HW-3

↳ Hardik Prajapati (2678294168)

1. $X_j | w_i \sim \text{Gamma}(\rho_i, \lambda_j)$

$$f_{X_j|w_i}(x_j|w_i) = \frac{1}{\Gamma(\rho_i)} \lambda_j^{\rho_i} x_j^{\rho_i - 1} e^{-\lambda_j x_j} \quad , \rho_i, \lambda_j > 0$$

\# of classes = c

\# of features = k $\rightarrow \{X_1, X_2 \ldots X_k\}$

a) Baye's optimal classifier's decision Rule:-

Decide w_i if $P(w_i|x) > P(w_r|x) \quad \forall r \neq i$

$$\Rightarrow \frac{f_{X_j|w_i}(x_j|w_i) P(w_i)}{f_{X_j}(x_j)} > \frac{f_{X_j|w_r}(x_j|w_r) P(w_r)}{f_{X_j}(x_j)}$$

$$\Rightarrow \frac{\frac{1}{\Gamma(\rho_i)} \lambda_j^{\rho_i} x_j^{\rho_i - 1} e^{-\lambda_j x_j}}{\frac{1}{\Gamma(\rho_r)} \lambda_j^{\rho_r} x_j^{\rho_r - 1} e^{-\lambda_j x_j}} > \frac{P(w_r)}{P(w_i)}$$

$$\Rightarrow \frac{\Gamma(\rho_r)}{\Gamma(\rho_i)} \lambda_j^{(\rho_i - \rho_r)} x_j^{(\rho_i - \rho_r)} > \frac{P(w_r)}{P(w_i)}$$

→ $\rho_r, \rho_i, \lambda_j, P(w_r), P(w_i)$ are known quantities & hence x_j^* can be calculated.

Q1 b) Decision boundaries are linear functions of $x_1, x_2, \cdots x_k$

↳ When features in each class are independent & have the same variance, Decision boundary is a hyperplane of $(k-1)$ dimensions

Hence, the decision boundary are linear function of $x_1, x_2, \cdots x_k$.

Q1. c) $\rho_1 = 4$, $\rho_2 = 2$, $C = 2$, $k = 4$, $\lambda_1 = \lambda_3 = 1$; $\lambda_2 = \lambda_4 = 2$.

$P(\omega_1) = P(\omega_2) = \frac{1}{2}$

$\vec{x} = (0.1, 0.2, 0.3, 4)$

$\Rightarrow$ Decide ω_1 if $P(\omega_1|\underline{x}) > P(\omega_2|\underline{x})$

$\Rightarrow f_{X|\omega_1}(\underline{x}|\omega_1)P(\omega_1) > f_{X|\omega_2}(\underline{x}|\omega_2)P(\omega_2)$

$$\Rightarrow \frac{f_{X_1|\omega_1}(x_1|\omega_1) \cdot f_{X_2|\omega_1}(x_2|\omega_1) \cdot f_{X_3|\omega_1}(x_3|\omega_1) \cdot f_{X_4|\omega_1}(x_4|\omega_1)}{f_{X_1|\omega_2}(x_1|\omega_2) \cdot f_{X_2|\omega_2}(x_2|\omega_2) \cdot f_{X_3|\omega_2}(x_3|\omega_2) \cdot f_{X_4|\omega_2}(x_4|\omega_2)} > \frac{P(\omega_2)}{P(\omega_1)}$$

$$\Rightarrow \frac{\left(\frac{1}{\Gamma(4)}(1)^4(0.1)^{4-1}e^{-(1)(0.1)}\right)\left(\frac{1}{\Gamma(4)}(2)^4(0.2)^{4-1}e^{-(2)(0.2)}\right) \times \left(\frac{1}{\Gamma(4)}(1)^4(0.3)^{4-1}e^{-(1)(0.3)}\right)\left(\frac{1}{\Gamma(4)}(2)^4(4)^{4-1}e^{-(2)(4)}\right)}{\left(\frac{1}{\Gamma(2)}(1)^2(0.1)^{2-1}e^{-(1)(0.1)}\right)\left(\frac{1}{\Gamma(2)}(2)^2(0.2)^{2-1}e^{-(2)(0.2)}\right) \times \left(\frac{1}{\Gamma(2)}(1)^2(0.3)^{2-1}e^{-(1)(0.3)}\right)\left(\frac{1}{\Gamma(2)}(2)^2(4)^{2-1}e^{-(2)(4)}\right)} \overset{?}{>} \frac{\frac{1}{2}}{\frac{1}{2}}$$

