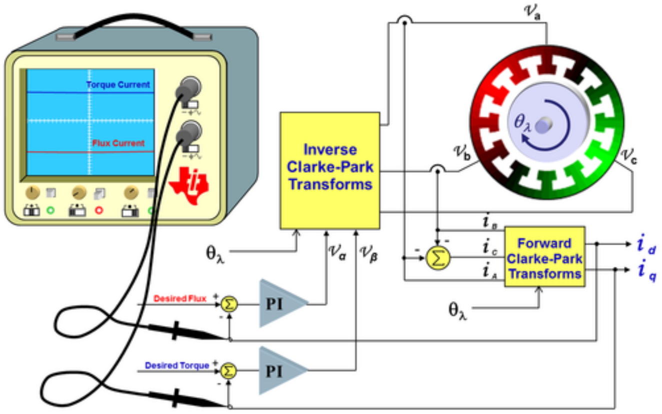
Chapter 4

Field Oriented Control (FOC) for PMSM



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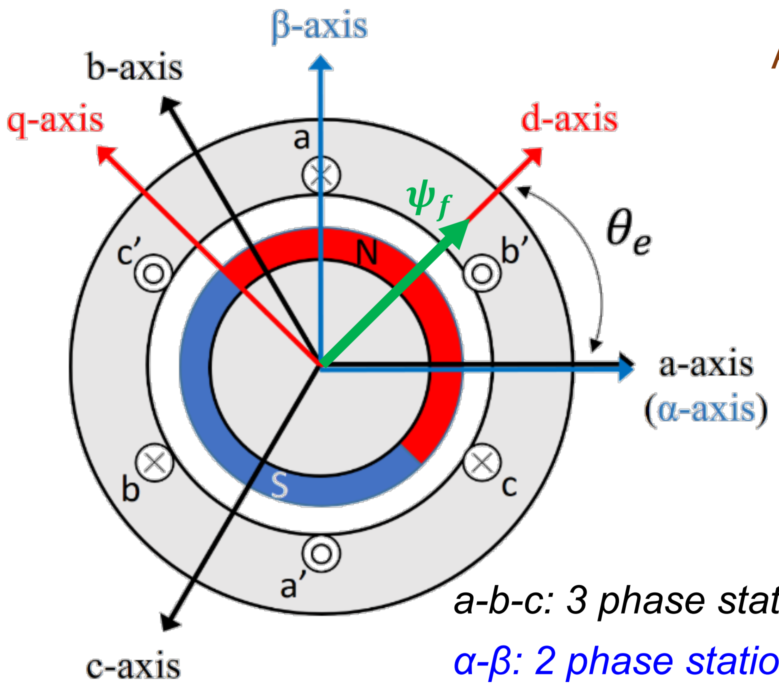
Professor Min-Fu Hsieh

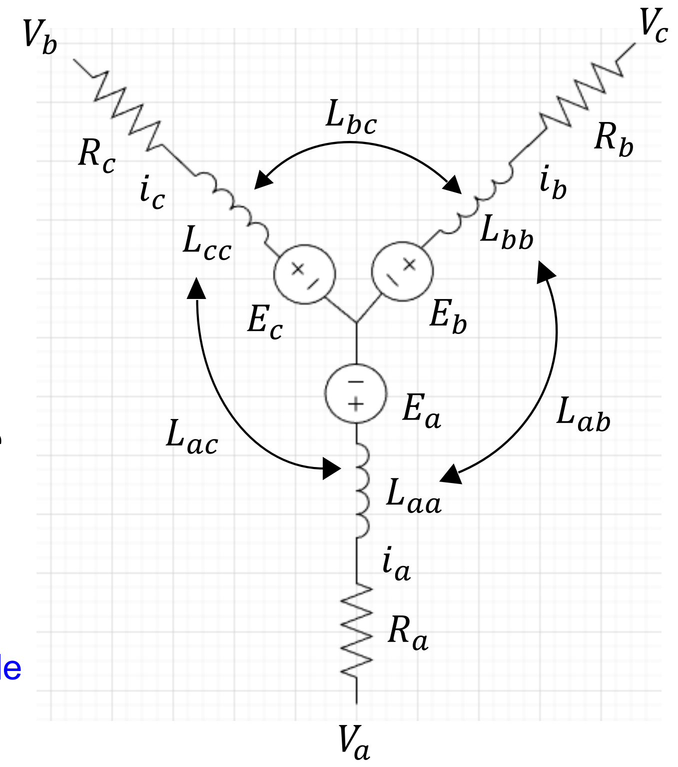
Fall Semester - 2022

Modeling and Equivalent Circuit of PMSM

Modelling and Reference Frames of PMSM



Assuming a balanced 3-phase system, the equivalent circuit of PMSM



a-b-c: 3 phase stationary frame

α-β: 2 phase stationary frame

d-q: rotating frame

Rotor position angle θ_e is defined in term of the angle between the magnetic axis of a-axis and d-axis

S.-Y. Wang, "**High Power Factor Control for PMSM Drives Using Reduced DC-link Capacitance**," Master Thesis, National Cheng Kung University, 2018

Mathematical Model of PMSM

Three-phase voltages of the stator winding in three-phase (a-b-c) reference frame are express as the following

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix}$$

where i_a, i_b, i_c are 3-phase currents and R_s is resistance of stator windings

Relationship of three-phase stator flux linkage and three-phase current can be expressed as follows

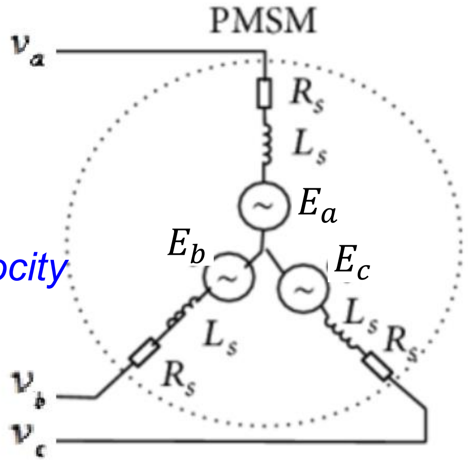
$$\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix} = L_s \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \Psi_f$$

where Ψ_f is permanent magnet flux linkage and L_s is stator inductance matrix

The electromotive forces E_a, E_b, E_c can be obtained

$$\begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = |\Psi_f| \omega_e \begin{bmatrix} \sin \theta_e \\ \sin\left(\theta_e - \frac{2}{3}\pi\right) \\ \sin\left(\theta_e + \frac{2}{3}\pi\right) \end{bmatrix}$$

where ω_e is electrical velocity



Mathematical Model of PMSM

L_s is stator inductance matrix, as follows

$$L_s = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}$$

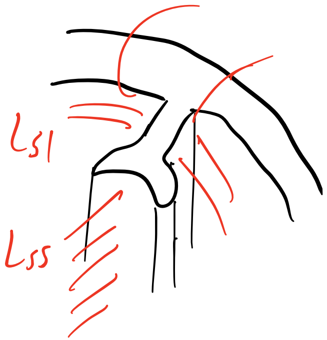
where L_{aa}, L_{bb}, L_{cc} are the self-inductances of three-phase stator winding,
$L_{ab}, L_{ac}, L_{ba}, L_{bc}, L_{cb}, L_{ca}$ are the mutual-inductances of three-phase stator winding

Assuming that the air gap of the motor is **uniform**, relations can be obtained, as follows

$$L_{aa} = L_{bb} = L_{cc} = L_{ss} + L_{sl}$$
$$L_{ab} = L_{ac} = L_{ba} = L_{bc} = L_{cb} = L_{ca}$$

where L_{sl} is stator leakage inductance and L_{ss} is the magnetizing inductance.

漏電感 磁化電感



Mathematical Model of PMSM

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix}$$

Clarke Transform

$$\begin{bmatrix} V_\alpha \\ V_\beta \\ V_0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

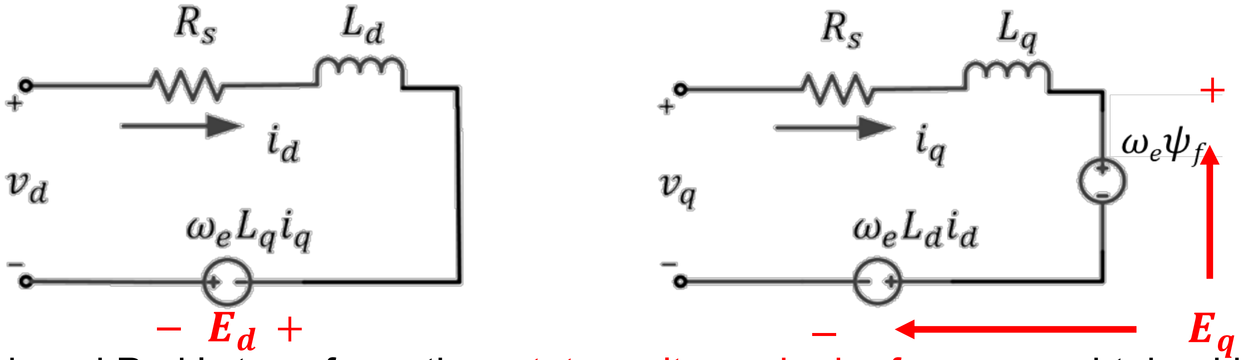
Park Transform

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$$

$$\begin{cases} V_d = R_s i_d + \dfrac{d\psi_d}{dt} - \omega_e \psi_q \\ V_q = R_s i_q + \dfrac{d\psi_q}{dt} + \omega_e \psi_d \end{cases}$$

Equivalent Circuit of PMSM

The dynamic equivalent circuit of a PMSM based on the d-q frame can be drawn:



After Clark and Park's transformation, stator voltages in d-q frame are obtained in the following:

$$v_d = R_s i_d + \frac{d\psi_d}{dt} - \omega_e \psi_q \qquad v_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_e \psi_d$$

Assuming that variation of the stator resistance R_s is neglected, d-q axis flux linkages is defined as follows:

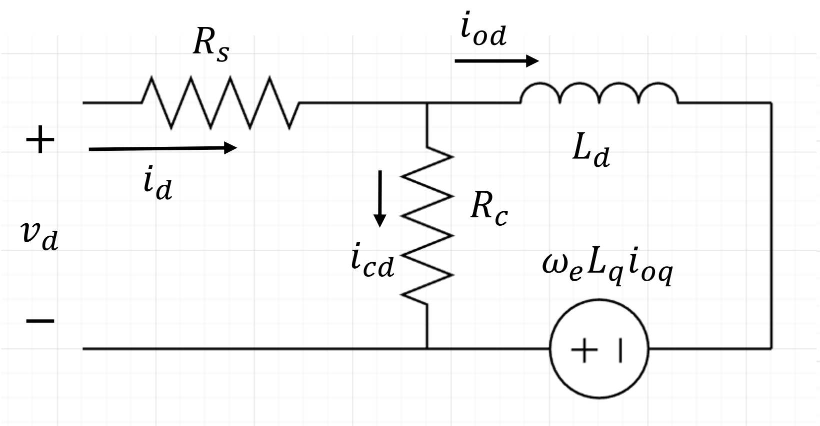
$$\Psi_d = L_d i_d + \Psi_f \qquad \Psi_q = L_q i_q$$

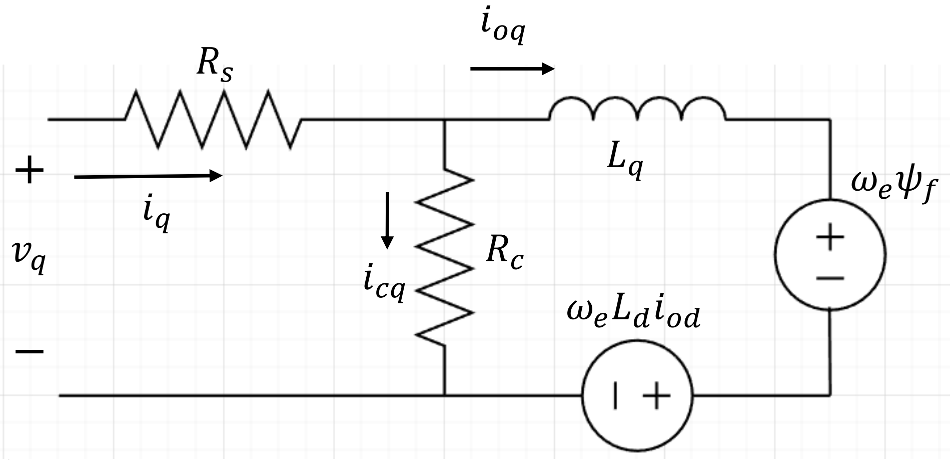
← 假設PM沒有供獻任何磁通給q軸

where L_d and L_q are corresponding to d-axis and q-axis inductances

Actual Equivalent Circuit of PMSM

The actual dynamic equivalent circuit of a PMSM based on the d-q frame including iron loss resistor R_s





