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April 6, 2025

Theory of backward optical parametric amplification in chiral media

In the following analysis, we will keep the model for pulsed wave propagation, as this easily is expressed for continuous waves by simply dropping the first and second order time derivatimes. The notations follow the separate document *Pulse propagation in chiral optical parametric processes*, as well as the manuscript *Pulsed optical parametric amplification in chiral media* by Fredrik Jonsson, Christos Flytzanis and Govind Agrawal, as submitted to J. Opt. Soc. Am. B in March 2025.

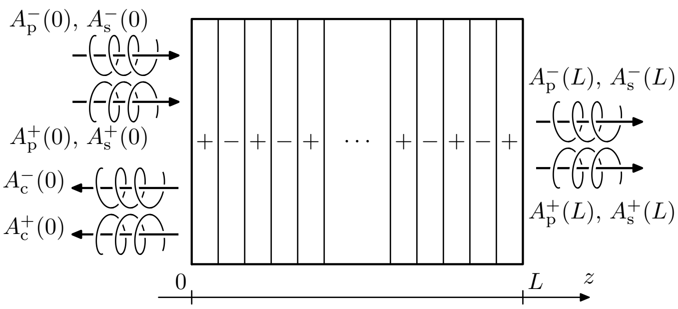

Figure 1. Schematic of the setup for backward-wave optical parametric amplification, in which the pump and signal waves are launched at $z = 0$, and where the idler is created in the opposite direction. The medium is in a quasi phase matching configuration with a periodic modulation of the sign of the nonlinear coupling coefficient, with spatial period of Λ.

Conventions for the fields

The natural time-harmonic oscillation of the quasi-monochromatic electric field is for the complex-valued amplitude of the total electric field taken as

$$\mathbf{E}(\mathbf{r}, t) = \sum_{\omega_\sigma} \mathrm{Re}[\mathbf{E}_{\omega_\sigma}(\mathbf{r}, t) \exp(-i\omega_\sigma t)].$$

We express this field in the circularly polarized base vectors $\mathbf{e}_+$ (left circular polarization, LCP) and $\mathbf{e}_-$ (right circular polarization, RCP),

$$\mathbf{e}_+ = \frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y), \qquad \mathbf{e}_- = \frac{1}{\sqrt{2}}(\mathbf{e}_x - i\mathbf{e}_y). \tag{1}$$

in terms of which the LCP and RCP components of the total electric field are projected by using the orthogonality conditions $\mathbf{e}_\pm^* \cdot \mathbf{e}_\pm = 1$ and $\mathbf{e}_\pm^* \cdot \mathbf{e}_\mp = 0$, to yield[1]

$$E_\omega^\pm = \mathbf{e}_\pm^* \cdot (E_\omega^+ \mathbf{e}_+ + E_\omega^- \mathbf{e}_-).$$

[1] We may equally well inverse these relations and express the fields in the Cartesian coordinate system in terms of these circularly polarized components, as

$$E_\omega^x = \frac{1}{\sqrt{2}}(E_\omega^+ + E_\omega^-), \qquad E_\omega^y = \frac{i}{\sqrt{2}}(E_\omega^+ - E_\omega^-).$$

Finally, the natural spatially harmonic oscillation of the field of forward and backward traveling components are formulated by

$$E^{\pm}_{\omega_k}(z,t) = E^{f\pm}_{\omega_k}(z,t)\exp(i(\omega n_k/c)z) + E^{b\mp}_{\omega_k}(z,t)\exp(-i(\omega n_k/c)z), \tag{2}$$

for $k = 1, 2, 3$ for the idler, signal and pump, respectively, in which we should note that any corrections to the wave vectors due to, say, circular dichroism or birefringence will show up in the wave equations to follow.

Wave equations for backward optical parametric amplification

From the separate document *Pulse propagation in chiral optical parametric processes*, using the notation therein, the relevant equations of propagation are, for the forward and backward propagating components of the idler at angular frequency ω_1,

$$\begin{aligned}
&\Big\{\frac{\partial E^{f\pm}_{\omega_1}}{\partial z} \pm i\frac{\omega_1^2\gamma_1}{2c^2}E^{f\pm}_{\omega_1} + \Big((k_1' \mp a_1')\frac{\partial}{\partial t} + \frac{i}{2}(k_1'' \mp b_1'')\frac{\partial^2}{\partial t^2}\Big)E^{f\pm}_{\omega_1}\Big\}\exp\Big(i\frac{\omega_1 n_1}{c}z\Big)\\
&\quad+\Big\{-\frac{\partial E^{b\mp}_{\omega_1}}{\partial z} \mp i\frac{\omega_1^2\gamma_1}{2c^2}E^{b\mp}_{\omega_1} + \Big((k_1' \pm a_1')\frac{\partial}{\partial t} + \frac{i}{2}(k_1'' \pm b_1'')\frac{\partial^2}{\partial t^2}\Big)E^{b\mp}_{\omega_1}\Big\}\exp\Big(-i\frac{\omega_1 n_1}{c}z\Big)\\
&\qquad= i\frac{\omega_1}{2cn_1}\Big(p_1 \pm iq_1\frac{\partial}{\partial z}\Big)\Big[E^{f\mp}_{\omega_3}E^{f\pm *}_{\omega_2}\exp\Big(i\Big(\frac{\omega_3 n_3}{c} - \frac{\omega_2 n_2}{c}\Big)z\Big)\\
&\qquad\qquad+ E^{f\mp}_{\omega_3}E^{b\mp *}_{\omega_2}\exp\Big(i\Big(\frac{\omega_3 n_3}{c} + \frac{\omega_2 n_2}{c}\Big)z\Big)\\
&\qquad\qquad+ E^{b\pm}_{\omega_3}E^{f\pm *}_{\omega_2}\exp\Big(-i\Big(\frac{\omega_3 n_3}{c} + \frac{\omega_2 n_2}{c}\Big)z\Big)\\
&\qquad\qquad+ E^{b\pm}_{\omega_3}E^{b\mp *}_{\omega_2}\exp\Big(-i\Big(\frac{\omega_3 n_3}{c} - \frac{\omega_2 n_2}{c}\Big)z\Big)\Big],
\end{aligned}\tag{3}$$

while the wave equation for the signal at angular frequency ω_2 is

$$\begin{aligned}
&\Big\{\frac{\partial E^{f\pm}_{\omega_2}}{\partial z} \pm i\frac{\omega_2^2\gamma_2}{2c^2}E^{f\pm}_{\omega_2} + \Big((k_2' \mp a_2')\frac{\partial}{\partial t} + \frac{i}{2}(k_2'' \mp b_2'')\frac{\partial^2}{\partial t^2}\Big)E^{f\pm}_{\omega_2}\Big\}\exp\Big(i\frac{\omega_2 n_2}{c}z\Big)\\
&\quad+\Big\{-\frac{\partial E^{b\mp}_{\omega_2}}{\partial z} \mp i\frac{\omega_2^2\gamma_2}{2c^2}E^{b\mp}_{\omega_2} + \Big((k_2' \pm a_2')\frac{\partial}{\partial t} + \frac{i}{2}(k_2'' \pm b_2'')\frac{\partial^2}{\partial t^2}\Big)E^{b\mp}_{\omega_2}\Big\}\exp\Big(-i\frac{\omega_2 n_2}{c}z\Big)\\
&\qquad= i\frac{\omega_2}{2cn_2}\Big(p_2 \pm iq_2\frac{\partial}{\partial z}\Big)\Big[E^{f\mp}_{\omega_3}E^{f\pm *}_{\omega_1}\exp\Big(i\Big(\frac{\omega_3 n_3}{c} - \frac{\omega_1 n_1}{c}\Big)z\Big)\\
&\qquad\qquad+ E^{f\mp}_{\omega_3}E^{b\mp *}_{\omega_1}\exp\Big(i\Big(\frac{\omega_3 n_3}{c} + \frac{\omega_1 n_1}{c}\Big)z\Big)\\
&\qquad\qquad+ E^{b\pm}_{\omega_3}E^{f\pm *}_{\omega_1}\exp\Big(-i\Big(\frac{\omega_3 n_3}{c} + \frac{\omega_1 n_1}{c}\Big)z\Big)\\
&\qquad\qquad+ E^{b\pm}_{\omega_3}E^{b\mp *}_{\omega_1}\exp\Big(-i\Big(\frac{\omega_3 n_3}{c} - \frac{\omega_1 n_1}{c}\Big)z\Big)\Big],
\end{aligned}\tag{4}$$

and, finally, the wave equation for the pump at angular frequency ω_3 is

$$\begin{aligned}
&\Big\{\frac{\partial E^{f\mp}_{\omega_3}}{\partial z} \mp i\frac{\omega_3^2\gamma_3}{2c^2}E^{f\mp}_{\omega_3} + \Big((k_3' \pm a_3')\frac{\partial}{\partial t} + \frac{i}{2}(k_3'' \pm b_3'')\frac{\partial^2}{\partial t^2}\Big)E^{f\mp}_{\omega_3}\Big\}\exp\Big(i\frac{\omega_3 n_3}{c}z\Big)\\
&\quad+\Big\{-\frac{\partial E^{b\pm}_{\omega_3}}{\partial z} \pm i\frac{\omega_3^2\gamma_3}{2c^2}E^{b\pm}_{\omega_3} + \Big((k_3' \mp a_3')\frac{\partial}{\partial t} + \frac{i}{2}(k_3'' \mp b_3'')\frac{\partial^2}{\partial t^2}\Big)E^{b\pm}_{\omega_3}\Big\}\exp\Big(-i\frac{\omega_3 n_3}{c}z\Big)\\
&\qquad= i\frac{\omega_3}{2cn_3}\Big(p_3 \mp iq_3\frac{\partial}{\partial z}\Big)\Big[E^{f\pm}_{\omega_1}E^{f\pm}_{\omega_2}\exp\Big(i\Big(\frac{\omega_1 n_1}{c} + \frac{\omega_2 n_2}{c}\Big)z\Big)\\
&\qquad\qquad+ E^{f\pm}_{\omega_1}E^{b\mp}_{\omega_2}\exp\Big(i\Big(\frac{\omega_1 n_1}{c} - \frac{\omega_2 n_2}{c}\Big)z\Big)\\
&\qquad\qquad+ E^{b\mp}_{\omega_1}E^{f\pm}_{\omega_2}\exp\Big(-i\Big(\frac{\omega_1 n_1}{c} - \frac{\omega_2 n_2}{c}\Big)z\Big)\\
&\qquad\qquad+ E^{b\mp}_{\omega_1}E^{b\mp}_{\omega_2}\exp\Big(-i\Big(\frac{\omega_1 n_1}{c} + \frac{\omega_2 n_2}{c}\Big)z\Big)\Big].
\end{aligned}\tag{5}$$

Projecting out linear gyrotropy from the envelopes

The wave equations (3)–(5) may be simplified considerably by only considering close to phase-matched components, say by multiplying by $\exp(i\omega_j n_j z/c)$ and averaging over a few spatial periods. However, before projecting out these matched terms, we may in Eqs. (3)–(5) straight away, from the appearance of the terms with coefficients γ_1, γ_2 and γ_3, find that the gyrotropic nature of the forward and backward traveling wave envelopes can be separated by the *ansatz*

$$E^{f\pm}_{\omega_k}(z,t) = A^{f\pm}_{\omega_k}(z,t)\exp\Big(\mp i\frac{\omega_k^2\gamma_k}{2c^2}z\Big), \tag{6a}$$

$$E^{b\mp}_{\omega_k}(z,t) = A^{b\mp}_{\omega_k}(z,t)\exp\Big(\mp i\frac{\omega_k^2\gamma_k}{2c^2}z\Big), \tag{6b}$$

with $k = 1, 2, 3$ for the idler, signal and pump, respectively. This way, carrying out the second step in the reduction of the coupled set of nonlinear PDE:s, the terms containing the gyration coefficients γ_k will be cancelled. Notice that in separating the field into forward and backward traveling components, the backward traveling LCP/RCP components will be connected to the *conjugated* basis vectors $\mathbf{e}^*_\pm$, hence the altered order of "$\mp$" in "$E^{b\mp}_{\omega_k}$". Thus, by inserting the ansatz of Eq. (6) into Eq. (3), we for the idler at angular frequency ω_1 obtain

