HOMEWORK 2

CS178

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In [1]:
```
from __future__ import division
import numpy as np
import mltools as ml

%matplotlib inline
import matplotlib.pyplot as plt

np.random.seed(0)
```

Problem 1: Linear Regression

In [2]:
```
data = np.genfromtxt("data/curve80.txt",delimiter=None) # load the data
X = data[:,0]
X = np.atleast_2d(X).T # code expects shape (M,N) so make sure it's 2-dimens
Y = data[:,1] # doesn't matter for Y
Xtr,Xte,Ytr,Yte = ml.splitData(X,Y,0.75) # split data set 75/25
```

Problem 1.1

In [3]:
```
print "Xtr: " + str(Xtr.shape) + " Xte: " + str(Xte.shape) + " Ytr: " + str(
```

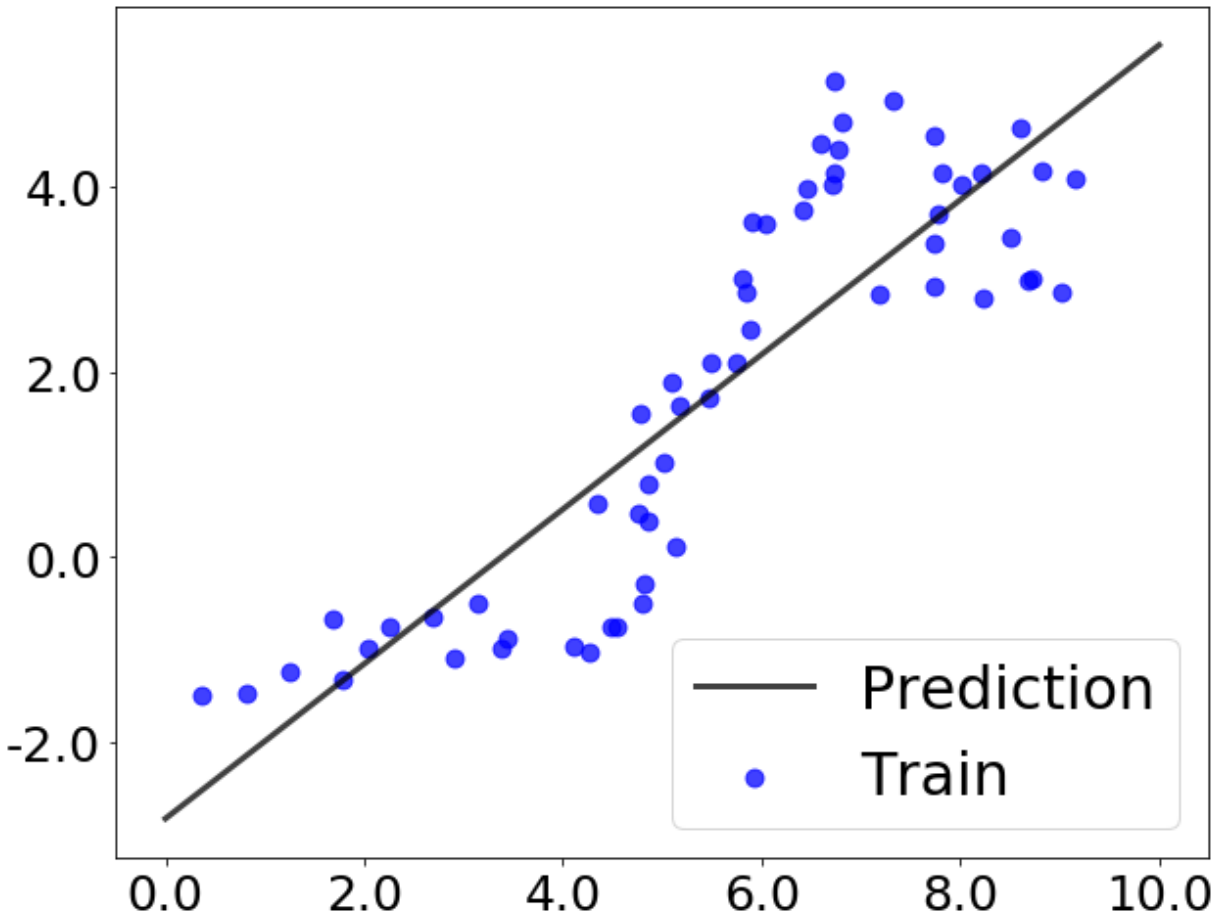
```
Xtr: (60, 1) Xte: (20, 1) Ytr: (60,) Yte: (20,)
```

Problem 1.2

In [4]:
```
lr = ml.linear.linearRegress( Xtr, Ytr ) # create and train model
xs = np.linspace(0,10,200) # densely sample possible x-values
xs = xs[:,np.newaxis] # force "xs" to be an Mx1 matrix (expected by our code
ys = lr.predict( xs ) # make predictions at xs
```

Problem 1.2 - A

In [5]:
```
# Plotting the data
f, ax = plt.subplots(1, 1, figsize=(10, 8))

ax.scatter(Xtr, Ytr, s=80, color='blue', alpha=0.75, label='Train')
ax.plot(xs, ys, lw=3, color='black', alpha=0.75, label='Prediction')

ax.set_xticklabels(ax.get_xticks(), fontsize=25)
ax.set_yticklabels(ax.get_yticks(), fontsize=25)

# Controlling the size of the legend and the location.
ax.legend(fontsize=30, loc=4)

plt.show()
```



Problem 1.2 - B

In [6]:
```
print lr.theta
print
print "SLOPE: " + str(lr.theta[0][1])
print "Y-INTERCEPT: " + str(lr.theta[0][0])
```

```
[[-2.82765049  0.83606916]]

SLOPE: 0.836069160262
Y-INTERCEPT: -2.82765048766
```

After observing my graph, the prediction function does indeed cross the y axis at -2.82765049, with a the slope moving upwards at a positive rate of 0.83606916.