Considering iron losses , stator voltages in d-q frame becomes

$$v_d = R_s i_d + \frac{d\psi_d}{dt} - \omega_e \psi_q \qquad v_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_e \psi_d$$

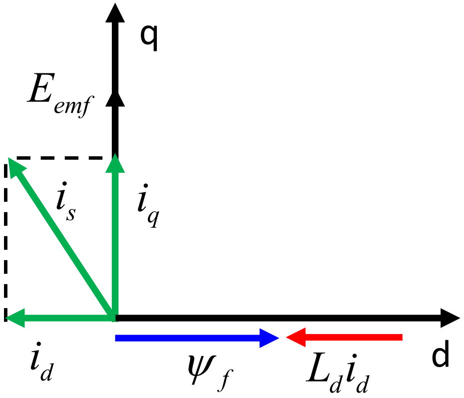
$$\Psi_q = L_q i_{oq} \qquad \Psi_d = L_d i_{od} + \Psi_f$$

$$\frac{d\psi_d}{dt} = L_d \frac{di_{od}}{dt}$$

$$\frac{d\psi_q}{dt} = L_q \frac{di_{oq}}{dt}$$

Phasor Diagram of PMSM

$I_d<0$ (flux weakening)

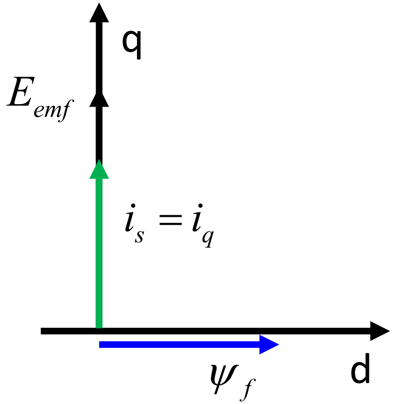


$$\psi_d = L_d i_d + \psi_f$$

$$E_{emf} = \psi_f \omega_e$$

$|\psi_d| < |\psi_f|$

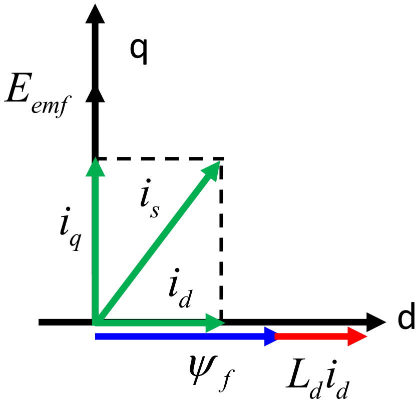
$I_d=0$



$$\psi_d = \psi_f$$

$$E_{emf} = \psi_f \omega_e$$

$I_d>0$ (flux intensifying)



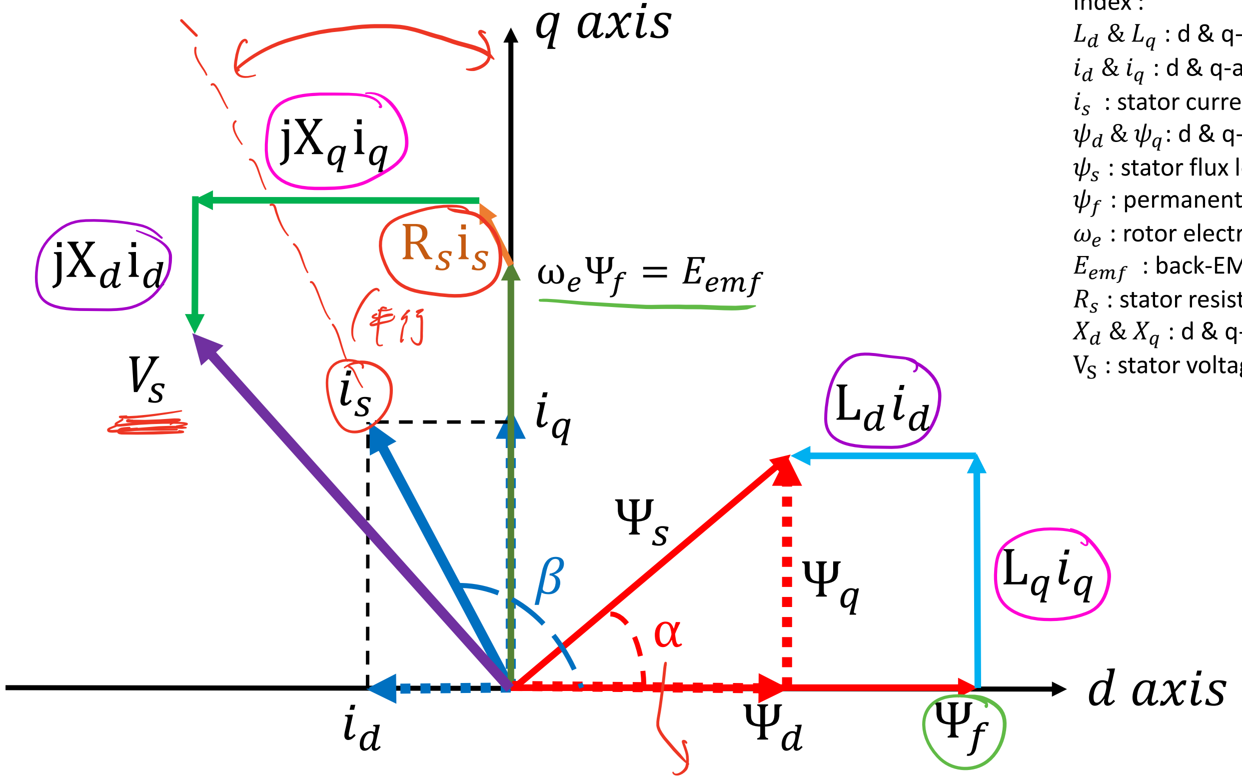
$$\psi_d = L_d i_d + \psi_f$$

$$E_{emf} = \psi_f \omega_e$$

$|\psi_d| > |\psi_f|$

Phasor Diagram of PMSM

γ → phase advance 超前角



load angle (負載角)

Index :
- L_d & L_q : d & q-axis inductance
- i_d & i_q : d & q-axis current
- i_s : stator current
- ψ_d & ψ_q: d & q-axis flux leakage
- ψ_s : stator flux leakage
- ψ_f : permanent magnet flux leakage
- ω_e : rotor electronic speed
- E_{emf} : back-EMF
- R_s : stator resistance
- X_d & X_q : d & q-axis reactance
- V_S : stator voltage

load ↑ α ↑
(i_q)

β : current angle

無感測器 or direct FOC 控制
則需用到α角

Equivalent Circuit of PMSM

In the d-q reference frame, electromotive force can be represented as follows:

$$E_d = -\omega_e L_q i_q \qquad E_q = \omega_e \psi_d = \omega_e \left(L_d i_d + \psi_f\right)$$

Assuming that the motor losses are negligible, the input power equals the output power, power P of motor as:

$$P = \frac{3}{2}\left(E_d i_d + E_q i_q\right)$$

Assuming that the motor losses are negligible, the input power equals the output power, power P of motor as:

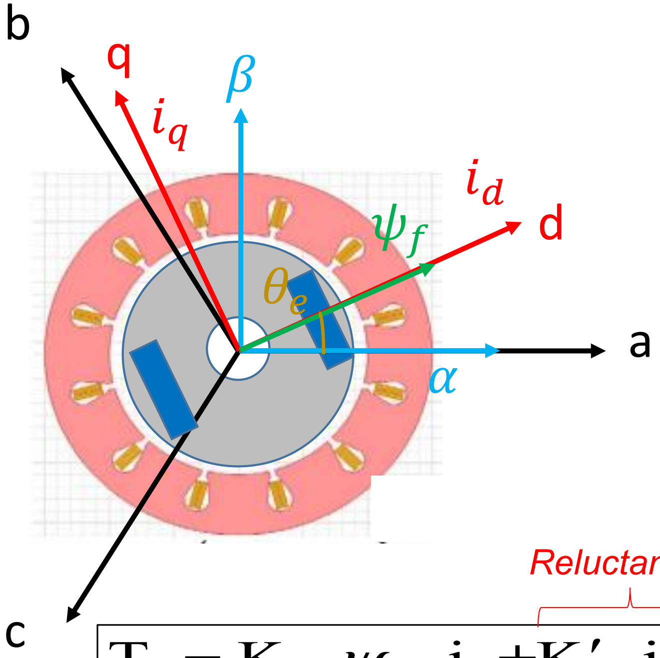
$$\omega_e = \frac{p}{2}\omega_m \Rightarrow P = \frac{3}{2}\frac{p}{2}\omega_m\left[\psi_f i_q + \left(L_d - L_q\right) i_d i_q\right]$$

Motor electromagnetic torque can be obtained as the following equation:

$$T_e = \frac{3}{2}\frac{p}{2}\left[\psi_f i_q + \left(L_d - L_q\right) i_d i_q\right]$$

where p is number of poles in motor

Torque Characteristic of PMSM

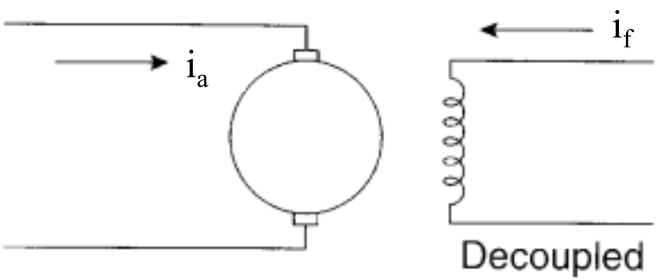


$$T_e = \underbrace{K_t \cdot \psi_f \cdot i_q}_{\text{Mutual torque (due to PM)}} + \overbrace{K_t' \cdot i_q \cdot i_d}^{\text{Reluctance torque}}$$