$$\begin{aligned}
&\Big\{\frac{\partial A^{f\pm}_{\omega_1}}{\partial z} + (k_1' \mp a_1')\frac{\partial A^{f\pm}_{\omega_1}}{\partial t} + \frac{i}{2}(k_1'' \mp b_1'')\frac{\partial^2 A^{f\pm}_{\omega_1}}{\partial t^2}\Big\}\exp\Big(i\Big(\frac{\omega_1 n_1}{c} \mp \frac{\omega_1^2\gamma_1}{2c^2}\Big)z\Big)\\
&\quad + \Big\{-\frac{\partial A^{b\mp}_{\omega_1}}{\partial z} + (k_1' \pm a_1')\frac{\partial A^{b\mp}_{\omega_1}}{\partial t} + \frac{i}{2}(k_1'' \pm b_1'')\frac{\partial^2 A^{b\mp}_{\omega_1}}{\partial t^2}\Big\}\exp\Big(-i\Big(\frac{\omega_1 n_1}{c}z \pm \frac{\omega_1^2\gamma_1}{2c^2}\Big)z\Big)\\
&\qquad = i\frac{\omega_1}{2cn_1}\Big(p_1 \pm iq_1\frac{\partial}{\partial z}\Big)\Big[A^{f\mp}_{\omega_3}A^{f\pm*}_{\omega_2}\exp\Big(i\Big(\frac{\omega_3 n_3}{c} - \frac{\omega_2 n_2}{c}\Big)z \pm i\Big(\frac{\omega_3^2\gamma_3}{2c^2} + \frac{\omega_2^2\gamma_2}{2c^2}\Big)z\Big)\\
&\qquad\qquad + A^{f\mp}_{\omega_3}A^{b\mp*}_{\omega_2}\exp\Big(i\Big(\frac{\omega_3 n_3}{c} + \frac{\omega_2 n_2}{c}\Big)z \pm i\Big(\frac{\omega_3^2\gamma_3}{2c^2} + \frac{\omega_2^2\gamma_2}{2c^2}\Big)z\Big)\\
&\qquad\qquad + A^{b\pm}_{\omega_3}A^{f\pm*}_{\omega_2}\exp\Big(-i\Big(\frac{\omega_3 n_3}{c} + \frac{\omega_2 n_2}{c}\Big)z \pm i\Big(\frac{\omega_3^2\gamma_3}{2c^2} + \frac{\omega_2^2\gamma_2}{2c^2}\Big)z\Big)\\
&\qquad\qquad + A^{b\pm}_{\omega_3}A^{b\mp*}_{\omega_2}\exp\Big(-i\Big(\frac{\omega_3 n_3}{c} - \frac{\omega_2 n_2}{c}\Big)z \pm i\Big(\frac{\omega_3^2\gamma_3}{2c^2} + \frac{\omega_2^2\gamma_2}{2c^2}\Big)z\Big)\Big],
\end{aligned} \tag{7}$$

while we for the signal at angular frequency ω_2 obtain

$$\begin{aligned}
&\Big\{\frac{\partial A^{f\pm}_{\omega_2}}{\partial z} + (k_2' \mp a_2')\frac{\partial A^{f\pm}_{\omega_2}}{\partial t} + \frac{i}{2}(k_2'' \mp b_2'')\frac{\partial^2 A^{f\pm}_{\omega_2}}{\partial t^2}\Big\}\exp\Big(i\Big(\frac{\omega_2 n_2}{c} \mp \frac{\omega_2^2\gamma_2}{2c^2}\Big)z\Big)\\
&\quad + \Big\{-\frac{\partial A^{b\mp}_{\omega_2}}{\partial z} + (k_2' \pm a_2')\frac{\partial A^{b\mp}_{\omega_2}}{\partial t} + \frac{i}{2}(k_2'' \pm b_2'')\frac{\partial^2 A^{b\mp}_{\omega_2}}{\partial t^2}\Big\}\exp\Big(-i\Big(\frac{\omega_2 n_2}{c}z \pm \frac{\omega_2^2\gamma_2}{2c^2}\Big)z\Big)\\
&\qquad = i\frac{\omega_2}{2cn_2}\Big(p_2 \pm iq_2\frac{\partial}{\partial z}\Big)\Big[A^{f\mp}_{\omega_3}A^{f\pm*}_{\omega_1}\exp\Big(i\Big(\frac{\omega_3 n_3}{c} - \frac{\omega_1 n_1}{c}\Big)z \pm i\Big(\frac{\omega_3^2\gamma_3}{2c^2} + \frac{\omega_1^2\gamma_1}{2c^2}\Big)z\Big)\\
&\qquad\qquad + A^{f\mp}_{\omega_3}A^{b\mp*}_{\omega_1}\exp\Big(i\Big(\frac{\omega_3 n_3}{c} + \frac{\omega_1 n_1}{c}\Big)z \pm i\Big(\frac{\omega_3^2\gamma_3}{2c^2} + \frac{\omega_1^2\gamma_1}{2c^2}\Big)z\Big)\\
&\qquad\qquad + A^{b\pm}_{\omega_3}A^{f\pm*}_{\omega_1}\exp\Big(-i\Big(\frac{\omega_3 n_3}{c} + \frac{\omega_1 n_1}{c}\Big)z \pm i\Big(\frac{\omega_3^2\gamma_3}{2c^2} + \frac{\omega_1^2\gamma_1}{2c^2}\Big)z\Big)\\
&\qquad\qquad + A^{b\pm}_{\omega_3}A^{b\mp*}_{\omega_1}\exp\Big(-i\Big(\frac{\omega_3 n_3}{c} - \frac{\omega_1 n_1}{c}\Big)z \pm i\Big(\frac{\omega_3^2\gamma_3}{2c^2} + \frac{\omega_1^2\gamma_1}{2c^2}\Big)z\Big)\Big],
\end{aligned} \tag{8}$$

and finally for the pump at angular frequency ω_3,

$$\begin{aligned}
&\Big\{\frac{\partial A^{f\mp}_{\omega_3}}{\partial z}+(k'_3\pm a'_3)\frac{\partial A^{f\mp}_{\omega_3}}{\partial t}+\frac{i}{2}(k''_3\pm b''_3)\frac{\partial^2 A^{f\mp}_{\omega_3}}{\partial t^2}\Big\}\exp\Big(i\Big(\frac{\omega_3 n_3}{c}\pm\frac{\omega_3^2\gamma_3}{2c^2}\Big)z\Big)\\
&\quad+\Big\{-\frac{\partial A^{b\pm}_{\omega_3}}{\partial z}+(k'_3\mp a'_3)\frac{\partial A^{b\pm}_{\omega_3}}{\partial t}+\frac{i}{2}(k''_3\mp b''_3)\frac{\partial^2 A^{b\pm}_{\omega_3}}{\partial t^2}\Big\}\exp\Big(-i\Big(\frac{\omega_3 n_3}{c}z\mp\frac{\omega_3^2\gamma_3}{2c^2}\Big)z\Big)\\
&\qquad=i\frac{\omega_3}{2cn_3}\Big(p_3\mp iq_3\frac{\partial}{\partial z}\Big)\Big[A^{f\pm}_{\omega_1}A^{f\pm}_{\omega_2}\exp\Big(i\Big(\frac{\omega_1 n_1}{c}+\frac{\omega_2 n_2}{c}\Big)z\mp i\Big(\frac{\omega_1^2\gamma_1}{2c^2}+\frac{\omega_2^2\gamma_2}{2c^2}\Big)z\Big)\\
&\qquad\qquad+A^{f\pm}_{\omega_1}A^{b\mp}_{\omega_2}\exp\Big(i\Big(\frac{\omega_1 n_1}{c}-\frac{\omega_2 n_2}{c}\Big)z\mp i\Big(\frac{\omega_1^2\gamma_1}{2c^2}+\frac{\omega_2^2\gamma_2}{2c^2}\Big)z\Big)\\
&\qquad\qquad+A^{b\mp}_{\omega_1}A^{f\pm}_{\omega_2}\exp\Big(-i\Big(\frac{\omega_1 n_1}{c}-\frac{\omega_2 n_2}{c}\Big)z\mp i\Big(\frac{\omega_1^2\gamma_1}{2c^2}+\frac{\omega_2^2\gamma_2}{2c^2}\Big)z\Big)\\
&\qquad\qquad+A^{b\mp}_{\omega_1}A^{b\mp}_{\omega_2}\exp\Big(-i\Big(\frac{\omega_1 n_1}{c}+\frac{\omega_2 n_2}{c}\Big)z\mp i\Big(\frac{\omega_1^2\gamma_1}{2c^2}+\frac{\omega_2^2\gamma_2}{2c^2}\Big)z\Big)\Big].
\end{aligned}\tag{9}$$

Separating out closely phase-matched terms

We will now narrow down the algebra to yield a configuration in which we have forward traveling pump and signal waves, while having a backward traveling idler wave. In this case, the idler wave at angular frequency ω_1 is from Eq. (7) obtained as

$$\begin{aligned}
&-\frac{\partial A^{b\mp}_{\omega_1}}{\partial z}+(k'_1\pm a'_1)\frac{\partial A^{b\mp}_{\omega_1}}{\partial t}+\frac{i}{2}(k''_1\pm b''_1)\frac{\partial^2 A^{b\mp}_{\omega_1}}{\partial t^2}\\
&\qquad=i\frac{\omega_1 p_1}{2cn_1}A^{f\mp}_{\omega_3}A^{f\pm *}_{\omega_2}\exp(i(k_3-k_2+k_1)z\pm i(\beta_3+\beta_2+\beta_1)z)\\
&\qquad+i\frac{\omega_1 p_1}{2cn_1}A^{f\mp}_{\omega_3}A^{b\mp *}_{\omega_2}\exp(i(k_3+k_2+k_1)z\pm i(\beta_3+\beta_2+\beta_1)z)\\
&\qquad+i\frac{\omega_1 p_1}{2cn_1}A^{b\pm}_{\omega_3}A^{f\pm *}_{\omega_2}\exp(-i(k_3+k_2-k_1)z\pm i(\beta_3+\beta_2+\beta_1)z)\\
&\qquad+i\frac{\omega_1 p_1}{2cn_1}A^{b\pm}_{\omega_3}A^{b\mp *}_{\omega_2}\exp(-i(k_3-k_2-k_1)z\pm i(\beta_3+\beta_2+\beta_1)z),
\end{aligned}\tag{10}$$

where we for the sake of simplicity ignored the nonlocal correction to the nonlinear coupling coefficients. In similar, the forward traveling signal wave at ω_2 is obtained as

$$\begin{aligned}
&\frac{\partial A^{f\pm}_{\omega_2}}{\partial z}+(k'_2\mp a'_2)\frac{\partial A^{f\pm}_{\omega_2}}{\partial t}+\frac{i}{2}(k''_2\mp b''_2)\frac{\partial^2 A^{f\pm}_{\omega_2}}{\partial t^2}\\
&\qquad=i\frac{\omega_2 p_2}{2cn_2}A^{f\mp}_{\omega_3}A^{f\pm *}_{\omega_1}\exp(i(k_3-k_1-k_2)z\pm i(\beta_3+\beta_2+\beta_1)z)\\
&\qquad+i\frac{\omega_2 p_2}{2cn_2}A^{f\mp}_{\omega_3}A^{b\mp *}_{\omega_1}\exp(i(k_3+k_1-k_2)z\pm i(\beta_3+\beta_2+\beta_1)z)\\
&\qquad+i\frac{\omega_2 p_2}{2cn_2}A^{b\pm}_{\omega_3}A^{f\pm *}_{\omega_1}\exp(-i(k_3+k_1+k_2)z\pm i(\beta_3+\beta_2+\beta_1)z)\\
&\qquad+i\frac{\omega_2 p_2}{2cn_2}A^{b\pm}_{\omega_3}A^{b\mp *}_{\omega_1}\exp(-i(k_3-k_1+k_2)z\pm i(\beta_3+\beta_2+\beta_1)z),
\end{aligned}\tag{11}$$

and, finally, the forward traveling pump wave at ω_3 as

$$\begin{aligned}
&\frac{\partial A^{f\mp}_{\omega_3}}{\partial z}+(k'_3\pm a'_3)\frac{\partial A^{f\mp}_{\omega_3}}{\partial t}+\frac{i}{2}(k''_3\pm b''_3)\frac{\partial^2 A^{f\mp}_{\omega_3}}{\partial t^2}\\
&\qquad=i\frac{\omega_3 p_3}{2cn_3}A^{f\pm}_{\omega_1}A^{f\pm}_{\omega_2}\exp(i(k_1+k_2-k_3)z\mp i(\beta_3+\beta_2+\beta_1)z)\\
&\qquad+i\frac{\omega_3 p_3}{2cn_3}A^{f\pm}_{\omega_1}A^{b\mp}_{\omega_2}\exp(i(k_1-k_2-k_3)z\mp i(\beta_3+\beta_2+\beta_1)z)\\
&\qquad+i\frac{\omega_3 p_3}{2cn_3}A^{b\mp}_{\omega_1}A^{f\pm}_{\omega_2}\exp(-i(k_1-k_2+k_3)z\mp i(\beta_3+\beta_2+\beta_1)z)\\
&\qquad+i\frac{\omega_3 p_3}{2cn_3}A^{b\mp}_{\omega_1}A^{b\mp}_{\omega_2}\exp(-i(k_1+k_2+k_3)z\mp i(\beta_3+\beta_2+\beta_1)z).
\end{aligned}\tag{12}$$

In these expressions, we defined

$$k_j \equiv \frac{\omega_j n_j}{c}, \qquad \beta_j \equiv \frac{\omega_j^2 \gamma_j}{2c^2}, \tag{13}$$

with, as previously, $j = 1, 2, 3$ denoting the idler, signal and pump, respectively. Notice the way the phase matching in Eqs. (12) appear, with the phase mismatch from the electric-dipolar parts occurring with various combinations of the wave vector magnitudes k_j, while the gyrotropic, non-local contributions all appear as the sum $\beta_1 + \beta_2 + \beta_3$.