Problem 1.2 - C

In [7]:
```
  def mse(y_true, y_hat):
    return np.mean((y_true - y_hat.reshape(y_true.shape))**2)
```

MSE of training data:

In [8]:
```
Ytrhat = lr.predict(Xtr)
mse(Ytr,Ytrhat)
```

Out[8]: 1.1277119556093911

MSE of test data:

In [9]:
```
Ytehat = lr.predict(Xte)
mse(Yte,Ytehat)
```

Out[9]: 2.2423492030101246

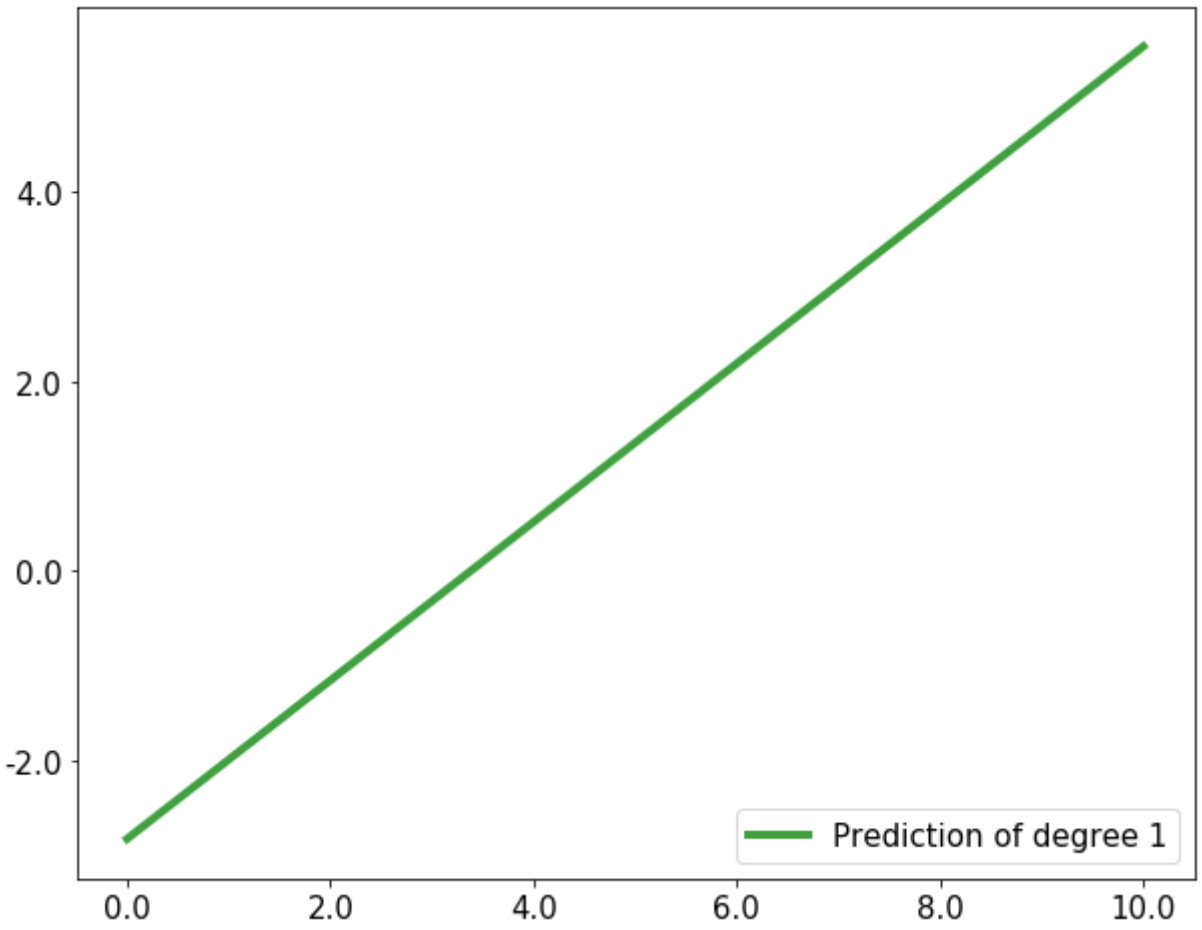
Problem 1.3

In [10]:
```
Xtr2 = np.zeros( (Xtr.shape[0],2) ) # create Mx2 array to store features
Xtr2[:,0] = Xtr[:,0] # place original "x" feature as X1
Xtr2[:,1] = Xtr[:,0]**2 # place "x^2" feature as X2
# Now, Xtr2 has two features about each data point: "x" and "x^2"
```

