HW1_Hardik_2678294168

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[128]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error
from sklearn.linear_model import Ridge
from sklearn.linear_model import LassoCV
from sklearn.model_selection import GridSearchCV
import math
import matplotlib.pyplot as plt
import warnings

warnings.filterwarnings('ignore')
```

[70]:
```
def readingdataset(filename):
    file = "../data/HW1_p2_material/"
    filead=file+filename
    dataset = np.load(filead)
    Xtrain, Ytrain, Xtest, Ytest=␣
 ↪dataset['X_train'],dataset['y_train'],dataset['X_test'],dataset['y_test']
    #Xtrain=np.concatenate((np.ones((len(Xtrain),1)),Xtrain),axis=1)
    #Xtest=np.concatenate((np.ones((len(Xtest),1)),Xtest),axis=1)
    return Xtrain,Ytrain,Xtest,Ytest
```

[119]:
```
Xtrain_d1,Ytrain_d1,Xtest_d1,Ytest_d1=readingdataset("dataset1_dim9_Ntr10.npz")
Xtrain_d2,Ytrain_d2,Xtest_d2,Ytest_d2=readingdataset("dataset2_dim9_Ntr100.npz")
Xtrain_d3,Ytrain_d3,Xtest_d3,Ytest_d3=readingdataset("dataset3_dim9_Ntr1000.
 ↪npz")
Xtrain_d4,Ytrain_d4,Xtest_d4,Ytest_d4=readingdataset("dataset4_dim2_Ntr10.npz")
Xtrain_d5,Ytrain_d5,Xtest_d5,Ytest_d5=readingdataset("dataset5_dim2_Ntr30.npz")
Xtrain_d6,Ytrain_d6,Xtest_d6,Ytest_d6=readingdataset("dataset6_dim2_Ntr100.npz")
Xtrain_d7,Ytrain_d7,Xtest_d7,Ytest_d7=readingdataset("dataset7_dim2_Ntr10.npz")
Xtrain_d8,Ytrain_d8,Xtest_d8,Ytest_d8=readingdataset("dataset8_dim2_Ntr30.npz")
Xtrain_d9,Ytrain_d9,Xtest_d9,Ytest_d9=readingdataset("dataset9_dim2_Ntr100.npz")
```

[72]:
```
reg_coeff=np.logspace(-10, 10, num=11, endpoint=True, base=2.0)
reg_coeff
```

[72]:
```
array([9.765625e-04, 3.906250e-03, 1.562500e-02, 6.250000e-02,
       2.500000e-01, 1.000000e+00, 4.000000e+00, 1.600000e+01,
       6.400000e+01, 2.560000e+02, 1.024000e+03])
```

[102]:
```
def linear_reg(Xtrain,Ytrain,Xtest,Ytest):
    clf=LinearRegression(fit_intercept=True, normalize=False)
    clf=clf.fit(Xtrain,Ytrain)
    w_p=clf.coef_
    w0=clf.intercept_
    w0=np.array(w0, ndmin=1)
    w= np.concatenate((w0,w_p),axis=0)
    Ytrain_pred=clf.predict(Xtrain)
    Ytest_pred=clf.predict(Xtest)
    mse_train=mean_squared_error(Ytrain, Ytrain_pred)
    mse_test=mean_squared_error(Ytest, Ytest_pred)
    l1_norm=np.linalg.norm(w,ord=1)
    l2_norm=np.linalg.norm(w,ord=2)
    spars=0
    for weight in w:
        if weight==0:
            spars=spars+1
    print ("Weight vector = {}".format(w))
    print ("MSE_train = {}".format(mse_train))
    print ("MSE_test = {}".format(mse_test))
    print ("L1_norm = {}".format(l1_norm))
    print ("L2_norm = {}".format(l2_norm))
    print ("Sparsity = {}".format(spars))

    return w,mse_train, mse_test, l1_norm, l2_norm, spars
```

[106]:
```
def linear_reg_lasso(Xtrain,Ytrain,Xtest,Ytest):
    reg_coeff=np.logspace(-10, 10, num=11, endpoint=True, base=2.0)
    clf=LassoCV( alphas=reg_coeff, fit_intercept=True, normalize=False,␣
 ↪max_iter=1000, cv=5, positive=False )
    clf=clf.fit(Xtrain,Ytrain)
    w_p=clf.coef_
    w0=clf.intercept_
    w0=np.array(w0, ndmin=1)
    w= np.concatenate((w0,w_p),axis=0)
    Ytrain_pred=clf.predict(Xtrain)
    Ytest_pred=clf.predict(Xtest)
    mse_train=mean_squared_error(Ytrain, Ytrain_pred)
    mse_test=mean_squared_error(Ytest, Ytest_pred)
    l1_norm=np.linalg.norm(w,ord=1)
    l2_norm=np.linalg.norm(w,ord=2)
    spars=0
    for weight in w:
```