If we now focus on terms in which the phase matching yields

$$k_3 - k_2 + k_1 \approx 0, \tag{14}$$

we will in the right-hand sides of Eqs. (10)–(12) only keep one of the terms, in this case reducing the set of equations to

$$\begin{aligned}
&-\frac{\partial A^{b\mp}_{\omega_1}}{\partial z} + (k'_1 \pm a'_1)\frac{\partial A^{b\mp}_{\omega_1}}{\partial t} + \frac{i}{2}(k''_1 \pm b''_1)\frac{\partial^2 A^{b\mp}_{\omega_1}}{\partial t^2} \\
&\qquad = i\frac{\omega_1 p_1}{2cn_1} A^{f\mp}_{\omega_3} A^{f\pm *}_{\omega_2} \exp(i(k_3 - k_2 + k_1)z \pm i(\beta_3 + \beta_2 + \beta_1)z),
\end{aligned} \tag{15a}$$

$$\begin{aligned}
&\frac{\partial A^{f\pm}_{\omega_2}}{\partial z} + (k'_2 \mp a'_2)\frac{\partial A^{f\pm}_{\omega_2}}{\partial t} + \frac{i}{2}(k''_2 \mp b''_2)\frac{\partial^2 A^{f\pm}_{\omega_2}}{\partial t^2} \\
&\qquad = i\frac{\omega_2 p_2}{2cn_2} A^{f\mp}_{\omega_3} A^{b\mp *}_{\omega_1} \exp(i(k_3 + k_1 - k_2)z \pm i(\beta_3 + \beta_2 + \beta_1)z),
\end{aligned} \tag{15b}$$

$$\begin{aligned}
&\frac{\partial A^{f\mp}_{\omega_3}}{\partial z} + (k'_3 \pm a'_3)\frac{\partial A^{f\mp}_{\omega_3}}{\partial t} + \frac{i}{2}(k''_3 \pm b''_3)\frac{\partial^2 A^{f\mp}_{\omega_3}}{\partial t^2} \\
&\qquad = i\frac{\omega_3 p_3}{2cn_3} A^{b\mp}_{\omega_1} A^{f\pm}_{\omega_2} \exp(-i(k_1 - k_2 + k_3)z \mp i(\beta_3 + \beta_2 + \beta_1)z).
\end{aligned} \tag{15c}$$

We adopt the short-hand notations[2]

$$\Delta k = k_3 - k_2 + k_1, \qquad \Delta\alpha = \beta_3 + \beta_2 + \beta_1, \tag{16}$$

and conclude the derivation of the equations for the field envelopes by summarizing them as

$$-\frac{\partial A^{b\mp}_{\omega_1}}{\partial z} + (k'_1 \pm a'_1)\frac{\partial A^{b\mp}_{\omega_1}}{\partial t} + \frac{i}{2}(k''_1 \pm b''_1)\frac{\partial^2 A^{b\mp}_{\omega_1}}{\partial t^2} = i\frac{\omega_1 p_1}{2cn_1} A^{f\mp}_{\omega_3} A^{f\pm *}_{\omega_2} \exp(i(\Delta k \pm \Delta\alpha)z), \tag{17a}$$

$$\frac{\partial A^{f\pm}_{\omega_2}}{\partial z} + (k'_2 \mp a'_2)\frac{\partial A^{f\pm}_{\omega_2}}{\partial t} + \frac{i}{2}(k''_2 \mp b''_2)\frac{\partial^2 A^{f\pm}_{\omega_2}}{\partial t^2} = i\frac{\omega_2 p_2}{2cn_2} A^{f\mp}_{\omega_3} A^{b\mp *}_{\omega_1} \exp(i(\Delta k \pm \Delta\alpha)z), \tag{17b}$$

$$\frac{\partial A^{f\mp}_{\omega_3}}{\partial z} + (k'_3 \pm a'_3)\frac{\partial A^{f\mp}_{\omega_3}}{\partial t} + \frac{i}{2}(k''_3 \pm b''_3)\frac{\partial^2 A^{f\mp}_{\omega_3}}{\partial t^2} = i\frac{\omega_3 p_3}{2cn_3} A^{b\mp}_{\omega_1} A^{f\pm}_{\omega_2} \exp(-i(\Delta k \pm \Delta\alpha)z). \tag{17c}$$

Continuous waves

For continuous waves, the time derivatives of the envelopes vanish, and we are left with the considerably simplified system

$$-\frac{\partial A^{b\mp}_{\omega_1}}{\partial z} = i\frac{\omega_1 p_1}{2cn_1} A^{f\mp}_{\omega_3} A^{f\pm *}_{\omega_2} \exp(i(\Delta k \pm \Delta\alpha)z), \tag{18a}$$

$$\frac{\partial A^{f\pm}_{\omega_2}}{\partial z} = i\frac{\omega_2 p_2}{2cn_2} A^{f\mp}_{\omega_3} A^{b\mp *}_{\omega_1} \exp(i(\Delta k \pm \Delta\alpha)z), \tag{18b}$$

$$\frac{\partial A^{f\mp}_{\omega_3}}{\partial z} = i\frac{\omega_3 p_3}{2cn_3} A^{b\mp}_{\omega_1} A^{f\pm}_{\omega_2} \exp(-i(\Delta k \pm \Delta\alpha)z). \tag{18c}$$

[2] Notice the difference to the usual co-propagating case of optical parametric amplification and oscillation, in which the electric dipolar part of the phase matching condition instead yields

$$\Delta k = k_3 - k_2 - k_1 \approx 0.$$

However, the nonlocal contribution $\Delta\alpha = \beta_3 + \beta_2 + \beta_1$ is identical in both cases.

A modulated sign of the nonlinear coupling coefficient

In quasi phase matching the idea is to periodically modulate the sign of the nonlinear coupling coefficient, in order to reverse the phase of the optical parametric process after having reached one coherence length of the interaction. In this case, we assume a periodic switching of the sign of the coupling coefficient without any other changes to its magnitude.

The Fourier decomposition of a function $s(z)$, which is defined by

$$s(z) = \begin{cases} 1, & \text{if } 0 \leq z < \Lambda/2, \\ 0, & \text{if } \Lambda/2 \leq z < \Lambda, \end{cases} \tag{19}$$

and is periodically extended by a period Λ, is given in a complex-valued exponential representation by[3]

$$s_M(z) = \frac{1}{2} + \sum_{m=1}^{M} \frac{1}{i\pi(2m-1)} \left[\exp\left(i\frac{2\pi(2m-1)}{\Lambda} z \right) - \exp\left(-i\frac{2\pi(2m-1)}{\Lambda} z \right) \right]. \tag{20}$$

The periodic expansion and the various terms are shown in Fig. 2 below.

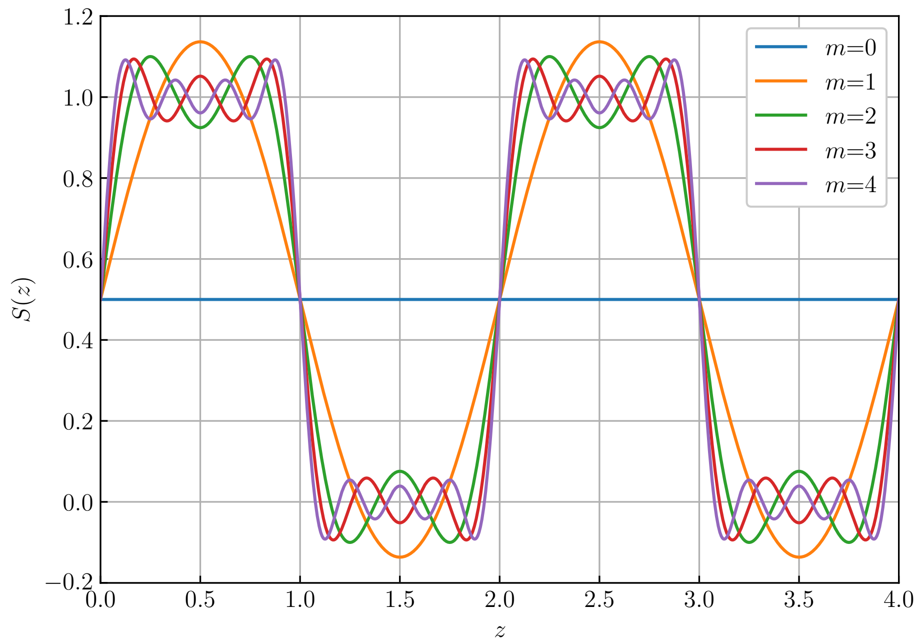

Figure 2. Fourier expansion of the periodic expansion of the rectangular (boxcar) function, for terms up to and including $m = 4$ and for a period of $\Lambda = 2$ and a duty cycle of 0.5 (50%).

Returning to the original purpose of this, we wish to construct a Fourier decomposition of the nonlinear coupling coefficient of the form

$$p_j(z) = \begin{cases} p_j, & \text{if } 0 \leq z < \Lambda/2, \\ -p_j, & \text{if } \Lambda/2 \leq z < \Lambda, \end{cases} \tag{21}$$

that is to say, by using the definition above for the boxcar function $s(z)$,

$$p_j(z) = 2(s(z) - 1/2)p_j, \tag{22}$$

[3] Or equivalently in real-valued terms,

$$s_M(z) = \frac{1}{2} + \sum_{m=1}^{M} \frac{(2/\pi)}{(2m-1)} \sin\left(\frac{2\pi(2m-1)}{\Lambda} z \right),$$

which, however, for the purpose of inclusion in the complex-valued wave equations is less suited.

a Λ-periodic modulation of the sign of the nonlinear coupling parameters $p_j(z)$ is expressed by the complex-valued Fourier decomposition

$$p_j(z) = 2p_j \sum_{m=1}^{M} \frac{1}{i\pi(2m-1)} \Big[\exp\Big(i\frac{2\pi(2m-1)}{\Lambda} z \Big) - \exp\Big(-i\frac{2\pi(2m-1)}{\Lambda} z \Big) \Big]. \tag{23}$$

The terms of the Fourier decomposition of $p_j(z)$ are hence

$$\begin{aligned}
m &= 1 \quad \rightarrow \quad -i(2/\pi)\big[\exp(i2\pi z/\Lambda) - \exp(-i2\pi z/\Lambda)\big]p_j, && \text{(24a)}\\
m &= 2 \quad \rightarrow \quad -i(2/3\pi)\big[\exp(i6\pi z/\Lambda) - \exp(-i6\pi z/\Lambda)\big]p_j, && \text{(24b)}\\
m &= 3 \quad \rightarrow \quad -i(2/5\pi)\big[\exp(i10\pi z/\Lambda) - \exp(-i10\pi z/\Lambda)\big]p_j, && \text{(24c)}\\
m &= 4 \quad \rightarrow \quad -i(2/7\pi)\big[\exp(i14\pi z/\Lambda) - \exp(-i14\pi z/\Lambda)\big]p_j, && \text{(24d)}\\
m &= 5 \quad \rightarrow \quad \cdots , && \text{(24e)}
\end{aligned}$$

where the missing $m = 0$ term stems from the fact that the sign-reversing form of Eq. (22) is perfectly balanced around zero, in contrary to the boxcar function as shown in Fig. 2. Admittedly, things are now sorted out in a somewhat reversed order, but if we allow for varying coupling coefficients $p_j(z)$ in (18), only keeping the lowest-order term in Eq. (24a) of the form (23) obtain

$$\begin{aligned}
-\frac{\partial A^{b\mp}_{\omega_1}}{\partial z} &= \frac{\omega_1 p_1}{\pi c n_1} A^{f\mp}_{\omega_3} A^{f\pm *}_{\omega_2} \big[\exp(i2\pi z/\Lambda) - \underbrace{\exp(-i2\pi z/\Lambda)}_{\text{keep this}}\big] \exp(i(\Delta k \pm \Delta\alpha)z), && \text{(25a)}\\
\frac{\partial A^{f\pm}_{\omega_2}}{\partial z} &= \frac{\omega_2 p_2}{\pi c n_2} A^{f\mp}_{\omega_3} A^{b\mp *}_{\omega_1} \big[\exp(i2\pi z/\Lambda) - \underbrace{\exp(-i2\pi z/\Lambda)}_{\text{keep this}}\big] \exp(i(\Delta k \pm \Delta\alpha)z), && \text{(25b)}\\
\frac{\partial A^{f\mp}_{\omega_3}}{\partial z} &= \frac{\omega_3 p_3}{\pi c n_3} A^{b\mp}_{\omega_1} A^{f\pm}_{\omega_2} \big[\underbrace{\exp(i2\pi z/\Lambda)}_{\text{keep this}} - \exp(-i2\pi z/\Lambda)\big] \exp(-i(\Delta k \pm \Delta\alpha)z). && \text{(25c)}
\end{aligned}$$