Torque Component

Flux Component

Separately Excited DC Motor



$$T_e = K_t \cdot \psi_a \cdot \psi_f = K_t' \cdot i_a \cdot i_f$$

Flux Component

Torque Component

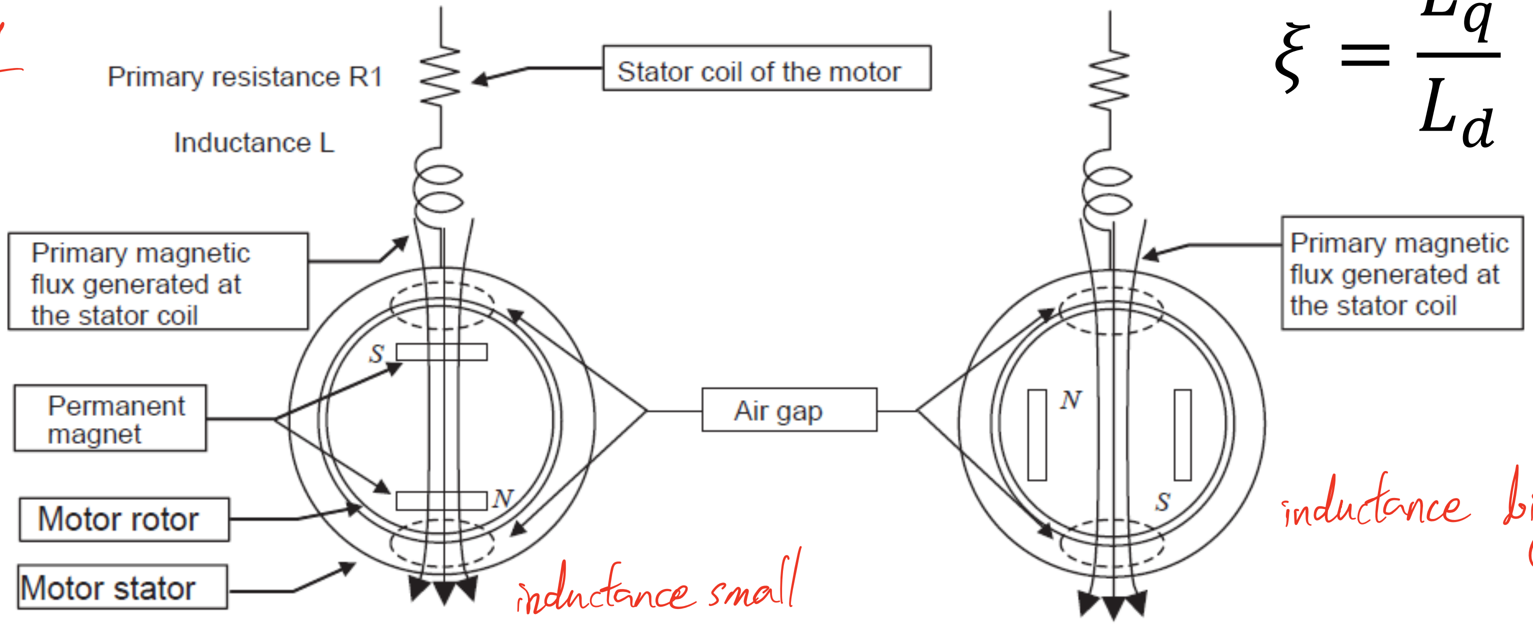
Simplified Motor Flux Diagram

$\psi = N\phi = Li$

$L = \frac{\psi}{i}$

saliency ratio

$$\xi = \frac{L_q}{L_d}$$



inductance small

inductance big

(a) (b)

磁鐵導磁率很差

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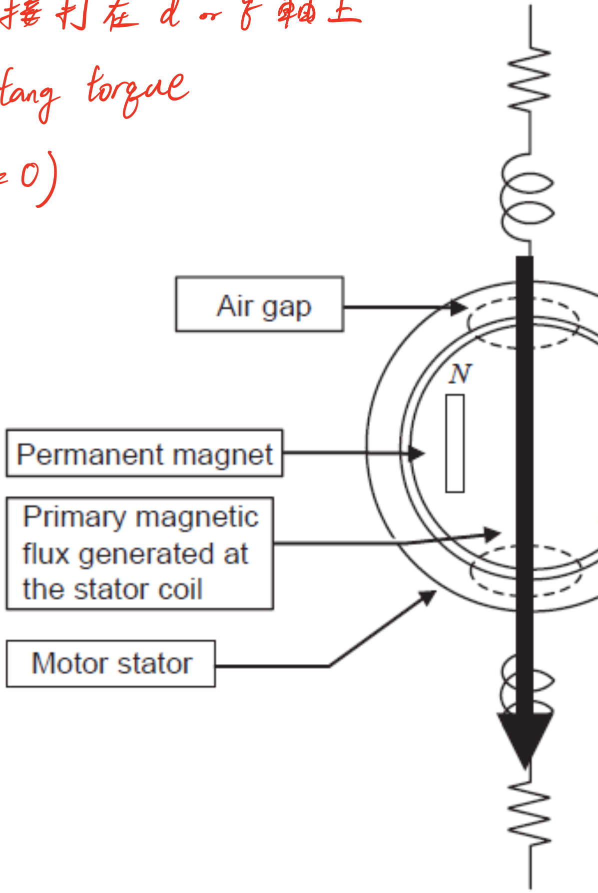
Reluctance Torque

電流不能直接打在 d or q 軸上
否則無 reluctang torque
($i_d = 0$ or $i_q = 0$)

(X)



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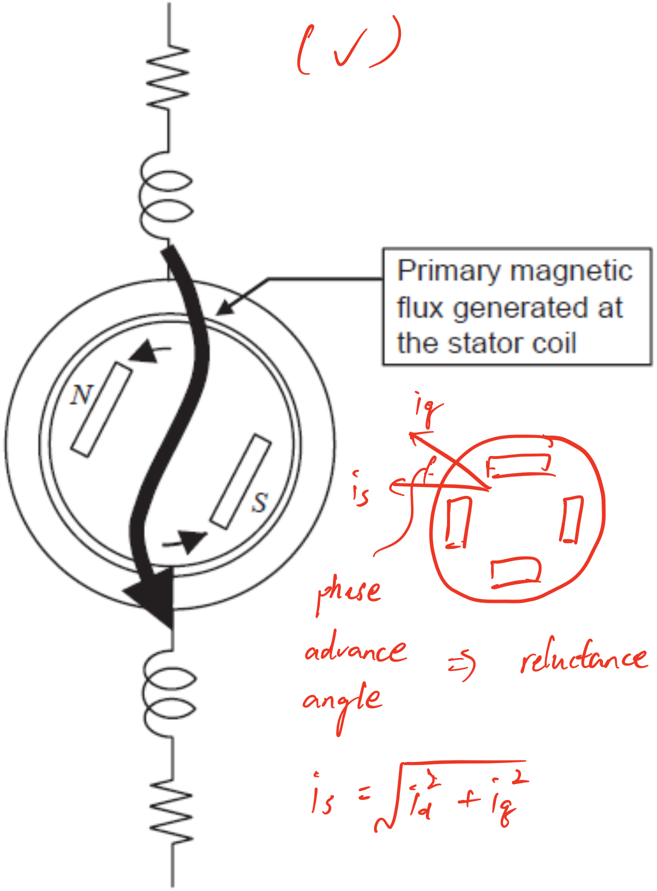

$T_e = \frac{3P}{4} \psi_f i_q$

phase advance angle ⇒ reluctance torque

$i_s = \sqrt{i_d^2 + i_q^2}$

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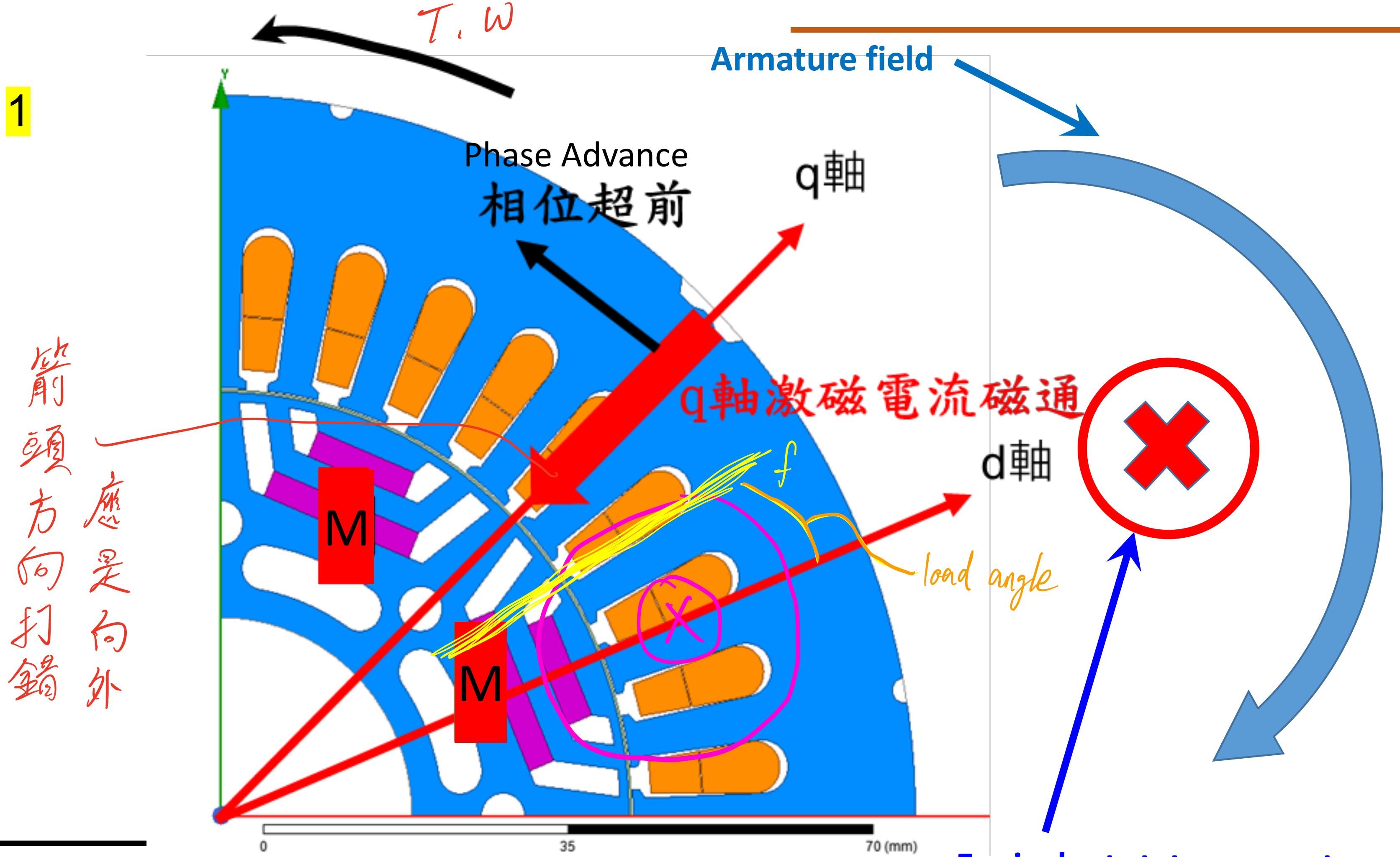
Phase Advance

- 1



Operation Characteristics of PMSM

The equation for the motor mechanical dynamics is shown as follows.

$$T_e = T_L + B_m \omega_r + J_m \frac{d\omega_r}{dt}$$

where T_L is load torque, B_m is damping coefficient, ω_r is mechanical rotor speed and J_m is moment of inertia of the motor.

The equation for the motor mechanical dynamics is shown as follows.

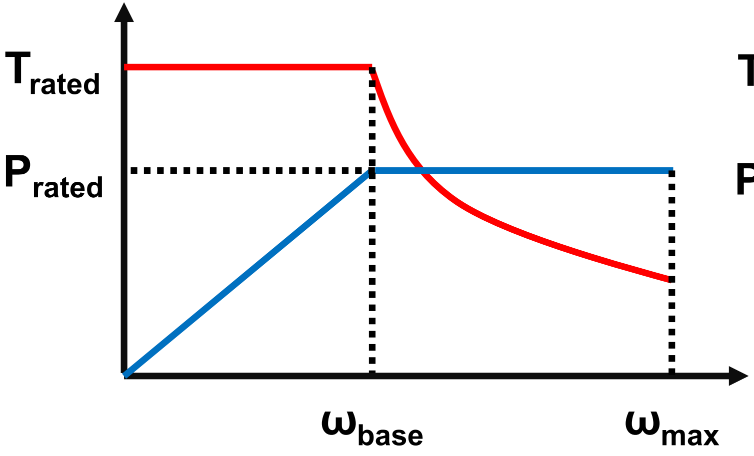
$CPSR = \frac{\omega_r}{\omega_b}$

FW,

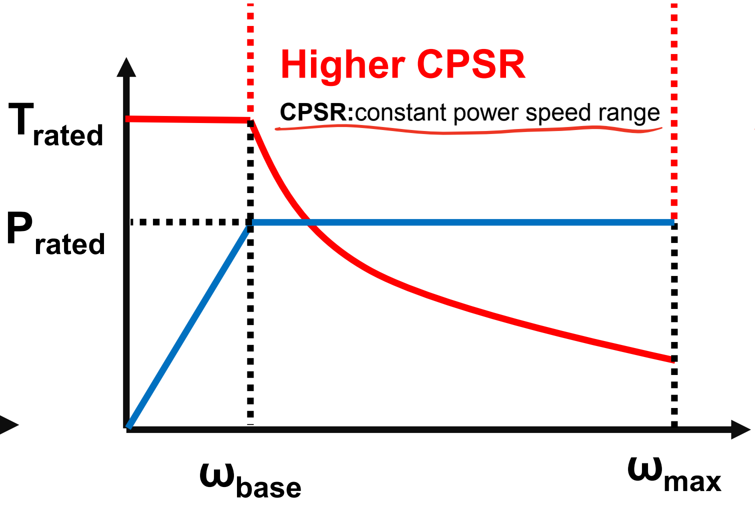
$I_{ch} = \frac{\psi_f}{L_d}$

$\psi_d = \underbrace{\psi_f + L_d i_d}_{=0}$

I_{ch} 特徵電流 加在 d 軸上 FW.



