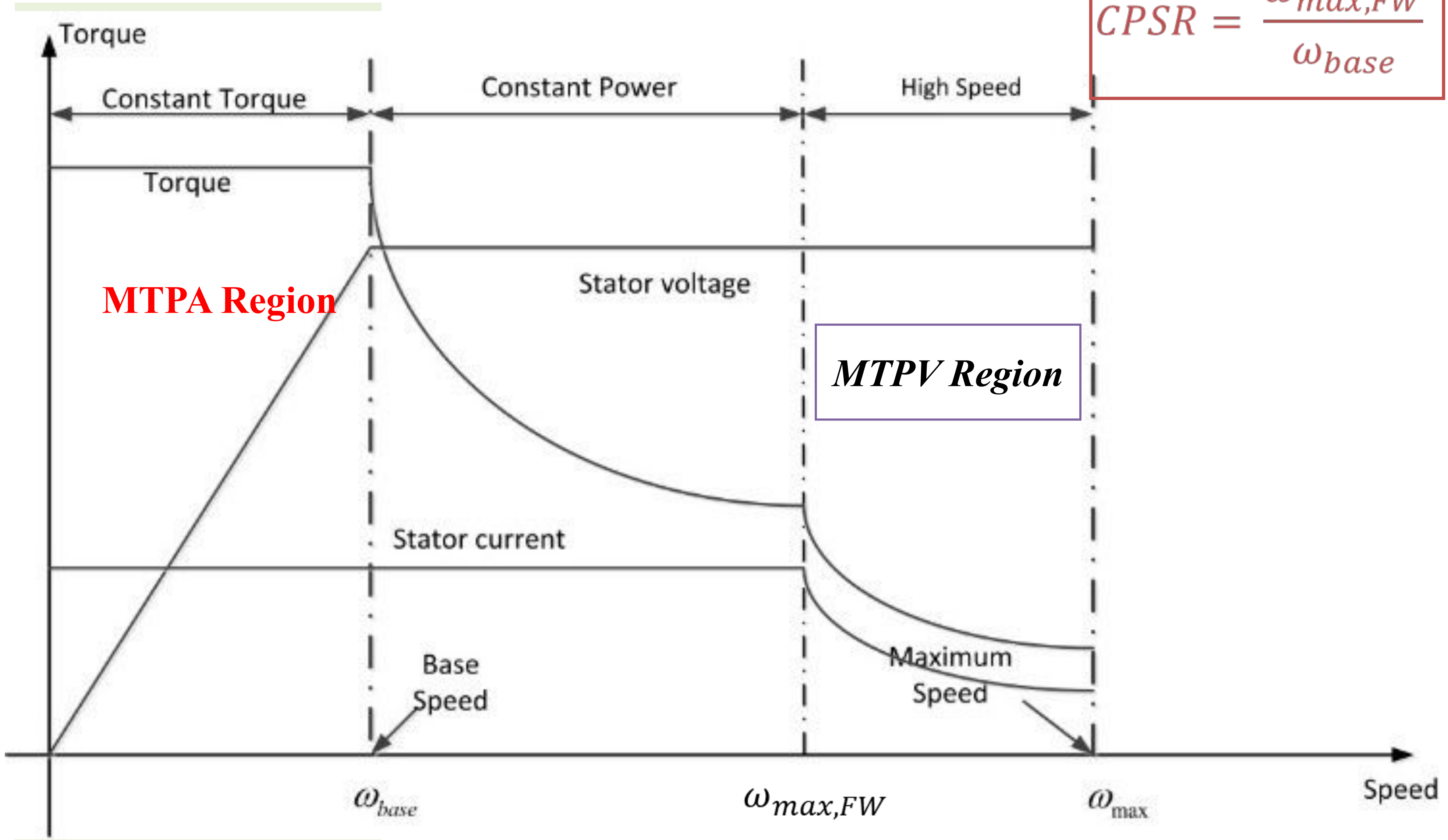
V_s *is* under ellipses



The operable region of the motor drive system

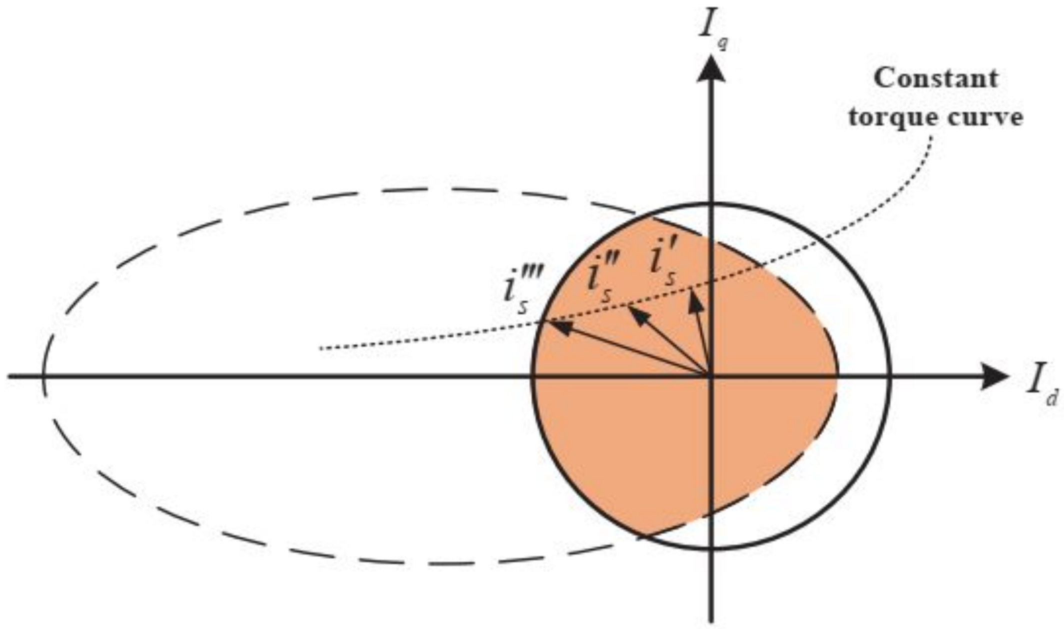
Maximum Torque Per Ampere Control (MTPA)

$$CPSR = \frac{\omega_{max,FW}}{\omega_{base}}$$



Control of MTPA Region

- A specific torque can be obtained by utilizing the innumerable combinations of i_d and i_q sets but there exists only a single pair that gives the torque with the minimum stator current.
- This will lead to the maximum torque in response to the minimum current or indirectly minimum losses. Hence, this approach is usually named as maximum toque per ampere or MTPA



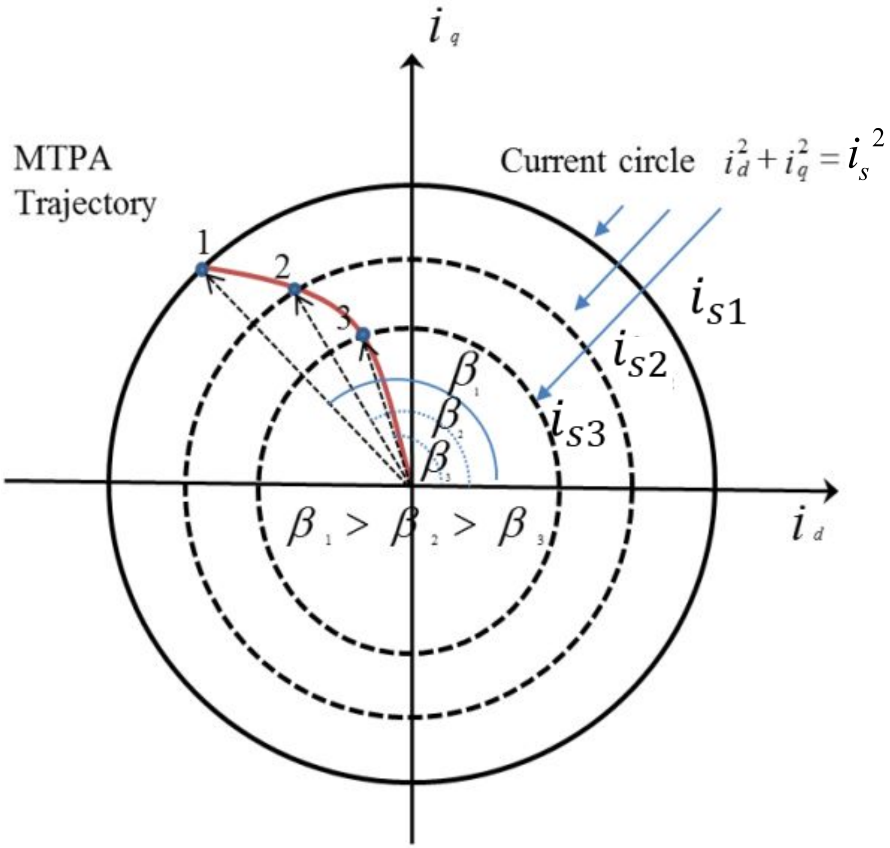
$$T_{e\_IPM} = \frac{3}{2}\frac{P}{2}(\psi_f i_q + (L_d - L_q) i_d i_q)$$

Ming-Shyan Wang*, Min-Fu Hsieh, Ika Noer Syamsiana, and Wei-Chin Fang, **"Fuzzy Maximum Torque per Ampere and Maximum Torque per Voltage Control of Interior Permanent Magnet Synchronous Motor Drive",** Sensors and Materials, Vol. 29, No. 4 (2017)

Control of MTPA Region

For each supply current value, there is a particular pair of d-axis current and q-axis current that results in the maximum torque under that condition.