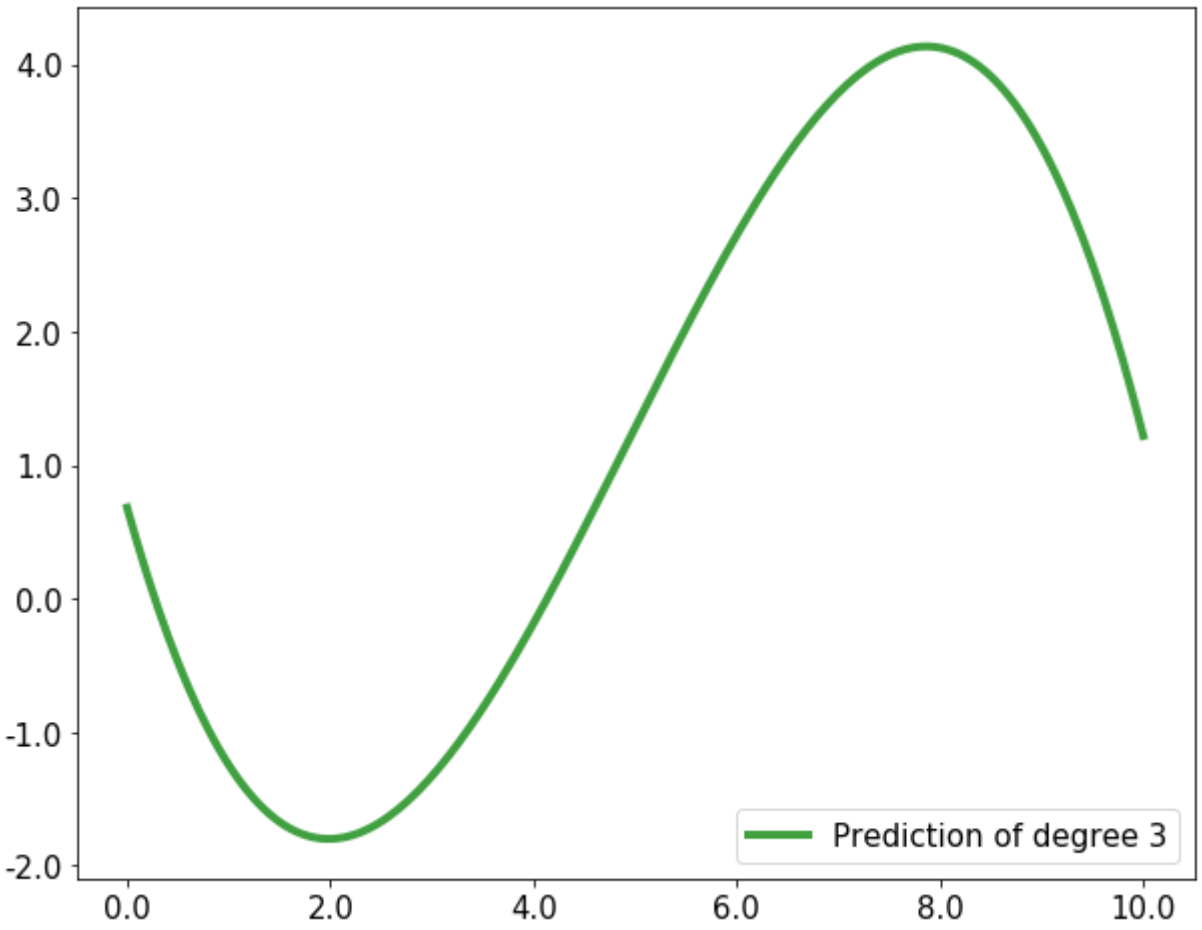
Problem 1.3 - A

In [11]:
```
def Phi(X):
    return ml.transforms.rescale( ml.transforms.fpoly(X,degree,False), paran

def LinRegressTset(degree):
    XtrP = ml.transforms.fpoly(Xtr, degree, bias=False)
    XtrP,params = ml.transforms.rescale(XtrP)
    XteP,_ = ml.transforms.rescale( ml.transforms.fpoly(Xte,degree,False), p
    XsP,_ = ml.transforms.rescale( ml.transforms.fpoly(xs,degree,False), par
    lr = ml.linear.linearRegress( XtrP, Ytr ) # create and train model

    YhatTrain = lr.predict( XtrP ) # predict on training data
    YhatTest  = lr.predict( XteP )
    yshat = lr.predict( XsP )
    return YhatTrain, YhatTest, yshat
```

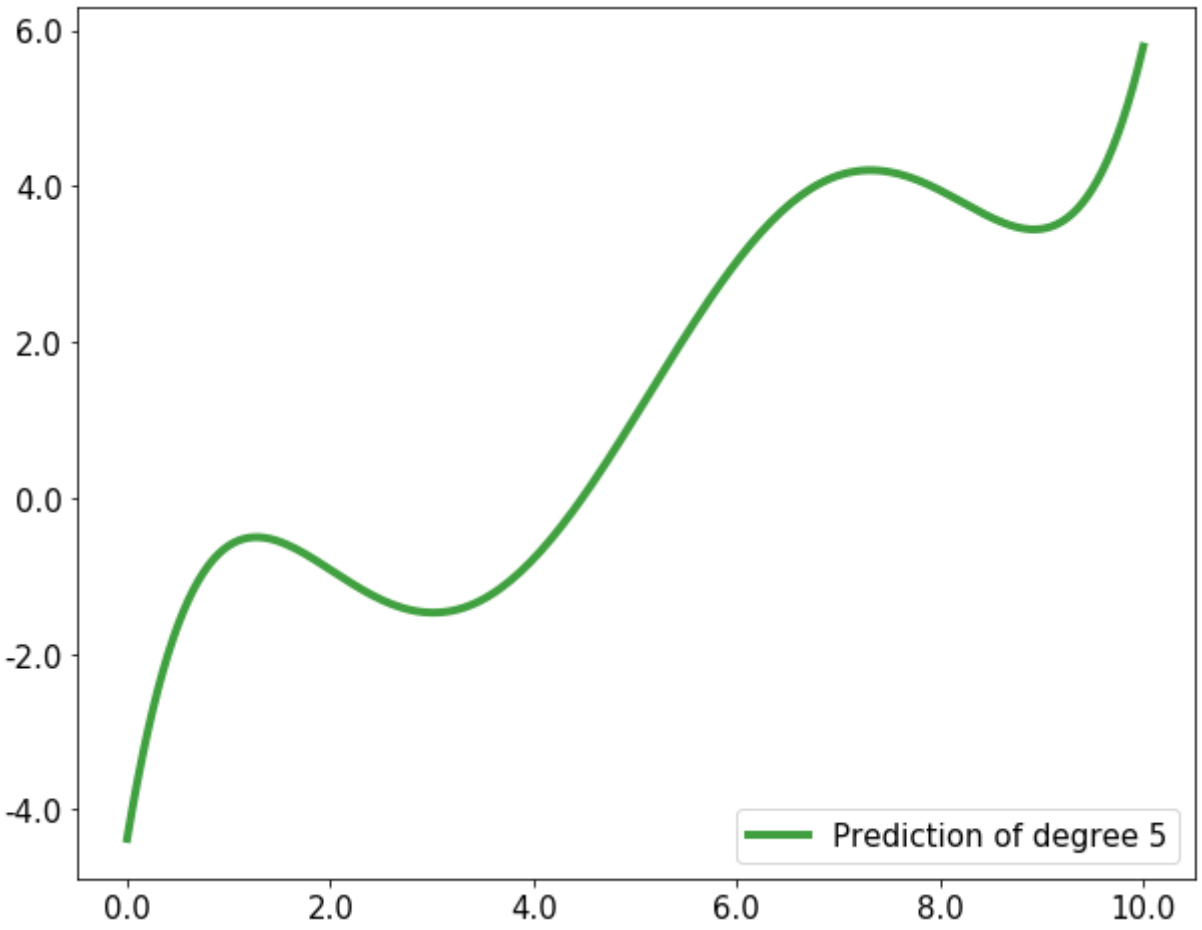
In [12]:
```
YhatTrain, YhatTest, yshat = LinRegressTset(1)
f, ax = plt.subplots(1, 1, figsize=(10, 8))
ax.plot(xs, yshat, lw=4, color='green', alpha=0.75, label='Prediction of deg
ax.set_xticklabels(ax.get_xticks(), fontsize=15)
ax.set_yticklabels(ax.get_yticks(), fontsize=15)
ax.legend(fontsize=15, loc=4)
plt.show()
```