Industry



Electric Vehicle

Control of PMSM

Brief Introduction of FOC

The concept of FOC was first invented in the beginning of 1970s. This method brought forward intensive efforts in investigating high performance control of ac drives because of the fact that an induction motor controlled by an FOC algorithm can be controlled in a similar manner to the control of a separately excited dc motor.

FOC is also known as vector control, decoupling control, and orthogonal control. In general, the principle of FOC schemes implies independent (decoupled) control of flux – current (i_d) and torque – current (i_q) components of a stator current through a coordinated change in the supply voltage amplitude, phase angle and frequency.

Current limit and Voltage limit

- In the real case, the current and voltage are subject to real constraints:

$Current\ limit\ equation$: $i_d^{\ 2} + i_q^{\ 2} = i_s^{\ 2} \le i_{max}^{\ 2}$ → *a continuous circle form*

$Voltage\ limit\ equation$: $v_d^{\ 2} + v_q^{\ 2} = v_s^{\ 2} \le v_{max}^{\ 2}$ → *difference of SPM and IPM*

- Assume that motor runs at high speed mode (and sufficiently high), so the phase resistance R_s is neglected because its voltage drop is much smaller than X_L , we have voltage on d-q axis as:

$$v_d = \omega_e L_q i_q$$

$$v_q = \omega_e L_d i_d + \omega_e \psi_f$$

$$\Rightarrow \quad \frac{\left(i_d + \frac{\psi_f}{L_d}\right)^2}{\left(\frac{V_S}{\omega_e L_d}\right)^2} + \frac{(i_q)^2}{\left(\frac{V_S}{\omega_e L_q}\right)^2} = 1 \qquad \text{(A)}$$

$V_d = V_R + V_L + E$

↑

高速不可略

$Z = R + jX$

↓

$W_e L$

$W_e L \gg R$

→ Taking into account (A) in case of SPM and IPM

Current limit and Voltage limit

SPM: $L_d = L_q = L_s$

Circle equation

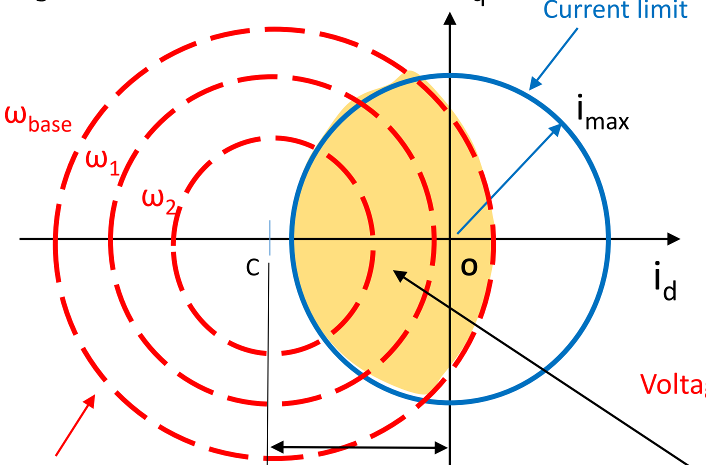
$$\frac{(i_d + \frac{\psi_f}{L_s})^2}{(\frac{V_S}{\omega_e L_s})^2} + \frac{(i_q)^2}{(\frac{V_S}{\omega_e L_s})^2} = 1$$

IPM: $L_d < L_q$

Ellipse equation

$$\frac{\left(i_d + \frac{\psi_f}{L_d}\right)^2}{\left(\frac{V_S}{\omega_e L_d}\right)^2} + \frac{(i_q)^2}{\left(\frac{V_S}{\omega_e L_q}\right)^2} = 1$$

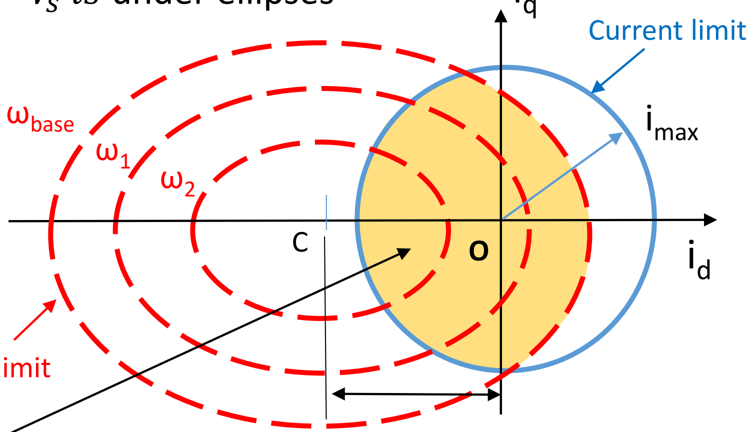
V_s *is* under circles



$i_{ch} = -\psi_f / L_s$

$\omega_{base} < \omega_1 < \omega_2$

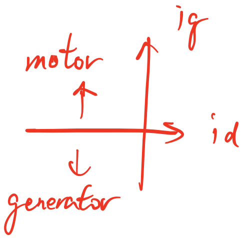
V_s *is* under ellipses



$i_{ch} = -\psi_f / L_d$

$\omega_{base} < \omega_1 < \omega_2$

The operable region of the motor drive system



Introduction to Direct FOC

Direct Field Oriented Control is a basic control method which can be easily applied either in IPM or SPM .The direct axis(d-axis) current is controlled to zero to make sure no field intensify or weakenking effect while q-axis current is controlled according to torque demand similar to DC motor.

easily controlled

Advantage : Output torque is ~~easy to control~~ due to the linear relationship to q-axis current.

Drawback : Reluctance torque cannot be produced.

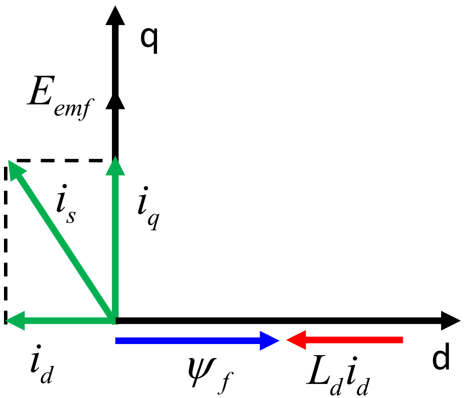
SPM no reluctance torque

IPM 用此控制有些浪費

Direct FOC Operating Condition

Direct FOC has no field intensify or weakenking effect(i_d =0), Magnetomotive force (MMF) is only established by permanent magnets mounted on rotor.

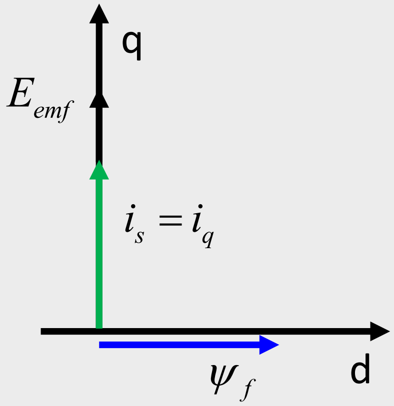
I_d<0 (demagnetizing)



$$\psi_d = -L_d i_d + \psi_f$$

$$E = \psi_f \omega_e$$

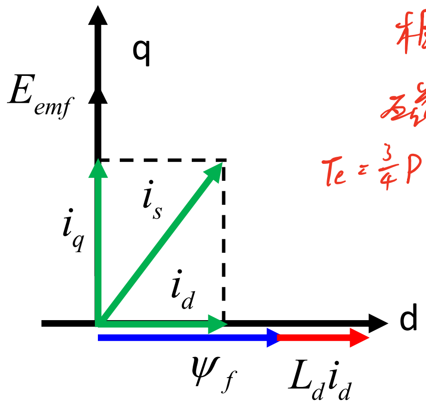
I_d=0



$$\psi_d = \psi_f$$

$$E = \psi_f \omega_e$$

I_d>0 (magnetizing)



$$\psi_d = L_d i_d + \psi_f$$

$$E = \psi_f \omega_e$$

IPM

相位落後

磁阻轉矩為負

$T_e = \frac{3}{4}P\left[\psi_f i_q + \underbrace{(L_d - L_q)}_{(-)}\underbrace{i_d}_{(+)} i_q\right]$

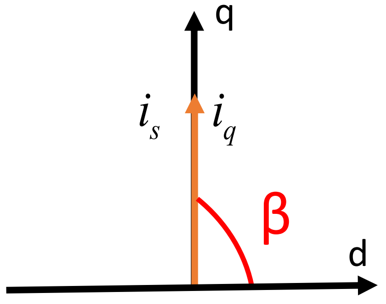
use in SynRM

FI-IPM

Touque of Direct FOC

Torque expression for PM motors for direct FOC .

phase advance angle start from q



With *current angle* $\beta = 90°$, *we have:*

$i_s = i_d + ji_q$

$i_d = i_s \sin(90^o - \beta) = i_s \cos\beta = 0$

$i_q = i_s \cos(90^o - \beta) = i_s \sin\beta = i_s$

$i_d = 0$

For SPM, no saliency **(L_d=L_q)**

→ *reluctance torque is zero*

Magnet torque (due to PM)

$$T_{e\_SPM} = \frac{3}{2}\frac{P}{2}\psi_f i_q$$

For IPM, (L_d<L_q)

$$T_{e\_IPM} = \frac{3}{2}\frac{P}{2}\left[\overbrace{\psi_f i_q}^{\text{Magnet torque (due to PM)}} + \underbrace{(L_d - L_q) i_d i_q}_{\text{Reluctance torque}}\right] = \boxed{\frac{3}{2}\frac{P}{2}(\psi_f i_q)}$$

No reluctance torque is produced in direct FOC

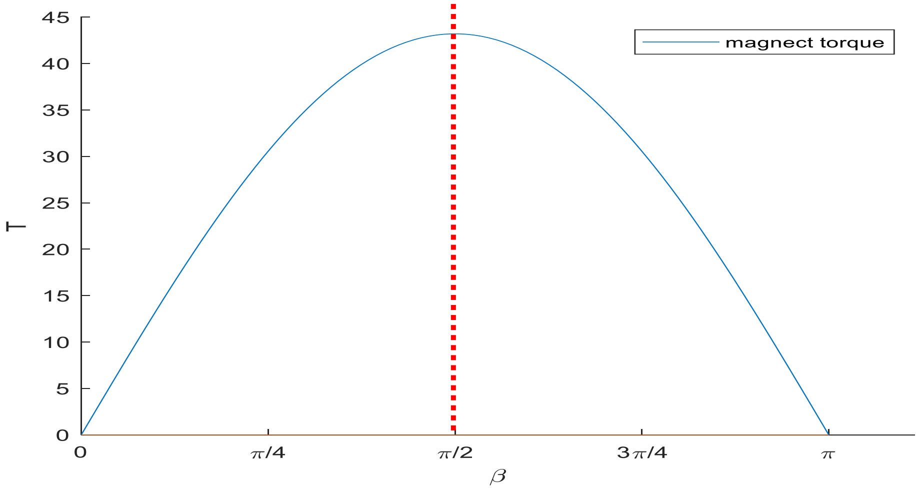
Touque of Direct FOC

$$T_{e\_IPM} = T_{e\_SPM} = \frac{3}{2}\frac{p}{2}(\psi_f i_s \sin\beta) = T_{magnet}$$

Either IPM or SPM, only magnet torque can be produced in Direct FOC.