$$\Rightarrow \frac{\{\Gamma(2)\}^4(0.1)^2(2)^2(0.2)^2(0.3)^2(2)^2(4)^2}{\{\Gamma(4)\}^4} \overset{?}{>} 1$$

$$\Rightarrow \quad 0.000256 \overset{?}{>} 1$$

$\Rightarrow$ For $\vec{x} = (0.1, 0.2, 0.3, 4)$, $Y = \omega_2$

Q1.(d) $\rho_1 = 3.2$, $\rho_2 = 8$, $c = 2$, $k = 1$, $\lambda_1 = 1$

$P(\omega_1) = P(\omega_2) = \frac{1}{2}$

*

Decide ω_1 if $P(\omega_1 | \underline{x}) > P(\omega_2 | \underline{x})$

$$\Rightarrow f_{X|\omega_1}(\underline{x} | \omega_1) P(\omega_1) > f_{X|\omega_2}(\underline{x} | \omega_2) P(\omega_2)$$

$$\Rightarrow \frac{\frac{1}{\Gamma(3.2)} (1)^{3.2} (x)^{3.2-1} e^{-(1)(x)}}{\frac{1}{\Gamma(8)} (1)^{8} (x)^{8-1} e^{-(1)(x)}} > \frac{1/2}{1/2}$$

$$\Rightarrow x^{2.2-7} > \frac{2.42}{5040}$$

$$\Rightarrow 0 < x < 4.91336.$$

$$\Rightarrow \text{Decide} \begin{cases} \rightarrow \omega_1 & \text{if } 0 < x < 4.91336 \\ \rightarrow \omega_2 & \text{otherwise.} \end{cases}$$

Type 1 error:- Decide ω for its true label is ω_1

$$\Rightarrow P(\text{error} | x) = \begin{cases} P(\omega_1 | x) & \text{if we decide } \omega_2 \\ P(\omega_2 | x) & \text{if we decide } \omega_1 \end{cases}$$

$$p(x) = \sum_{j=1}^{2} p(x|\omega_j) P(\omega_j)$$

$$\Rightarrow p(x) = p(x|\omega_1)P(\omega_1) + p(x|\omega_2)P(\omega_2)$$

$$= \frac{1}{\Gamma(3.2)}(1)(x)^{2.2}e^{-x}\left(\frac{1}{2}\right) + \frac{1}{\Gamma(8)}(1)(x)^{7}e^{-x}\left(\frac{1}{2}\right)$$

$$= \frac{e^{-x}}{2}\left\{\frac{x^{2.2}}{\Gamma(3.2)} + \frac{x^{7}}{\Gamma(8)}\right\}$$

$$\Rightarrow P(\text{error}) = \int_{-\infty}^{\infty} P(\text{error}|x)\,p(x)\,dx$$

$$= \int_{-\infty}^{0} P(\omega_1|x)\,p(x)\,dx + \int_{0}^{4.9} P(\omega_2|x)\,p(x)\,dx$$

$$+ \int_{4.9}^{\infty} P(\omega_1|x)\,p(x)\,dx$$

$$= \int_{-\infty}^{0} \frac{p(x|\omega_1)P(\omega_1)}{p(x)}\,p(x)\,dx + \int_{0}^{4.9} \frac{p(x|\omega_2)P(\omega_2)}{p(x)}\,p(x)\,dx$$

$$+ \int_{4.9}^{\infty} \frac{p(x|\omega_1)P(\omega_1)}{p(x)}\,p(x)\,dx$$

$$= \int_{-\infty}^{0} \frac{1}{\Gamma(3.2)}(1)(x)^{2.2}e^{-x}\left(\frac{1}{2}\right)dx + \int_{0}^{4.9} \frac{1}{\Gamma(8)}(1)(x)^{7}e^{-x}\left(\frac{1}{2}\right)dx$$

$$+ \int_{4.9}^{\infty} \frac{1}{\Gamma(3.2)}(1)(x)^{2.2}e^{-x}\left(\frac{1}{2}\right)dx$$