```
        if weight==0:
            spars=spars+1
    mse_lambda_array=clf.mse_path_
    mse_lambda_mean=np.mean(mse_lambda_array)
    mse_lambda_std=np.std(mse_lambda_array)
    best_alpha=math.log(clf.alpha_,2)
    print ("Best param log_lambda = {}".format(best_alpha))
    print ("Mean of MSE = {}".format(mse_lambda_mean))
    print ("STD of MSE = {}".format(mse_lambda_std))
    print ("Weight vector = {}".format(w))
    print ("MSE_train = {}".format(mse_train))
    print ("MSE_test = {}".format(mse_test))
    print ("L1_norm = {}".format(l1_norm))
    print ("L2_norm = {}".format(l2_norm))
    print ("Sparsity = {}".format(spars))
    return w,mse_train, mse_test, l1_norm, l2_norm, spars
```

[114]:
```
def linear_reg_ridge(Xtrain,Ytrain,Xtest,Ytest):
    model=Ridge(fit_intercept=True, max_iter=1000)
    parameters = {'alpha':np.logspace(-10, 10, num=11, endpoint=True, base=2.0)}
    clf=GridSearchCV(model, parameters, scoring='neg_mean_squared_error',␣
 ↪cv=5,return_train_score=True)
    clf=clf.fit(Xtrain,Ytrain)
    best_model = clf.best_estimator_
    w_p=best_model.coef_
    w0=best_model.intercept_
    w0=np.array(w0, ndmin=1)
    w= np.concatenate((w0,w_p),axis=0)
    Ytrain_pred=best_model.predict(Xtrain)
    Ytest_pred=best_model.predict(Xtest)
    mse_train=mean_squared_error(Ytrain, Ytrain_pred)
    mse_test=mean_squared_error(Ytest, Ytest_pred)
    l1_norm=np.linalg.norm(w,ord=1)
    l2_norm=np.linalg.norm(w,ord=2)
    spars=0
    for weight in w:
        if weight==0:
            spars=spars+1
    mse_lambda_array=clf.cv_results_['mean_test_score']
    mse_lambda_mean=np.mean(mse_lambda_array)
    mse_lambda_std=np.std(mse_lambda_array)
    best_alpha=math.log(clf.best_params_['alpha'],2)
    print ("Best param log_lambda = {}".format(best_alpha))
    print ("Mean of MSE = {}".format(mse_lambda_mean))
    print ("STD of MSE = {}".format(mse_lambda_std))
    print ("Weight vector = {}".format(w))
    print ("MSE_train = {}".format(mse_train))
```

```
    print ("MSE_test = {}".format(mse_test))
    print ("L1_norm = {}".format(l1_norm))
    print ("L2_norm = {}".format(l2_norm))
    print ("Sparsity = {}".format(spars))
    return w,mse_train, mse_test, l1_norm, l2_norm, spars
```

[103]:
```
#dataset1-9 feat - no regularizer
w_d1,mse_train_d1, mse_test_d1, l1_norm_d1, l2_norm_d1, spars_d1=␣
↪linear_reg(Xtrain_d1,Ytrain_d1,Xtest_d1,Ytest_d1)
```

```
Weight vector = [ -7.01477582   3.20265861  -2.01056618   4.61891474
-8.48679639
   5.34513234  -1.36854253 -20.00142649  13.2641012    3.11232438]
MSE_train = 5.275901734118127e-28
MSE_test = 480.8978021950014
L1_norm = 68.42523867301641
L2_norm = 27.802696240879936
Sparsity = 0
```

[107]:
```
#dataset1-9 feat - lasso regularizer
w_d1_lasso,mse_train_d1_lasso, mse_test_d1_lasso, l1_norm_d1_lasso,␣
↪l2_norm_d1_lasso, spars_d1_lasso=␣
↪linear_reg_lasso(Xtrain_d1,Ytrain_d1,Xtest_d1,Ytest_d1)
```

```
Best param log_lambda = 2.0
Mean of MSE = 1067.508880995702
STD of MSE = 1492.7569038540025
Weight vector = [ 0.12578696  2.26001059  0.         -3.34237423 -0.
5.01163416
  0.         -5.93509725 -0.          1.43300028]
MSE_train = 14.1078838492251
MSE_test = 233.38359844231735
L1_norm = 18.107903459292224
L2_norm = 8.870754296600296
Sparsity = 4
```

[115]:
```
#dataset1-9 feat - Ridge regularizer
w_d1_ridge,mse_train_d1_ridge, mse_test_d1_ridge, l1_norm_d1_ridge,␣
↪l2_norm_d1_ridge, spars_d1_ridge=␣
↪linear_reg_ridge(Xtrain_d1,Ytrain_d1,Xtest_d1,Ytest_d1)
```