Keeping only the phase-matching terms with negative signs of the exponents of $2\pi/\Lambda$, Eqs. (25) are reduced to

$$\begin{aligned}
-\frac{\partial A^{b\mp}_{\omega_1}}{\partial z} &= -\frac{\omega_1 p_1}{\pi c n_1} A^{f\mp}_{\omega_3} A^{f\pm *}_{\omega_2} \exp\big(i(\Delta k \pm \Delta\alpha - 2\pi/\Lambda)z\big), && \text{(26a)}\\
\frac{\partial A^{f\pm}_{\omega_2}}{\partial z} &= -\frac{\omega_2 p_2}{\pi c n_2} A^{f\mp}_{\omega_3} A^{b\mp *}_{\omega_1} \exp\big(i(\Delta k \pm \Delta\alpha - 2\pi/\Lambda)z\big), && \text{(26b)}\\
\frac{\partial A^{f\mp}_{\omega_3}}{\partial z} &= \frac{\omega_3 p_3}{\pi c n_3} A^{b\mp}_{\omega_1} A^{f\pm}_{\omega_2} \exp\big(-i(\Delta k \pm \Delta\alpha - 2\pi/\Lambda)z\big). && \text{(26c)}
\end{aligned}$$

We assign the short-hand notation

$$\kappa_\pm = \Delta k \pm \Delta\alpha - 2\pi/\Lambda \tag{27}$$

and clean up Eqs. (26) from signs, to produce the equation for the spatial evolution of the idler (ω_1), signal (ω_2) and pump (ω_3) envelopes as

$$\begin{aligned}
\frac{\partial A^{b\mp}_{\omega_1}}{\partial z} &= \frac{\omega_1 p_1}{\pi c n_1} A^{f\mp}_{\omega_3} A^{f\pm *}_{\omega_2} \exp(i\kappa_\pm z), && \text{(28a)}\\
\frac{\partial A^{f\pm}_{\omega_2}}{\partial z} &= -\frac{\omega_2 p_2}{\pi c n_2} A^{f\mp}_{\omega_3} A^{b\mp *}_{\omega_1} \exp(i\kappa_\pm z), && \text{(28b)}\\
\frac{\partial A^{f\mp}_{\omega_3}}{\partial z} &= \frac{\omega_3 p_3}{\pi c n_3} A^{b\mp}_{\omega_1} A^{f\pm}_{\omega_2} \exp(-i\kappa_\pm z). && \text{(28c)}
\end{aligned}$$

The concept of quasi phase matching

In Eqs. (27) and (28), we see how the concept of the *coherence length* naturally appear in the theory of quasi phase matching. If we for the moment drop the differential contribution $\Delta\alpha$ to the phase, perfect phase matching of the OPA process requires that

$$\kappa_{\pm} = \Delta k - 2\pi/\Lambda = 0. \tag{28d}$$

Since Λ is the *period* of the modulation, containing two layers of opposite sign for their respective nonlinear coupling coefficient, this means that each layer of "constant sign"[4] of the nonlinear coupling coefficient has the physical length $\Lambda/2$. In other words, in order to provide perfect phase matching, or rather *quasi phase matching*, each layer should have a physical thickness of half this period, that is to say

$$L_{\mathrm{C}} = \Lambda/2 = \pi/\Delta k. \tag{28e}$$

This quantity is actually the very definition of the classically used *coherence length* L_{C}, which if we return to the form of Eq. (27) including the differential contribution from the nonlocal interaction yields[5]

$$L_{\mathrm{C}}^{\pm} \equiv \pi/(\Delta k \pm \Delta\alpha). \tag{28f}$$

Thus, the concept of quasi phase matching can be summarized by that each layer of constant sign of the nonlinear coupling coefficient (that is to say, "$\chi^{(2)}$") should be chosen of physical thickess corresponding to one coherence length. In the case of an optically active medium, we have the choice to design either for the LCP coherence length L_{C}^{+} or for the RCP coherence length L_{C}^{-}, providing different premises for optimal phase matching.

Solutions to the envelopes under the non-depleted pump approximation

Whenever the pump may be considered as close to constant over the optical parametric amplification, the system described by Eqs. (28) is reduced to

$$\frac{\partial A^{b\mp}_{\omega_1}}{\partial z} = \gamma_1 A^{f\mp}_{\omega_3} A^{f\pm *}_{\omega_2} \exp(i\kappa_{\pm} z), \tag{29a}$$

$$\frac{\partial A^{f\pm}_{\omega_2}}{\partial z} = -\gamma_2 A^{f\mp}_{\omega_3} A^{b\mp *}_{\omega_1} \exp(i\kappa_{\pm} z), \tag{29b}$$

where we defined the short-hand notation for the coupling coefficients

$$\gamma_j = \frac{\omega_j p_j}{\pi c n_j}, \qquad j = 1, 2, \tag{30}$$

and where the pump envelope $A^{f\mp}_{\omega_3} = \text{const.}$, and where the idler and signal are subject to the boundary conditions

$$A^{b\mp}_{\omega_1}(z = L) = 0, \qquad A^{f\pm}_{\omega_2}(z = 0) = A^{f\pm}_{\omega_2}(0). \tag{30a}$$

By differentiating Eq. (29b) with respect to z and substituting Eq. (29a) for the idler into Eq. (29b), we get the ordinary differential equation for the forward traveling signal (ω_2) envelope as

$$\begin{aligned}
\frac{\partial^2 A^{f\pm}_{\omega_2}}{\partial z^2} &= -\gamma_2 A^{f\mp}_{\omega_3} \frac{\partial}{\partial z}\left[A^{b\mp *}_{\omega_1} \exp(i\kappa_{\pm} z)\right] \\
&= -\gamma_2 A^{f\mp}_{\omega_3} \left(\frac{\partial A^{b\mp *}_{\omega_1}}{\partial z} + i\kappa_{\pm} A^{b\mp *}_{\omega_1}\right) \exp(i\kappa_{\pm} z) \\
&= -\gamma_2 A^{f\mp}_{\omega_3} \left\{\gamma_1 A^{f\mp *}_{\omega_3} A^{f\pm}_{\omega_2} \exp(-i\kappa_{\pm} z) - i\kappa_{\pm}\left[\frac{\dfrac{\partial A^{f\pm}_{\omega_2}}{\partial z}\exp(-i\kappa_{\pm} z)}{\gamma_2 A^{f\mp}_{\omega_3}}\right]\right\} \exp(i\kappa_{\pm} z) \\
&= -\gamma_1\gamma_2 |A^{f\mp}_{\omega_3}|^2 A^{f\pm}_{\omega_2} + i\kappa_{\pm} \frac{\partial A^{f\pm}_{\omega_2}}{\partial z}.
\end{aligned} \tag{31}$$

---

[4] Keep in mind that we currently actually are looking at the first-order term of the Fourier decomposition in Eq. (24a), which provides a *sinusoidal* variation of the coupling coefficient, rather than a periodically extended boxcar function. Hence the quotation marks.

[5] See the manuscript *Pulsed optical parametric amplification in chiral media* as submitted to J. Opt. Soc. Am. B in March 2025.

In similar, we by differentiating Eq. (29a) with respect to z and substituting Eq. (29b) for the idler into Eq. (29a), we get the ordinary differential equation for the backward traveling idler (ω_1) envelope as

$$\begin{aligned}\frac{\partial^2 A^{f\pm}_{\omega_1}}{\partial z^2} &= \gamma_1 A^{f\mp}_{\omega_3}\frac{\partial}{\partial z}\big[A^{f\pm*}_{\omega_2}\exp(i\kappa_\pm z)\big]\\ &= \gamma_1 A^{f\mp}_{\omega_3}\left(\frac{\partial A^{f\pm*}_{\omega_2}}{\partial z} + i\kappa_\pm A^{f\pm*}_{\omega_2}\right)\exp(i\kappa_\pm z)\\ &= \gamma_1 A^{f\mp}_{\omega_3}\left\{-\gamma_2 A^{f\mp*}_{\omega_3}A^{b\mp}_{\omega_1}\exp(-i\kappa_\pm z) + i\kappa_\pm\left[\frac{\frac{\partial A^{b\mp}_{\omega_1}}{\partial z}\exp(-i\kappa_\pm z)}{\gamma_1 A^{f\mp}_{\omega_3}}\right]\right\}\exp(i\kappa_\pm z)\\ &= -\gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2 A^{b\mp}_{\omega_1} + i\kappa_\pm\frac{\partial A^{b\mp}_{\omega_1}}{\partial z}.\end{aligned} \tag{32}$$

Thus, to summarize, we for the backward traveling idler and forward traveling signal envelopes have the identical forms

$$\boxed{\begin{aligned}&\frac{\partial^2 A^{b\mp}_{\omega_1}}{\partial z^2} - i\kappa_\pm\frac{\partial A^{b\mp}_{\omega_1}}{\partial z} + \gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2 A^{b\mp}_{\omega_1} = 0, &(33a)\\ &\frac{\partial^2 A^{f\pm}_{\omega_2}}{\partial z^2} - i\kappa_\pm\frac{\partial A^{f\pm}_{\omega_2}}{\partial z} + \gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2 A^{f\pm}_{\omega_2} = 0. &(33b)\end{aligned}}$$

That these equations are identical, despite that one is for a backward traveling wave and the other for a forward traveling component, should not be surprising as they origin from the same wave equation, which always is invariant regardless of the direction of propagation, with the solutions and directions determined by the boundary conditions for launching of the waves.

General solution

In order to solve the second-order ordinary differential equation for the idler and signal envelopes, we notice that they both share the characteristic polynomial

$$r^2 - i\kappa_\pm r + \gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2 = 0, \tag{34}$$

with two distinct roots for each polarization state,

$$r_1 = i\kappa_\pm/2 + ib_\pm, \qquad r_2 = i\kappa_\pm/2 - ib_\pm, \tag{35}$$

where we defined

$$b_\pm = (\kappa^2_\pm/4 + \gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2)^{1/2}. \tag{36}$$

From the characteristic roots of Eq. (34), we immediately find the general solutions for envelopes of the idler signal as

$$\begin{aligned}A^{b\mp}_{\omega_1}(z) &= C'_1\exp(i(\kappa_\pm/2 + ib_\pm)z) + C'_2\exp(i(\kappa_\pm/2 - ib_\pm)z)\\ &= (C_1\cos(b_\pm z) + C_2\sin(b_\pm z))\exp(i\kappa_\pm z/2), &(37a)\\ A^{f\pm}_{\omega_2}(z) &= (D_1\cos(b_\pm z) + D_2\sin(b_\pm z))\exp(i\kappa_\pm z/2), &(37b)\end{aligned}$$

where C_j and D_j, $j = 1, 2$, are constants of integration.