Q1. e.

$$p_1 = p_2 = 4, \quad c = 2, \quad k = 2, \quad \lambda_1 = 8, \quad \lambda_2 = 0.3$$

$$P(\omega_1) = \frac{1}{4}, \quad P(\omega_2) = \frac{3}{4}$$

Decide ω_1 if $P(\omega_1 | \underline{x}) > P(\omega_2 | \underline{x})$

$$\Rightarrow \frac{f_{X|\omega_1}(\underline{x}|\omega_1) P(\omega_1)}{f_X(\underline{x})} \overset{?}{>} \frac{f_{X|\omega_2}(\underline{x}|\omega_2) P(\omega_2)}{f_X(\underline{x})}$$

$$\Rightarrow \frac{\left(\frac{1}{\Gamma(4)} (8)^4 (x_1)^{4-1} e^{-8x_1}\right)\left(\frac{1}{\Gamma(4)} (0.3)^4 (x_2)^{4-1} e^{-0.3x_2}\right)}{\left(\frac{1}{\Gamma(4)} (8)^4 (x_1)^{4-1} e^{-8x_1}\right)\left(\frac{1}{\Gamma(4)} (0.3)^4 (x_2)^{4-1} e^{-0.3x_2}\right)} \overset{?}{>} \frac{\left(\frac{3}{4}\right)}{\left(\frac{1}{4}\right)}$$

$$\Rightarrow 1 \overset{?}{>} 3$$

$\Rightarrow$ It will always decide ω_2 irrespective of feature values (x_1, x_2).

Q2.

$$x_i | w_j \sim Lap(m_{ij}, \lambda_i)$$

$$P_{x_i|w_j}(x_i|w_j) = \frac{\lambda_i}{2} e^{-\lambda_i |x_i - m_{ij}|} \qquad , \lambda_i > 0$$

$$P(w_j) = \frac{1}{c} \qquad \begin{array}{l} i \in \{1, 2, \cdots k\} \\ j \in \{1, \cdots c\} \end{array}$$

Minimum Error rate classifier:-

Decide w_i if $\quad P(w_i|\underline{x}) > P(w_j|\underline{x}) \quad \forall\, i \neq j$

$$\Rightarrow P_{x_1|w_i}(x_1|w_i) \cdot P_{x_2|w_i}(x_2|w_i) \cdots P_{x_k|w_i}(x_k|w_i) \cdot \cancel{P(w_i)}^{1/c}$$

$$\overset{w_i}{>} \qquad \forall\, j \neq i$$

$$P_{x_1|w_j}(x_1|w_j) \cdot P_{x_2|w_j}(x_2|w_j) \cdots P_{x_k|w_j}(x_k|w_j) \cdot \cancel{P(w_j)}^{1/c}$$

$$\Rightarrow \frac{\lambda_1}{2} e^{-\lambda_1 |x_1 - m_{1i}|} \cdot \frac{\lambda_2}{2} e^{-\lambda_2 |x_2 - m_{2i}|} \cdots \frac{\lambda_k}{2} e^{-\lambda_k |x_k - m_{ki}|}$$

$$\overset{w_i}{>} \qquad \forall\, j \neq i$$

$$\frac{\lambda_1}{2} e^{-\lambda_1 |x_1 - m_{1j}|} \cdot \frac{\lambda_2}{2} e^{-\lambda_2 |x_2 - m_{2j}|} \cdots \frac{\lambda_k}{2} e^{-\lambda_k |x_k - m_{ki}|}$$

$$\Rightarrow e^{\sum_{b=1}^{k} -\lambda_b |x_b - m_{bi}|} \overset{\omega_i}{>} e^{\sum_{b=1}^{k} -\lambda_b |x_b - m_{bj}|} \quad \forall j \neq i$$