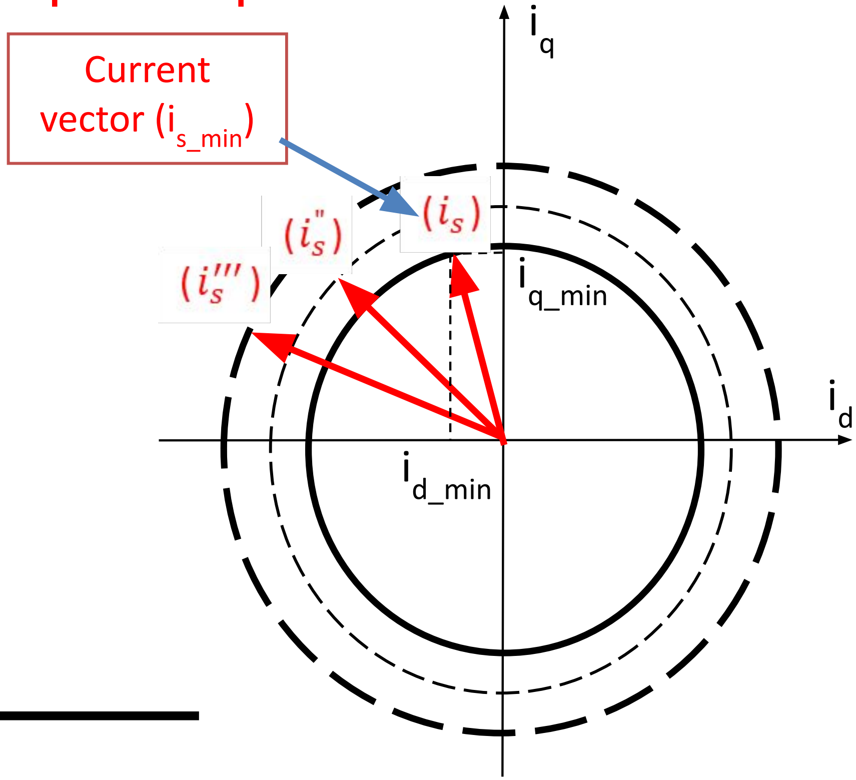
The torque angle for the MTPA operation is determined not only by motor parameters but also by supply stator current value (i_s). For different i_s, there are different cross points between the current circles and the MTPA trajectory which yields different torque angles for MTPA operations



Torque angle β for maximum torque condition

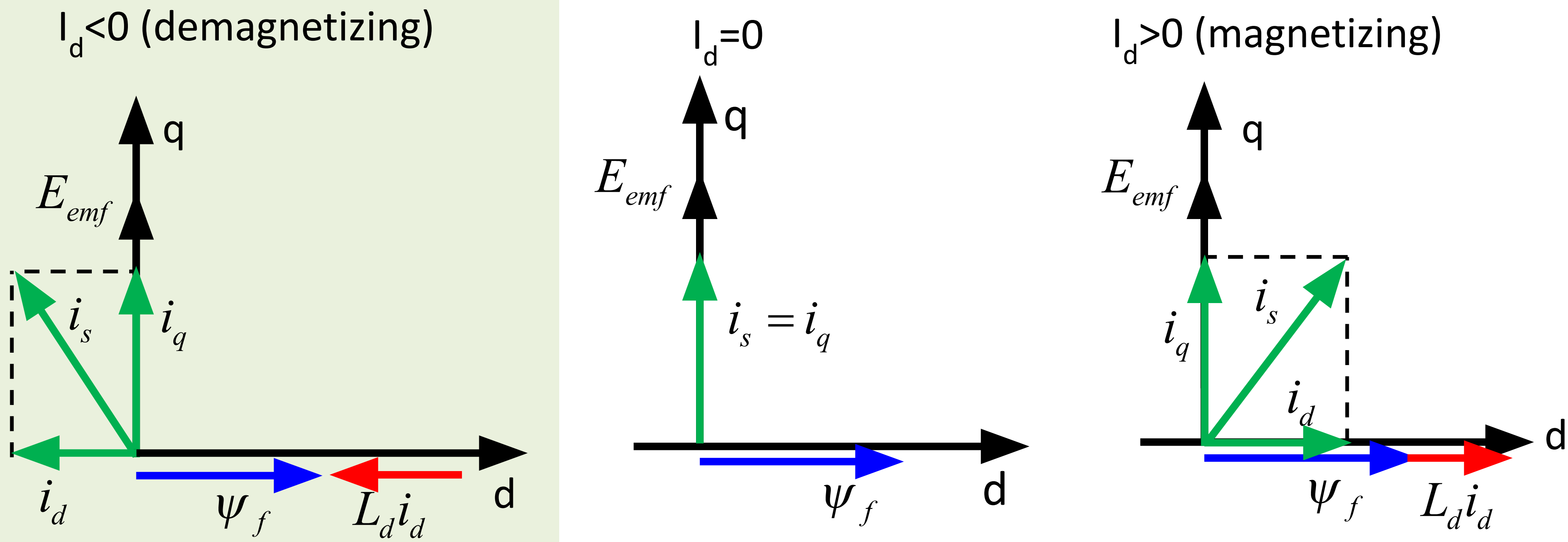
Control of MTPA Region

- A specific torque can be obtained by utilizing the innumerable combinations of i_d and i_q sets but there exists only a single pair that gives the torque with the minimum stator current.
- This will lead to the maximum torque in response to the minimum current or indirectly minimum losses. Hence, this approach is usually named as maximum toque per ampere or MTPA



Control of MTPA Region

During the FW operation region, a demagnetizing MMF is established by the stator currents and winding to counteract the “apparent” MMF established by PMs mounted on the rotor.

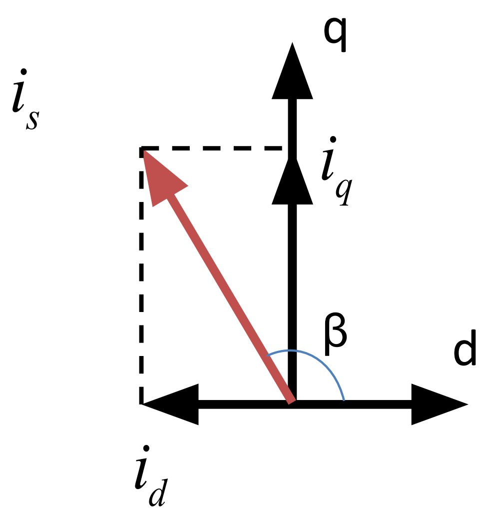


$$\psi_d = \cancel{+} L_d i_d + \psi_f \qquad \psi_d = \psi_f \qquad \psi_d = L_d i_d + \psi_f$$

$$E = \psi_f \omega_e \qquad E = \psi_f \omega_e \qquad E = \psi_f \omega_e$$

Control of MTPA Region

- In order to produce the maximum torque at a given current value, the torque expression for PM motors has to be analyzed



With β is current angle, we have:

$$i_s = i_d + ji_q$$

$$i_d = i_s \sin(90^o - \beta) = i_s \cos\beta$$

$$i_q = i_s \cos(90^o - \beta) = i_s \sin\beta$$

$$(180^o \geq \beta \geq 0^o)$$

For SPM, no saliency **(L_d=L_q)**
 reluctance torque is zero

$$T_{e\_SPM} = \frac{3}{2}\frac{P}{2}\psi_f i_q$$

(i_d=0, i_q=i_s , β=90°), there is the maximum torque

For IPM, (L_d<L_q)

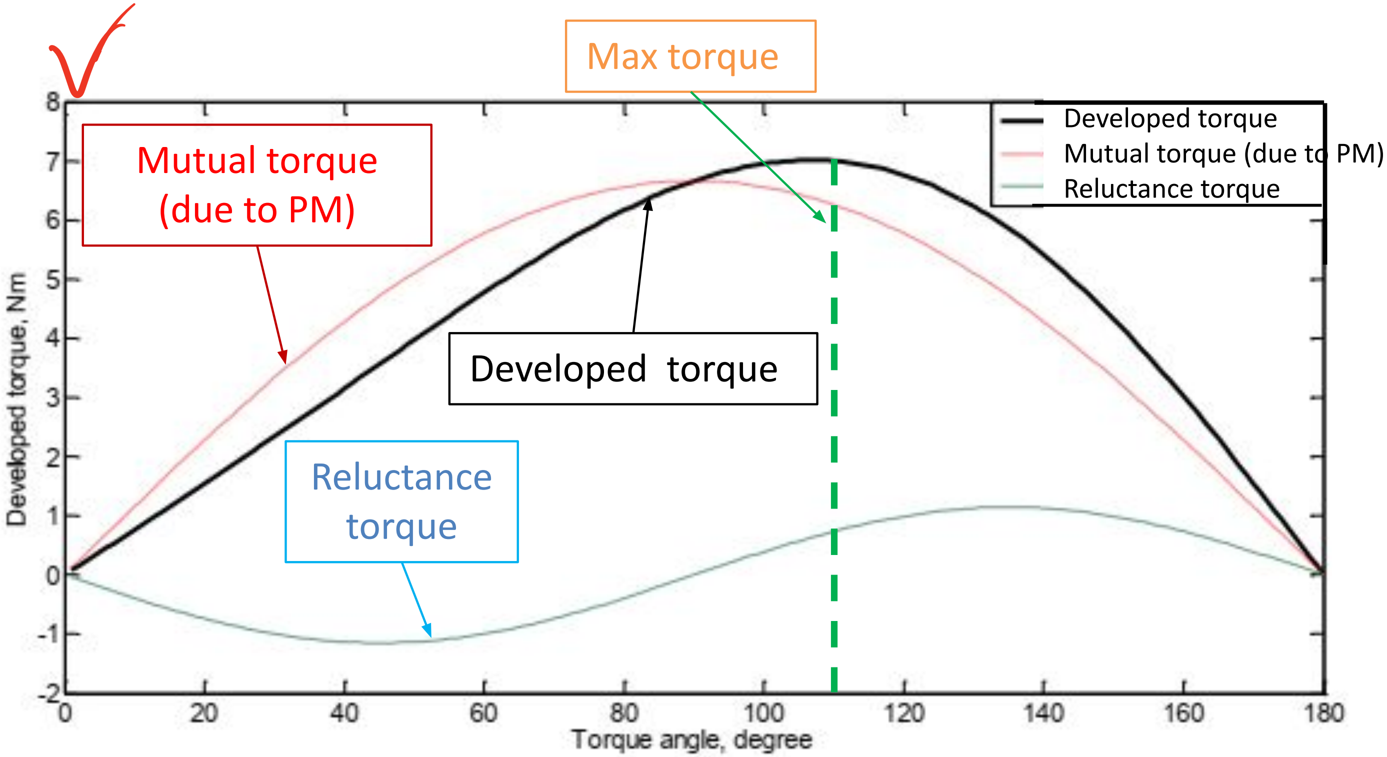
$$T_{e\_IPM} = \frac{3}{2}\frac{P}{2}(\overbrace{\psi_f i_q}^{\text{Mutual torque (due to PM)}} + \underbrace{(L_d - L_q) i_d i_q}_{\text{Reluctance torque}})$$

Control of MTPA Region

$$T_{e\_IPM} = \frac{3}{2}\frac{P}{2}(\psi_f i_s \sin\beta + (L_d - L_q) i_s \cos\beta i_s \sin\beta)$$

$$T_{e\_IPM} = \frac{3}{2}\frac{P}{2}(\psi_f i_s \sin\beta + (L_d - L_q) i_s^2 \frac{\sin 2\beta}{2}) = T_{magnet} + T_{reluctance}$$

Torque angle β changes, the variation of the mutual torque (due to PM) and the reluctance torque



Since,

$T_{e\_SPM} = T_{magnet}$

$T_{e\_IPM} = T_{magnet} + T_{reluctance}$

→ $T_{e\_IPM} > T_{e\_SPM}$

Realize that,

→ To obtain maximum torque, $T_{reluctance} > 0$ and should be kept $\beta > 90^o$

https://epublications.marquette.edu/cgi/viewcontent.cgi?referer=https://www.google.com.tw/&httpsredir=1&article=1285&context=theses_open

Control of MTPA Region

The MTPA operation is favored in the control of interior-PM synchronous motor(IPMSM) drive since it is capable of achieving the optimal efficiency by controlling the current vector at specific load conditions.

