

4 HW #1

- Hardik Prayapati (2678294) 68

~~Pb.1. Let a point (x, y, z) be projected on a line given by (a, b, c) parameters. Then, point projected satisfies the given equation.~~

~~Please ORIT~~

~~$$ax + by + cz = 0$$~~

~~\Rightarrow Equation of plane, containing all points whose projections lie on a line given by parameters (a, b, c) is~~

~~$$ax + by + cz = 0.$$~~

Pb1. Let a point (x, y, z) be projected on a line by (a, b, c) parameters. Then, projected point satisfies

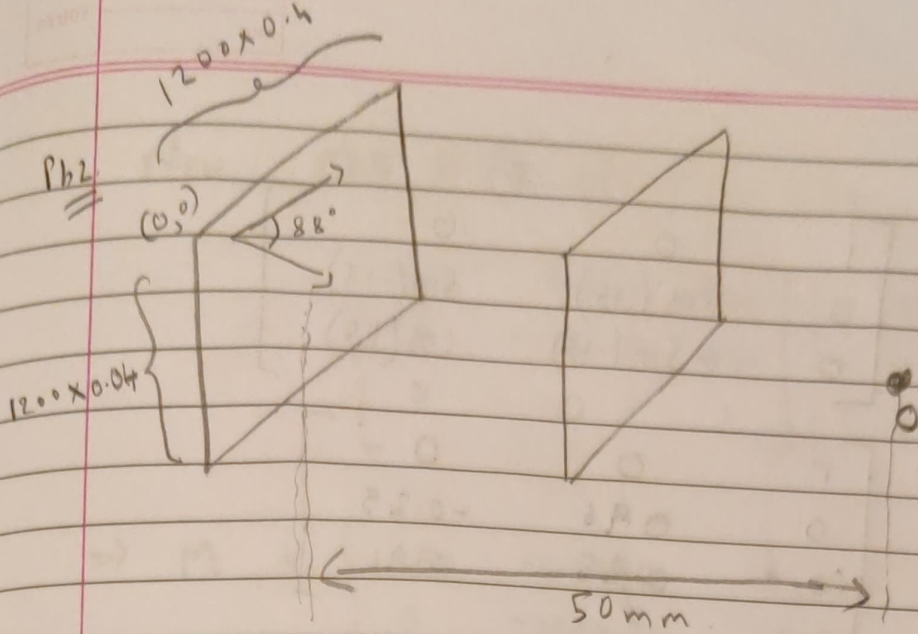
$$a \left\{ \alpha \frac{x}{z} - \alpha \cot \theta \frac{y}{z} + x_0 \right\} + b \left\{ \beta \frac{y}{z} + y_0 \right\} + c = 0$$

$$\Rightarrow x \left\{ \frac{a\alpha}{z} \right\} + y \left\{ -\frac{a\alpha \cot \theta}{z} + \frac{b\beta}{\sin \theta} \right\} + \{ax_0 + by_0 + c\} = 0$$

$$\Rightarrow x \{a\alpha\} + y \left\{ -a\alpha \cot \theta + \frac{b\beta}{\sin \theta} \right\} + z \{ax_0 + by_0 + c\} = 0$$

\uparrow (Given K , intrinsic parameters)

Equation of plane containing the 3-D line casting image on line $l(a, b, c)$



a) $p = K \hat{p}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & -\alpha \cot \theta & x_0 \\ 0 & \frac{\beta}{\sin \theta} & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ 1 \end{bmatrix}$$

$$x_0 = -600$$

$$y_0 = 600$$

$$\theta = -88^\circ$$

$$\alpha = \beta = \frac{50}{0.04} = 1250$$

$$\Rightarrow K = \begin{bmatrix} 1250 & -1250 \cot(-88) & -600 \\ 0 & \frac{1250}{\sin(-88)} & 600 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1250 & 43.65 & -600 \\ 0 & -1250.76 & 600 \\ 0 & 0 & 1 \end{bmatrix}$$

Pb2. b)

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-15) & \sin(-15) \\ 0 & -\sin(-15) & \cos(15) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.96 & -0.25 \\ 0 & 0.25 & 0.96 \end{bmatrix}$$

$$O_c = [4, -3, 2] \quad \leftarrow \text{In world coordinate system (meter)}$$

$$O_c = [0, 0, 0] \quad \leftarrow \text{In camera coordinate system}$$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} 4000 \\ -3000 \\ 2000 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_1 \\ 0 & 0.96 & -0.25 & t_2 \\ 0 & 0.25 & 0.96 & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4000 \\ -3000 \\ 2000 \\ 1 \end{bmatrix}$$

$$\Rightarrow 0 = 4000 + t_1 \quad \Rightarrow t_1 = -4000$$

$$\Rightarrow 0 = 0.96(-3000) - 0.25(2000) + t_2 \quad \Rightarrow t_2 = 3380$$

$$\Rightarrow 0 = 0.25(-3000) + 0.96(2000) + t_3 \quad \Rightarrow t_3 = -1170$$

$$\Rightarrow \vec{t} = (-4000, 3380, -1170)$$

Now, $M = K [R \ t]$

$$= \begin{bmatrix} 1250 & 43.65 & -600 \\ 0 & -1250.76 & 600 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -4000 \\ 0 & 0.96 & -0.25 & 3380 \\ 0 & 0.25 & 0.96 & -1170 \end{bmatrix}$$

$$\Rightarrow M = \begin{bmatrix} 1250 & -108.096 & -586.913 & -4150463 \\ 0 & -1050.73 & 888.69 & -4.92957 \times 10^6 \\ 0 & 0.25 & 0.96 & -1170 \end{bmatrix}$$

Pb.2 c)

Set of parallel lines (in $x_w - z_w$ plane)

→ The infinite point in direction of lines is given by $(x_0 \ 0 \ z_0 \ 0)^T$

⇒ Homogenous coordinates of vanishing point

$$= M P_w$$

$$= \begin{bmatrix} 1250 & -108.096 & -586.913 & -4150463 \\ 0 & -1050.73 & 888.69 & -4.92957 \times 10^6 \\ 0 & 0.25 & 0.96 & -1170 \end{bmatrix} \begin{bmatrix} x_0 \\ 0 \\ z_0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1250x_0 - 586.913z_0 \\ 888.69z_0 \\ 0.96z_0 \end{bmatrix}$$

⇒ Vanishing point = $\left[\frac{1250x_0 - 586.913z_0}{0.96z_0}, \frac{888.69z_0}{0.96z_0} \right]$
 in image coordinate

$$\Rightarrow \text{Vanishing point} = \left[\frac{1302 x_0}{z_0} - 611.36, 925.72 \right]$$

