H.W 7.

↳ Hardik Prajapati (2678294168)

1. To prove:- $w_0 = y_i - \varepsilon - \sum_{i \in S} (\lambda_i^+ - \lambda_i^-) K(x_i, x_j)$ where x_j is a Support Vector

Now, According to KKT condition,

$$\lambda_i^+ (\varepsilon + \xi_j^+ + \hat{y}_i - y_i) = 0$$

Vectors that are support vectors, $\lambda_i^+ \neq 0$

$$\Rightarrow \varepsilon + \xi_j^+ + \hat{y}_i - y_i = 0$$

Now, by optimizing Lagrange optimizer, we get

$$W = \sum_{i=1}^{N} (\lambda_i^+ - \lambda_i^-) \phi(x_i)$$

Also, $\hat{y}_i = \omega^T \phi(x_i) + w_0$

$$\Rightarrow \varepsilon + \omega^T \phi(x_i) + \omega_0 - y_i = 0$$

$$\Rightarrow \omega_0 = y_i - \varepsilon - \omega^T \phi(x_i)$$

$$\Rightarrow \omega_0 = y_i - \varepsilon - \left(\sum_{i \in S} (\lambda_i^+ - \lambda_i^-) \phi(x_i) \right)^T \phi(x_j)$$

$$\Rightarrow \omega_0 = y_i - \varepsilon - \sum_{i \in S} (\lambda_i^+ - \lambda_i^-) \underbrace{\phi(x_i)^T \phi(x_j)}_{K(x_i, x_j)}$$

$$\Rightarrow \omega_0 = y_i - \varepsilon - \sum_{i \in S} (\lambda_i^+ - \lambda_i^-) K(x_i, x_j)$$

Hence proved.

2.

$D_{\omega_1} = \left\{ x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

$D_{\omega_2} = \left\{ x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

$D_{\omega_3} = \left\{ x_3 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\}$

Encoding $\omega_1, \omega_2, \omega_3$:-

$y_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ $y_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ $y_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

$n(i) = 0.5$

a) $W(0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$W(i+1) = W(i) + n(i)\, x(i)\, e^T$

$g(x) = w^T x + \cancel{w_0}^{\,0}$

$\Rightarrow g(0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow e(0) = y_1 - g(0) = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

$\Rightarrow w(1) = W(0) + n(i)\, x(0)\, e^T$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + 0.5 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}^T$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + 0.5 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 \end{bmatrix}$

$$\therefore w(1) = \begin{bmatrix} 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 \end{bmatrix}$$

Now, $f(w^T(1)\,x(1))$

$$= \text{Sign}\left(\begin{bmatrix} 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$= \text{Sign}\left(\begin{bmatrix} 0.5 & 0.5 \\ -0.5 & -0.5 \\ -0.5 & -0.5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$= \text{Sign}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$e(1) = y_2 - g(1) = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$w(2) = w(1) + n(i)\,x(1)\,e^T$$

$$= \begin{bmatrix} 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 \end{bmatrix} + 0.5 \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 \end{bmatrix} + 0.5 \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow w(2) = \begin{bmatrix} 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 \end{bmatrix} + \begin{bmatrix} -0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

Now, $f\left(w^T(2)\,x(2)\right)$

$$= \text{Sign}\left(\begin{bmatrix} 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}^T \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right)$$

$$= \text{Sign}\left(\begin{bmatrix} 0 & 1 \\ 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right)$$

$$= \text{Sign}\left(\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$e(2) = y_3 - g(2) = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$$

$$w(3) = w(2) + \eta(i)\,x(2)\,e^T$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} + 0.5 \begin{bmatrix} -1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}^T$$

$$\Rightarrow W(3) = \begin{bmatrix} 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} + 0.5 \begin{bmatrix} -1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Q2. b.

$$W(0) = \begin{bmatrix} w_1(0) & w_2(0) & w_3(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$w_1(1) = w_1(0) + \eta e_1 x(1)$$

$$e_i = y_{i1} - \text{Sign}(w_i(0)^T x(1))$$

$$\Rightarrow e_1 = y_{11} - \text{Sign}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$

$$= 1 - \text{Sign}\left(\begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$

$$= 1 - \text{Sign}(0)$$

$$= 1 - 0$$

$$= 1$$

$$\Rightarrow e_2 = y_{21} - \text{Sign}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$

$$= -1 - \text{Sign}(0)$$

$$= -1$$

$$\Rightarrow e_3 = y_{31} - \text{Sign}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$$
$$= -1 - \text{Sign}(0)$$
$$= -1.$$

$$\Rightarrow w_1(1) = w_1(0) + \eta e_1 x(1)$$
$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + (0.5)(1)\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$\Rightarrow w_2(1) = w_2(0) + \eta e_2 x(1)$$
$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + (0.5)(-1)\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$$

$$\Rightarrow w_3(1) = \hat{w}_3(0) + \eta e_3 x(1)$$
$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + (0.5)(-1)\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$$

$$\Rightarrow w(1) = \begin{bmatrix} 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 \end{bmatrix}$$ ← Iteration 1 Epoch #1