By setting the derivate of T_e_IPM=0, torque angle which results in the maximum torque of IPM motors can be derived

$$\frac{dT_e}{d\beta} = \frac{3}{2}\frac{P}{2}(\psi_f i_s \cos\beta + (L_d - L_q) i_s^2 \cos 2\beta) = 0$$

For simplicity, can be rewritten:

$$\frac{dT_e}{d\beta} = \frac{3}{2}\frac{P}{2}(\psi_f i_s \cos\beta + (L_d - L_q)\left[\overbrace{(i_s \cos\beta)^2}^{i_d^2} - \overbrace{(i_s \sin\beta)^2}^{i_q^2}\right] = 0$$

$$i_d^2 + i_q^2 = i_s^2 \rightarrow i_q = \sqrt{i_s^2 - i_d^2}$$

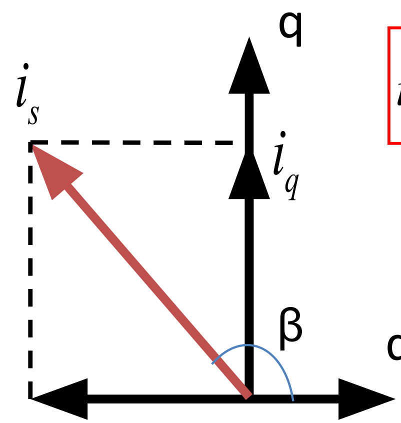
□ $$\boxed{2(L_d - L_q) i_d^2 + \psi_f i_d - (L_d - L_q) i_s^2 = 0} \qquad \text{(B)}$$

Control of MTPA Region

From (B), the d- and q-axis currents for MTPA control of IPM motors can be expressed as follows:

$$i_{d,MTPA} = \frac{-\psi_f + \sqrt{\psi_f^2 + 8(L_d - L_q)^2 i_s^2}}{4(L_d - L_q)} < 0$$

$$i_{q,MTPA} = \sqrt{i_s^2 - i_{d,MTPA}^2}$$



$$\boxed{i_q = \sqrt{i_s^{\,2} - i_d^{\,2}} \quad , i_q > 0}$$

$$\boxed{Keep\ i_d < 0}$$

As aforementioned, to obtain maximum torque, $T_{reluctance}$ must be positive and should be kept β>90^o .Therefore, the projection of is on d-axis should have a negative value (i_d<0)

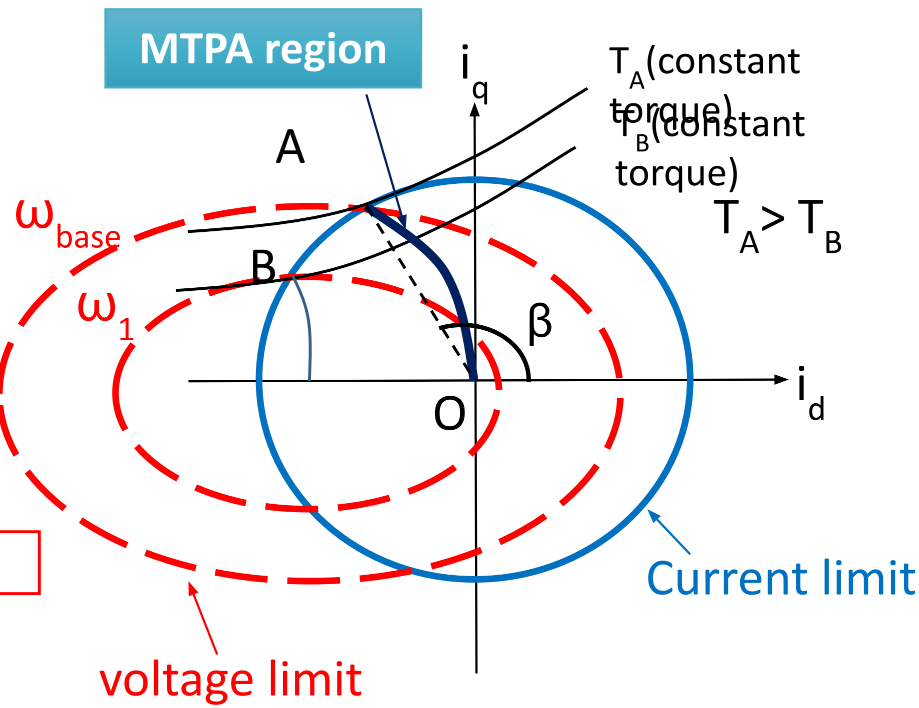
Control of MTPA Region

In the constant torque region, motor can be accelerated by the maximum torque until the terminal voltage of such motor reaches its limit value at $\omega = \omega_{base}$. Such base speed is the highest speed of a PM motor controlled by the

$$\omega_{base} = \frac{v_{max}}{\sqrt{(\psi_f + L_d i_d)^2 + (L_q i_q)^2}}$$

$$\frac{\left(i_d + \frac{\psi_f}{L_d}\right)^2}{\left(\frac{V_S}{\omega_e L_d}\right)^2} + \frac{(i_q)^2}{\left(\frac{V_S}{\omega_e L_q}\right)^2} = 1$$

$\omega_{base} < \omega_1$



O to A curve is MTPA region The point A is at max torque and max speed for I_{max} and V_{max}

Ld, Lq also changes by current

Examples (MTPA)