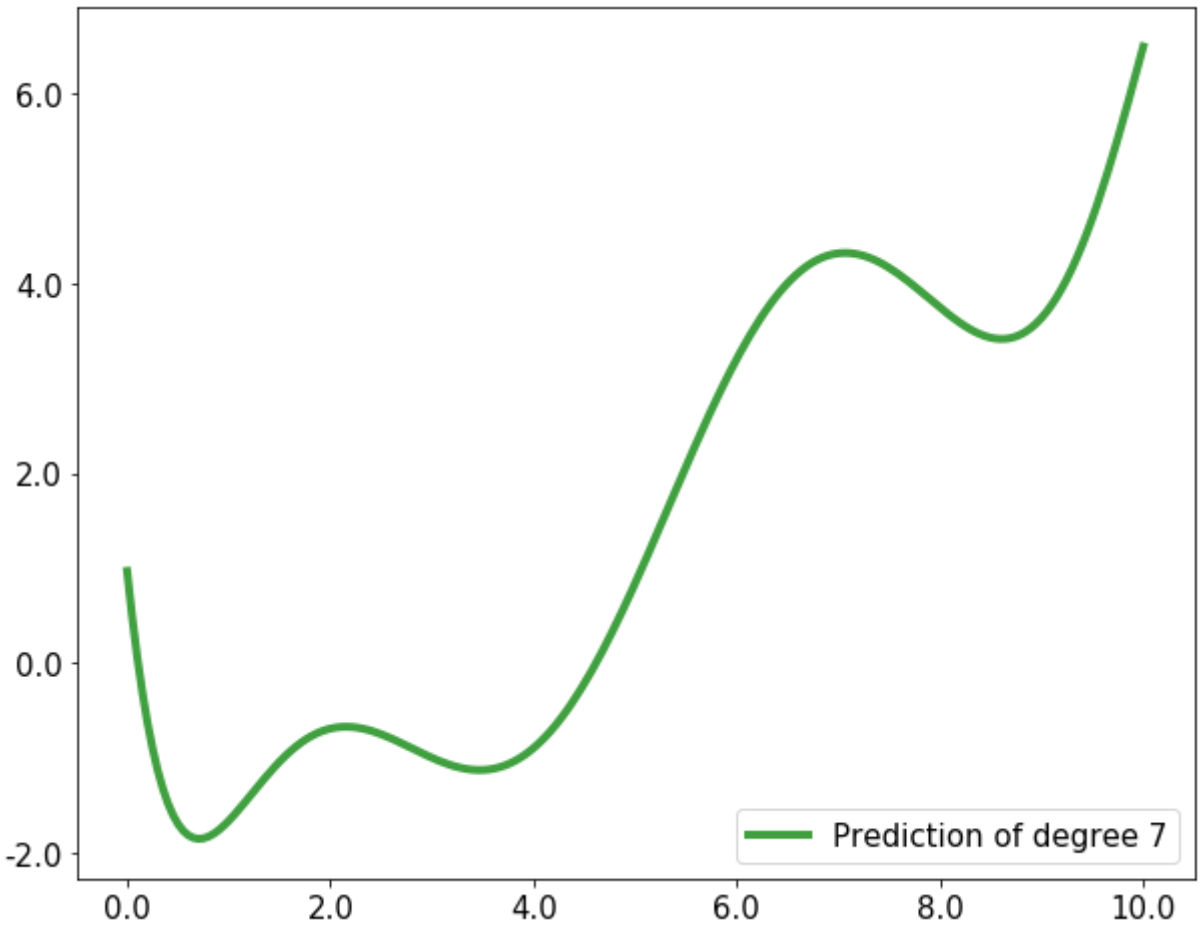
In [13]:
```
YhatTrain, YhatTest, yshat = LinRegressTset(3)
f, ax = plt.subplots(1, 1, figsize=(10, 8))
ax.plot(xs, yshat, lw=4, color='green', alpha=0.75, label='Prediction of deg
ax.set_xticklabels(ax.get_xticks(), fontsize=15)
ax.set_yticklabels(ax.get_yticks(), fontsize=15)
ax.legend(fontsize=15, loc=4)
plt.show()
```