Taking natural log on both sides

$$\Rightarrow \sum_{b=1}^{k} -\lambda_b |x_b - m_{bi}| \overset{\omega_i}{>} \sum_{b=1}^{k} -\lambda_b |x_b - m_{bj}| \quad \forall_j \neq i$$

$$\Rightarrow \sum_{b=1}^{k} \lambda_b |x_b - m_{bi}| \overset{\omega_i}{<} \sum_{b=1}^{k} \lambda_b |x_b - m_{bj}| \quad \forall_j \neq i$$

$\Rightarrow$ Hence, it can be seen that Minimum Error Rate Classifier is also a minimum weighted Manhattan distance (or weighted L_1-distance) classifier.

If $\lambda_b = 1 \quad \forall \ b \in \{1, \dots k\}$

Then,

$$\sum_{b=1}^{k} |x_b - m_{bi}| \overset{\omega_i}{<} \sum_{b=1}^{k} |x_b - m_{bj}| \quad \forall j \neq i$$

becomes the minimum Manhattan distance classifier.

Q3.

x	$p(x\|\omega_1)$	$p(x\|\omega_2)$	$p(x\|\omega_3)$	$p(x\|\omega_4)$
1	$1/3$	$1/2$	$1/6$	$2/5$
2	$1/3$	$1/4$	$1/3$	$2/5$
3	$1/3$	$1/4$	$1/2$	$1/5$

	ω_1	ω_2	ω_3	ω_4
α_1	0	2	3	4
α_2	1	0	1	8
α_3	3	2	0	2
α_4	5	3	1	0

$\alpha_i \rightarrow$ Decide Pattern ω_i (class)

$P(\omega_1) = \frac{1}{10}$, $P(\omega_2) = \frac{1}{5}$, $P(\omega_3) = \frac{1}{2}$, $P(\omega_4) = \frac{1}{5}$

a) $R(\alpha_i|x) = \sum_{j=1}^{4} \lambda(\alpha_i|\omega_j) p(\omega_j|x) = \sum_{j=1}^{4} \lambda(\alpha_i|\omega_j) \frac{p(x|\omega_j)P(\omega_j)}{p(x)}$

$\rightarrow R(\alpha_1|x=1) = \frac{(0)(\frac{1}{3})(\frac{1}{10}) + (2)(\frac{1}{2})(\frac{1}{5}) + (3)(\frac{1}{6})(\frac{1}{2}) + (4)(\frac{2}{5})(\frac{1}{5})}{(\frac{1}{3})(\frac{1}{10}) + (\frac{1}{2})(\frac{1}{5}) + (\frac{1}{6})(\frac{1}{2}) + (\frac{2}{5})(\frac{1}{5})}$

$= \frac{231}{89}$

$= 2.59$

$$\rightarrow R(\alpha_1|x=2) = \frac{(0)(\frac{1}{3})(\frac{1}{10}) + (2)(\frac{1}{4})(\frac{1}{5}) + (3)(\frac{1}{3})(\frac{1}{2}) + (4)(\frac{2}{3})(\frac{1}{5})}{(\frac{1}{3})(\frac{1}{10}) + (\frac{1}{4})(\frac{1}{5}) + (\frac{1}{3})(\frac{1}{2}) + (\frac{2}{3})(\frac{1}{5})}$$

$$= \frac{92}{33}$$

$$= 2.78$$

$$\rightarrow R(\alpha_1|x=3) = \frac{(0)(\frac{1}{3})(\frac{1}{10}) + (2)(\frac{1}{4})(\frac{1}{5}) + (3)(\frac{1}{2})(\frac{1}{2}) + (4)(\frac{1}{5})(\frac{1}{5})}{(\frac{1}{3})(\frac{1}{10}) + (\frac{1}{4})(\frac{1}{5}) + (\frac{1}{2})(\frac{1}{2}) + (\frac{1}{5})(\frac{1}{5})}$$