Application of boundary conditions I – The idler envelope

Starting with the backward traveling idler, we from Eq. (37) have that

$$(C_1 \cos(b_\pm L) + C_2 \sin(b_\pm L)) \exp(i\kappa_\pm L/2) = A^{b\mp}_{\omega_1}(L) = \text{known}.$$

Meanwhile, by differentiating Eq. (37) with respect to z, evaluating the expression at $z = 0$ and interpreting the result against Eq. (29a), we obtain a second relation for the coefficients C_1 and C_2 as

$$\begin{aligned} &b_\pm(-C_1 \sin(b_\pm 0) + C_2 \cos(b_\pm 0)) \exp(i\kappa_\pm 0/2) \\ &\qquad + i(\kappa_\pm/2)(C_1 \cos(b_\pm 0) + C_2 \sin(b_\pm 0)) \exp(i\kappa_\pm 0/2) \\ &\qquad = \gamma_1 A^{f\mp}_{\omega_3} A^{f\pm *}_{\omega_2}(0) \exp(i\kappa_\pm 0), \end{aligned} \tag{39}$$

that is to say,

$$i(\kappa_\pm/2)C_1 + b_\pm C_2 = \gamma_1 A^{f\mp}_{\omega_3}(0) A^{f\pm *}_{\omega_2}(0), \tag{40}$$

where we recall that this analysis entirely is done under the assumption $A^{f\mp}_{\omega_3}(z) = \text{const}$. Solving Eqs. (38) and (39) for C_1 and C_2 then yields

$$C_1 = \frac{A^{b\mp}_{\omega_1}(L) \exp(-i\kappa_\pm L/2) - (\gamma_1/b_\pm) \sin(b_\pm L) A^{f\mp}_{\omega_3}(0) A^{f\pm *}_{\omega_2}(0)}{\cos(b_\pm L) - i(\kappa_\pm/2b_\pm) \sin(b_\pm L)}, \tag{41a}$$

$$C_2 = \frac{(\gamma_1/b_\pm) \cos(b_\pm L) A^{f\mp}_{\omega_3}(0) A^{f\pm *}_{\omega_2}(0) - i(\kappa_\pm/2b_\pm) A^{b\mp}_{\omega_1}(L) \exp(-i\kappa_\pm L/2)}{\cos(b_\pm L) - i(\kappa_\pm/2b_\pm) \sin(b_\pm L)}, \tag{41b}$$

where we already at this stage took the opportunity to normalize the parameters $b_\pm$, γ_j and $\kappa_\pm$ by multiplying the numerators and denominators in Eqs. (41) by the length L. Thus, just to summarize without entering a rather messy algebra in attempting to formulate an explicit form, the solution for the envelope of the backward traveling idler wave from Eqs. (37a) and (41) is

$$A^{b\mp}_{\omega_1}(z) = (C_1 \cos(b_\pm z) + C_2 \sin(b_\pm z)) \exp(i\kappa_\pm z/2). \tag{42}$$

Application of boundary conditions II – The signal envelope

Continuing with the forward traveling signal, we from Eq. (37) at $z = 0$ have that

$$(D_1 \cos(b_\pm 0) + D_2 \sin(b_\pm 0)) \exp(i\kappa_\pm 0/2) = D_1 = A^{f\pm}_{\omega_2}(0) = \text{known}. \tag{43}$$

In other words, the coefficient D_1 is trivially obtained as the amplitude of the initial signal at $z = 0$. Meanwhile, in similar to the process of obtaining the coefficients for the idler, by differentiating Eq. (37b) with respect to z, evaluating the expression at $z = L$ and interpreting the result against Eq. (29b) under the assumption that we do not launch an idler at $z = L$ ($A^{b\mp *}_{\omega_1}(L) = 0$), we obtain a second relation for the coefficients D_1 and D_2 as

$$\begin{aligned} &b_\pm(-D_1 \sin(b_\pm L) + D_2 \cos(b_\pm L)) \exp(i\kappa_\pm L/2) \\ &\qquad + i(\kappa_\pm/2)(D_1 \cos(b_\pm L) + D_2 \sin(b_\pm L)) \exp(i\kappa_\pm L/2) \\ &\qquad = -\gamma_2 A^{f\mp}_{\omega_3} A^{b\mp *}_{\omega_1}(L) \exp(i\kappa_\pm L) \\ &\qquad = 0, \end{aligned} \tag{44}$$

that is to say, since we previously found that $D_1 = A^{f\pm}_{\omega_2}(0)$, we may solve Eq. (44) for D_2 to obtain[6]

$$D_2 = \frac{\sin(b_\pm L) - i(\kappa_\pm/2b_\pm)\cos(b_\pm L)}{\cos(b_\pm L) + i(\kappa_\pm/2b_\pm)\sin(b_\pm L)} A^{f\pm}_{\omega_2}(0), \tag{45}$$

To summarize, the envelope of the forward traveling signal is from Eqs. (37b) and (43)–(45) explicitly given as

$$\boxed{A^{f\pm}_{\omega_2}(z) = A^{f\pm}_{\omega_2}(0)\bigg(\cos(b_\pm z) + \frac{\sin(b_\pm L) - i(\kappa_\pm/2b_\pm)\cos(b_\pm L)}{\cos(b_\pm L) + i(\kappa_\pm/2b_\pm)\sin(b_\pm L)}\sin(b_\pm z)\bigg)\exp(i\kappa_\pm z/2). \qquad (46)}$$

As a simple sanity check on this expression, we see that it trivially fulfils the initial condition when $z = 0$.

Signal amplification over a full passage of the medium

When passing over the medium from $z = 0$ to $z = L$, the LCP/RCP modes of the signal expressed by Eq. (46) experience the gain

$$\begin{aligned}
G^\pm_{\rm s} &\equiv \frac{|A^{f\pm}_{\omega_2}(L)|^2}{|A^{f\pm}_{\omega_2}(0)|^2}\\
&= \left|\cos(b_\pm L) + \frac{\sin(b_\pm L) - i(\kappa_\pm/2b_\pm)\cos(b_\pm L)}{\cos(b_\pm L) + i(\kappa_\pm/2b_\pm)\sin(b_\pm L)}\sin(b_\pm L)\right|^2\\
&= \left|\frac{\cos(b_\pm L) + i(\kappa_\pm/2b_\pm)\sin(b_\pm L)}{\cos(b_\pm L) + i(\kappa_\pm/2b_\pm)\sin(b_\pm L)}\cos(b_\pm L) + \frac{\sin(b_\pm L) - i(\kappa_\pm/2b_\pm)\cos(b_\pm L)}{\cos(b_\pm L) + i(\kappa_\pm/2b_\pm)\sin(b_\pm L)}\sin(b_\pm L)\right|^2\\
&= \left|\frac{[\cos(b_\pm L) + i(\kappa_\pm/2b_\pm)\sin(b_\pm L)]\cos(b_\pm L) + [\sin(b_\pm L) - i(\kappa_\pm/2b_\pm)\cos(b_\pm L)]\sin(b_\pm L)}{\cos(b_\pm L) + i(\kappa_\pm/2b_\pm)\sin(b_\pm L)}\right|^2\\
&= \frac{1}{|\cos(b_\pm L) + i(\kappa_\pm/2b_\pm)\sin(b_\pm L)|^2}\\
&= \frac{1}{\cos^2(b_\pm L) + (\kappa_\pm/2b_\pm)^2\sin^2(b_\pm L)}.
\end{aligned} \tag{47}$$

In perfect phase matching, $\kappa_\pm = 0$ for whichever mode LCP/RCP, and the gain under this circumstance takes the form

$$G^\pm_{\rm s} = \frac{1}{\cos^2(b_\pm L)}, \tag{48}$$

which goes to infinity whenever

$$b_\pm L = \pi/2 \quad \Rightarrow \quad G^\pm_{\rm s} \to \infty, \tag{49}$$

or from Eq. (36), keeping in mind that $\kappa_\pm = 0$, equivalently

$$\gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2 L^2 = (\pi/2)^2 \quad \Rightarrow \quad G^\pm_{\rm s} \to \infty. \tag{50}$$

In other words, the threshold is reached whenever the pump intensity reaches

$$\gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2 = \left(\frac{\pi}{2L}\right)^2. \tag{51}$$

[6] We may here ask ourselves the very basic question: *Why is it that the coupling coefficient γ_2 does not show up neither in Eq. (41)–(42) nor Eq. (46)?* For example, γ_1 appear explicitly in Eq. (41), so why not γ_1? Is something wrong here? Obviously, the only way that γ_2 would enter the expressions explicitly is through Eq. (44); however, there γ_2 is coupled to the fact that $A^{b\mp*}_{\omega_1}(L) = 0$, and γ_2 hence never enter the expressions at this stage. However, γ_2 is present through the definition of $b_\pm$ in Eq. (36), $b_\pm = (\kappa^2_\pm/4 + \gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2)^{1/2}$, so both coupling coefficients are actually present, hence solving this paradox.

As the intensity is defined as $I = c_0 n\varepsilon_0|E|^2$, where c_0 is the vacuum speed of light, n is the index of refraction of the medium and ε_0 the vacuum permittivity, this means that the corresponding LCP/RCP pump threshold intensities $I^{\pm}_{\rm th}$ expressed in SI units become[7]

$$I^{\pm}_{\rm th} = \frac{cn_3\varepsilon_0}{\gamma_1\gamma_2}\Big(\frac{\pi}{2L}\Big)^2, \tag{52}$$

Interpretation in normalized and dimensionless variables

In the expressions for the explicit solutions and in the definition of $b_{\pm} = (\kappa^2_{\pm}/4 + \gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2)^{1/2}$ in Eq. (36), we find that there are a set of naturally appearing pairs of variables, such as

$$b_{\pm}L, \qquad \kappa_{\pm}L/2, \qquad \kappa_{\pm}/2b_{\pm}, \qquad \text{and} \qquad \gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2L^2. \tag{53}$$

Let us therefore attempt to identify suitable normalized and dimensionless forms involving these parameters as far as possible, starting with the pump intensity $\gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2L^2$, which we via the threshold of Eq. (52) find can be written as

$$\gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2L^2 = \Big(\frac{\pi}{2}\Big)^2\frac{I_{\rm pump}}{I_{\rm th}}. \tag{54}$$

As for the term $\kappa_{\pm}L/2$, we should here be somewhat careful, as this for the backward OPA contains ΔkL which for the backward configuration is huge in numerical value as such; the idea in QPM is that this should be very close to the term $2\pi L/\Lambda$, and we should hence separarate out this pair as the effective phase mismatch. Hence, the term $\kappa_{\pm}L/2$ can be formulated in terms of normalized and dimensionless quotes as[8]

$$\begin{aligned}\kappa_{\pm}L/2 &\equiv (\Delta k \pm \Delta\alpha - 2\pi/\Lambda)L/2\\ &= \Big(\frac{\Delta kL}{2} - \frac{2\pi L}{2\Lambda}\Big)\Big(1 \pm \frac{\Delta\alpha}{\Delta k - 2\pi/\Lambda}\Big).\end{aligned} \tag{55}$$

Meanwhile, the term $b_{\pm}L$ can similarly be formulated in terms of the normalized and dimensionless quotes, including the pump intensity vs threshold, as[9]

$$\begin{aligned}b_{\pm}L &\equiv (\kappa^2_{\pm}/4 + \gamma_1\gamma_2|A^{f\mp}_{\omega_3}|^2)^{1/2}L\\ &= \big[(\kappa_{\pm}L/2)^2 + (\pi/2)^2(I_{\rm pump}/I_{\rm th})\big]^{1/2}\\ &= \big[((\Delta k \pm \Delta\alpha - 2\pi/\Lambda)L/2)^2 + (\pi/2)^2(I_{\rm pump}/I_{\rm th})\big]^{1/2}\\ &= \Big[\Big(\frac{\Delta kL}{2} - \frac{2\pi L}{2\Lambda}\Big)^2\Big(1 \pm \frac{\Delta\alpha}{\Delta k - 2\pi/\Lambda}\Big)^2 + \Big(\frac{\pi}{2}\Big)^2\frac{I_{\rm pump}}{I_{\rm th}}\Big]^{1/2},\end{aligned} \tag{56}$$

and finally and trivially,

$$\frac{\kappa_{\pm}}{2b_{\pm}} = \frac{\kappa_{\pm}L/2}{b_{\pm}L} = \frac{\Big(\dfrac{\Delta kL}{2} - \dfrac{2\pi L}{2\Lambda}\Big)\Big(1 \pm \dfrac{\Delta\alpha}{\Delta k - 2\pi/\Lambda}\Big)}{\Big[\Big(\dfrac{\Delta kL}{2} - \dfrac{2\pi L}{2\Lambda}\Big)^2\Big(1 \pm \dfrac{\Delta\alpha}{\Delta k - 2\pi/\Lambda}\Big)^2 + \Big(\dfrac{\pi}{2}\Big)^2\dfrac{I_{\rm pump}}{I_{\rm th}}\Big]^{1/2}}. \tag{57}$$

---

[7] We here ignore the fact that LCP/RCP in chiral media in fact have different refractive indices; also notice that we from Eq. (30) may reformulate this as the rather silly expression

$$I^{\pm}_{\rm th} = \frac{\pi^4c^3n_1n_2n_3\varepsilon_0}{4\omega_1\omega_2p_1p_2L^2} \quad \ldots$$

[8] See Eq. (27) for the definition of $\kappa_{\pm}$.

[9] Notice that at perfect quasi phase matching against the electric dipolar mismatch Δk, we have $2\pi/(\Delta k\Lambda) = 1$, and hence the expression for $b_{\pm}$ becomes *symmetric* in the chiral phase mismatch parameter $\Delta\alpha$.