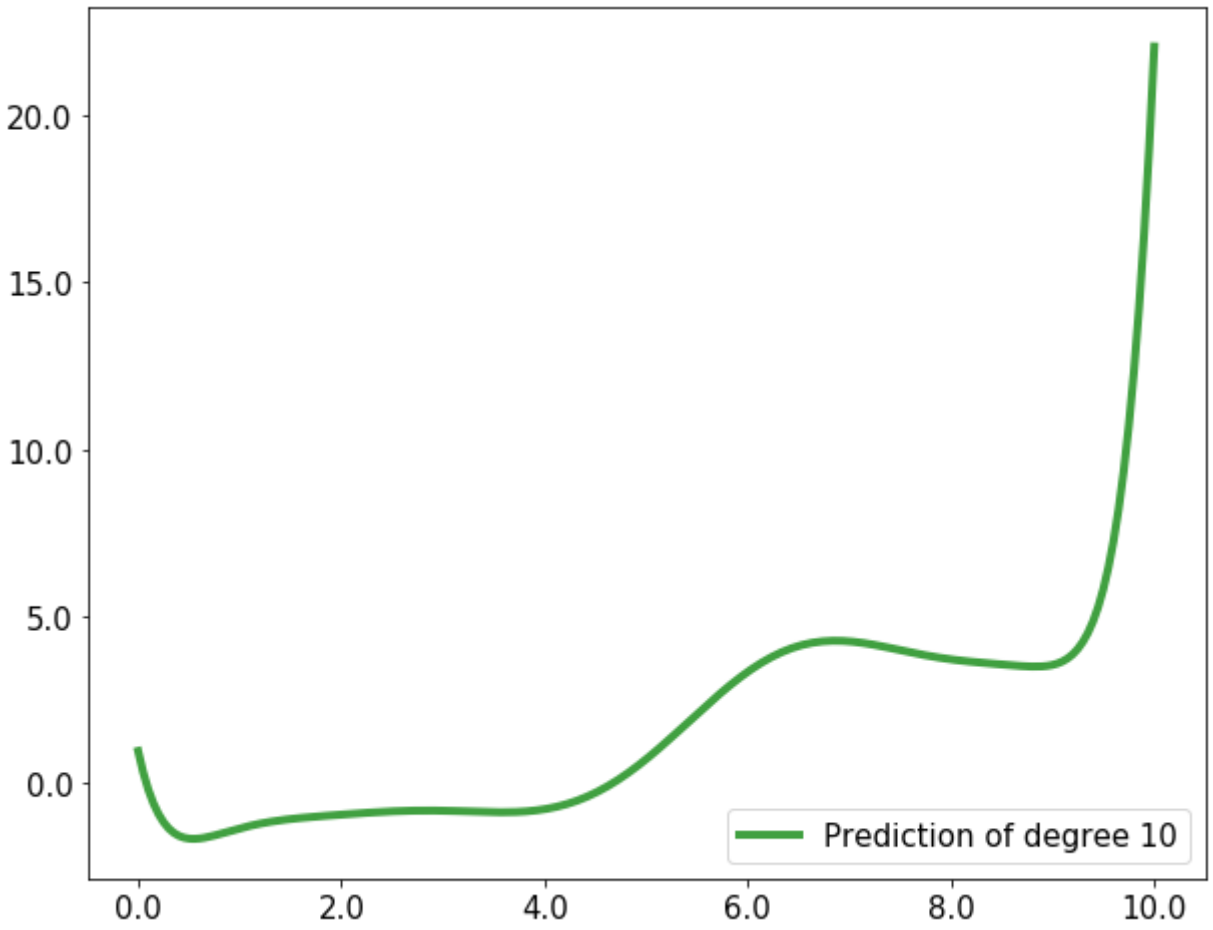
In [14]:
```
YhatTrain, YhatTest, yshat = LinRegressTset(5)
f, ax = plt.subplots(1, 1, figsize=(10, 8))
ax.plot(xs, yshat, lw=4, color='green', alpha=0.75, label='Prediction of deg
ax.set_xticklabels(ax.get_xticks(), fontsize=15)
ax.set_yticklabels(ax.get_yticks(), fontsize=15)
ax.legend(fontsize=15, loc=4)
plt.show()
```



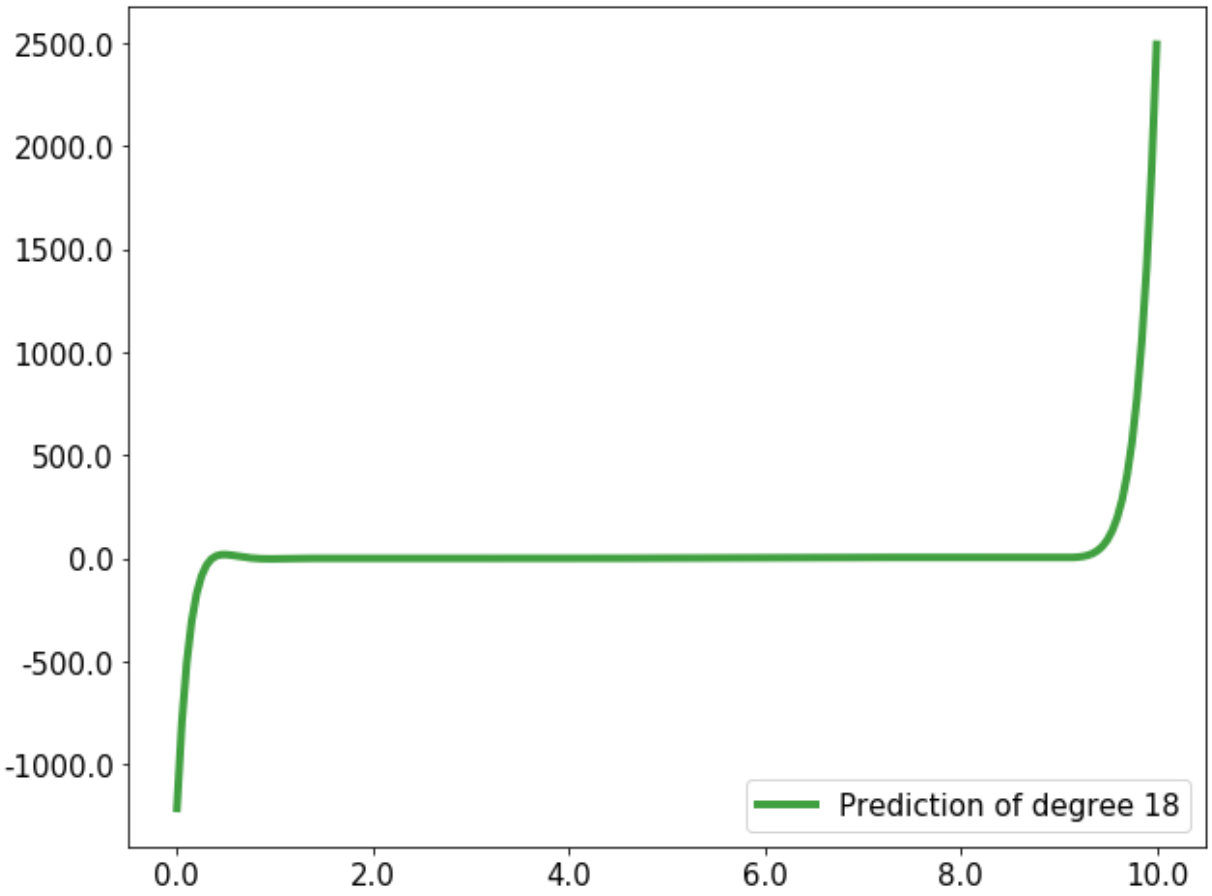
In [15]:
```
YhatTrain, YhatTest, yshat = LinRegressTset(7)
f, ax = plt.subplots(1, 1, figsize=(10, 8))
ax.plot(xs, yshat, lw=4, color='green', alpha=0.75, label='Prediction of deg
ax.set_xticklabels(ax.get_xticks(), fontsize=15)
ax.set_yticklabels(ax.get_yticks(), fontsize=15)
ax.legend(fontsize=15, loc=4)
plt.show()
```