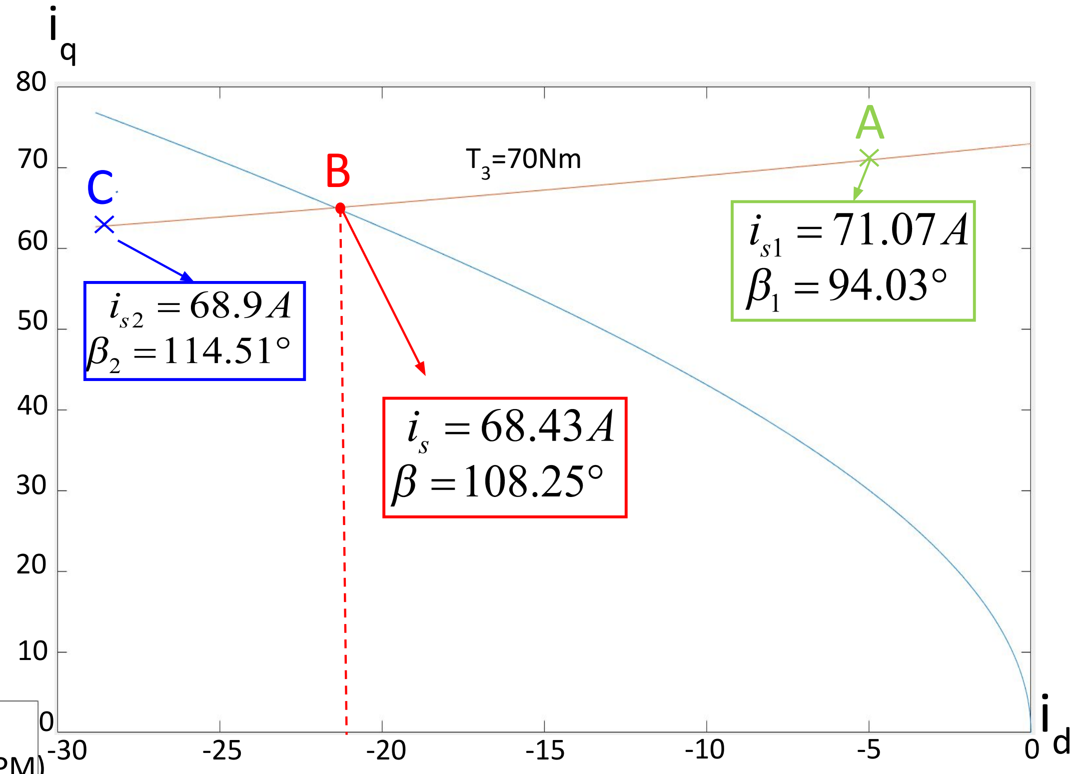
IPM

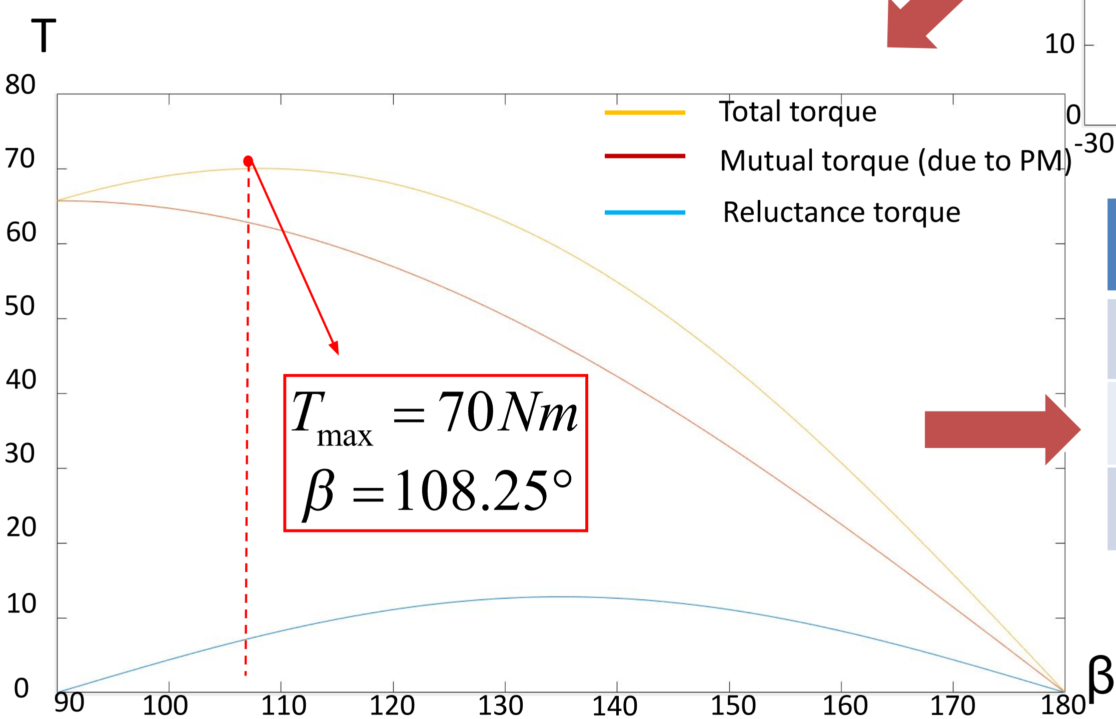
Pole	8
R_S	41.31mΩ
L_d	0.619mH
L_q	1.53mH
Ψ_f	0.16Wb

V_{smax}=450V

I_{smax}=81A

$$\beta - 90^{\circ} = \sin^{-1}(\frac{i_d}{i_s})$$





	T	i_s	β
A	70N-m	71.07A	94.03°
B	70N-m	68.43A	108.25°
C	70N-m	68.9A	114.51°

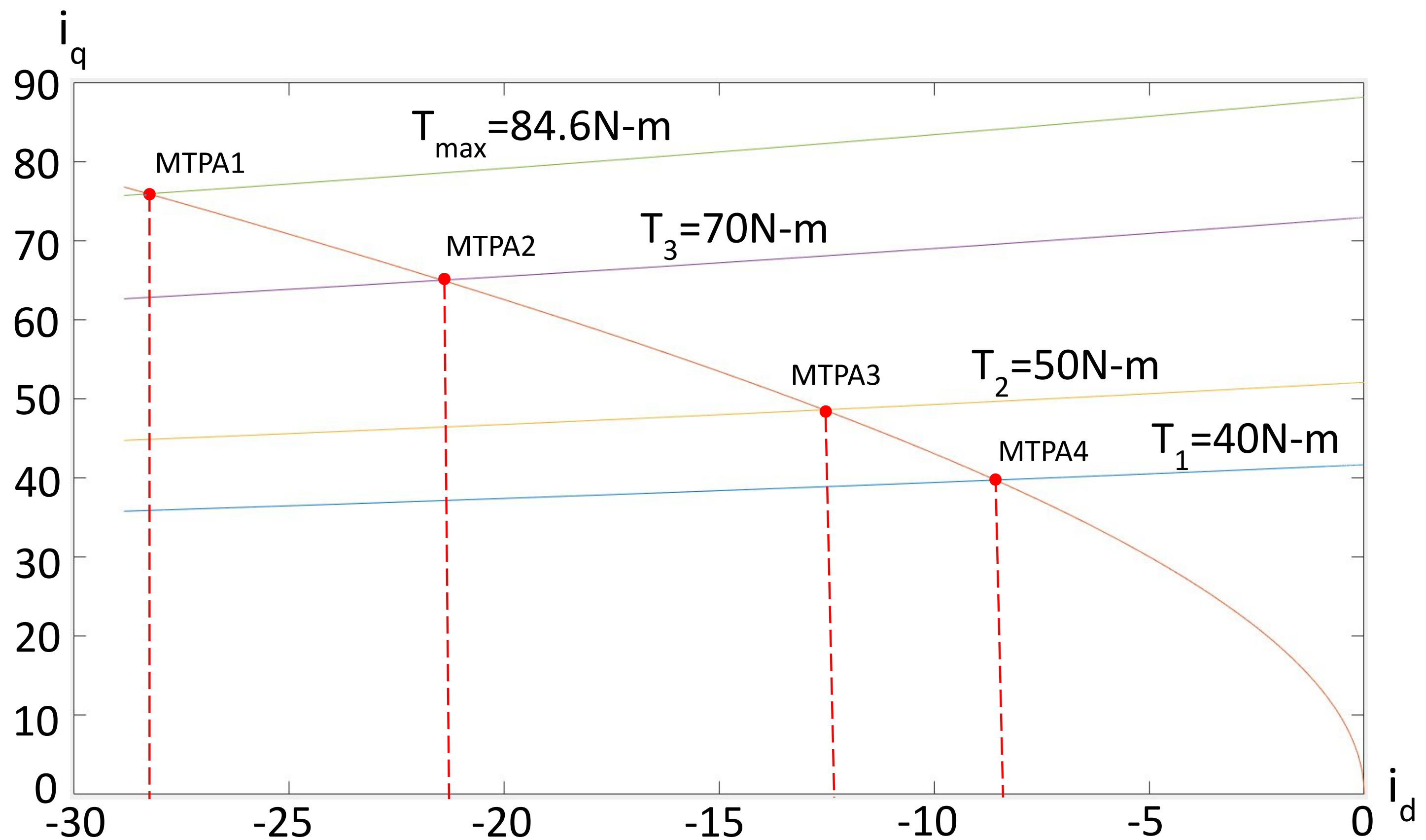
Examples (MTPA)

IPM

Pole	8
R_S	41.31mΩ
L_d	0.619mH
L_q	1.53mH
Ψ_f	0.16Wb

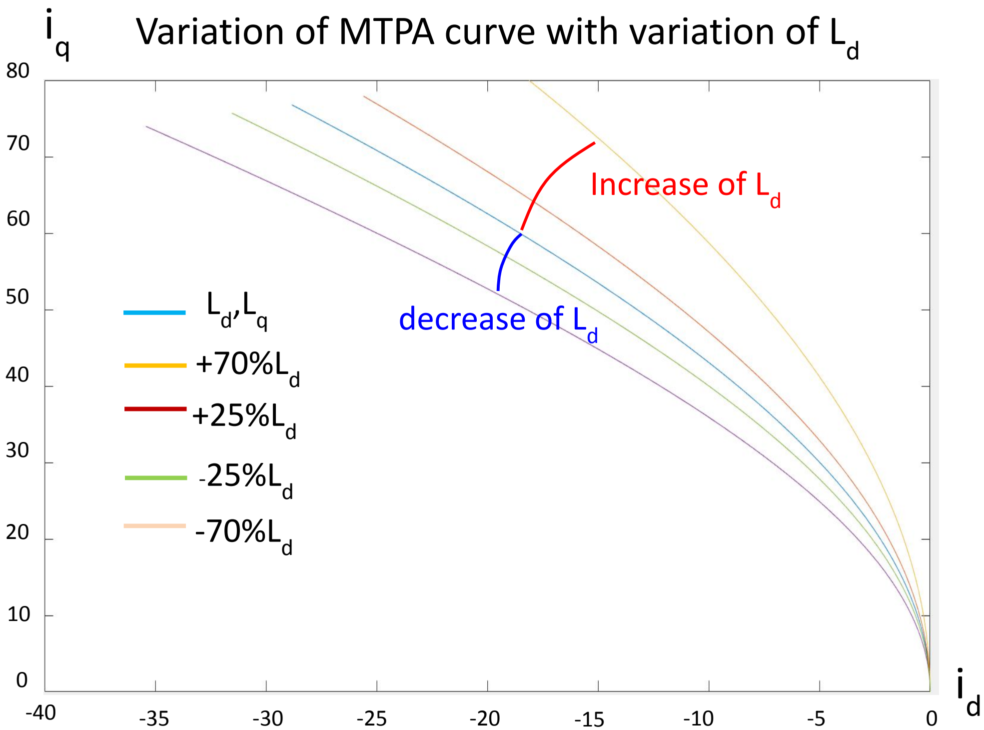
V_{smax}=450V

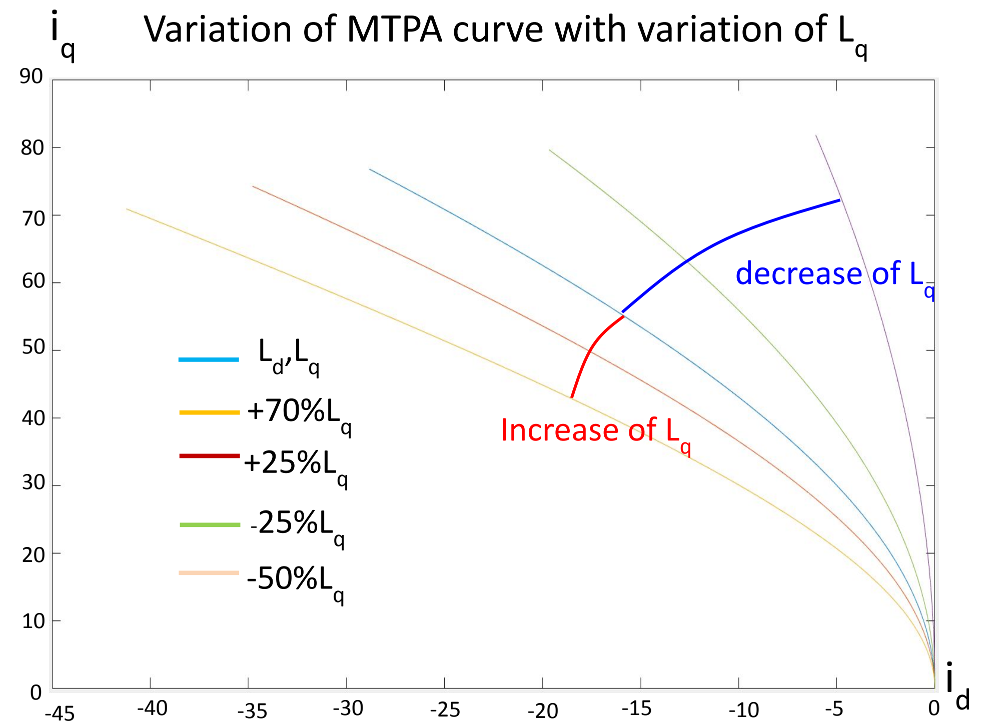
I_{smax}=81A



	MTPA1	MTPA2	MTPA3	MTPA4
T	84.6N-m	70N-m	50N-m	40N-m
i_s	81A	68.43A	50.21A	40.65A
β	110.42°	108.25°	104.48°	102.17°