To find the maximum torque, $\frac{\partial T_e}{\partial \beta} = \frac{3}{2}\frac{p}{2}(\psi_f i_s \cos\beta) = 0$

Maximum torque can only be obtained when β equal to 90^o



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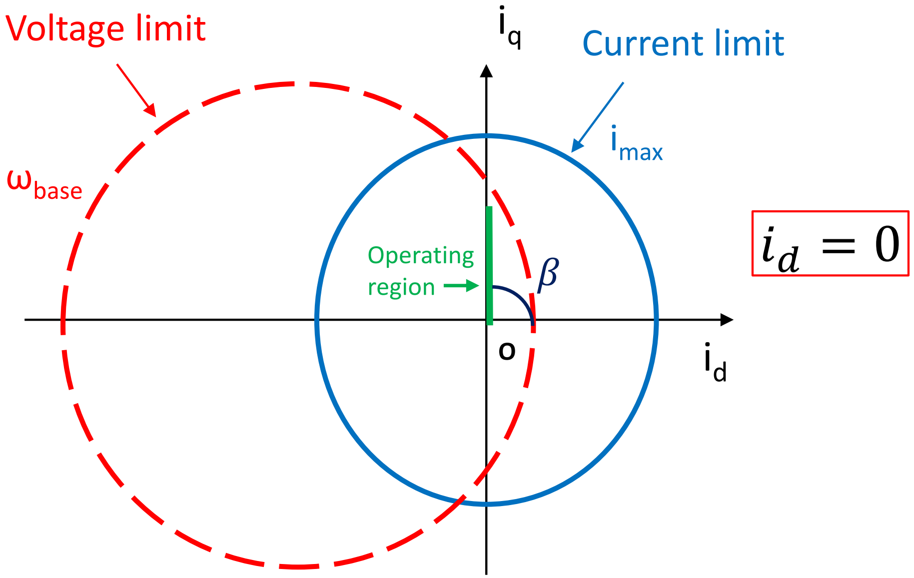
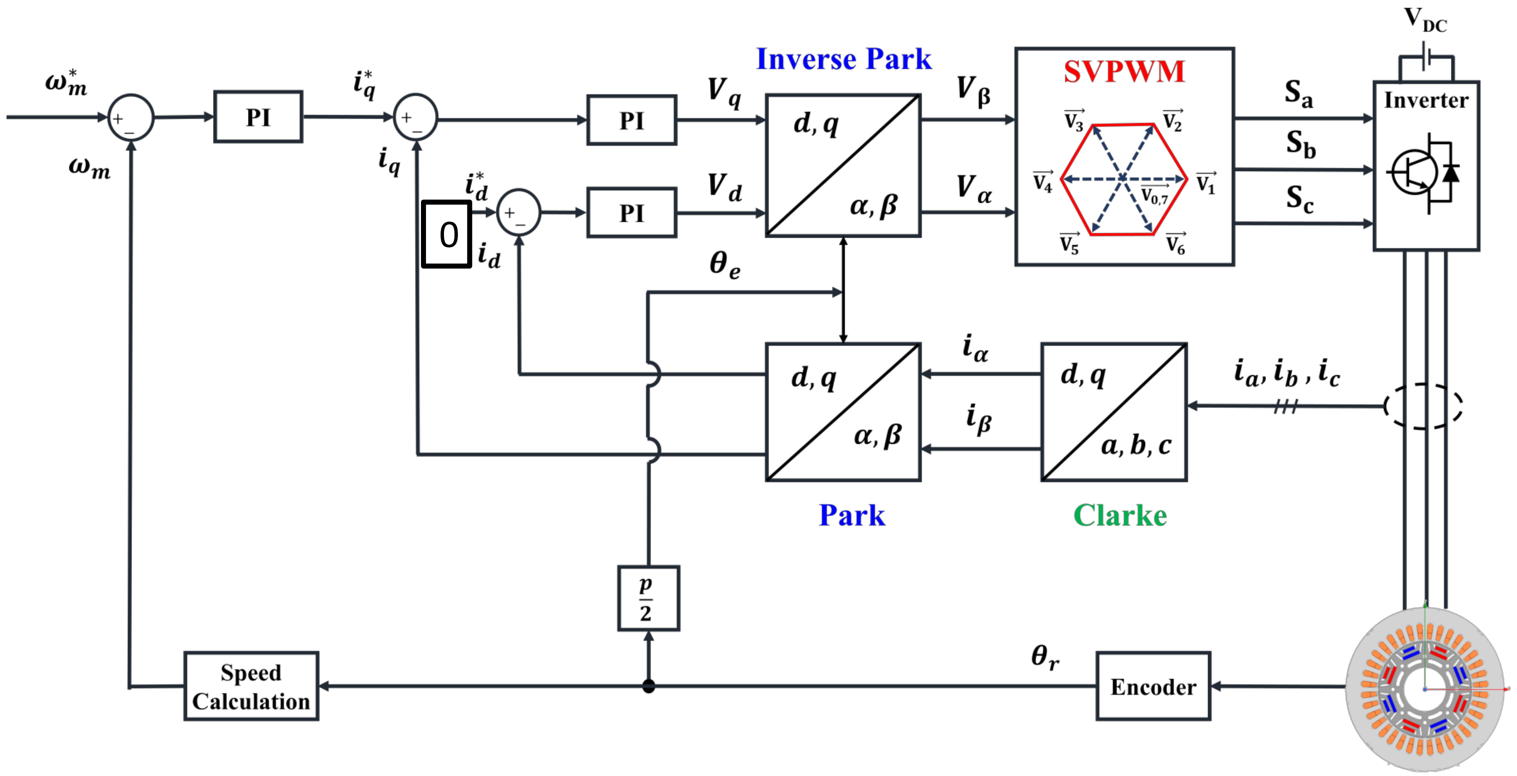
Direct FOC Operating Region

The operating region of Direct FOC is a simple straight line. Without the effect of i_d ,output torque $T_e = \frac{3}{2}\frac{p}{2}\psi_f i_q$ can be easily controlled by $\mathrm{i_q}$ due to the linear relation.

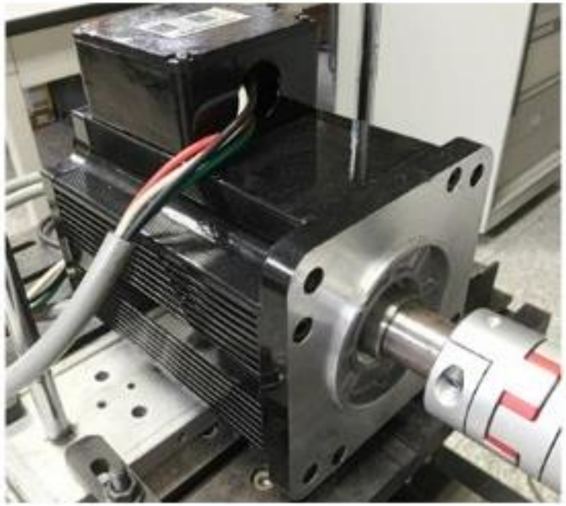


Control Architecture of FOC for PMSM

Control block diagram for Direct FOC



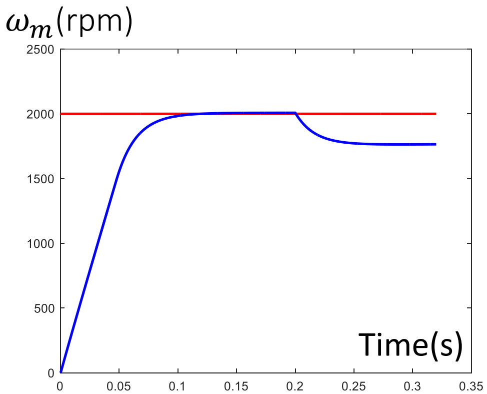
Simulation IPMSM model

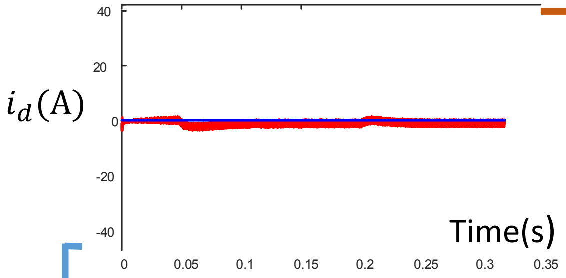


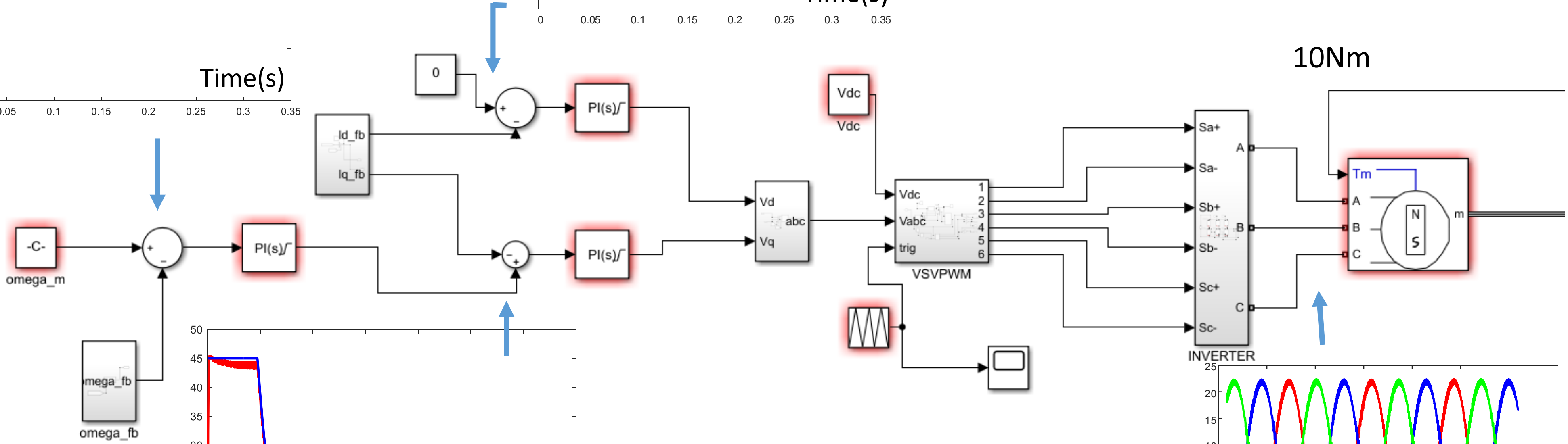
Specification	Value
Rated Torque T_{rated} (N.m)	11.78
Max. Torque T_{max} (N.m)	29.4
Rated Voltage V_{rated} (V_{pk})	220
Rated Current I_{rated} (A_{rms})	18.5
Max. Current I_{max} (A_{rms})	45
Rated Speed $N_{e,rated}$ (RPM)	3000
Rated Power P_{rated} (kW)	3.7
Pole	8
Torque Constant K_T ($N \cdot m/A_{pk}$, FOC $i_d = 0$)	0.48
Voltage Constant K_e ($V_{pk} \cdot s/rad$)	0.08
Inertia J_m ($kg \cdot m^2$)	0.00633
resolution	128
Parameter	**Value**
D Axis Inductance L_d (mH)	0.76
Q Axis Inductance L_q (mH)	1.61
Resistance of Stator Windings R_s (mΩ)	141.6
Flux Linkage λ_m (mWb)	80