In [16]:
```
YhatTrain, YhatTest, yshat = LinRegressTset(10)
f, ax = plt.subplots(1, 1, figsize=(10, 8))
ax.plot(xs, yshat, lw=4, color='green', alpha=0.75, label='Prediction of deg
ax.set_xticklabels(ax.get_xticks(), fontsize=15)
ax.set_yticklabels(ax.get_yticks(), fontsize=15)
ax.legend(fontsize=15, loc=4)
plt.show()
```

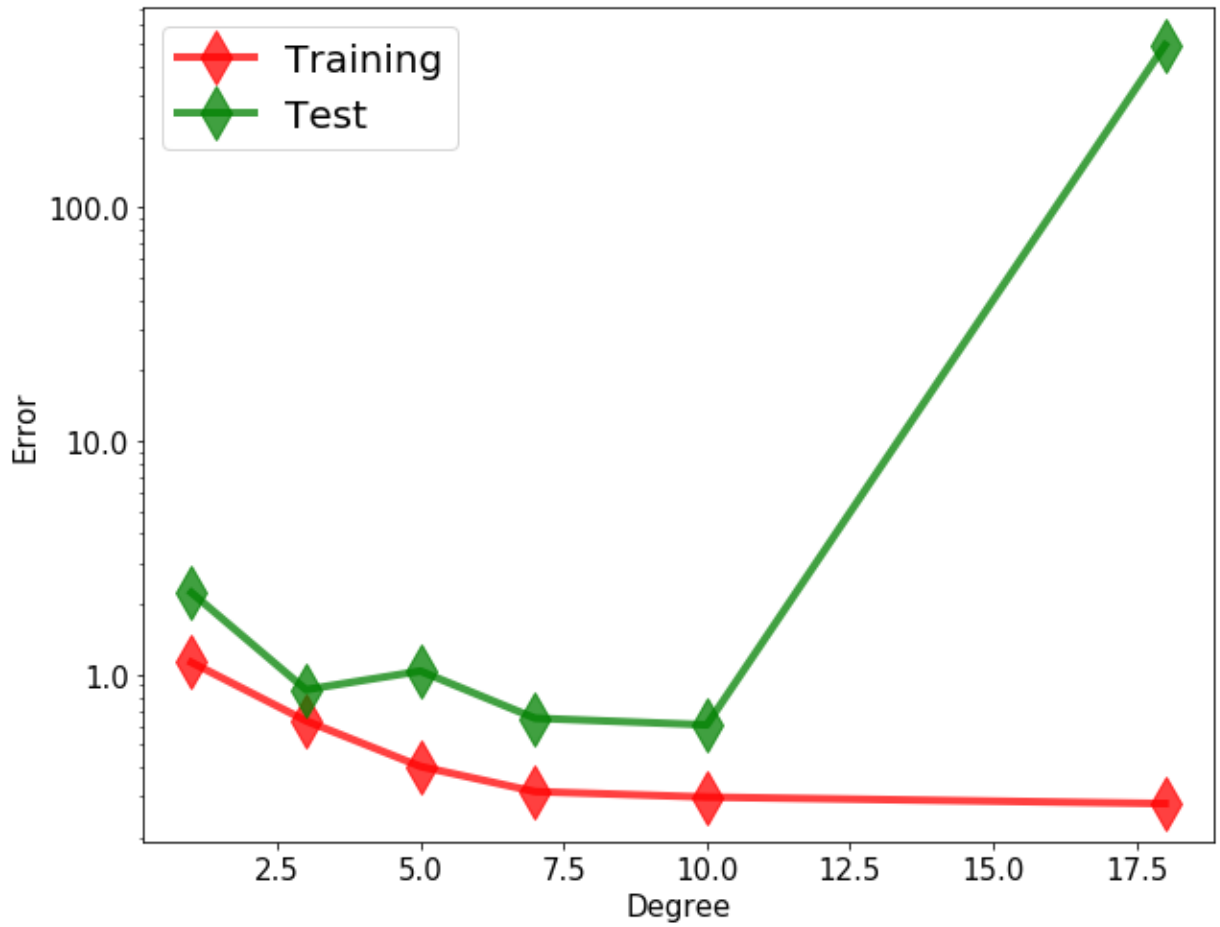


In [17]:
```
YhatTrain, YhatTest, yshat = LinRegressTset(18)
f, ax = plt.subplots(1, 1, figsize=(10, 8))
ax.plot(xs, yshat, lw=4, color='green', alpha=0.75, label='Prediction of deg
ax.set_xticklabels(ax.get_xticks(), fontsize=15)
ax.set_yticklabels(ax.get_yticks(), fontsize=15)
ax.legend(fontsize=15, loc=4)
plt.show()
```