Examples (MTPA)





$$T_{IPM} = \frac{3}{2}\frac{P}{2}(\psi_f i_q + (L_d - L_q) i_d i_q)$$

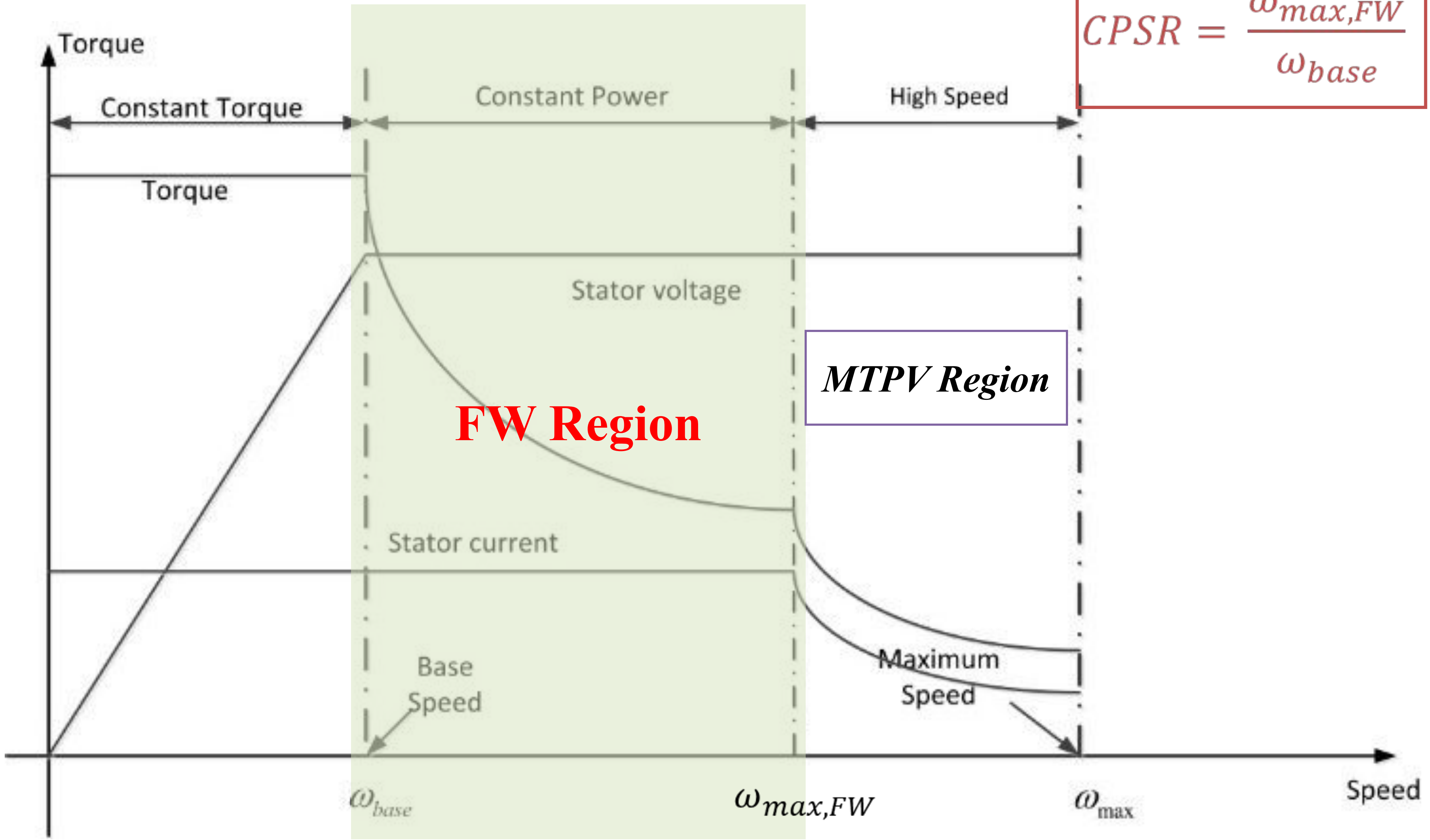
$$\begin{cases} i_d = \dfrac{-\psi_f + \sqrt{8 i_s^{\,2}(L_d - L_q)^2 + \psi_f^{\,2}}}{4(L_d - L_q)} \\ i_q = \sqrt{i_s^{\,2} - i_d^{\,2}} \end{cases}$$

In the same torque:

$L_d \uparrow$	$i_d \downarrow$	$i_q \uparrow$	$L_q \uparrow$	$i_d \uparrow$	$i_q \downarrow$
$L_d \downarrow$	$i_d \uparrow$	$i_q \downarrow$	$L_q \downarrow$	$i_d \downarrow$	$i_q \uparrow$