Thus, by adopting the short-hand notation for the normalized and dimensionless variables

$$\boxed{\beta \equiv \frac{\Delta\alpha}{\Delta k - 2\pi/\Lambda}, \qquad \delta \equiv \Big(\frac{\Delta k L}{2} - \frac{2\pi L}{2\Lambda}\Big), \qquad \eta = (\pi/2)^2 I_{\mathrm{pump}}/I_{\mathrm{th}},} \tag{58}$$

the expressions $\kappa_\pm L/2$, $b_\pm L$ and $\kappa_\pm/2b_\pm$ are compactified to

$$\boxed{\kappa_\pm L/2 = \delta(1\pm\beta), \qquad b_\pm L = (\delta^2(1\pm\beta)^2+\eta)^{1/2}, \qquad \frac{\kappa_\pm}{2b_\pm} = \frac{\delta(1\pm\beta)}{(\delta^2(1\pm\beta)^2+\eta)^{1/2}}.} \tag{59}$$

In particular, in terms of these parameters the gain expressed by Eq. (47) becomes

$$\begin{aligned} G_{\mathrm{s}}^{\pm} &= \frac{1}{\cos^2(b_\pm L) + (\kappa_\pm/2b_\pm)^2 \sin^2(b_\pm L)}. \\ &= \frac{1}{\cos^2((\delta^2(1\pm\beta)^2+\eta)^{1/2}) + \Big(\frac{\delta(1\pm\beta)}{(\delta^2(1\pm\beta)^2+\eta)^{1/2}}\Big)^2 \sin^2((\delta^2(1\pm\beta)^2+\eta)^{1/2})}. \\ &= \frac{(\delta^2(1\pm\beta)^2+\eta)^{1/2}}{(\delta^2(1\pm\beta)^2+\eta)^{1/2}\cos^2((\delta^2(1\pm\beta)^2+\eta)^{1/2}) + \delta^2(1\pm\beta)^2\sin^2((\delta^2(1\pm\beta)^2+\eta)^{1/2})}. \end{aligned} \tag{60}$$

Interpretation in case of perfect QPM against electric dipolar phase mismatch

Of particular interest is the interpretation of the terms $\kappa_\pm L/2$, $b_\pm L$ and their quote $\kappa_\pm/2b_\pm$ in the case of perfect quasi phase matching against the electric dipolar phase mismatch Δk, that is to say whenever

$$\Delta k = \frac{2\pi}{\Lambda}. \tag{61}$$

In this particular case of interest,

$$\kappa_\pm L/2 = \Big(\frac{\Delta k L}{2}\Big)\Big(1 \pm \frac{\Delta\alpha}{\Delta k} - \underbrace{\frac{2\pi}{\Delta k\Lambda}}_{=1}\Big) = \pm\frac{\Delta\alpha L}{2} \tag{62}$$

Meanwhile, the term $b_\pm L$ in the perfect QPM case becomes

$$b_\pm L = \Big[\Big(\frac{\Delta k L}{2}\Big)^2\Big(1 \pm \frac{\Delta\alpha}{\Delta k} - \underbrace{\frac{2\pi}{\Delta k\Lambda}}_{=1}\Big)^2 + \Big(\frac{\pi}{2}\Big)^2\frac{I_{\mathrm{pump}}}{I_{\mathrm{th}}}\Big]^{1/2} = \Big[\Big(\frac{\Delta\alpha L}{2}\Big)^2 + \Big(\frac{\pi}{2}\Big)^2\frac{I_{\mathrm{pump}}}{I_{\mathrm{th}}}\Big]^{1/2}, \tag{63}$$

and finally and trivially,

$$\frac{\kappa_\pm}{2b_\pm} = \pm\frac{\Big(\frac{\Delta\alpha L}{2}\Big)}{\Big(\Big(\frac{\Delta\alpha L}{2}\Big)^2 + \Big(\frac{\pi}{2}\Big)^2\frac{I_{\mathrm{pump}}}{I_{\mathrm{th}}}\Big)^{1/2}} \tag{64}$$

Now notice that in the fully quasi phase matched case against the electric dipolar phase mismatch, $\kappa_\pm L/2$ is fully *antisymmetric* with respect to chiral coefficient $\Delta\alpha$ for the phase mismatch, while $b_\pm L$ is fully *symmetric* with respect to the same parameter. Thus, in this particular case of perfect phase matching, the single-pass gain is from Eq. (60) expressed as

$$
\begin{aligned}
G_{\rm s}^{\pm} &= \frac{1}{\cos^2(b_{\pm}L) + (\kappa_{\pm}/2b_{\pm})^2\sin^2(b_{\pm}L)}.\\
&= \Bigg\{\cos^2\Big(\Big(\Big(\frac{\Delta\alpha L}{2}\Big)^2 + \Big(\frac{\pi}{2}\Big)^2\frac{I_{\rm pump}}{I_{\rm th}}\Big)^{1/2}\Big)\\
&\qquad + \frac{\Big(\frac{\Delta\alpha L}{2}\Big)^2}{\Big(\Big(\frac{\Delta\alpha L}{2}\Big)^2 + \Big(\frac{\pi}{2}\Big)^2\frac{I_{\rm pump}}{I_{\rm th}}\Big)}\sin^2\Big(\Big(\Big(\frac{\Delta\alpha L}{2}\Big)^2 + \Big(\frac{\pi}{2}\Big)^2\frac{I_{\rm pump}}{I_{\rm th}}\Big)^{1/2}\Big)\Bigg\}^{-1}.
\end{aligned}
\tag{65}
$$

This result might indeed be rather surprising, as this expression for the gain is independent of the sign of $\Delta\alpha$; that is to say, the signal gain $G_{\pm}$ is completely symmetric against the chiral phase mismatch $\Delta\alpha$. *In other words, the LCP and RCP modes are affected equally as functions of the chiral phase mismatch parameter, without any distinction.*

Reasonable parameter values

In the backward OPA configuration, the phase mismatch parameters are radically different than in the classical forward configuration. To start with, the expressions in Eqs. (55)–(57) all contain the electric dipolar phase mismatch factor $\Delta kL/2$ which from Eq. (16) yields

$$\frac{\Delta kL}{2} \equiv (k_3 - k_2 + k_1)L/2, \qquad \Delta\alpha = \beta_3 + \beta_2 + \beta_1, \tag{66}$$

where $k_j = n_j\omega_j/c = n_j 2\pi/\lambda_0$ are the magnitude of the corresponding wave vectors. If the idler and signal are of comparable wavelengths, say in the order of 1200 nm, and the pump (fulfilling the requirement $\omega_3 = \omega_2 + \omega_1$) in the order of 600 nm, and that we furthermore assume the refractive index for the idler and signal being in the order of $n_{1,2} \sim 1.7$ and the pump $n_{1,2} \sim 1.9$, we then from Eq. (66) obtain the typical order of the phase mismatch over a crystal of length $L = 10$ mm as

$$\frac{\Delta kL}{2} = \frac{1}{2}\Big(\frac{2\pi\times 1.9}{600\times 10^{-9}\ \text{m}} - \frac{2\pi\times 1.7}{1200\times 10^{-9}\ \text{m}} + \frac{2\pi\times 1.7}{1200\times 10^{-9}\ \text{m}}\Big)\times(10\times 10^{-3}\ \text{m}) \approx 99\times 10^3. \tag{67}$$

In order to match this with quasi phase matching configuration, we from Eq. (55) hence require that the spatial period Λ is chosen as

$$1 \pm \frac{\Delta\alpha}{\Delta k} - \frac{2\pi}{\Delta k\Lambda} = 0 \quad\Leftrightarrow\quad \Lambda = \frac{2\pi}{\Delta k\Big(1 \pm \frac{\Delta\alpha}{\Delta k}\Big)} \quad\Leftrightarrow\quad \Lambda = \frac{\pi L}{\Big(\frac{\Delta kL}{2}\Big)\Big(1 \pm \frac{\Delta\alpha}{\Delta k}\Big)}, \tag{68}$$

that is to say, if we design this period to just match the electric dipolar mismatch Δk, ignoring $\Delta\alpha/\Delta k$,

$$\Lambda = \frac{\pi(10\times 10^{-3}\ \text{m})}{(99\times 10^3)} \approx 0.32\times 10^{-6}\ \text{m} = 320\ \text{nm}. \tag{69}$$

Thus, we may at first conclude that a reasonable value for the dimensionless "direct" electric dipolar phase mismatch parameter is

$$\frac{\Delta kL}{2} = 100\times 10^3. \tag{70}$$

Thus, in order to achieve phase matching as described by $\kappa_{\pm} = 0$ in Eq. (55),[10] we have three

[10] Eq. (55) simply states that

$$\kappa_{\pm}L/2 = \Big(\frac{\Delta kL}{2}\Big)\Big(1 \pm \frac{\Delta\alpha}{\Delta k} - \frac{2\pi}{\Delta k\Lambda}\Big),$$

which should equal to zero at perfect phase matching.

primary choices for the quasi phase matching period Λ:

$$\begin{aligned}
\frac{2\pi}{\Delta k\Lambda} &= 1+\frac{\Delta\alpha}{\Delta k} &\quad\Rightarrow\quad& \kappa_+ = 0 &\quad& \text{(LCP phase matched)}\\
\frac{2\pi}{\Delta k\Lambda} &= 1-\frac{\Delta\alpha}{\Delta k} &\quad\Rightarrow\quad& \kappa_- = 0 &\quad& \text{(RCP phase matched)}\\
\frac{2\pi}{\Delta k\Lambda} &= 1 &\quad\Rightarrow\quad& \kappa_+ + \kappa_- = 0 &\quad& \text{(phase matched at zero gyrotropy)}
\end{aligned}$$

Graphs and visual interpretations

We will in the following focus on the dependence of backward-wave optical parametric amplification on the electric dipolar phase mismatch against the nominal quasi phase matching period Λ, using the normalized set of parameters as defined in Eq. (58),

$$\beta \equiv \frac{\Delta\alpha}{\Delta k - 2\pi/\Lambda}, \qquad \delta \equiv \Big(\frac{\Delta kL}{2} - \frac{2\pi L}{2\Lambda}\Big), \qquad \eta = (\pi/2)^2 I_{\rm pump}/I_{\rm th}. \tag{71}$$

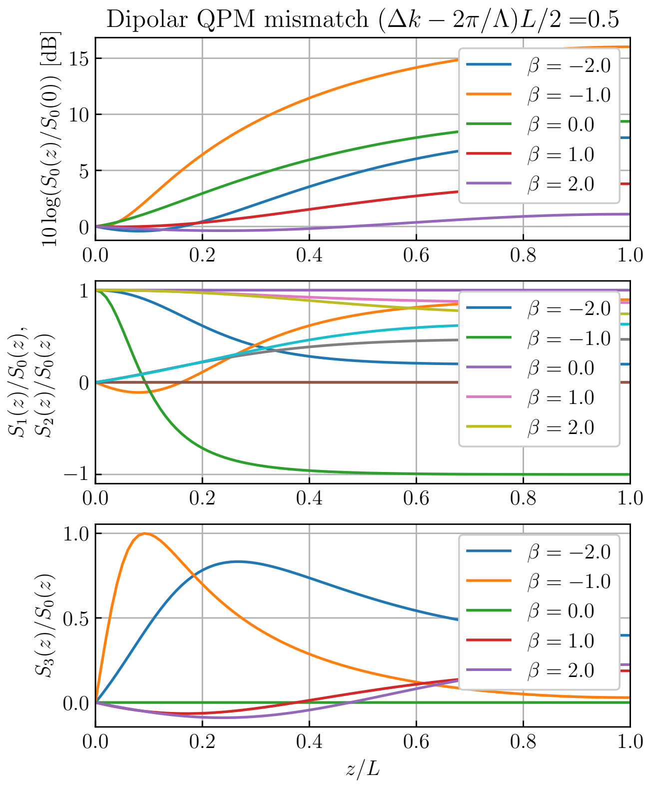

Figure 3. Stokes parameters for the forward traveling signal envelope, for the case of an electric dipolar phase mismatch of $(\Delta k = 2\pi/\Lambda)L/2 = 0.5$ and with a pump intensity to threshold quote of $\eta = (\pi/2)^2 I_{\rm pump}/I_{\rm th} = 2.0$. [Filename: `python/graphs/graph-01-delta-0.50.eps`]

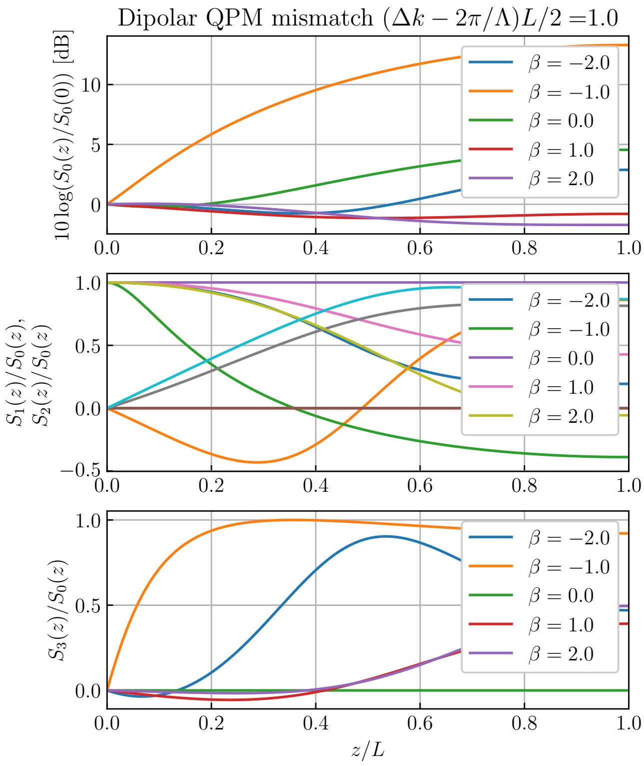

Figure 4. Identical to Fig. 3 but with an electric dipolar phase mismatch against nominal QPM period $(\Delta k = 2\pi/\Lambda)L/2 = 1.0$. [Filename: `python/graphs/graph-01-delta-1.00.eps`]