Problem 1.3 - B

In [18]:
```
degrees = np.array([1, 3, 5, 7, 10, 18])
train_error = np.zeros(degrees.shape[0])
test_error = np.zeros(degrees.shape[0])
fig,ax=plt.subplots(1,1, figsize=(10, 8))

for i, degree in enumerate(degrees):
    YhatTrain, YhatTest, yshat = LinRegressTset(degree)
    train_error[i] = mse(Ytr, YhatTrain)
    test_error[i] = mse(Yte, YhatTest)

ax.semilogy(degrees,train_error,'r-',lw=4, marker='d', markersize=20, alpha=
ax.semilogy(degrees,test_error,'g-',lw=4, marker='d', markersize=20, alpha=(
ax.set_xlabel('Degree',fontsize=15)
ax.set_ylabel('Error',fontsize=15)
ax.set_xticklabels(ax.get_xticks(), fontsize=15)
ax.set_yticklabels(ax.get_yticks(), fontsize=15)

ax.legend(fontsize=20, loc=0)
plt.show()
```



Problem 1.3 - C

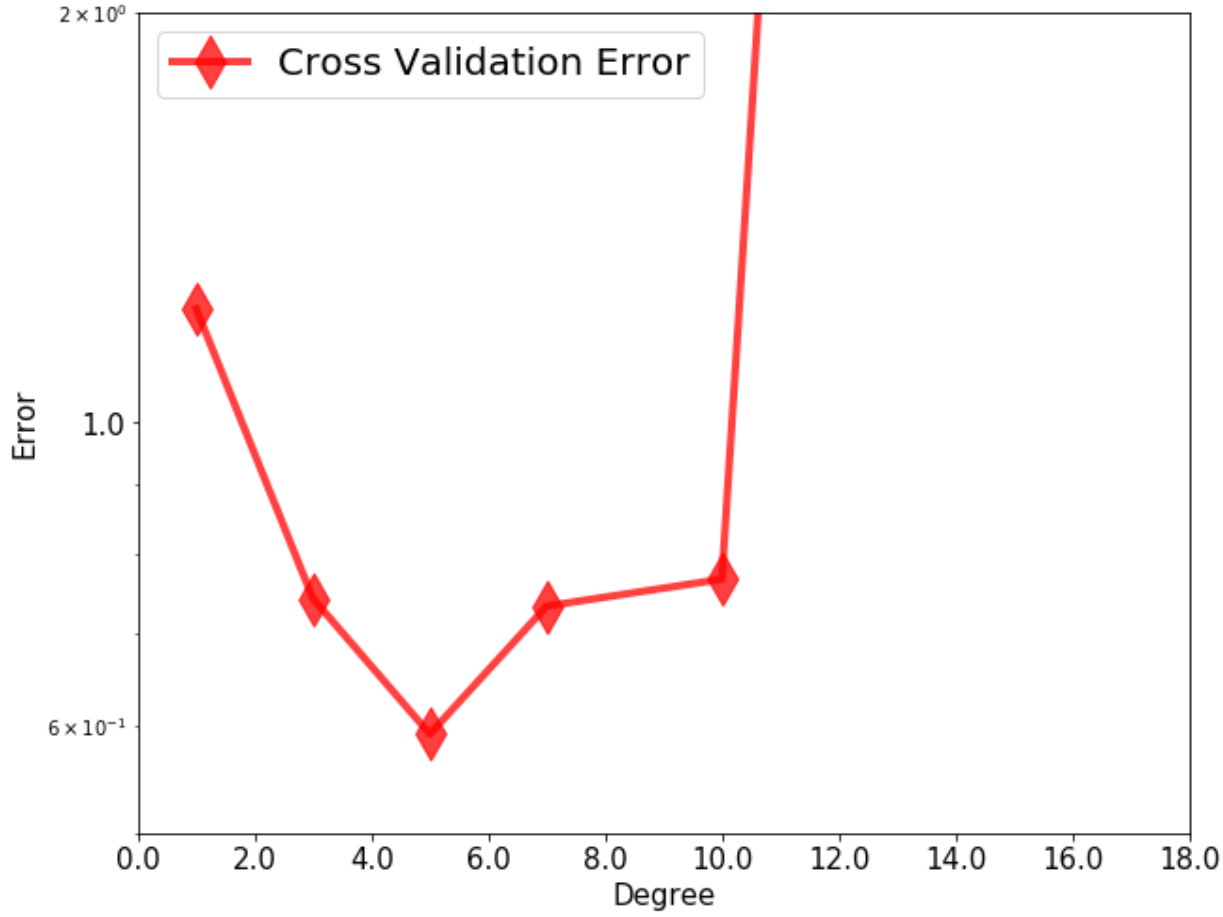
Recommended Degree: 10

Problem 2: Cross Validation

In [19]:
```
def Crossvalidation(degree, nFolds):
    J = np.zeros(nFolds)
    for iFold in range(nFolds):
        Xti,Xvi,Yti,Yvi = ml.crossValidate(Xtr,Ytr,nFolds,iFold) # use ith l
        XtiP = ml.transforms.fpoly(Xti, degree, bias=False)
        XtiP,params = ml.transforms.rescale(XtiP)
        XviP,_ = ml.transforms.rescale( ml.transforms.fpoly(Xvi,degree,False
        learner = ml.linear.linearRegress( XtiP, Yti )# TODO: train on Xti,
        YhatTest = learner.predict( XviP )
        J[iFold] = mse(Yvi,YhatTest)
    return np.mean(J)
```