$$= \frac{303}{112}$$

$$= 2.705$$

$$\rightarrow R(\alpha_2|x=1) = \frac{(1)(\frac{1}{3})(\frac{1}{10}) + (0)(\frac{1}{2})(\frac{1}{5}) + (1)(\frac{1}{6})(\frac{1}{2}) + (8)(\frac{2}{5})(\frac{1}{5})}{(\frac{1}{3})(\frac{1}{10}) + (\frac{1}{2})(\frac{1}{5}) + (\frac{1}{6})(\frac{1}{2}) + (\frac{2}{5})(\frac{1}{5})}$$

$$= \frac{227}{889}$$

$$= 2.55$$

$$\rightarrow R(\alpha_2|x=2) = \frac{(1)(\frac{1}{3})(\frac{1}{10}) + (0)(\frac{1}{4})(\frac{1}{5}) + (1)(\frac{1}{3})(\frac{1}{2}) + (8)(\frac{2}{5})(\frac{1}{5})}{(\frac{1}{3})(\frac{1}{10}) + (\frac{1}{4})(\frac{1}{5}) + (\frac{1}{3})(\frac{1}{2}) + (\frac{2}{5})(\frac{1}{5})}$$

$$= \frac{28}{11}$$

$$= 2.54$$

$$\rightarrow R(\alpha_2 \mid x=3) = \frac{(1)\left(\frac{1}{3}\right)\left(\frac{1}{10}\right) + (0)\left(\frac{1}{4}\right)\left(\frac{1}{5}\right) + (1)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + (8)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)}{\left(\frac{1}{3}\right)\left(\frac{1}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)}$$

$$= \frac{181}{112}$$

$$= 1.616$$

$$\rightarrow R(\alpha_3 \mid x=1) = \frac{(3)\left(\frac{1}{3}\right)\left(\frac{1}{10}\right) + (2)\left(\frac{1}{2}\right)\left(\frac{1}{5}\right) + (0)\left(\frac{1}{6}\right)\left(\frac{1}{2}\right) + (2)\left(\frac{2}{5}\right)\left(\frac{1}{5}\right)}{\left(\frac{1}{3}\right)\left(\frac{1}{10}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{5}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{5}\right)\left(\frac{1}{5}\right)}$$

$$= \frac{138}{89}$$

$$= 1.55$$

$$\rightarrow R(\alpha_3 \mid x=2) = \frac{12}{11}$$

$$= 1.09$$

$$\rightarrow R(\alpha_3 \mid x=3) = \frac{3}{4}$$

$$= 0.75$$

$$\rightarrow R(\alpha_4 \mid x=1) = \frac{(5)\left(\frac{1}{3}\right)\left(\frac{1}{10}\right) + (3)\left(\frac{1}{2}\right)\left(\frac{1}{5}\right) + (1)\left(\frac{1}{6}\right)\left(\frac{1}{2}\right) + (0)\left(\frac{2}{5}\right)\left(\frac{1}{5}\right)}{\left(\frac{1}{3}\right)\left(\frac{1}{10}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{5}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{5}\right)\left(\frac{1}{5}\right)}$$

$$= \frac{165}{89}$$

$$= 1.85$$

$\rightarrow R(\alpha_4 | X=2) = \frac{145}{99}$

$= 1.46$

$\rightarrow R(\alpha_4 | X=3) = \frac{265}{224}$

$= 1.18$

Q3. b) $R = \sum_{i=1}^{3} R(\alpha(x_i) | x_i) p(x_i)$

$= R(\alpha_3(x_1) | x_1) p(x_1) + R(\alpha_3(x_2) | x_2) p(x_2)$
$+ R(\alpha_3(x_3) | x_3) p(x_3)$

$= (1.55)\left(\frac{89}{300}\right) + (1.09)\left(\frac{33}{100}\right) + (0.75)\left(\frac{28}{75}\right)$

$= 1.099$

Q4.