```
Best param log_lambda = 4.0
Mean of MSE = -840.6303690331886
STD of MSE = 628.9771239660494
Weight vector = [-0.24139412  2.5665958  -0.28155627 -1.71113314 -1.61199141
2.81003838
  2.21325862 -3.03423719 -2.75553818  1.62177076]
MSE_train = 18.11719332610634
MSE_test = 264.8044400260602
```

```
L1_norm = 18.847513865491255
L2_norm = 6.669033494963576
Sparsity = 0
```

[112]:
```
#dataset2-9 feat - no regularizer
w_d2,mse_train_d2, mse_test_d2, l1_norm_d2, l2_norm_d2, spars_d2=␣
↪linear_reg(Xtrain_d2,Ytrain_d2,Xtest_d2,Ytest_d2)
```

```
Weight vector = [ 0.43392102  2.397075    0.5682055  -3.87069203  0.8554485
2.25097789
  2.04197312 -6.17726984 -1.80441184  1.25424529]
MSE_train = 86.3366112987716
MSE_test = 112.65154328000659
L1_norm = 21.654220029500788
L2_norm = 8.613675992794825
Sparsity = 0
```

[108]:
```
#dataset2-9 feat - lasso regularizer
w_d2_lasso,mse_train_d2_lasso, mse_test_d2_lasso, l1_norm_d2_lasso,␣
↪l2_norm_d2_lasso, spars_d2_lasso=␣
↪linear_reg_lasso(Xtrain_d2,Ytrain_d2,Xtest_d2,Ytest_d2)
```

```
Best param log_lambda = 0.0
Mean of MSE = 735.8136444968056
STD of MSE = 1269.9358862344855
Weight vector = [ 0.43208693  2.33887343  0.43071582 -2.94652499  0.
2.36208926
  1.92436118 -6.33525333 -1.61782688  1.14532181]
MSE_train = 87.63581771635758
MSE_test = 110.19640754070862
L1_norm = 19.533053620667054
L2_norm = 8.238431033695141
Sparsity = 1
```

[116]:
```
#dataset2-9 feat - Ridge regularizer
w_d2_ridge,mse_train_d2_ridge, mse_test_d2_ridge, l1_norm_d2_ridge,␣
↪l2_norm_d2_ridge, spars_d2_ridge=␣
↪linear_reg_ridge(Xtrain_d2,Ytrain_d2,Xtest_d2,Ytest_d2)
```

```
Best param log_lambda = 6.0
Mean of MSE = -121.8610900745945
STD of MSE = 27.571406540496987
Weight vector = [ 0.45904623  2.25533004  0.55844399 -2.57037539 -0.32269209
2.23048119
  2.05571123 -4.14875114 -3.77234633  1.17294188]
MSE_train = 89.14761102319815
MSE_test = 111.42028497489191
L1_norm = 19.54611950615057
L2_norm = 7.371538310088324
```

```
Sparsity = 0
```

[105]:
```
#dataset3-9 feat - no regularizer
w_d3,mse_train_d3, mse_test_d3, l1_norm_d3, l2_norm_d3, spars_d3=␣
↪linear_reg(Xtrain_d3,Ytrain_d3,Xtest_d3,Ytest_d3)
```

```
Weight vector = [ 1.71594731  1.90468457  0.41212604 -3.17204863  0.25311452
4.87289258
 -0.25297342 -8.71299177  0.80571383  0.89176542]
MSE_train = 98.21301479826998
MSE_test = 109.12481315987687
L1_norm = 22.994258103832294
L2_norm = 10.864521591717066
Sparsity = 0
```

[109]:
```
#dataset3-9 feat - lasso regularizer
w_d3_lasso,mse_train_d3_lasso, mse_test_d3_lasso, l1_norm_d3_lasso,␣
↪l2_norm_d3_lasso, spars_d3_lasso=␣
↪linear_reg_lasso(Xtrain_d3,Ytrain_d3,Xtest_d3,Ytest_d3)
```

```
Best param log_lambda = -2.0
Mean of MSE = 674.2737984543094
STD of MSE = 1094.9497290230072
Weight vector = [ 1.70180095  1.88836961  0.37895217 -2.91361253  0.
4.60921066
  0.         -7.90337403 -0.          0.87016587]
MSE_train = 98.45022018747636
MSE_test = 109.07211761162876
L1_norm = 20.265485823458295
L2_norm = 9.977982841156606
Sparsity = 3
```

[118]:
```
#dataset3-9 feat - Ridge regularizer
w_d3_ridge,mse_train_d3_ridge, mse_test_d3_ridge, l1_norm_d3_ridge,␣
↪l2_norm_d3_ridge, spars_d3_ridge=␣
↪linear_reg_ridge(Xtrain_d3,Ytrain_d3,Xtest_d3,Ytest_d3)
```