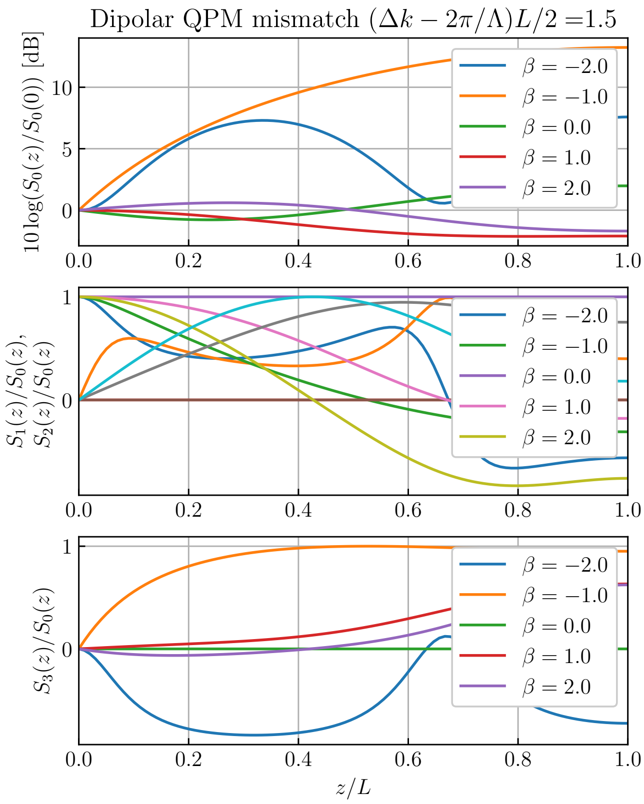

Figure 5. Identical to Fig. 3 but with an electric dipolar phase mismatch against nominal QPM period $(\Delta k = 2\pi/\Lambda)L/2 = 1.5$. [Filename: `python/graphs/graph-01-delta-1.50.eps`]

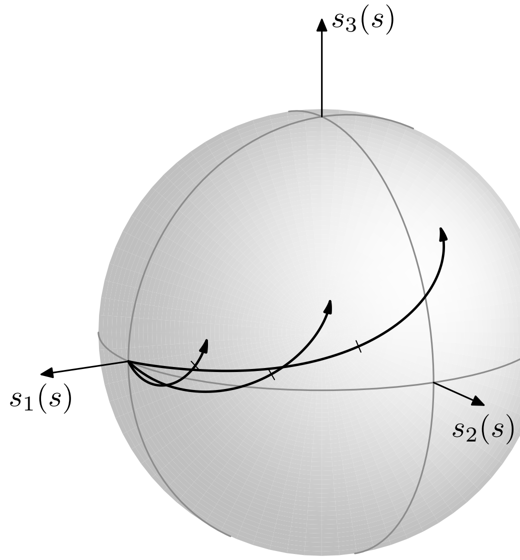

Figure 6. Poincaré map of the Stokes trajectories described by Figs. 1–3. [Filename: `poincare/graph-01-poincare.eps`]

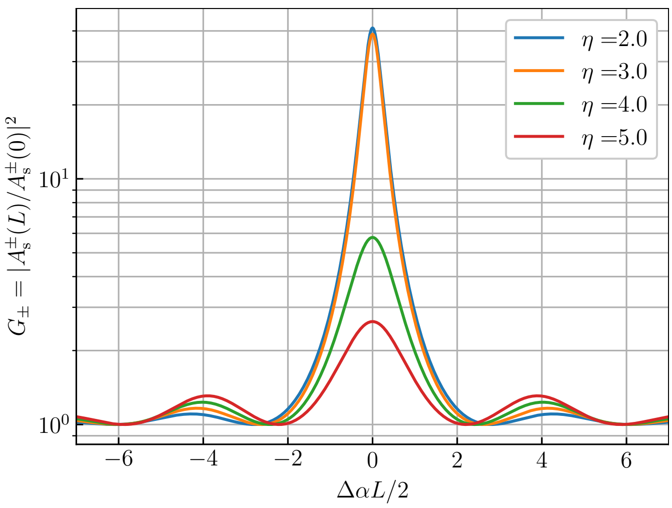

Figure 7. Signal gain $G_\pm$ as function of the chiral contribution $\Delta\alpha$ to the phase mismatch, for the case when the quasi phase matching period Λ is chosen to perfectly match the electric dipolar phase mismatch, that is to say with $\Delta k = 2\pi/\Lambda$. The gain curves are here mapped for a set of pump intensity quotes vs threshold, $\eta = (\pi/2)^{I}_{\text{pump}}/I_{\text{th}}$. Notice the completely symmetric dependence of gain against $\Delta\alpha$. [Filename: `python/graphs/graph-02.eps`]

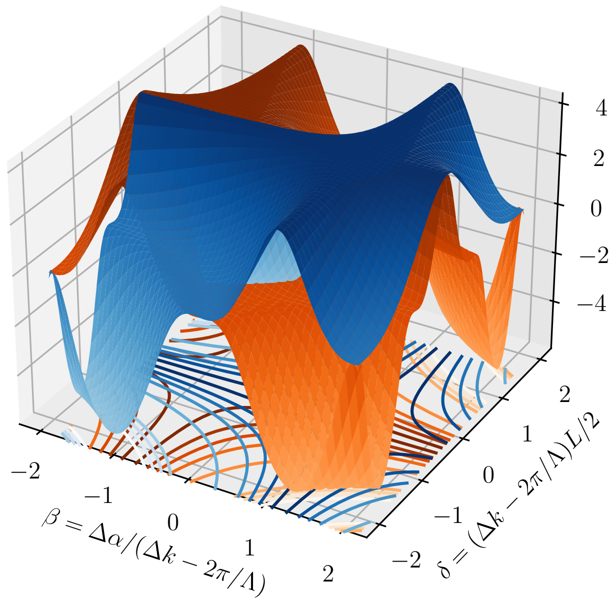

Figure 8. The signal gain G_+ (LCP, orange surface) and G_- (RCP, blue surface) mapped as functions of the electric dipolar phase mismatch $\delta = (\Delta k - 2\pi/\Lambda)L/2$, against the nominal quasi phase matching period Λ, and the chiral contribution to the phase matching $\beta = \Delta\alpha/(\Delta k - 2\pi/\Lambda)$. Levels of constant gain are at the bottom plane mapped for G_+ (LCP, orange contours) and G_- (RCP, blue contours). [Filename: `python/graphs/graph-03-surf.eps`]

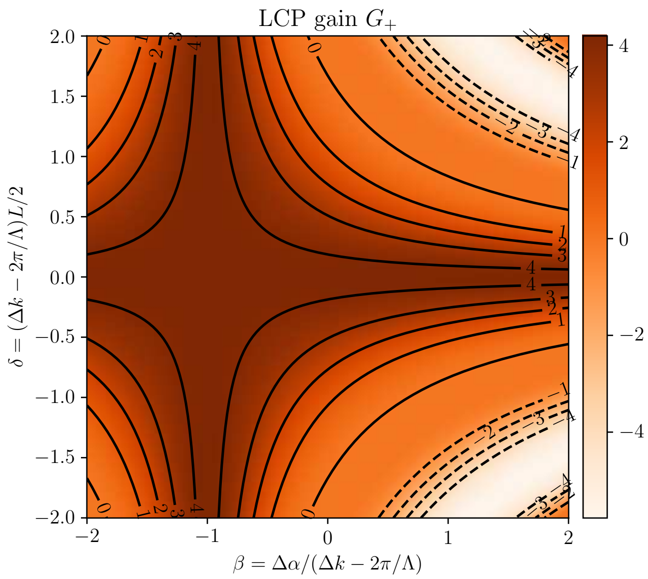

Figure 9. Constant levels of the signal gain G_+ (LCP), showing the dependence of the electric dipolar phase mismatch $\delta = (\Delta k - 2\pi/\Lambda)L/2$ relative the chiral contribution to the phase matching $\beta = \Delta\alpha/(\Delta k - 2\pi/\Lambda)$. [Filename: `python/graphs/graph-03-gplus-image.eps`]

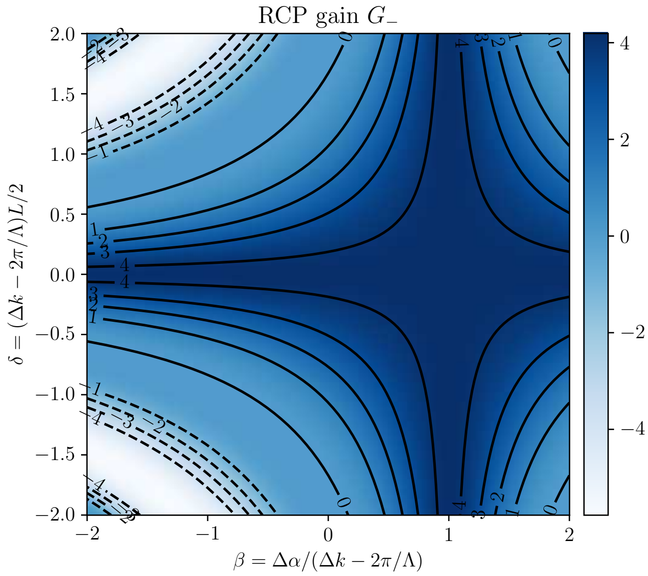

Figure 10. Same as Fig. 9 but for G_- (RCP), being antisymmetric to the LCP case with respect to the chirality mismatch term $\Delta\alpha$. [Filename: `python/graphs/graph-03-gminus-image.eps`]

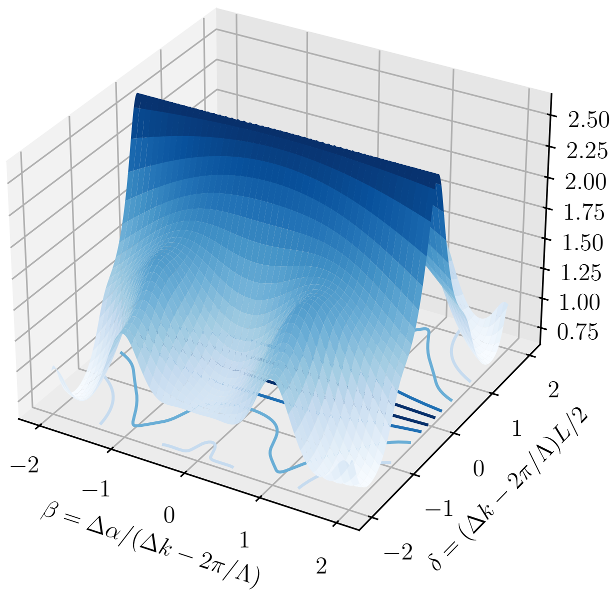

Figure 11. Map of the intensity gain $S_0(L)/S_0(0)$ vs electric dipolar phase mismatch $\delta = (\Delta k - 2\pi/\Lambda)L/2$ and the chiral contribution to the phase matching $\beta = \Delta\alpha/(\Delta k - 2\pi/\Lambda)$. The map is constructed for a pump intensity to threshold quote of $\eta = (\pi/2)^2 I_{\text{pump}}/I_{\text{th}} = 2.0$. Contours at constant intensity gain are mapped at the bottom plane. [Filename: `python/graphs/graph-04-s0-surface.eps`]

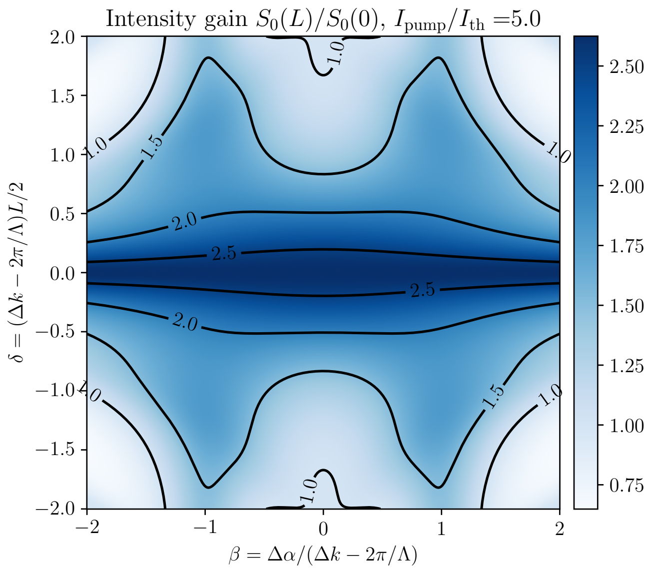

Figure 12. Map of the levels of constant intensity gain $S_0(L)/S_0(0)$, showing the complex interconnection between the electric dipolar phase mismatch $\delta = (\Delta k - 2\pi/\Lambda)L/2$ and the chiral contribution to the phase matching $\beta = \Delta\alpha/(\Delta k - 2\pi/\Lambda)$. The mapped contours at constant intensity gain are equal to those mapped in Fig. 8, for a pump intensity to threshold quote of $\eta = (\pi/2)^2 I_{\rm pump}/I_{\rm th} = 2.0$. [Filename: `python/graphs/graph-04-s0-5.00-image.eps`]