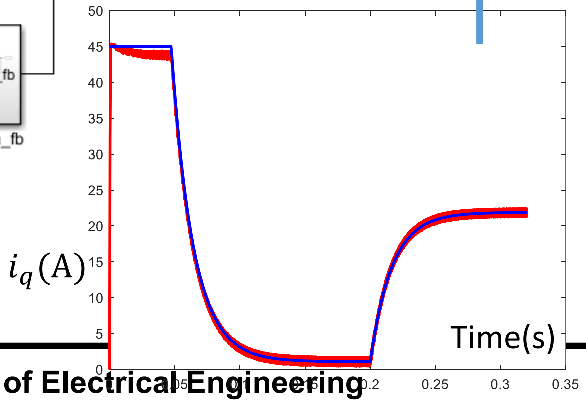
Direct FOC simulation result

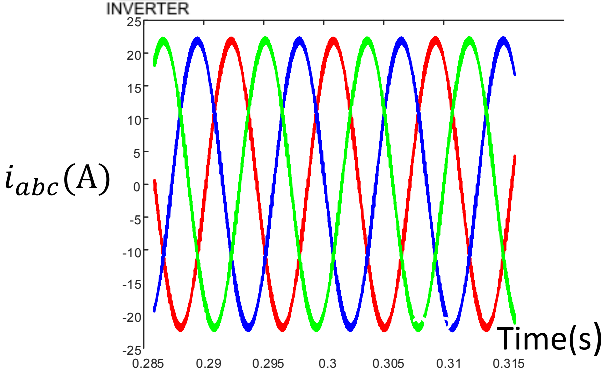
Test condition:
Accelerating to 2000rpm unloaded and then loading 10Nm at 0.2s











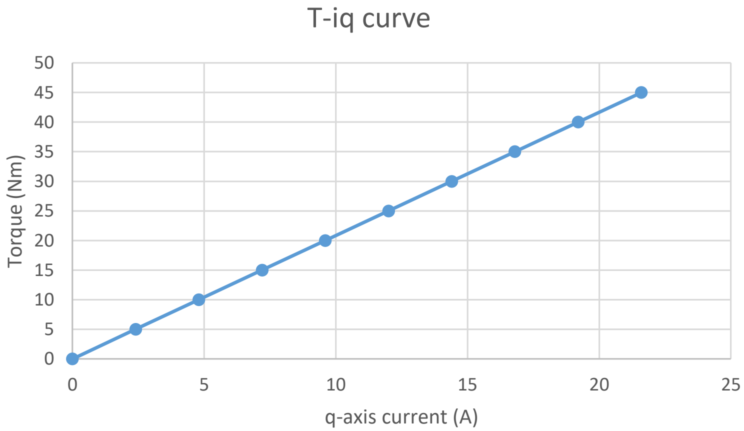
$i_q/i_d/\omega_m$ feedback(red line)
$i_q/i_d/\omega_m$ reference(blue line)

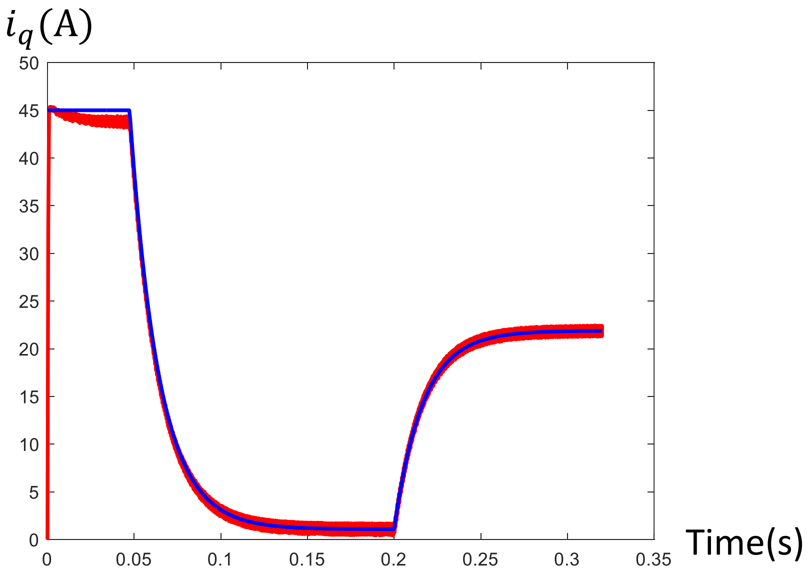
Direct FOC Simulation Result

Test condition : Speed up to 2000rpm with unload from 0 to 0.2 sec
Adding 10Nm load at t=0.2s

i_q dominates the torque output , so when accelerating or loading , i_q rises significantly for high torque demand.

Torque(Nm)	i_q(A)
21.6	45
19.2	40
16.8	35
14.4	30
12	25
9.6	20
7.2	15
4.8	10
2.4	5
0	0

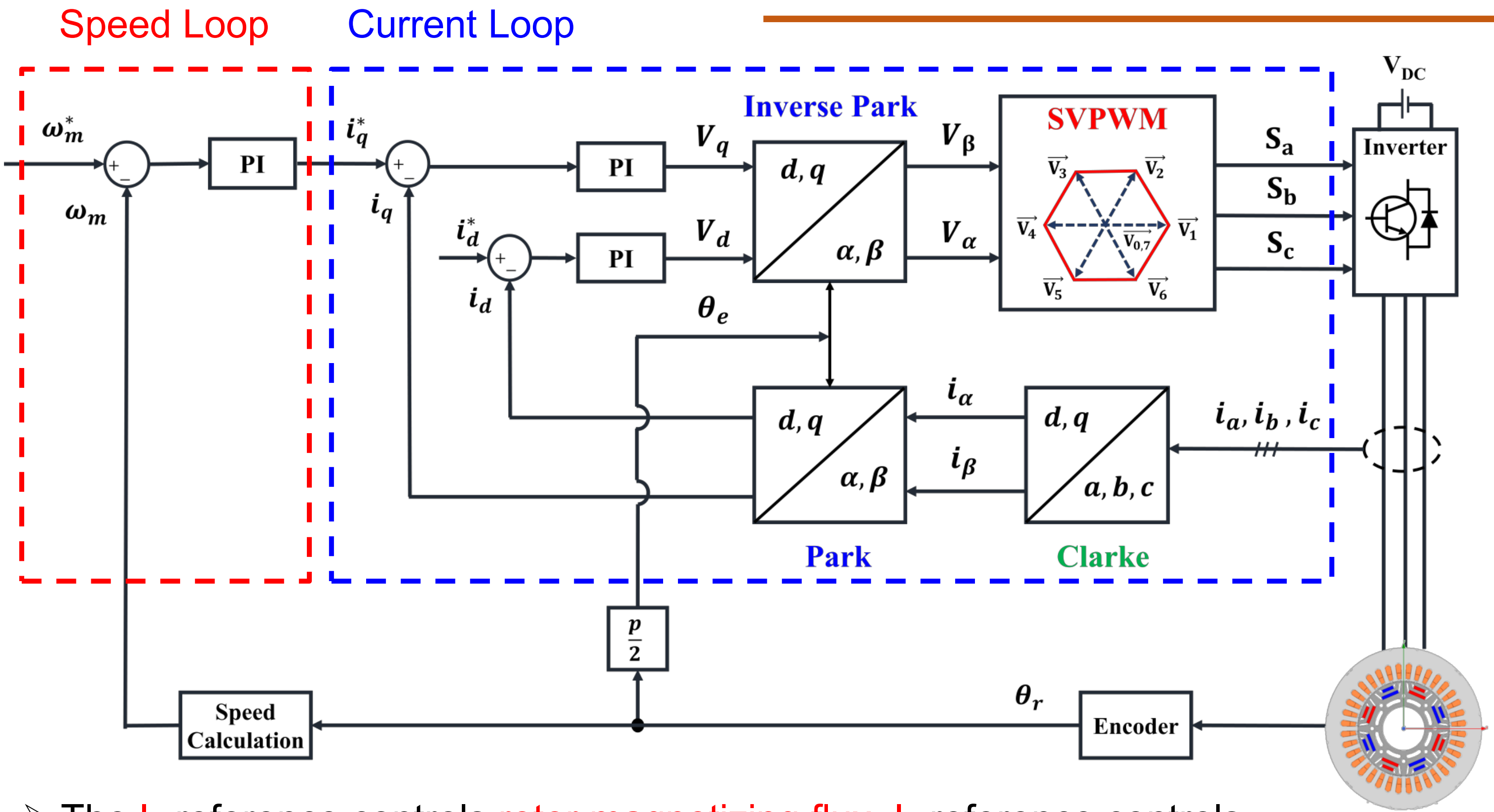




i_q current feedback(red line)
i_q current reference(blue line)

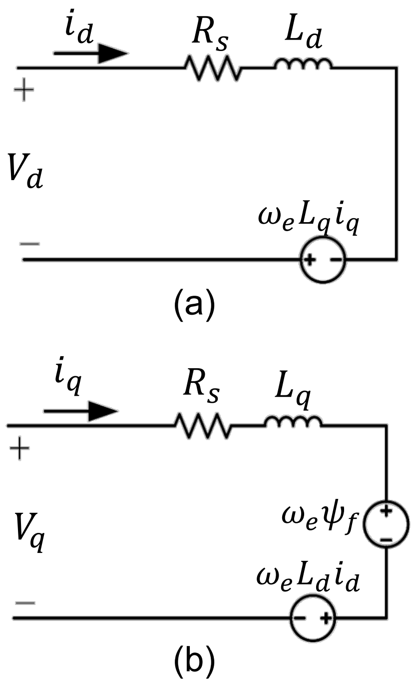
Controller Design of FOC

Control Architecture of FOC for PMSM



- The I_d reference controls rotor magnetizing flux, I_q reference controls the torque output of the motor
- I_d and I_q are only time-invariant under steady-state load conditions

PMSM Dynamic Model



The equivalent circuit of PMSM under the d-q axis
(a) d axis (b) q axis

Voltage equation

$$\begin{cases} V_d = R_s i_d + \dfrac{d}{dt}\psi_d - \omega_e \psi_q \\ V_q = R_s i_q + \dfrac{d}{dt}\psi_q + \omega_e \psi_d \end{cases} \quad (1)$$

$$\begin{cases} \psi_d = L_d i_d + \psi_f \\ \psi_q = L_q i_q \end{cases} \quad (2)$$

Electromagnetic torque equation

$$T_e = \frac{3}{2}\frac{p}{2}(\psi_d i_q - \psi_q i_d) = \frac{3}{2}\frac{p}{2}[\lambda_m i_q + (L_d - L_q) i_d i_q] \quad (3)$$

(p=number of pole)

Dynamic equation

$$T_e - T_L = J_m \frac{d\omega_m}{dt} + B_m \omega_m \quad (4)$$

(T_L : Load Torque , B_m : Friction Torque , J_m : Inertia Torque ,
$\omega_e = \omega_m * p/2$)

PMSM Dynamic Model

t- domain

According to voltage equations (1) & (2)

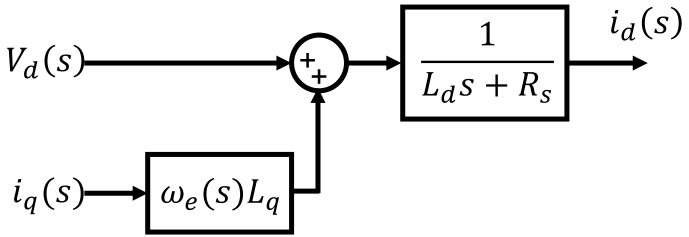
$$\begin{cases} \dfrac{di_d}{dt} = \dfrac{-R_s}{L_d} i_d + \dfrac{1}{L_d} V_d + \dfrac{\omega_e L_q}{L_d} i_q \\ \dfrac{di_q}{dt} = \dfrac{-R_s}{L_q} i_q + \dfrac{1}{L_q} V_q - \dfrac{\omega_e L_d}{L_q} i_d - \dfrac{\omega_e \psi_f}{L_q} \end{cases} \tag{5}$$

s - domain

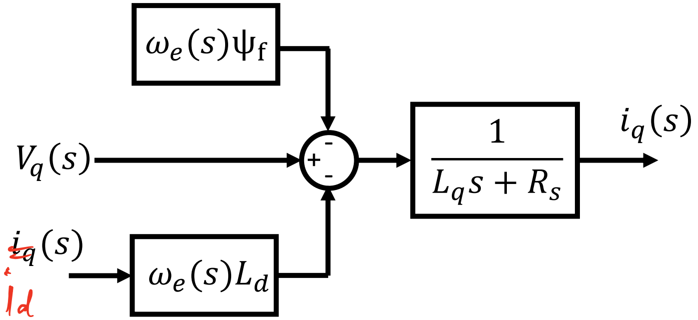
Laplace Transform (5)

$$\begin{cases} s i_d(s) = \dfrac{-R_s}{L_d} i_d(s) + \dfrac{1}{L_d} V_d(s) + \dfrac{\omega_e(s) L_q}{L_d} i_q(s) \\ s i_q(s) = \dfrac{-R_s}{L_q} i_q(s) + \dfrac{1}{L_q} V_q(s) - \dfrac{\omega_e(s) L_d}{L_q} i_d(s) - \dfrac{\omega_e(s) \psi_f}{L_q} \end{cases} \tag{6}$$