Epoch 1, Iteration #2. $y_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ $x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$e_1 = -1 - \text{Sign}\left(\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$

$$= -1 - \text{Sign}\left(\begin{bmatrix} 0 \end{bmatrix}\right)$$

$$= -1$$

$$e_2 = 1 - \text{Sign}\left(\begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$

$$= 1 - \text{Sign}(0)$$

$$= 1$$

$$e_3 = -1 - \text{Sign}\left(\begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$

$$= -1 - \text{Sign}(0)$$

$$= -1$$

$$\Rightarrow w_1(2) = w_1(1) + \eta e_1 x(2)$$

$$= \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} + (0.5)(-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow w_2(2) = w_2(1) + \eta e_2 x(2)$$

$$= \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix} + (0.5)(1)\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\Rightarrow w_3(2) = w_3(1) + \eta e_3 x(2)$$

$$= \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix} + (0.5)(-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\Rightarrow W(2) = \begin{bmatrix} 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

Epoch 1, Iteration # 3. $y_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ $x_3 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$$e_1 = -1 - \text{Sign}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \begin{bmatrix} -1 \\ -1 \end{bmatrix}\right)$$

$$= -1 - \text{Sign}(-1)$$

$$= -1 + (-1)$$

$$= 0.$$

$$e_2 = -1 - \text{Sign}\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}^T \begin{bmatrix} -1 \\ -1 \end{bmatrix}\right)$$

$$= -1 - \text{Sign}(1)$$

$$= -2.$$

$$e_3 = 1 - \text{Sign}\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}^T \begin{bmatrix} -1 \\ -1 \end{bmatrix}\right)$$

$$= 1 - \text{Sign}(1)$$

$$= 0.$$

$$\Rightarrow w_1(3) = w_1(2) + \eta e_1 x(3)$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + (0.5)(0)\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow w_2(3) = w_2(2) + \eta e_2 x(3)$$

$$= \begin{bmatrix} 0 \\ -1 \end{bmatrix} + (0.5)(-2)\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow w_3(3) = w_3(2) + \eta e_3 x(3)$$

$$= \begin{bmatrix} -1 \\ 0 \end{bmatrix} + (0.5)(0)\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\Rightarrow W(3) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Epoch 2, iteration 1. $y = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$e_1 = 1 - \text{Sign}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$
$$= 0$$

$$e_2 = -1 - \text{Sign}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$
$$= -2$$

$$e_3 = -1 - \text{Sign}\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$
$$= 0$$

$$w_1(4) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + (0.5)(0) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$w_2(4) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (0.5)(-2) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$w_3(4) = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + (0.5)(0) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$W(4) = \begin{bmatrix} 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

Epoch 2, Iteration 2. $y = \begin{bmatrix} -1 \\ +1 \\ -1 \end{bmatrix}$ $x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$e_1 = -1 - \text{Sign}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$
$$= 0.$$

$$e_2 = +1 - \text{Sign}\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$
$$= 0$$

$$e_3 = -1 - \text{Sign}\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$
$$= 0.$$

$$w_1(5) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + (0.5)(0)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$w_2(5) = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + 0 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$w_3(5) = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + 0 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\Rightarrow W(5) = \begin{bmatrix} 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

Epoch 2, iteration 3 $\quad y = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad x_3 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$$e_1 = -1 - \text{Sign}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right)$$
$$= 0$$

$$e_2 = -1 - \text{Sign}\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}^T \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right)$$
$$= -2.$$

$$e_3 = 1 - \text{Sign}\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}^T \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right)$$
$$= 0$$

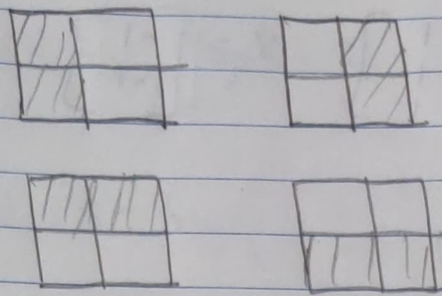
$$W_1(6) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + (0.5)(0)\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$W_2(6) = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + (0.5)(-2)\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$W_3(6) = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + (0.5)(0)\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\Rightarrow W(6) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Q3.