Control of Field Weakening (FW) Region

$$CPSR = \frac{\omega_{max,FW}}{\omega_{base}}$$

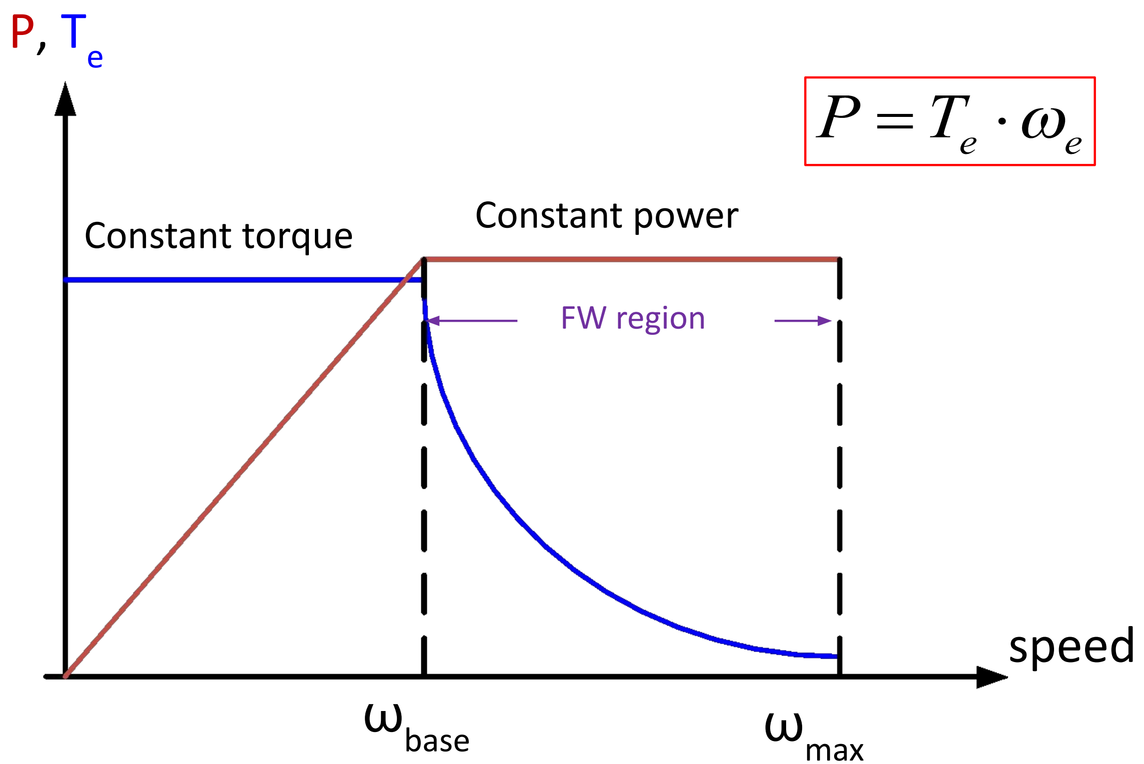


Department of Electrical Engineering

Control of FW Region

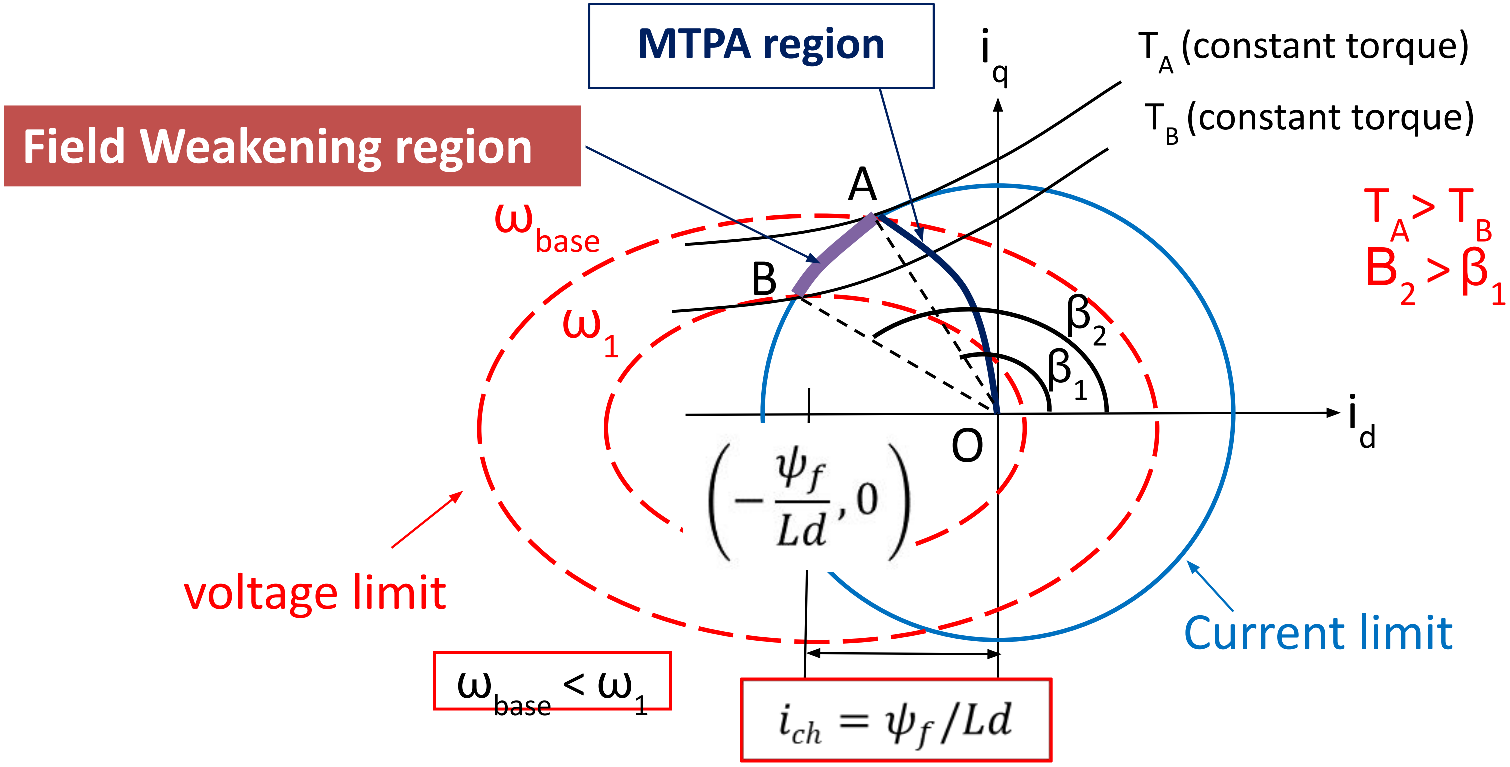
Above the base speed, the voltage cannot be increased further therefore the flux has to be decreased in this case and hence, the torque will also be decreased in turn.

However, the power will remain constant in this region since the flux or torque will decrease proportionally to the increase in the speed of machine. Unlike in the MTPA control where the torque is only subjected to the current limit constraints, both the voltage and current constraints limit the torque production during the FW region



Control of FW Region

It is evident from Fig. that as the speed of the motor increases, the voltage ellipse shrinks towards its center at ($-\psi_f/L_d$,0). The current references derived from the MTPA calculations are incapable of satisfying both the current and voltage constraints above the base speed



Field Weakening happens to start from point A

Control of FW Region

$$\frac{\left(i_d + \frac{\psi_f}{L_d}\right)^2}{\left(\frac{V_S}{\omega_e L_d}\right)^2} + \frac{(i_q)^2}{\left(\frac{V_S}{\omega_e L_q}\right)^2} = 1$$

$$i_q = \sqrt{i_s^{\,2} - i_d^{\,2}}$$

- Therefore, new current references are now derived by simultaneous solution of current limit and the voltage limit

$$(L_d^{\,2} - L_q^{\,2})i_d^2 + 2\psi_f L_d i_d + (L_q^{\,2} i_s^2 + \psi_f^2 - \frac{V_{max}^2}{\omega_e^2}) = 0$$

- From the above analysis , the d- and q-axis currents for the FW control of IPM motors can be expressed as follows:

$$i_{d,FW} = \frac{-\psi_f L_d + \sqrt{(\psi_f L_d)^2 - (L_d^2 - L_q^2)(L_q^{\,2} i_s^2 + \psi_f^2 - v_{max}^2 / \omega_e^2)}}{(L_d^2 - L_q^2)} < 0$$

$$i_{q,FW} = \sqrt{i_{max}^2 - i_{d,FW}^2}$$

- The equation stated above provides the current references in the voltage and current limited FW region. PM drives can be classified into two types based on their speed capabilities. There are **“Finite Speed Drives”** and **“Infinite Speed Drives”.**
- The distinction comes from the comparison of drives characteristic current i_{ch} and drives maximum current i_{max}.

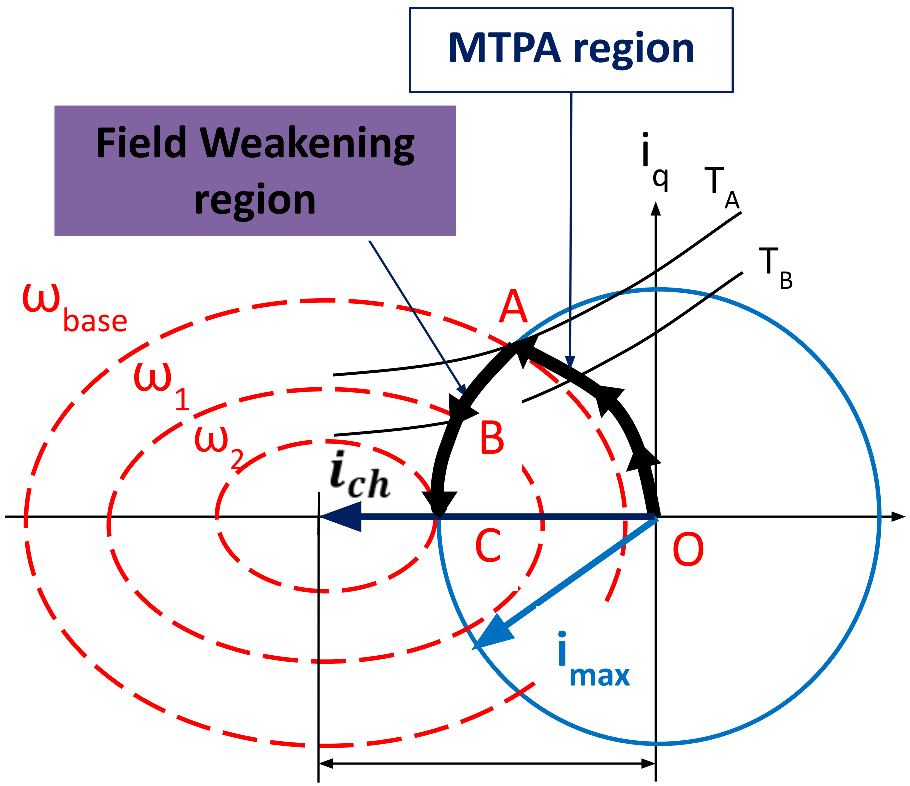
$$i_{ch} = \psi_f / Ld$$

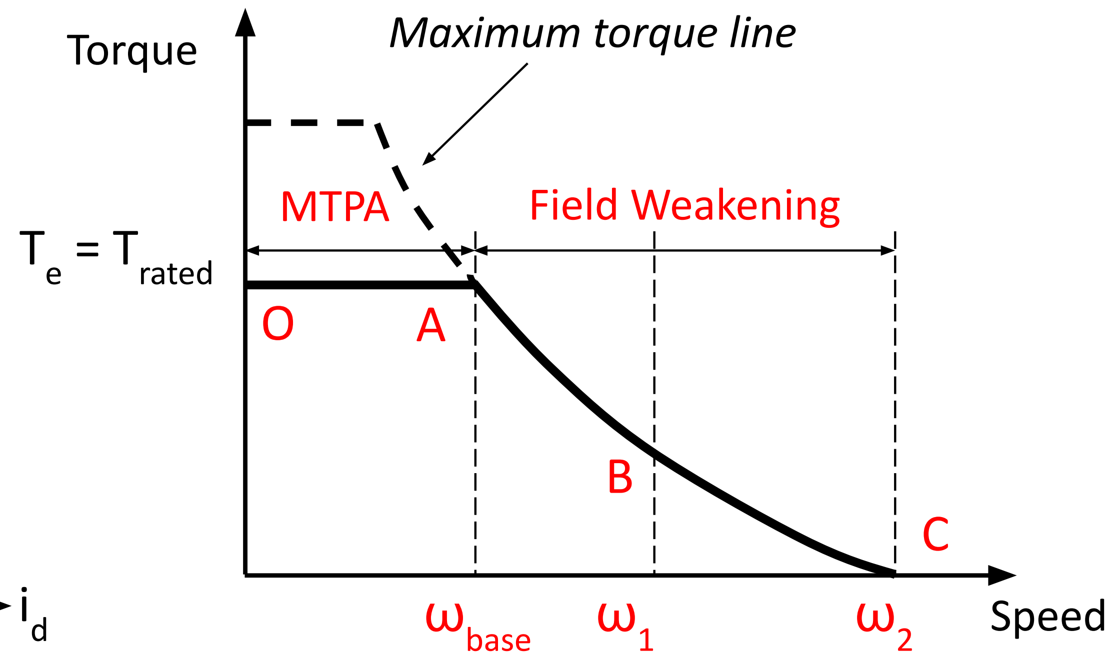
Control of FW Region (Finite Speed Drives)