Problem 2.1

In [20]:
```
nFolds = 5
degrees = np.array([1, 3, 5, 7, 10, 18])
CVE = np.zeros(degrees.shape[0])
fig,ax=plt.subplots(1,1, figsize=(10, 8))

for i, degree in enumerate(degrees):
    CVE[i] = Crossvalidation(degree,nFolds)

ax.semilogy(degrees,CVE,'r-',lw=4, marker='d', markersize=20, alpha=0.75,lab
ax.set_xlim(0,18)
ax.set_ylim(.5,2)
ax.set_xlabel('Degree',fontsize=15 )
ax.set_ylabel('Error',fontsize=15)
ax.set_xticklabels(ax.get_xticks(), fontsize=15)
ax.set_yticklabels(ax.get_yticks(), fontsize=15)

ax.legend(fontsize=20, loc=0)
plt.show()
```



Problem 2.2

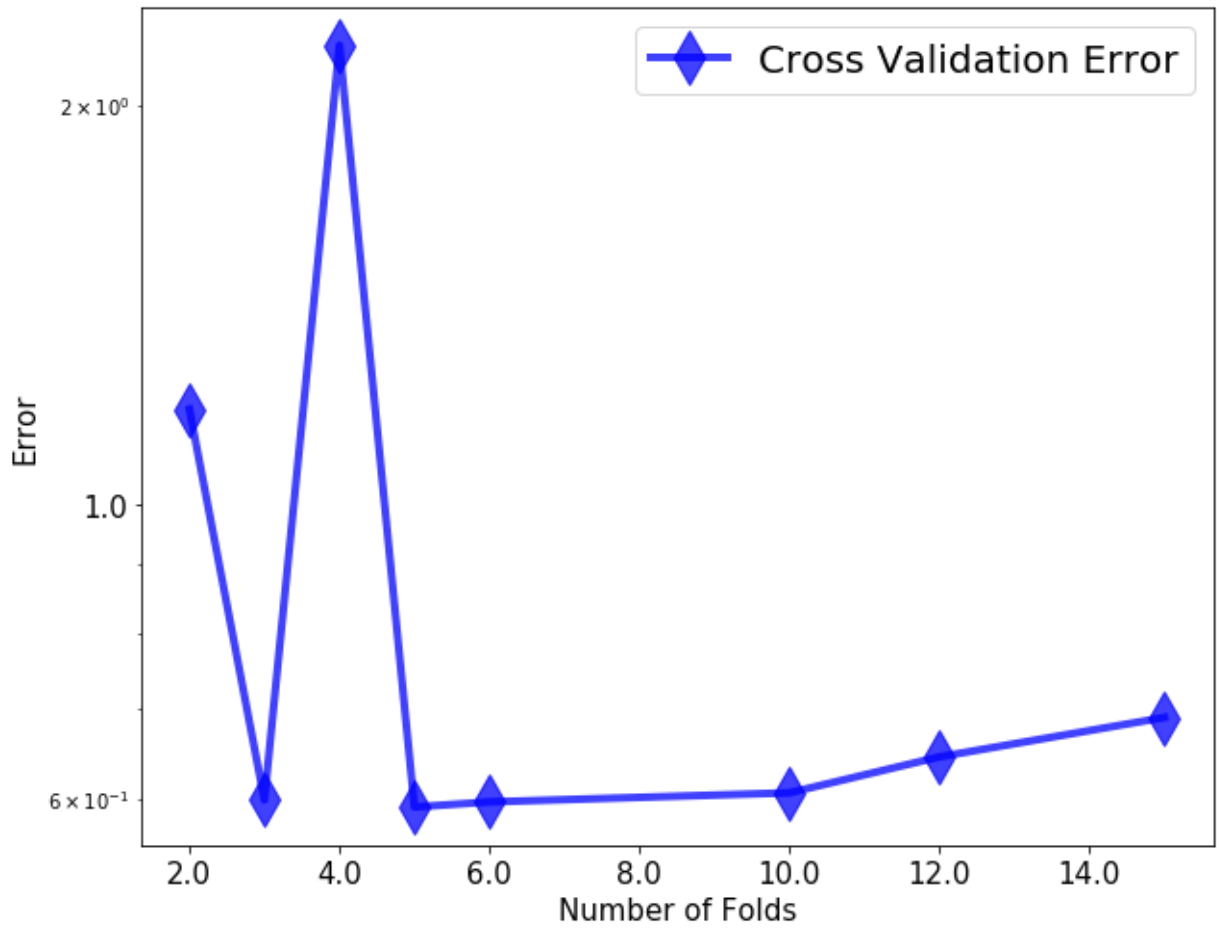
The MSE estimates of the five-fold cross validation data were very similar to the actual MSE's evaluated on the actual test data. Both graphs seemed to follow similar patterns given certain degrees (except degree 5). Both also ran into issues with overfitting when degree reached higher than 10.

Problem 2.3

Recommended Degree: 5

Problem 2.4

In [21]:
```
degree = 5
nFolds = np.array([2, 3, 4, 5, 6, 10, 12, 15])
CVE = np.zeros(nFolds.shape[0])
fig,ax=plt.subplots(1,1, figsize=(10, 8))

for i, nFold in enumerate(nFolds):
    CVE[i] = Crossvalidation(degree, nFold)

ax.semilogy(nFolds,CVE,'b-',lw=4, marker='d', markersize=20, alpha=0.75, lab
ax.set_xlabel('Number of Folds',fontsize=15)
ax.set_ylabel('Error',fontsize=15)
ax.set_xticklabels(ax.get_xticks(), fontsize=15)
ax.set_yticklabels(ax.get_yticks(), fontsize=15)

ax.legend(fontsize=20, loc=0)
plt.show()
```



I did notice that given a degree, as the number of folds increased, the cross validation error

gradually increased as well. I believe this happens because when the number of folds increases, the number of training data also increases while the number of test data decreases. So there's more variance.

Statement of Collaboration

I collabed with Sergey Kochetov and Chad Lei to discuss different aspects of linear regression and cross validation that we didn't understand from class. For homework assistance I relied on piazza to answer any logistical questions about the given code or any understanding of the code that I needed help on. But the homework was solely worked on my own.