```
Best param log_lambda = 2.0
Mean of MSE = -102.13749808847291
STD of MSE = 2.223683320142983
Weight vector = [ 1.71554948  1.90414688  0.41169323 -3.16239173  0.24355474
4.83535943
 -0.21617519 -8.46183322  0.55511326  0.8910176 ]
MSE_train = 98.22296899901596
MSE_test = 108.98663170734264
L1_norm = 22.396834768429468
L2_norm = 10.626877909890354
Sparsity = 0
```

[120]:
```
#dataset4-2 feat - no regularizer
w_d4,mse_train_d4, mse_test_d4, l1_norm_d4, l2_norm_d4, spars_d4=␣
↪linear_reg(Xtrain_d4,Ytrain_d4,Xtest_d4,Ytest_d4)
```

```
Weight vector = [ 6.77265711 -2.4928513   7.23801612]
MSE_train = 95.38019904643618
MSE_test = 163.48761227397387
L1_norm = 16.503524531665292
L2_norm = 10.221157922129628
Sparsity = 0
```

[121]:
```
#dataset4-2 feat - lasso regularizer
w_d4_lasso,mse_train_d4_lasso, mse_test_d4_lasso, l1_norm_d4_lasso,␣
↪l2_norm_d4_lasso, spars_d4_lasso=␣
↪linear_reg_lasso(Xtrain_d4,Ytrain_d4,Xtest_d4,Ytest_d4)
```

```
Best param log_lambda = 2.0
Mean of MSE = 256.25146952709184
STD of MSE = 249.16970913724862
Weight vector = [5.53704387 0.         4.00141645]
MSE_train = 98.92934420027042
MSE_test = 134.48864379047166
L1_norm = 9.538460320163804
L2_norm = 6.831558271694046
Sparsity = 1
```

[122]:
```
#dataset4-2 feat - Ridge regularizer
w_d4_ridge,mse_train_d4_ridge, mse_test_d4_ridge, l1_norm_d4_ridge,␣
↪l2_norm_d4_ridge, spars_d4_ridge=␣
↪linear_reg_ridge(Xtrain_d4,Ytrain_d4,Xtest_d4,Ytest_d4)
```

```
Best param log_lambda = 4.0
Mean of MSE = -196.33770706675338
STD of MSE = 50.48813742731631
Weight vector = [4.61680832 1.55139276 2.16748412]
MSE_train = 102.92593770240387
MSE_test = 127.90734292133521
L1_norm = 8.335685196859789
L2_norm = 5.331015470702115
Sparsity = 0
```

[123]:
```
#dataset5-2 feat - no regularizer
w_d5,mse_train_d5, mse_test_d5, l1_norm_d5, l2_norm_d5, spars_d5=␣
↪linear_reg(Xtrain_d5,Ytrain_d5,Xtest_d5,Ytest_d5)
print(" ")
print("######")

#dataset5-2 feat - lasso regularizer
```

```
w_d5_lasso,mse_train_d5_lasso, mse_test_d5_lasso, l1_norm_d5_lasso,␣
 ↪l2_norm_d5_lasso, spars_d5_lasso=␣
 ↪linear_reg_lasso(Xtrain_d5,Ytrain_d5,Xtest_d5,Ytest_d5)
print(" ")
print("#####")

#dataset5-2 feat - Ridge regularizer
w_d5_ridge,mse_train_d5_ridge, mse_test_d5_ridge, l1_norm_d5_ridge,␣
 ↪l2_norm_d5_ridge, spars_d5_ridge=␣
 ↪linear_reg_ridge(Xtrain_d5,Ytrain_d5,Xtest_d5,Ytest_d5)
```

```
Weight vector = [ 4.05510307  2.74884213 -0.29784002]
MSE_train = 87.12261767437649
MSE_test = 114.70433167932684
L1_norm = 7.101785217832998
L2_norm = 4.908024311927913
Sparsity = 0
#####
Best param log_lambda = 2.0
Mean of MSE = 139.29912344057894
STD of MSE = 75.98257910583324
Weight vector = [ 3.7532937   2.47956376 -0.        ]
MSE_train = 88.72353370540213
MSE_test = 105.57083378205994
L1_norm = 6.232857452937665
L2_norm = 4.498383042243042
Sparsity = 1
#####
Best param log_lambda = 6.0
Mean of MSE = -115.6039596435588
STD of MSE = 14.631848794699655
Weight vector = [ 3.79650626  2.41954902 -0.18839922]
MSE_train = 88.86631773419617
MSE_test = 106.00172830717882
L1_norm = 6.4044545005619415
L2_norm = 4.505904073798311
Sparsity = 0
```

[124]:
```
#dataset6-2 feat - no regularizer
w_d6,mse_train_d6, mse_test_d6, l1_norm_d6, l2_norm_d6, spars_d6=␣
 ↪linear_reg(Xtrain_d6,Ytrain_d6,Xtest_d6,Ytest_d6)
print(" ")
print("#####")