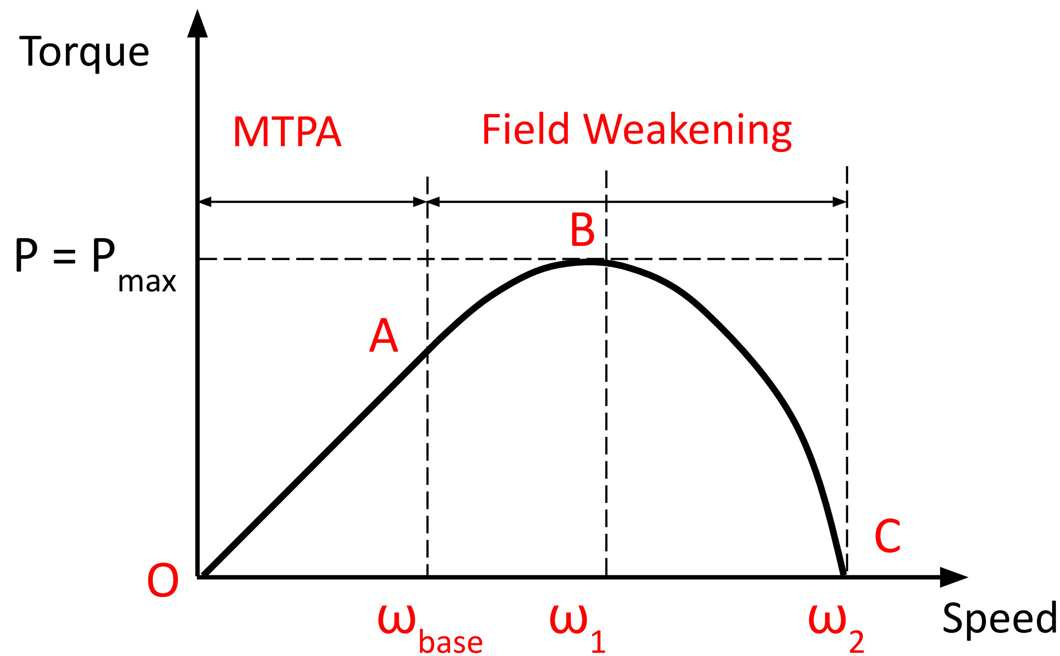
- The drives having $i_{ch} > i_{max}$ are known as **"finite speed drives"**. They are able to achieve very high speeds using a proper FW control strategy. The operation of a finite speed drive from zero to maximum speed
- Firstly, motor is accelerated during speed interval 0-ω_{base}, maximum torque is obtained by operating with rated current at the MTPA (trajectory O-A). The speed increases with the voltage and beyond $u_{s,\,max}$, the FW control takes over.
- Secondly, current cannot be obtained beyond point B by moving on the torque locus because of i_{max} limit, maximum torque is now determined by the intersection of both the voltage and the current limit indicated by trajectory A-B.
- FW algorithm decreases the output torque to increase the motor speed beyond ω_1. The decrease results in an increase in power indicated by trajectory B-C in power speed characteristics. Point C indicates that the maximum speed has been reached as the current and voltage limits are tangential to each other.

(illustrated in the next figures)

Control of FW Region (Finite Speed Drives)



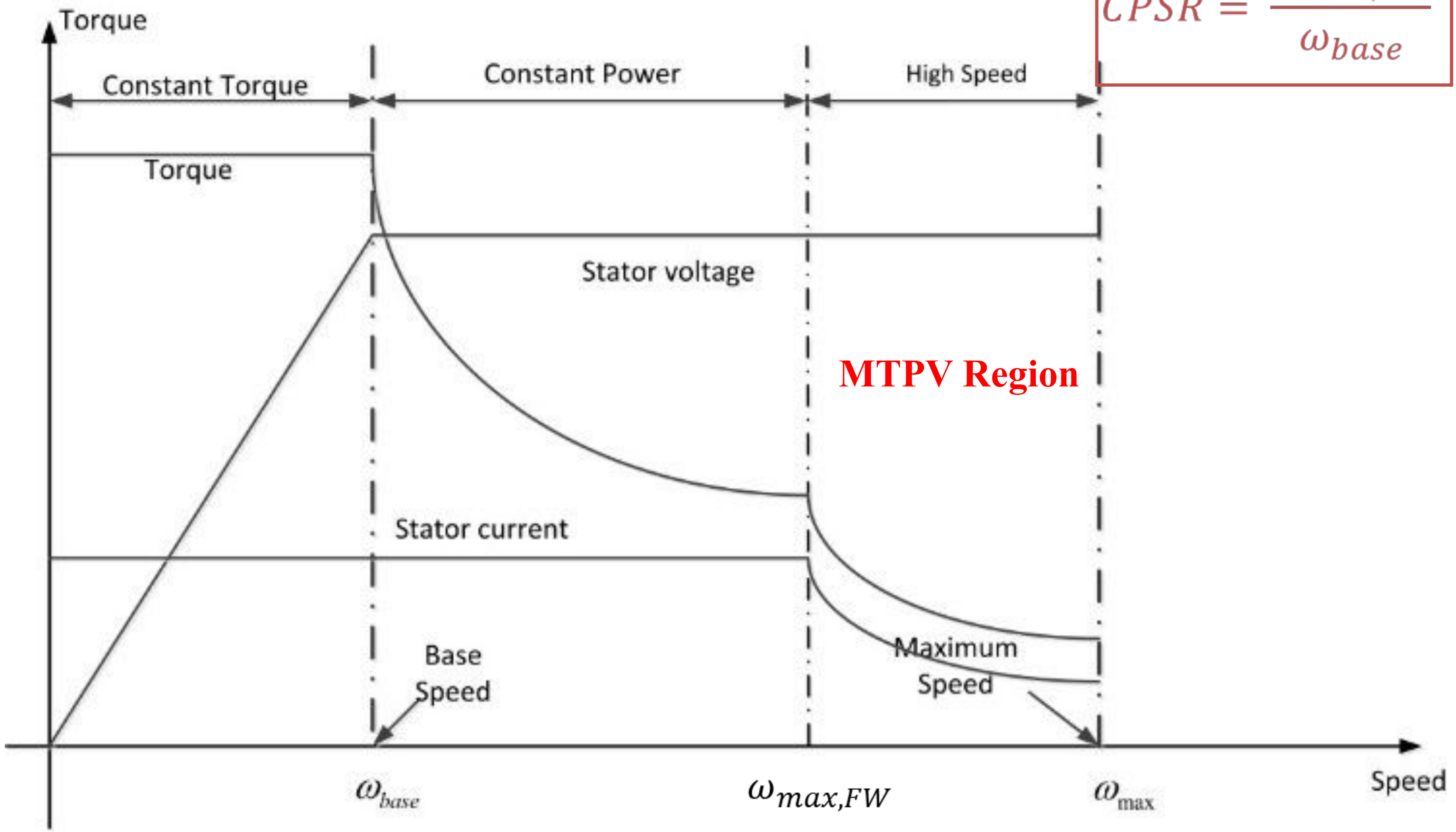




$$Case:\ i_{ch} = \frac{\psi_f}{L_d} > i_{max}$$

Control of MTPV Region

$$CPSR = \frac{\omega_{max,FW}}{\omega_{base}}$$

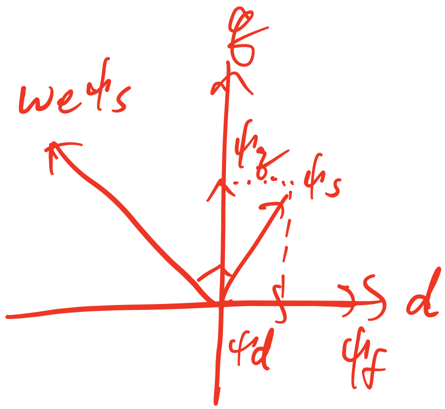


MTPV Control - Infinite Speed Drives

- Drives having $i_{ch} < i_{max}$ are known as **"infinite speed drives"**, no upper limit. Such drives need to obey the maximum torque per voltage (MTPV) limit at "VERY" high speeds.
- Keeping the voltage limit, after particular speed has been reached maximum torque cannot be obtained by exploiting full inverter current. In such a condition, current are derived based on tangential intersection between the torque curves and the shrinking voltage ellipses

(illustrated in the next figures)

$V_s = \omega_e \psi_s$



- The relationship between id and iq for MTPV is given

$$i_d = \frac{\psi_d - \psi_f}{L_d}, \quad i_q = \frac{\sqrt{\psi_s^2 - \psi_d^2}}{L_q}$$

- Moreover, from projection of ψ_s on d-q axis and voltage by eq.(A)

$$\begin{aligned} \psi_d &= L_d i_d + \psi_f \\ \psi_q &= L_q i_q \end{aligned} \rightarrow \psi_s = \sqrt{\psi_d^2 + \psi_q^2} = \frac{V_s}{\omega_e}$$

$$\omega_e^{\,2}(\psi_f + L_d i_d)^2 + \omega_e^{\,2}(L_q i_q)^2 = V_s^2$$

Control of MTPV Region - Infinite Speed Drives

- From ψ_s, v_s equation, the new torque's equation is:

$$T_e = \frac{3P}{4}\left[\psi_d\left(\frac{\sqrt{\psi_s^2-\psi_d^2}}{L_q}\right)+\sqrt{\psi_s^2-\psi_d^2}\left(\frac{\psi_f-\psi_d}{L_d}\right)\right] \qquad \textbf{Eq.(C)}$$

- Differentiating Eq.(C) with respect to ψ_d and setting it to zero, that is $\frac{\partial T_e}{\partial \psi_d} = 0$, we have the d-axis flux and current for MTPV

$$\psi_{d,MTPV} = \frac{-L_q\psi_f + \sqrt{\left(L_q\psi_f\right)^2 + 8\left(L_d-L_q\right)^2\left(\frac{v_s}{\omega_e}\right)^2}}{4\left(L_d-L_q\right)}$$