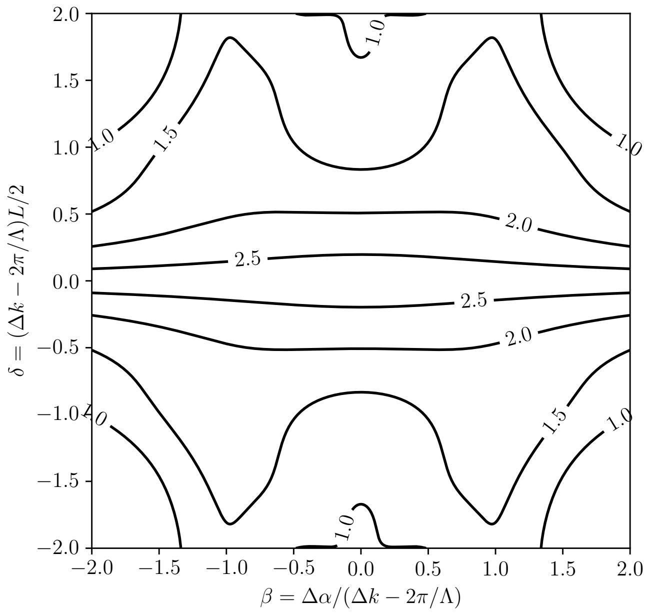

Figure 13. Identical to Fig. 12 but in plain black and white, possibly more suitable for printing in a journal. [Filename: `python/graphs/graph-04-s0-contour-5.00-black.eps`]

Signal ellipticity dependence on electric dipolar and chiral phase mismatch, pump intensity-to-threshold quote 2.0

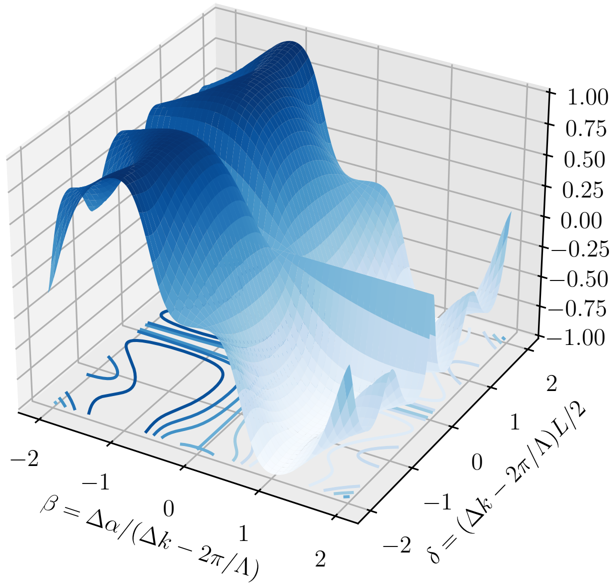

Figure 14. Map of the transmitted signal ellipticity $S_3(L)/S_0(L)$ vs electric dipolar phase mismatch $\delta = (\Delta k - 2\pi/\Lambda)L/2$ and the chiral contribution to the phase matching $\beta = \Delta\alpha/(\Delta k - 2\pi/\Lambda)$. The polarization state of the signal and pump at $z = 0$ were linearly polarized, and the map is constructed for a pump intensity to threshold quote of $\eta = (\pi/2)^2 I_{\text{pump}}/I_{\text{th}} = 2.0$. Contours at constant intensity gain are mapped at the bottom plane. [Filename: `python/graphs/graph-05-s3-2.00-surface.eps`]

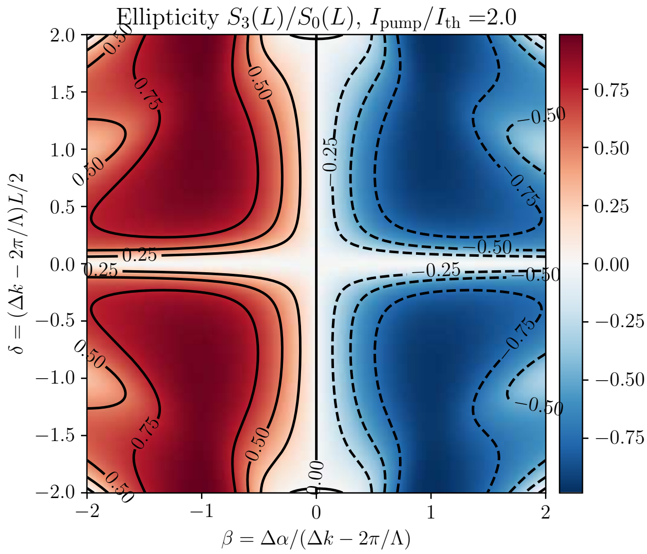

Figure 15. Map of levels of constant transmitted signal ellipticity $S_3(L)/S_0(L)$, showing the interconnection between electric dipolar phase mismatch $\delta = (\Delta k - 2\pi/\Lambda)L/2$ and the chiral contribution to the phase matching $\beta = \Delta\alpha/(\Delta k - 2\pi/\Lambda)$. The polarization state of the signal and pump at $z = 0$ were linearly polarized, and the map is constructed for a pump intensity to threshold quote of $\eta = (\pi/2)^2 I_{\text{pump}}/I_{\text{th}} = 2.0$. These contours are identical to the ones mapped at the bottom plane of Fig. 14. [Filename: `python/graphs/graph-05-s3-2.00-image.eps`]

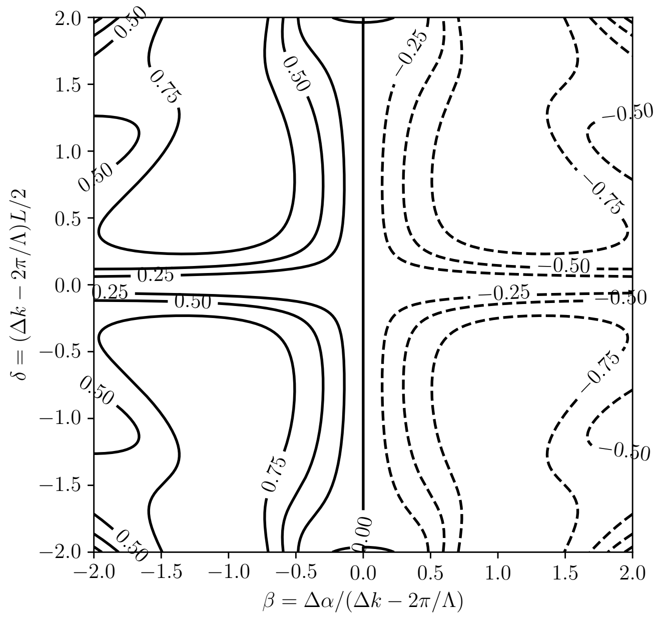

Figure 16. Levels identical to the ones shown in Fig. 15, but in plain black and white, suitable for printing. [Filename: `python/graphs/graph-05-s3-2.00-contour-black.eps`]

Signal ellipticity dependence on electric dipolar and chiral phase mismatch, pump intensity-to-threshold quote 5.0

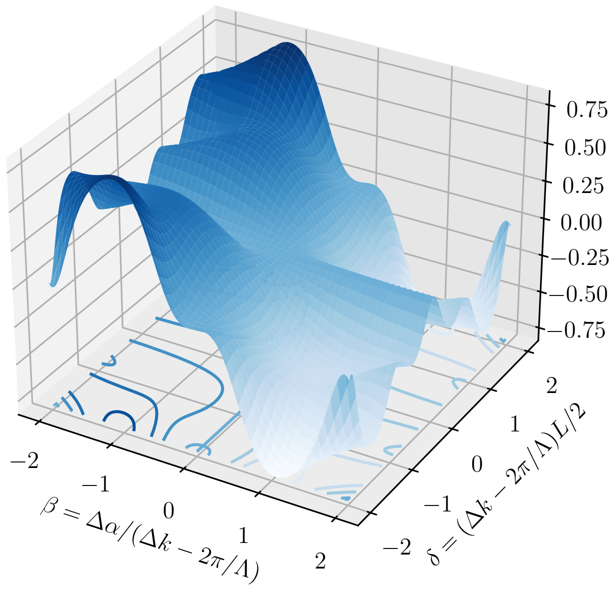

Figure 17. Map of the transmitted signal ellipticity $S_3(L)/S_0(L)$ vs electric dipolar phase mismatch $\delta = (\Delta k - 2\pi/\Lambda)L/2$ and the chiral contribution to the phase matching $\beta = \Delta\alpha/(\Delta k - 2\pi/\Lambda)$. The polarization state of the signal and pump at $z = 0$ were linearly polarized, and the map is constructed for a pump intensity to threshold quote of $\eta = (\pi/2)^2 I_{\text{pump}}/I_{\text{th}} = 3.0$. Contours at constant intensity gain are mapped at the bottom plane. [Filename: `python/graphs/graph-05-s3-5.00-surface.eps`]

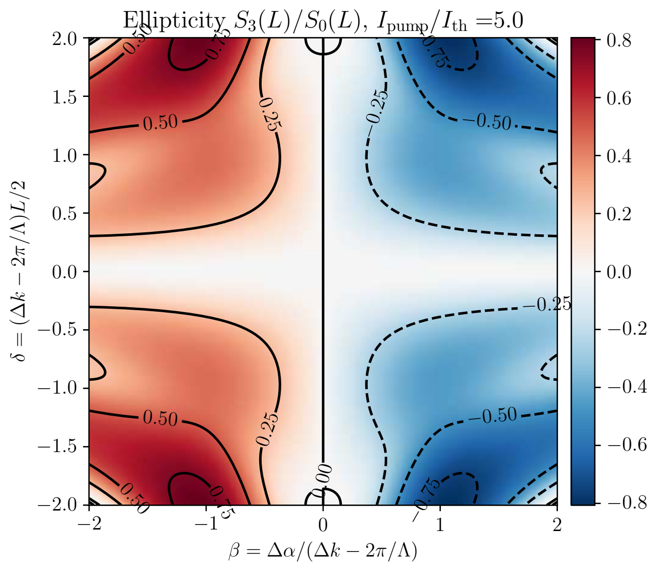

Figure 18. Map of levels of constant transmitted signal ellipticity $S_3(L)/S_0(L)$, showing the interconnection between electric dipolar phase mismatch $\delta = (\Delta k - 2\pi/\Lambda)L/2$ and the chiral contribution to the phase matching $\beta = \Delta\alpha/(\Delta k - 2\pi/\Lambda)$. The polarization state of the signal and pump at $z = 0$ were linearly polarized, and the map is constructed for a pump intensity to threshold quote of $\eta = (\pi/2)^2 I_{\text{pump}}/I_{\text{th}} = 2.0$. These contours are identical to the ones mapped at the bottom plane of Fig. 17. [Filename: `python/graphs/graph-05-s3-5.00-image.eps`]

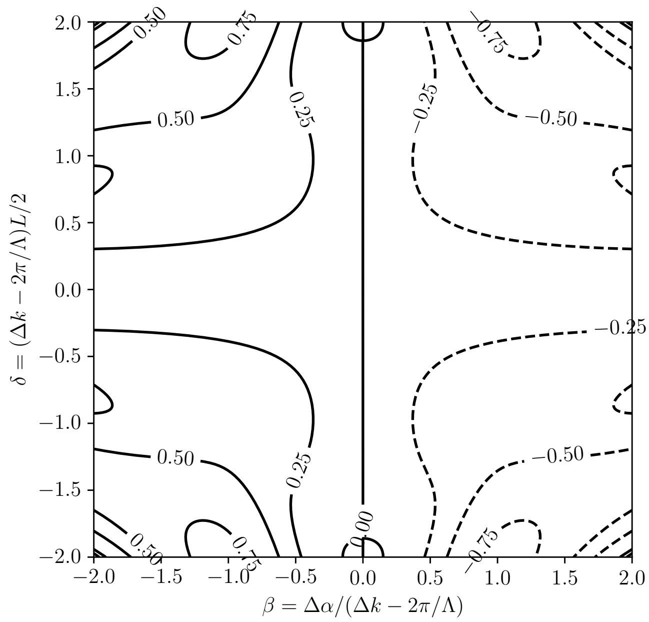

Figure 19. Levels identical to the ones shown in Fig. 18, but in plain black and white, suitable for printing. [Filename: `python/graphs/graph-05-s3-5.00-contour-black.eps`]