Tax ID	Refund	Marital Status	Taxable income	Evade
1	Yes	Single	122K	No
2	No	Married	77K	No
3	No	Married	106K	No
4	No	Single	88 K	Yes
5	Yes	Divorced	210 K	No
6	No	Single	72 K	No
7	Yes	Married	117 K	No
8	No	Married	60 K	No
9	No	Divorced	90 K	Yes
10	No	Single	85 K	Yes

a) $\omega_1 = \text{Yes}$, $\omega_2 = \text{No}$

$P(\omega_1) = \frac{3}{10}$ $\quad P(\omega_2) = \frac{7}{10}$ ← Prior class probabilities

b) X_1 - Taxable Income (Continuous R.V).
Assuming Conditional Gaussianity

For $\omega_1 = \text{Yes}$,

$$\bar{x}_1 = \frac{88 + 90 + 85}{3}$$
$$= 87.67$$

$$\sigma_1^2 = \frac{(88 - 87.67)^2 + (90 - 87.67)^2 + (85 - 87.67)^2}{3 - 1}$$
$$= 6.33$$

$$\Rightarrow p(\underline{x} | \omega_1) \rightarrow N(87.67, 6.33)$$

For $\omega_2 = No$,

$$\bar{x}_2 = \frac{122 + 77 + 106 + 210 + 72 + 117 + 60}{7}$$

$$= 109.14$$

$$\sigma_2^2 = \frac{\sum_{i=1}^{7} (x_i - \bar{x}_2)^2}{n-1} \quad , n = 7$$

$$= 2539.48$$

$$\Rightarrow p(\underline{x} | \omega_2) \rightarrow N(109.14, 2539.48)$$

Q4. c) $X_2 \rightarrow$ Refund (Discrete R.V)

For $\omega_1 = Yes$,

$$P_{x|\omega_i}(x_j = Yes | \omega_i = Yes) = \frac{0}{3} = 0$$

$$P_{x|\omega_i}(x_j = No | \omega_i = Yes) = \frac{3}{3} = 1.$$

For $\omega_2 = No$,

$$P_{x|\omega_i}(x_j = Yes | \omega_i = No) = \frac{3}{7}$$

$$P_{x|\omega_i}(x_j = No | \omega_i = No) = \frac{4}{7}$$

$X_3 \rightarrow$ Marital Status (Discrete R.V).

For $\omega_1 = $ Yes,

$$P_{x|\omega_i}(x_j = \text{Single} \mid \omega_i = \text{Yes}) = \frac{2}{3}$$

$$P_{x|\omega_i}(x_j = \text{Married} \mid \omega_i = \text{Yes}) = \frac{0}{3}$$

$$P_{x|\omega_i}(x_j = \text{Divorced} \mid \omega_i = \text{Yes}) = \frac{1}{3}$$

For $\omega_2 = $ No,

$$P_{x|\omega_i}(x_j = \text{Single} \mid \omega_i = \text{No}) = \frac{2}{7}$$

$$P_{x|\omega_i}(x_j = \text{Married} \mid \omega_i = \text{No}) = \frac{4}{7}$$

$$P_{x|\omega_i}(x_j = \text{Divorced} \mid \omega_i = \text{No}) = \frac{1}{7}$$

$\rightarrow$ This is not entirely a valid estimate of PMF because it gives us a probability 0 for certain labels which actually exist. Instead there should be a small non-zero probability assigned to such labels

Q4 d. For $\omega_1 = \text{Yes}$, $X_2 \to$ Refund

$$P_{X_2|\omega_i}(x_j = \text{Yes} \mid \omega_i = \text{Yes}) = \frac{1}{3+2} = \frac{1}{5}$$

$$P_{X_2|\omega_i}(x_j = \text{No} \mid \omega_i = \text{Yes}) = \frac{3+1}{3+2} = \frac{4}{5}$$