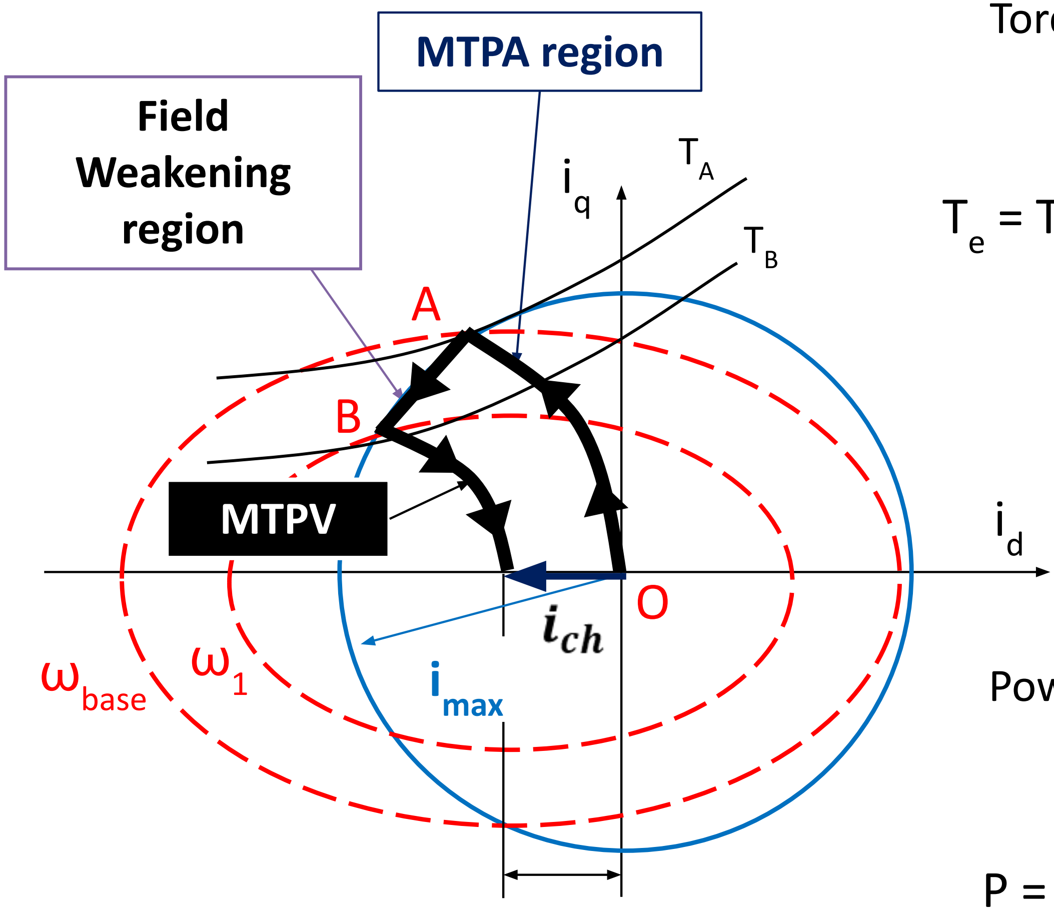
$$i_{d,MTPV} = -\frac{\psi_f-\psi_{d,MTPV}}{L_d}$$

Control of MTPV Region - Infinite Speed Drives

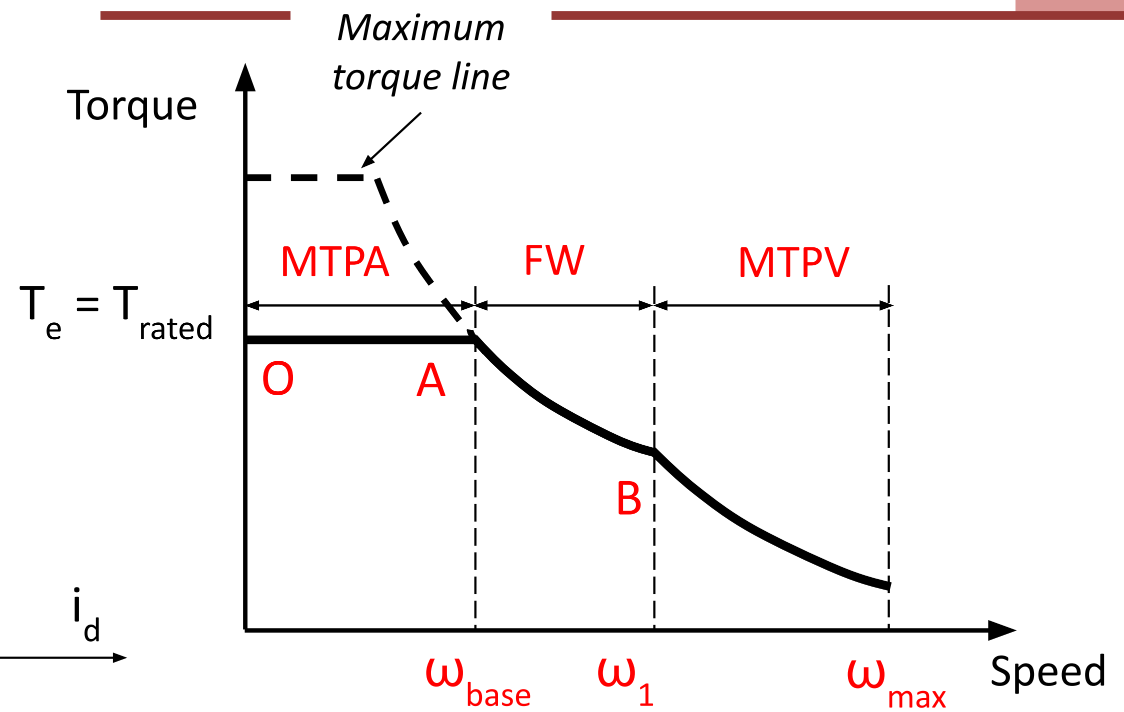
(Based on illustrated in the next figures)

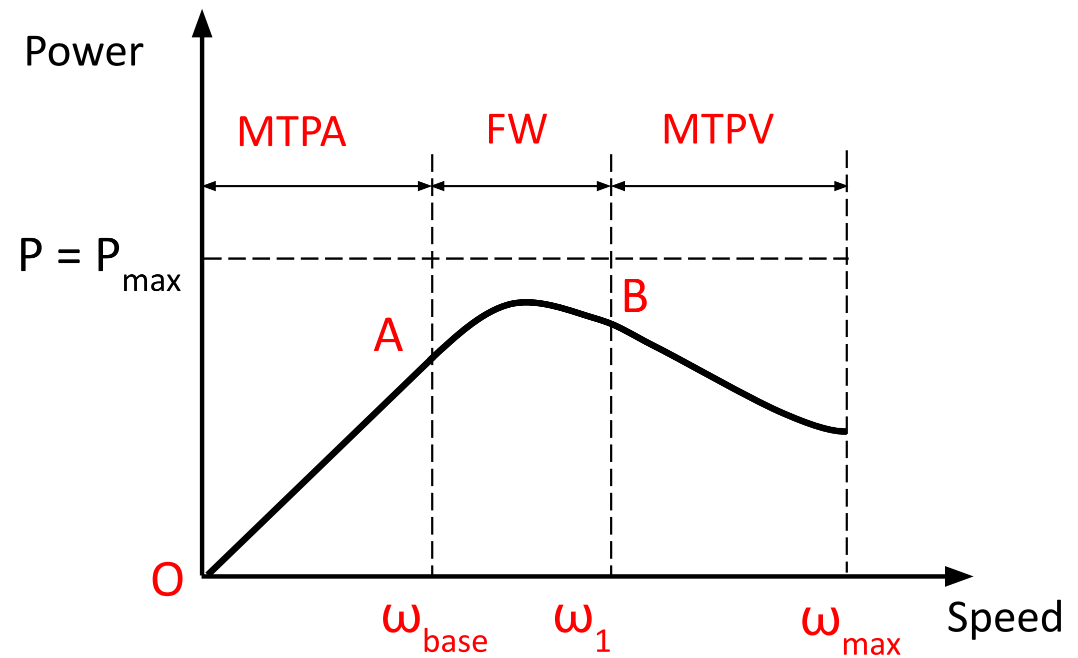
- MTPV limit similar to the MTPA limit is a hyperbola in the id-iq plane for an IPM machine. Up to point B the behavior of an infinite speed IPM drive is exactly similar to a finite speed drive.
- MTPV limitation is activated at point B and the optimal current references are now selected by solution of (D) . During the MTPV operation, the input current of the drive is lower than the rated current while voltage is limited to $u_{s,max}$. The output power of the drive falls inversely with the speed and is very small at very high speeds.

Control of MTPV Region - Infinite Speed Drives



$$Case: i_{ch} = \frac{\psi_f}{L_d} < i_{max}$$





Appendix

Derivation of FW Region Current

From current and voltage limit equation,

$$\frac{\left(i_d + \frac{\psi_f}{L_d}\right)^2}{\left(\frac{V_S}{\omega_e L_d}\right)^2} + \frac{(i_q)^2}{\left(\frac{V_S}{\omega_e L_q}\right)^2} = 1$$ We can get $$L_d^{\,2} \times \left(i_d + \frac{\psi_f}{L_d}\right)^2 + L_q^{\,2} \times (i_q)^2 = \left(\frac{V_S}{\omega_e}\right)^2$$

$$\rightarrow \left(L_d i_d + \psi_f\right)^2 + L_q^{\,2} i_q^{\,2} - \left(\frac{V_S}{\omega_e}\right)^2 = 0 \rightarrow L_d^{\,2} i_d^{\,2} + 2L_d i_d \psi_f + \psi_f^{\,2} + L_q^{\,2} i_q^{\,2} - \left(\frac{V_S}{\omega_e}\right)^2 = 0$$

We can create

$$\rightarrow L_d^{\,2} i_d^{\,2} \boxed{+ L_q^{\,2} i_d^{\,2} - L_q^{\,2} i_d^{\,2}} + 2L_d i_d \psi_f + \psi_f^{\,2} + L_q^{\,2} i_q^{\,2} - \left(\frac{V_S}{\omega_e}\right)^2 = 0$$

Combine these to i_s

$$\rightarrow L_d^{\,2} i_d^{\,2} - L_q^{\,2} i_d^{\,2} + 2L_d i_d \psi_f + \psi_f^{\,2} + \boxed{L_q^{\,2} i_q^{\,2} + L_q^{\,2} i_d^{\,2}} - \left(\frac{V_S}{\omega_e}\right)^2 = 0$$

$$\rightarrow i_d^{\,2}(L_d^{\,2} - L_q^{\,2}) + 2L_d i_d \psi_f + (\psi_f^{\,2} + \boxed{L_q^{\,2} i_s^{\,2}} - \left(\frac{V_S}{\omega_e}\right)^2) = 0$$ It is a Quadratic equation of i_d

We can get solution

$$\mathrm{i_{d,FW}} = \frac{-\psi_f L_d + \sqrt{(\psi_f L_d)^2 - (L_d^2 - L_q^2)(L_q^{\,2} i_s^2 + \psi_f^2 - v_{max}^2 / \omega_e^2)}}{(L_d^2 - L_q^2)} < 0$$

$$\mathrm{i_{q,FW}} = \sqrt{i_{max}^2 - i_{d,FW}^2}$$

Additional References

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[2] Thanh Anh Huynh and Min-Fu Hsieh, **"Comparative Study of PM-Assisted SynRM and IPMSM on Constant Power Speed Range for EV Applications"** IEEE Transactions on Magnetics, Vol. 53, No. 11, November 2017.

[3] Muyang Li, ***"Flux-Weakening Control for Permanent-Magnet Synchronous Motors Based on Z-Source Inverters",*** Master's Theses (2009). *(available online)*

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