COMPUTATIONAL COMPLEXITY.

REGULAR EXPRESSIONS RESEARCH ASSIGNMENT.

1. Regular expressions for languages over $\Sigma = \{a, b\}$.

@ All strings starting with 'a' and ending with 'b'.

Keys: — A string must start with 'a'.
- the string must end with 'b'.
- Any string / characters may take the middle part. $\{a, b\}^*$
- the middle can be empty hence ab.

$$a \left(\{a, b\}^* \right) b.$$

Hence the regular expression, $R = a(a+b)^* b$

ⓑ All strings containing exactly 3 a's.
from $\{a, b\}$.

strings $= \{a, b, aa, ab, ba, bb, aab, \ldots, aaa, aab, \ldots, ababa, \ldots\}$

Keys: — A string can consist of only a's. $\{aaa\}$.
- strings of all the 3 a's together. $\{baaa, aaa, baaab, \ldots\}$.
- At most 2 a's together $\{abaa, bbaaba, aaba, \ldots\}$.
- No consecutive a's. $\{ababa, bbababba, \ldots\}$.
- the string can begin and end with any character.

$$b^* (a) \, b^* (a) \, b^* (a) b^*$$

Hence a regular expression, $R = b^* a b^* a b^* a b^*$, taking only 3 a's with any number of 'b's (including λ).

ⓒ All strings containing no consecutive identical letters.
Strings $= \{a, b, aa, ba, ab, bb, aab, aba, \ldots\}$.

Keys: A string cannot have 2 'a's or 2 'b's in a row.
- Pattern alternates between a and b.

$$R_1 = (ab)^* \qquad R_2 = (ba)^*$$

$L_{R_1} = \{ab, abab, ababab, \ldots\}$, $L_{R_2} = \{ba, baba, bababa, \ldots\}$

L_{R_1} and L_{R_2} only contains strings of even length.

$R_3 = (a|b) \rightarrow$ a string of either a or b.

$R = (ab)^* + (ba)^* + (ab)^*(a) + (ba)^* b + (a+b).$

$R = (ab)^* (\lambda+a) + (ba)^* (\lambda+b) + (a+b)$

ⓓ All strings with length of at least 2.
Strings = {a, b, aa, ab, ba, bb, aba, abab, ... },

Key: - string must contain a minimum of 2 character.
 - string can be any combination of 'a's and 'b's.

$R_1 = (a+b)(a+b)$
$L_{R_1} = \{aa, ab, ba, bb\}$ exactly of length 2.

$R_2 = (a+b)^*$
$L_{R_2} = \{\lambda, a, b, aa, ab, ba, bb, ---\}.$

final regular expression, $R = R_1 R_2.$
$$R = (a+b)(a+b)(a+b)^*$$

2. Languages under Regular Expressions.

ⓐ $r = a(baba)^*$
 $La = \{a\}.$
 $L_{(baba)^*} = \{\lambda, baba, babababa, ---\}.$

Language, $L_r = \{a, ababa, abababa, ---\}$ ⟹ all strings sta
with a followed by 0 or more repetition of 'baba'.

ⓑ $r = (a+b) abb (a+b).$
 $r = \{a, b\} \{abb\} \{a, b\}.$

 $L_r = \{aabb, babb\} \{a, b\}.$
Language, $L_r = \{aabba, aabbb, babba, babbb\}.$

Ⓒ $r = (ab + ba)^* (a+b+\lambda)$.

let $r_1 = (ab+ba)^*$ and $r_2 = (a+b+\lambda)$.

$L_{r_1} = \{\lambda, ab, ba, abab, baba, abba, ababba, bababa, \cdots\}$
$L_{r_2} = \{a, b, \lambda\}$.

$\Rightarrow L_r = L_{r_1} \cdot L_{r_2}$.

language, $L_r = \{\lambda, a, b, aab, aba, ab, ba, bab, baba, abab, \cdots\}$.

Ⓓ $r = 1(0101)^*$

$r = \{1\}\{\lambda, 0101, 01010101, 010101010101, \cdots\}$.

hence, Language of r, L_r.

$L_r = \{1, 10101, 101010101, 1010101010101, \cdots\}$.

All strings in the language of r starts with 1 followed by 0 or more repetition of '0101'.

Ⓔ $r = (aa + bb)^* b$.

$r = (\{aa, bb\}^*) \{b\}$.

language of r.

$L_r = \{\lambda, aa, bb, aabb, aabbaa, \cdots\}.\{b\}$.

$L_r = \{b, aab, bbb, aabbb, aabbaab, \cdots\}$.

All strings in the language ends with 'b' but starts with 0 or more repetition of '(aa + bb)'.