#dataset6-2 feat - lasso regularizer
```

```
w_d6_lasso,mse_train_d6_lasso, mse_test_d6_lasso, l1_norm_d6_lasso,␣
 ↪l2_norm_d6_lasso, spars_d6_lasso=␣
 ↪linear_reg_lasso(Xtrain_d6,Ytrain_d6,Xtest_d6,Ytest_d6)
print(" ")
print("#####")

#dataset6-2 feat - Ridge regularizer
w_d6_ridge,mse_train_d6_ridge, mse_test_d6_ridge, l1_norm_d6_ridge,␣
 ↪l2_norm_d6_ridge, spars_d6_ridge=␣
 ↪linear_reg_ridge(Xtrain_d6,Ytrain_d6,Xtest_d6,Ytest_d6)
```

```
Weight vector = [1.21077089 2.30240071 0.23152547]
MSE_train = 101.35833777888638
MSE_test = 101.44570933707959
L1_norm = 3.74469706915353
L2_norm = 2.6116315269679835
Sparsity = 0

#####
Best param log_lambda = -10.0
Mean of MSE = 137.055342060517
STD of MSE = 63.836299120708816
Weight vector = [1.2107991  2.30236633 0.23141756]
MSE_train = 101.35833791468285
MSE_test = 101.44595275106839
L1_norm = 3.744582986648264
L2_norm = 2.6116047303958663
Sparsity = 0

#####
Best param log_lambda = 6.0
Mean of MSE = -112.14403331581853
STD of MSE = 4.0246773291225395
Weight vector = [1.22012634 2.2180724  0.24090883]
MSE_train = 101.47660067151448
MSE_test = 101.30261474052405
L1_norm = 3.6791075627301373
L2_norm = 2.542949176508624
Sparsity = 0
```

[125]:
```
#dataset7-2 feat - no regularizer
w_d7,mse_train_d7, mse_test_d7, l1_norm_d7, l2_norm_d7, spars_d7=␣
 ↪linear_reg(Xtrain_d7,Ytrain_d7,Xtest_d7,Ytest_d7)
print(" ")
print("#####")

#dataset7-2 feat - lasso regularizer
```

```
w_d7_lasso,mse_train_d7_lasso, mse_test_d7_lasso, l1_norm_d7_lasso,␣
 ↪l2_norm_d7_lasso, spars_d7_lasso=␣
 ↪linear_reg_lasso(Xtrain_d7,Ytrain_d7,Xtest_d7,Ytest_d7)
print(" ")
print("#####")

#dataset7-2 feat - Ridge regularizer
w_d7_ridge,mse_train_d7_ridge, mse_test_d7_ridge, l1_norm_d7_ridge,␣
 ↪l2_norm_d7_ridge, spars_d7_ridge=␣
 ↪linear_reg_ridge(Xtrain_d7,Ytrain_d7,Xtest_d7,Ytest_d7)
```

```
Weight vector = [ 1.6193184   4.35846137 -2.05316003]
MSE_train = 25.417551693358597
MSE_test = 116.51141337592615
L1_norm = 8.030939797861016
L2_norm = 5.082700433707281
Sparsity = 0

#####
Best param log_lambda = 0.0
Mean of MSE = 80.56417433998845
STD of MSE = 88.39221271457387
Weight vector = [ 1.49398093  3.84541442 -1.4616377 ]
MSE_train = 26.522508540648026
MSE_test = 108.51285152596137
L1_norm = 6.801033058092814
L2_norm = 4.37670833943422
Sparsity = 0

#####
Best param log_lambda = 2.0
Mean of MSE = -70.89051049847157
STD of MSE = 13.911391429017938
Weight vector = [ 1.59580358  3.78666035 -1.49465134]
MSE_train = 26.617548498116584
MSE_test = 109.29761312532439
L1_norm = 6.877115268463562
L2_norm = 4.372569988665177
Sparsity = 0
```

[126]:
```
#dataset8-2 feat - no regularizer
w_d8,mse_train_d8, mse_test_d8, l1_norm_d8, l2_norm_d8, spars_d8=␣
 ↪linear_reg(Xtrain_d8,Ytrain_d8,Xtest_d8,Ytest_d8)
print(" ")
print("#####")

#dataset8-2 feat - lasso regularizer
```

```
w_d8_lasso,mse_train_d8_lasso, mse_test_d8_lasso, l1_norm_d8_lasso,␣
 ↪l2_norm_d8_lasso, spars_d8_lasso=␣
 ↪linear_reg_lasso(Xtrain_d8,Ytrain_d8,Xtest_d8,Ytest_d8)
print(" ")
print("#####")