For $\omega_1 = \text{No}$,

$$P_{X_2|\omega_i}(x_j = \text{Yes} \mid \omega_i = \text{No}) = \frac{3+1}{7+2} = \frac{4}{9}$$

$$P_{X_2|\omega_i}(x_j = \text{No} \mid \omega_i = \text{No}) = \frac{4+1}{7+2} = \frac{5}{9}$$

$X_3 \to$ Marital Status

For $\omega_1 = \text{Yes}$,

$$P_{X_3|\omega_i}(x_j = \text{Single} \mid \omega_i = \text{Yes}) = \frac{2+1}{3+3} = \frac{3}{6}$$

$$P_{X_3|\omega_i}(x_j = \text{Married} \mid \omega_i = \text{Yes}) = \frac{0+1}{3+3} = \frac{1}{6}$$

$$P_{X_3|\omega_i}(x_j = \text{Divorced} \mid \omega_i = \text{Yes}) = \frac{1+1}{3+3} = \frac{2}{6}$$

For $\omega_2 = \text{No}$,

$$P_{X_3|\omega_i}(x_j = \text{Single} \mid \omega_i = \text{No}) = \frac{2+1}{7+3} = \frac{3}{10}$$

$$P_{X_3|\omega_i}(x_j = \text{Married} \mid \omega_i = \text{No}) = \frac{4+1}{7+3} = \frac{5}{10}$$

$$P_{X_3|\omega_i}(x_j = \text{Divorced} \mid \omega_i = \text{No}) = \frac{1+1}{7+3} = \frac{2}{10}$$

→ This is relatively more valid Estimate of PMF
→

e.g. Let $\vec{x} = \{x_1 = 100K, x_2 = Yes, x_3 = Single\}$

Then, $P(Y = Yes \mid \vec{x}) = \dfrac{P(\vec{x} \mid Yes) \cdot P(Yes)}{P(x)}$

Checking the Numerator $\Rightarrow \left\{\dfrac{1}{\sqrt{2\pi}(\sqrt{6.33})} e^{-\frac{(100-87.67)^2}{(2)(6.33)}}\right\}\{0\}\left\{\dfrac{2}{3}\right\}$

$= 0.$

⇒ This creates a problem which gets solved by Laplace smoothing.

Qn. e) Decide ω_1 if $P(\omega_1 \mid \underline{x}) > P(\omega_2 \mid x)$

$$\Rightarrow \frac{P(\underline{x} \mid \omega_1) P(\omega_1)}{\cancel{P(\underline{x})}} > \frac{P(\underline{x} \mid \omega_2) P(\omega_2)}{\cancel{P(\underline{x})}}$$

$$\Rightarrow \frac{f_{x_1|\omega_1}(x_1 \mid \omega_1) \cdot f_{x_2|\omega_1}(x_2 \mid \omega_1)\, f_{x_3|\omega_1}(x_3 \mid \omega_1)}{f_{x_1|\omega_2}(x_1 \mid \omega_2)\, f_{x_2|\omega_2}(x_2 \mid \omega_2) \cdot f_{x_3|\omega_2}(x_3 \mid \omega_2)} \overset{\omega_1}{>} \frac{P(\omega_2)}{P(\omega_1)}$$

$$\Rightarrow \frac{\left\{\frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sqrt{6.33}} e^{-\frac{(x_1-87.67)^2}{(2)(6.33)}}\right\} P_{x_2|\omega_1}(x_2 \mid \omega_1)\, P_{x_3|\omega_1}(x_3 \mid \omega_1)}{\left\{\frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sqrt{2539.48}} e^{-\frac{(x_1-109.14)^2}{(2)(2539.48)}}\right\} P_{x_2|\omega_2}(x_2 \mid \omega_2)\, P_{x_3|\omega_2}(x_3 \mid \omega_2)} \overset{\omega_1}{>} \frac{7/10}{3/10}$$