Class I

Class II

Let,

x_1	x_2
x_3	x_4

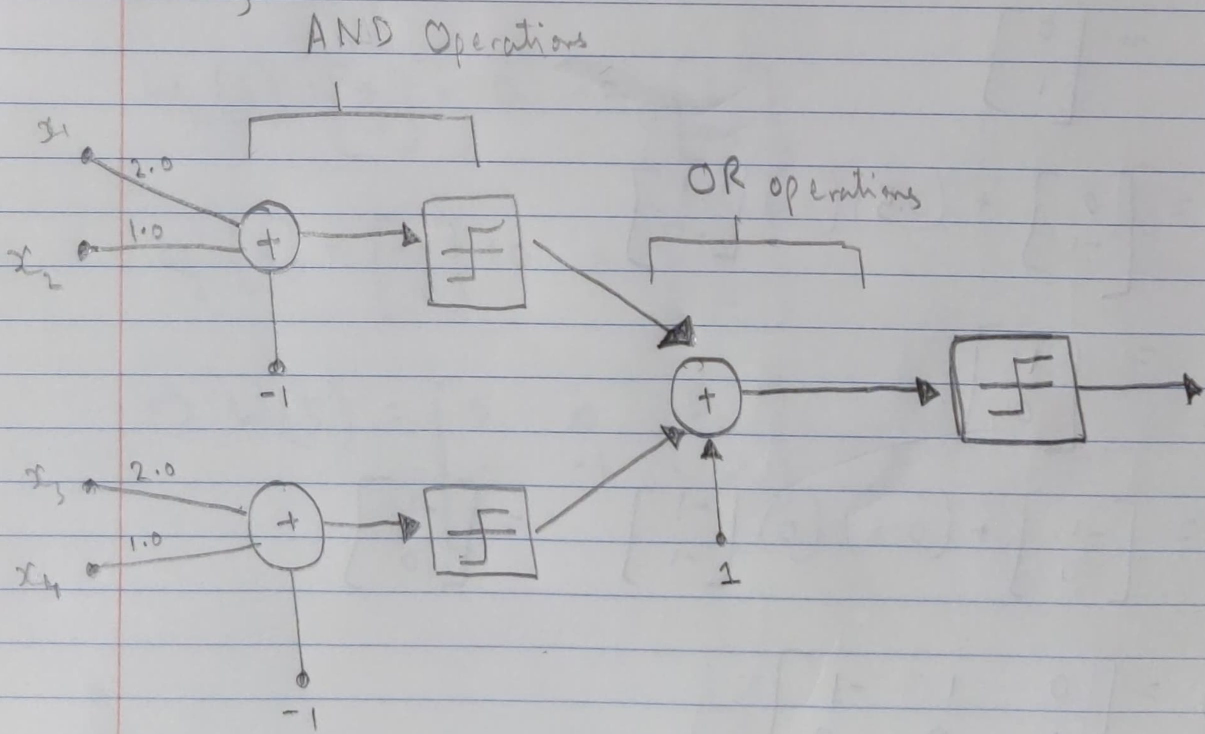
& Class I = −1 &  = 1 (Shaded)

Class II = +1  = 0 (Unshaded)

Then,

x_1	x_2	x_3	x_4	Y
1	0	1	0	−1
0	1	0	1	−1
1	1	0	0	1
0	0	1	1	1

Hence,



Illustrations :-

Case (i)

x_1	x_2	x_3	x_4
1	0	1	0

$$\text{Output} = \left[\underbrace{\{(2.0)\,x_1\}\{(1.0)\,x_2\}}_{\text{AND operation}} + (-1) \right] \underbrace{+}_{\text{OR operation}} \left[\underbrace{\{(2.0)\,x_3\}\{(1.0)\,x_4\}}_{\text{AND operation}} + (-1) \right] + 1$$

$$\text{Output (case i)} = \{(2)(1)(1)(0) + (-1)\} + \{(2)(1)(1)(0) + (-1)\} + 1$$
$$= (-1) + (-1) + 1$$
$$= -1.$$
$$= \text{Class I.}$$

Case (ii)

x_1	x_2	x_3	x_4
0	0	1	1

$$\text{Output (case ii)} = \{(2)(0)(1)(0) + (-1)\} + \{(2)(1)(1)(1) + (-1)\} + 1$$
$$= (-1) + (1) + 1$$
$$= 1$$
$$= \text{Class II}$$