#dataset8-2 feat - Ridge regularizer
w_d8_ridge,mse_train_d8_ridge, mse_test_d8_ridge, l1_norm_d8_ridge,␣
 ↪l2_norm_d8_ridge, spars_d8_ridge=␣
 ↪linear_reg_ridge(Xtrain_d8,Ytrain_d8,Xtest_d8,Ytest_d8)
```

```
Weight vector = [3.58068323 1.91863829 0.60434473]
MSE_train = 95.15432277075584
MSE_test = 109.24257017878496
L1_norm = 6.10366625566634
L2_norm = 4.107030295969647
Sparsity = 0

#####
Best param log_lambda = 0.0
Mean of MSE = 155.94480304519337
STD of MSE = 88.25024220759911
Weight vector = [3.51599517 1.88223308 0.593483  ]
MSE_train = 95.20146796600213
MSE_test = 109.46763164804094
L1_norm = 5.991711258559851
L2_norm = 4.032027469140141
Sparsity = 0

#####
Best param log_lambda = 6.0
Mean of MSE = -115.91126727230889
STD of MSE = 10.698199618203887
Weight vector = [3.20005109 1.41174703 0.97725369]
MSE_train = 95.90349733909382
MSE_test = 110.51198296942115
L1_norm = 5.589051809257596
L2_norm = 3.6315811189650824
Sparsity = 0
```

[127]:
```
#dataset9-2 feat - no regularizer
w_d9,mse_train_d9, mse_test_d9, l1_norm_d9, l2_norm_d9, spars_d9=␣
 ↪linear_reg(Xtrain_d9,Ytrain_d9,Xtest_d9,Ytest_d9)
print(" ")
print("#####")

#dataset9-2 feat - lasso regularizer
```

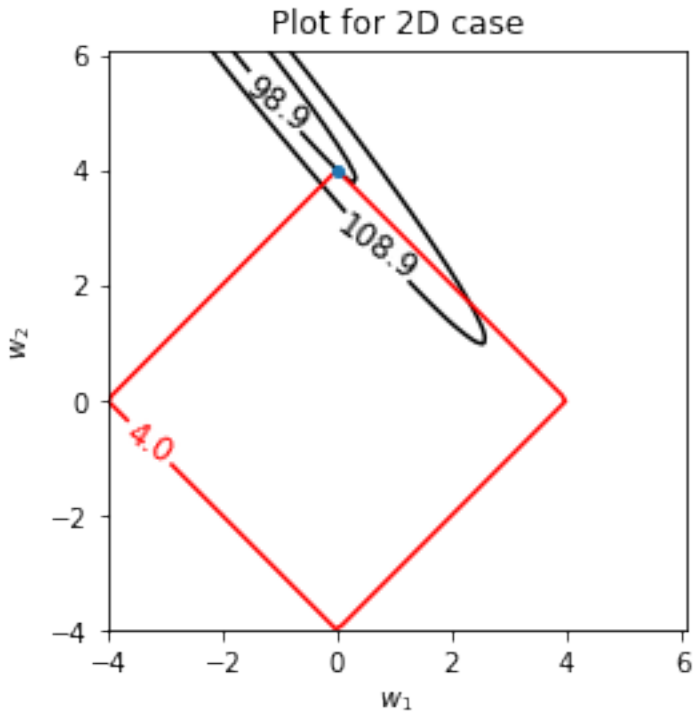
```
w_d9_lasso,mse_train_d9_lasso, mse_test_d9_lasso, l1_norm_d9_lasso,␣
↪l2_norm_d9_lasso, spars_d9_lasso=␣
↪linear_reg_lasso(Xtrain_d9,Ytrain_d9,Xtest_d9,Ytest_d9)
print(" ")
print("#####")