Yields:



PMSM d-axis current



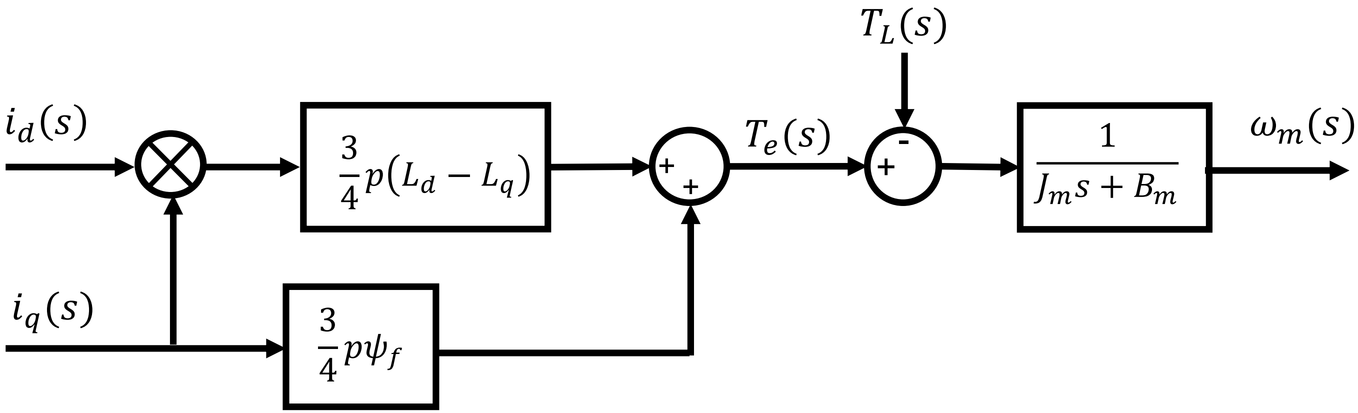
PMSM q-axis current

PMSM Dynamic Model

According to torque and dynamic equation (3) & (4)

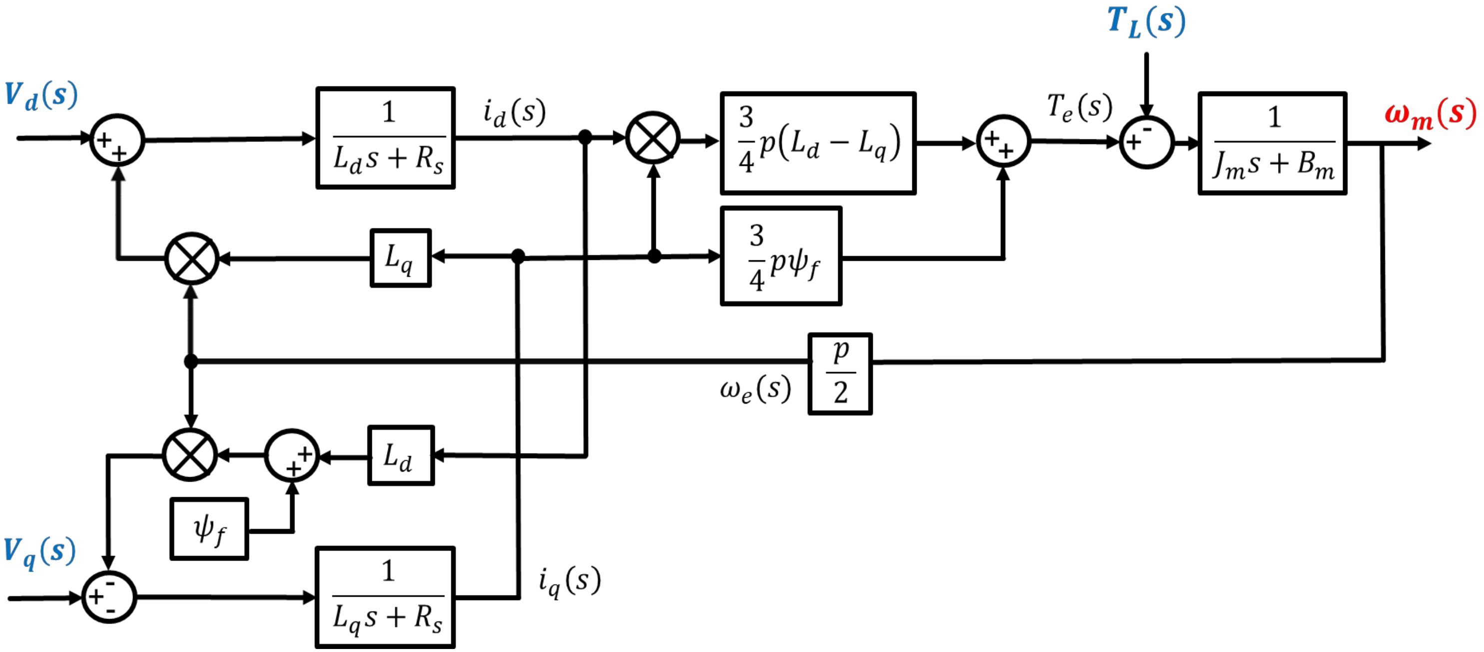
$$\begin{cases} T_e(s) = \frac{3}{4}p[\psi_f i_q(s) + (L_d - L_q)i_d(s)i_q(s)] \\ T_e(s) - T_L(s) = sJ_m\omega_m(s) + B_m\omega_m(s) \end{cases} \quad (7)$$

Yields:



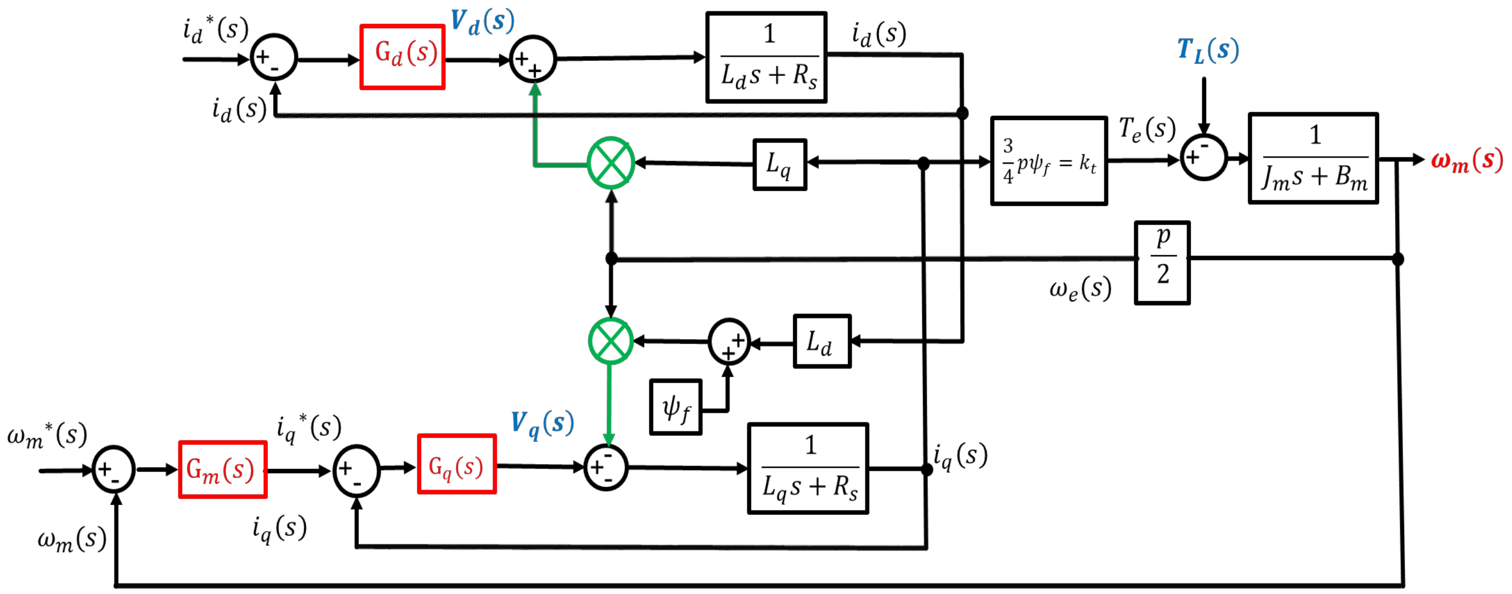
PMSM Speed diagram

PMSM Dynamic Model



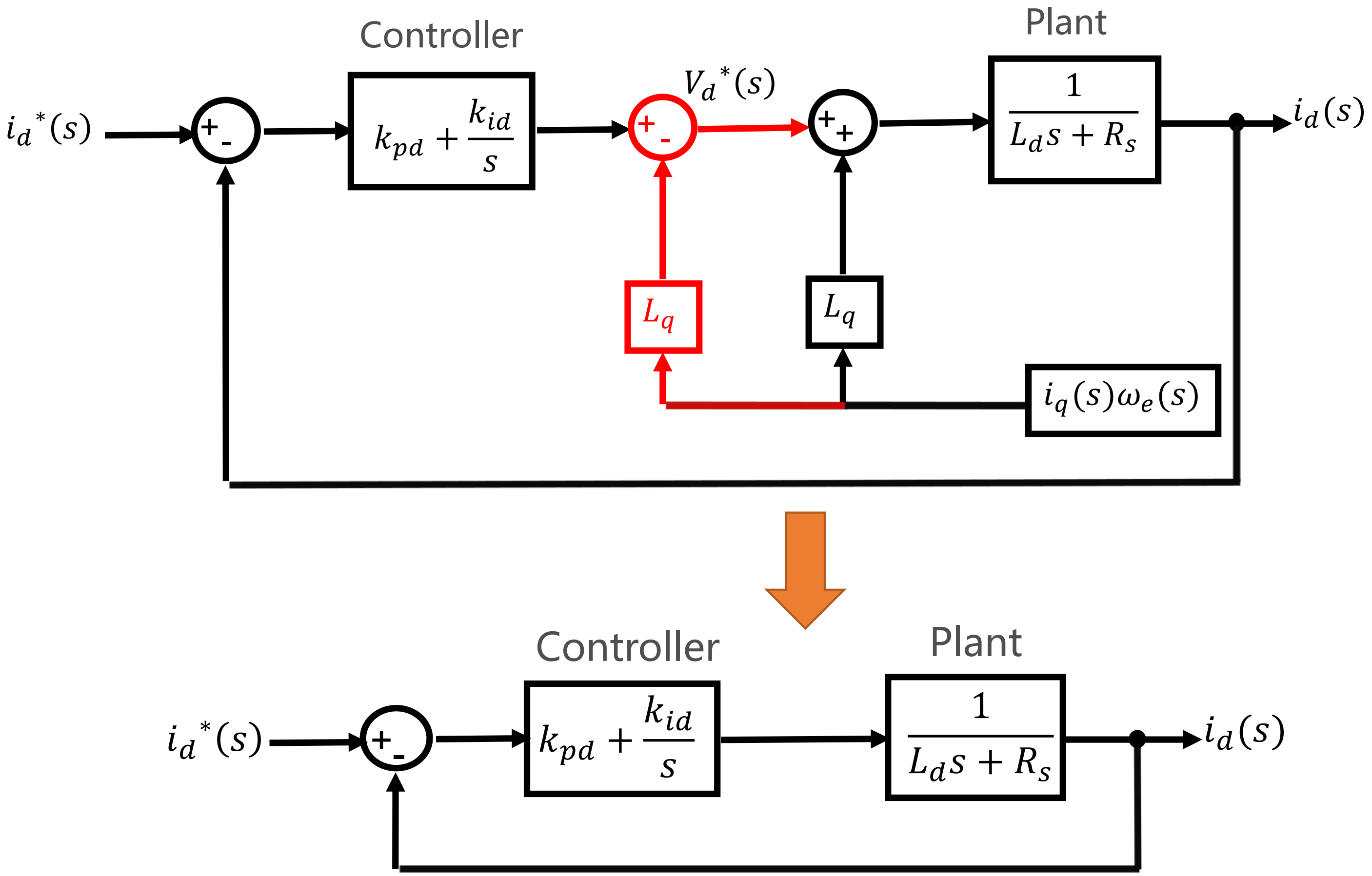
PMSM Dynamic Model

PMSM Dynamic Model

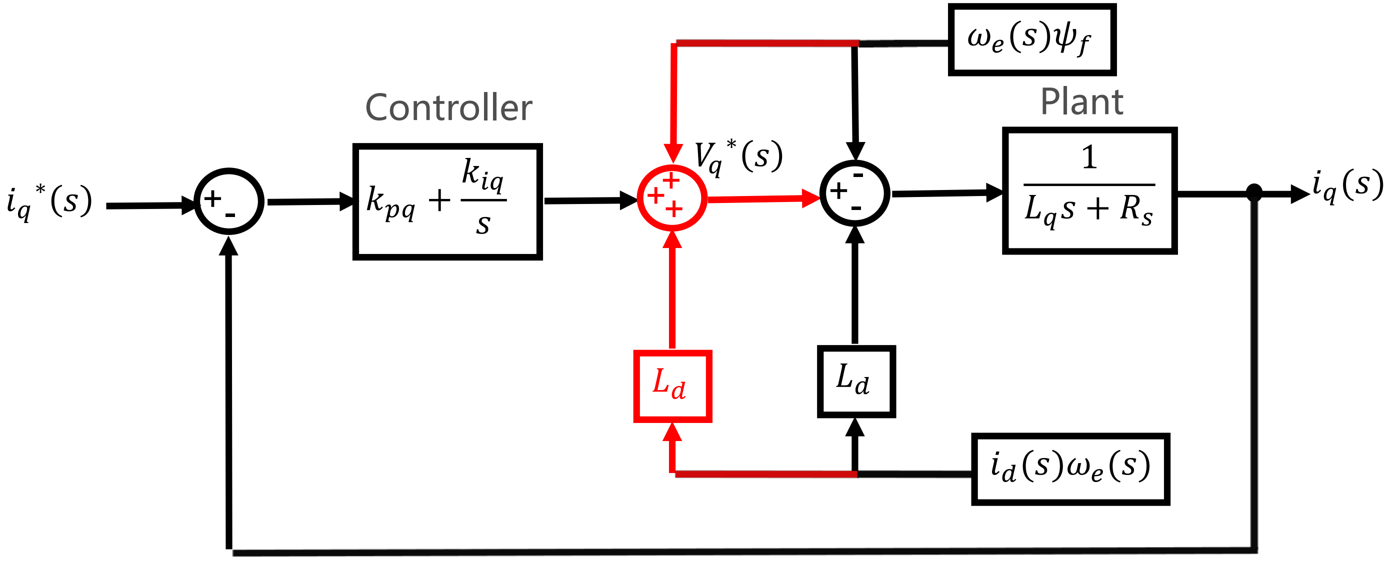


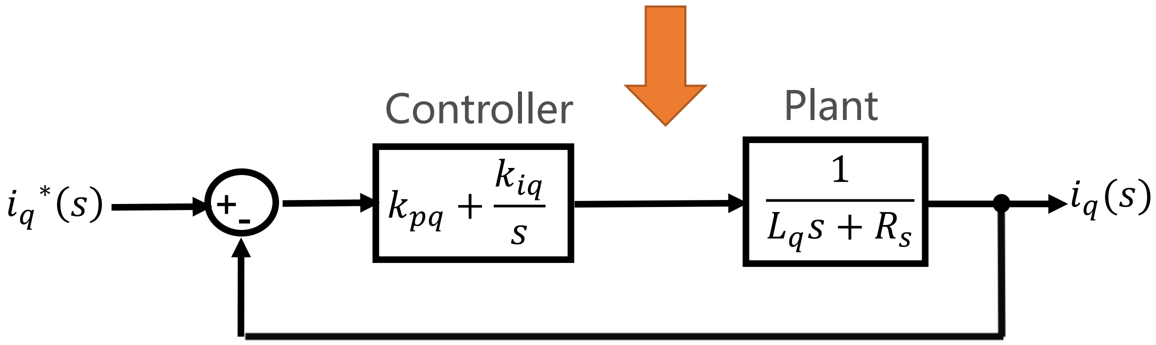
PMSM Dynamic Model including speed and current controllers

PI controller - D-axis current loop PI



PI controller - Q-axis current loop PI



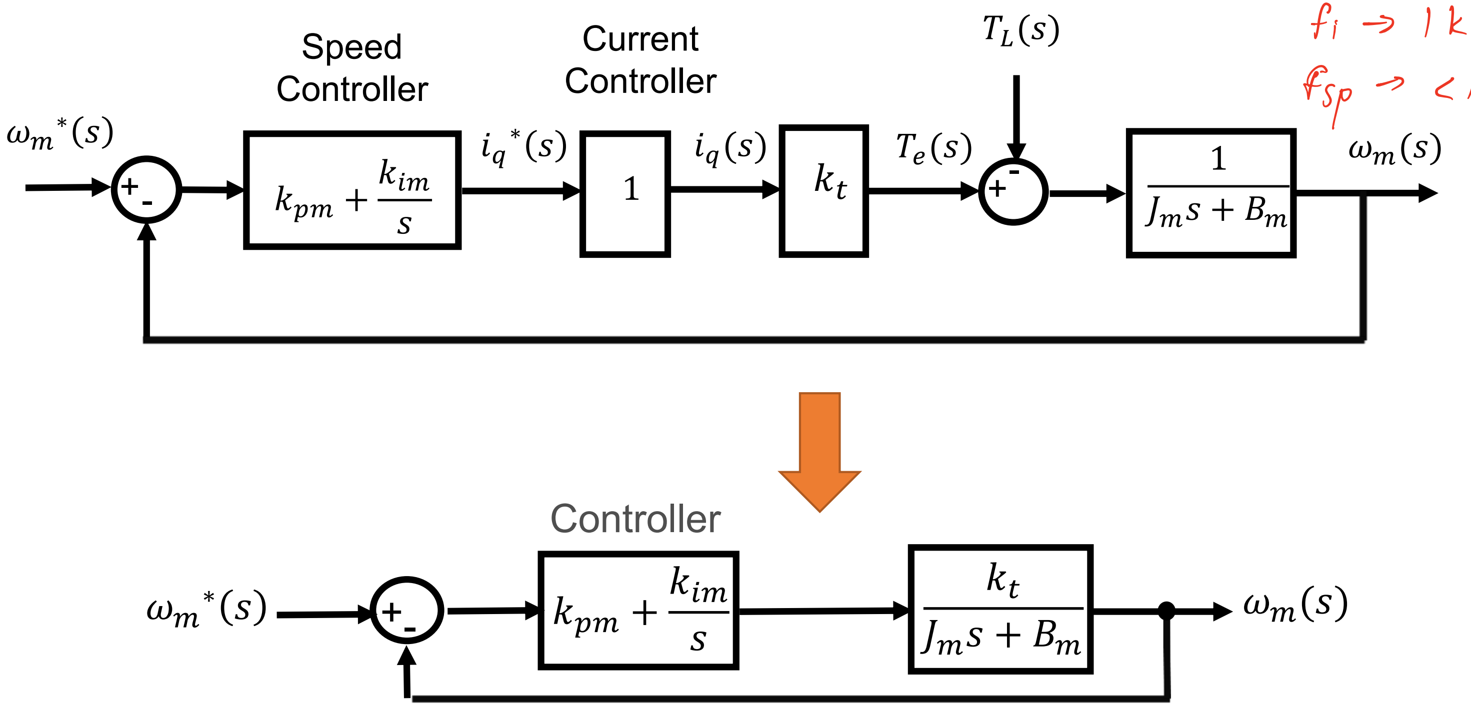


Design PI controller- Q-axis speed loop PI

$f_{PWM} \to 20\,kHz$

$f_i \to 1\,kHz$

$f_{sp} \to < 100\ Hz$



Design PI controller - D-axis current loop PI

From above PI introduction, we can set transfer function $G_d(s)$:

$$G_d(s) = k_{pd} + \frac{k_{id}}{s}$$

k_{pd} is Proportional constant, k_{id} is intergral constant

Transfer function of d-axis current closed loop is :

$$\frac{i_d(s)}{i_d^*(s)} = \frac{(k_{pd} + \frac{k_{id}}{s}) \cdot \frac{1}{sL_d + R_s}}{1 + (k_{pd} + \frac{k_{id}}{s}) \cdot \frac{1}{sL_d + R_s}}$$

Using zero pole canceling, we set $\frac{k_{pd}}{k_{id}} = \frac{L_d}{R_s}$

Then we can get

$$\frac{i_d(s)}{i_d^*(s)} = \frac{1}{\frac{R_s}{k_{id}} s + 1} = \frac{1}{\frac{1}{2\pi f_{BWd}} s + 1}$$

Generally, the bandwidth f_{BWd} is equal to 0.1*f_{sw} (switching frequency)

As a result,

$$k_{pd} = 2\pi f_{BWd} L_d$$

$$k_{id} = 2\pi f_{BWd} R_s$$

Design PI controller - Q-axis current loop PI

From above PI introduction, we can set transfer function $G_q(s)$:

$$G_q(s) = k_{pq} + \frac{k_{iq}}{s}$$

k_{pq} is Proportional constant, k_{iq} is intergral constant

Transfer function of q-axis current closed loop is :

$$\frac{i_q(s)}{i_q^*(s)} = \frac{(k_{pq} + \frac{k_{iq}}{s}) \cdot \frac{1}{sL_q + R_s}}{1 + (k_{pq} + \frac{k_{iq}}{s}) \cdot \frac{1}{sL_q + R_s}}$$

Using zero pole canceling, we set $\frac{k_{pq}}{k_{iq}} = \frac{L_q}{R_s}$

Then we can get

$$\frac{i_q(s)}{i_q^*(s)} = \frac{1}{\frac{R_s}{k_{iq}} s + 1} = \frac{1}{\frac{1}{2\pi f_{BWq}} s + 1}$$

Generally, the bandwidth f_{BWd} is equal to 0.1*f_{sw} (switching frequency)

As a result,

$$k_{pq} = 2\pi f_{BWq} L_q$$

$$k_{iq} = 2\pi f_{BWq} R_s$$

Design PI controller speed loop PI

From above PI introduction, we can set transfer function $G_m(s)$:

$$G_m(s) = k_{pm} + \frac{k_{im}}{s}$$

k_{pm} is Proportional constant, k_{im} is intergral constant

Transfer function of speed closed loop is :

$$\frac{\omega_m(s)}{\omega_m^*(s)} = \frac{(k_{pm} + \frac{k_{im}}{s}) \cdot \frac{1}{sJ_m + B_m} \cdot k_t}{1 + (k_{pm} + \frac{k_{im}}{s}) \cdot \frac{1}{sJ_m + B_m} \cdot k_t}$$

Using zero pole canceling, we set $\frac{k_{pm}}{k_{im}} = \frac{J_m}{B_m}$

Then we can get

$$\frac{\omega_m(s)}{\omega_m^*(s)} = \frac{1}{\frac{B_m}{k_{im}k_t} s + 1} = \frac{1}{\frac{1}{2\pi f_{BWm}} s + 1}$$

Generally, the bandwidth f_{BWd} is equal to 0.01*f_{sw} (switching frequency)

As a result,

$$k_{pm} = \frac{2\pi f_{BWm} J_m}{k_t} \qquad k_{im} = \frac{2\pi f_{BWm} B_m}{k_t}$$