#dataset9-2 feat - Ridge regularizer
w_d9_ridge,mse_train_d9_ridge, mse_test_d9_ridge, l1_norm_d9_ridge,␣
↪l2_norm_d9_ridge, spars_d9_ridge=␣
↪linear_reg_ridge(Xtrain_d9,Ytrain_d9,Xtest_d9,Ytest_d9)
```

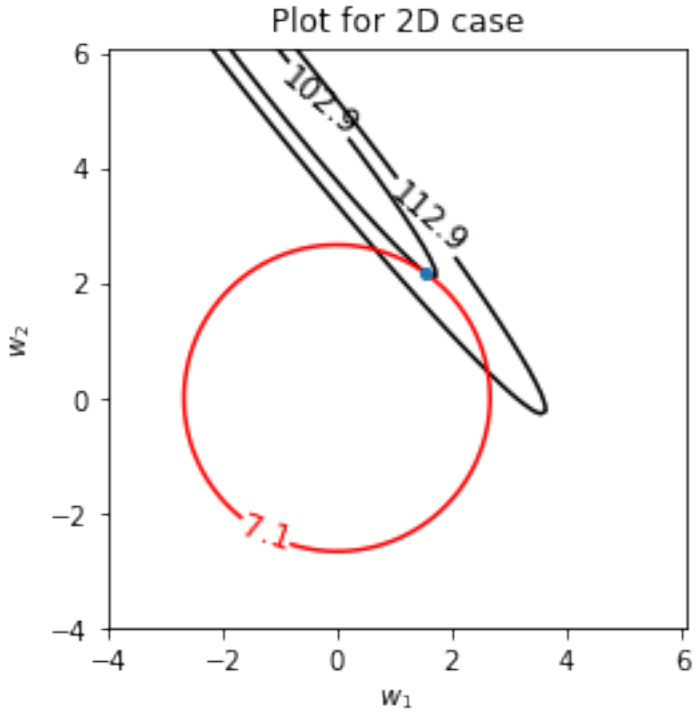
```
Weight vector = [ 4.11404128  3.04009919 -0.51630424]
MSE_train = 83.3240152571577
MSE_test = 111.41165530589785
L1_norm = 7.6704447096754755
L2_norm = 5.141411166840956
Sparsity = 0

#####
Best param log_lambda = 0.0
Mean of MSE = 123.04568773619322
STD of MSE = 57.15983617651494
Weight vector = [ 4.10151715  2.50447238 -0.        ]
MSE_train = 83.90836860287891
MSE_test = 109.50398425369923
L1_norm = 6.605989527505749
L2_norm = 4.805707525211649
Sparsity = 1

#####
Best param log_lambda = 4.0
Mean of MSE = -90.84543283804109
STD of MSE = 3.3415442744863117
Weight vector = [ 4.10848705  2.79878652 -0.28354144]
MSE_train = 83.44263614295154
MSE_test = 110.36009501840029
L1_norm = 7.19081501224555
L2_norm = 4.979283841239782
Sparsity = 0
```

[131]:
```
def MSE(X,Y,w):
    return np.mean(np.square(np.dot(X,w)-Y))

def display(w,Xtest,Ytest,norm,levels=None,w1_range=(-4.0, 6.1, 100),␣
↪w2_range=(-4.0, 6.1, 100)):
    w = np.array(w)
    Xtest=np.concatenate((np.ones((len(Xtest),1)),Xtest),axis=1)
```

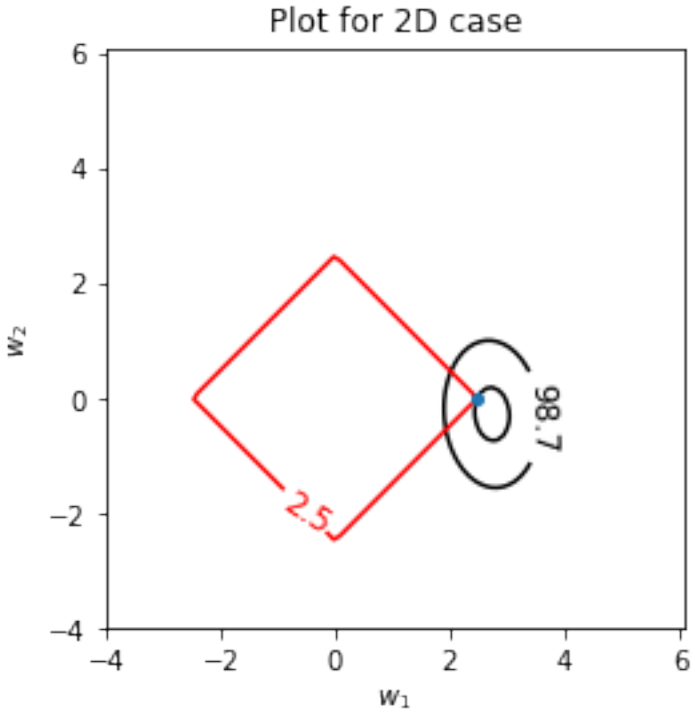
```
    w1list = np.linspace(w1_range[0], w1_range[1], w1_range[2])
    w2list = np.linspace(w2_range[0], w2_range[1], w2_range[2])
    W1, W2 = np.meshgrid(w1list, w2list)

    Z = np.stack((w[0]*np.ones(W1.shape),W1,W2),axis=0)
    Z = Z.reshape((Z.shape[0],-1))
    Z = np.matmul(Xtest,Z) - Ytest.reshape((len(Ytest),1))
    Z = np.square(Z)
    Z = np.sum(Z, axis=0, keepdims=False)/Xtest.shape[0]
    Z = Z.reshape(W1.shape)
    if norm == 'l2':
        W_norm = np.square(W1) + np.square(W2)
    elif norm == 'l1':
        W_norm = np.abs(W1) + np.abs(W2)
    else:
        raise RuntimeError('Unimplemented norm. Please enter "l1" or "l2".')
    plt.figure()

    mse_ori = MSE(Xtest,Ytest,w)
    levels = [mse_ori, mse_ori+10]
    contour = plt.contour(W1, W2, Z, levels, colors='k')
    plt.clabel(contour, colors = 'k', fmt = '%2.1f', fontsize=12)

    if norm == 'l2':
        levels = [np.sum(np.square(w[1:]))]
    elif norm == 'l1':
        levels = [np.sum(abs(w[1:]))]
    else:
        raise RuntimeError('Unimplemented norm. Please enter "l1" or "l2".')
    contour = plt.contour(W1, W2, W_norm, levels, colors='r')
    plt.clabel(contour, colors = 'r', fmt = '%2.1f', fontsize=12)
    plt.plot(w[1],w[2],marker = ".",markersize=8)

    plt.title('Plot for 2D case')
    plt.xlabel('$w_1$')
    plt.ylabel('$w_2$')
    plt.axis('square')
    return
```

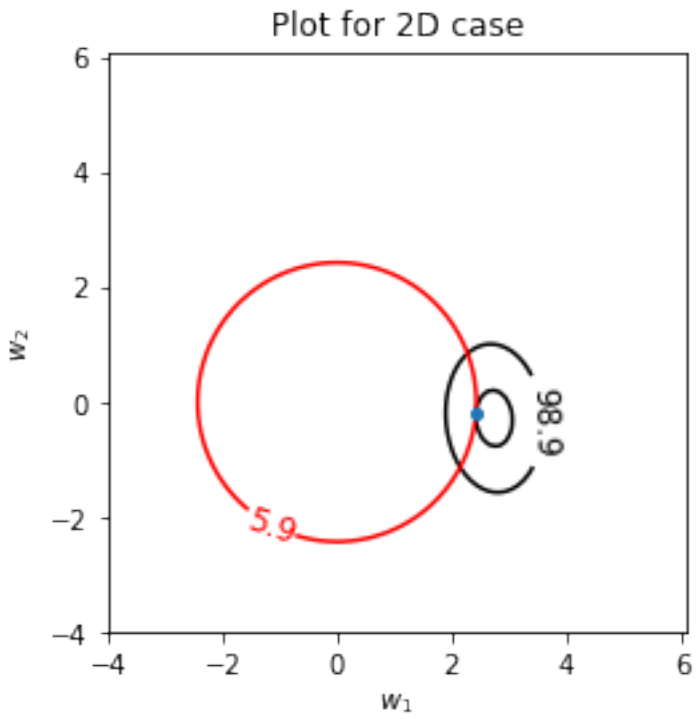
[132]:
```
# D4-l1 results
display(w_d4_lasso,Xtrain_d4,Ytrain_d4,norm='l1')

# D4-l2 results
display(w_d4_ridge,Xtrain_d4,Ytrain_d4,norm='l2')
plt.show()
```


Plot for 2D case
98.9
108.9
4.0
w_1
w_2

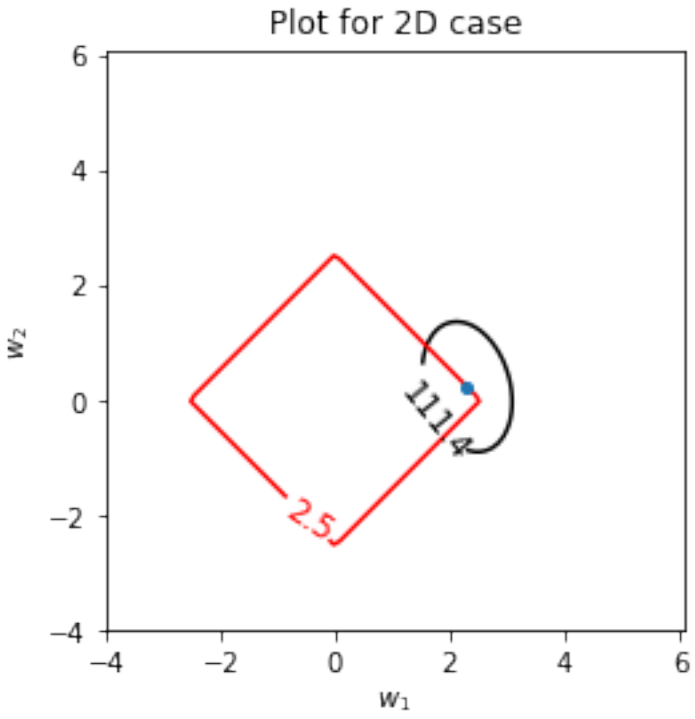

Plot for 2D case
102.9
112.9
7.1
w_1
w_2

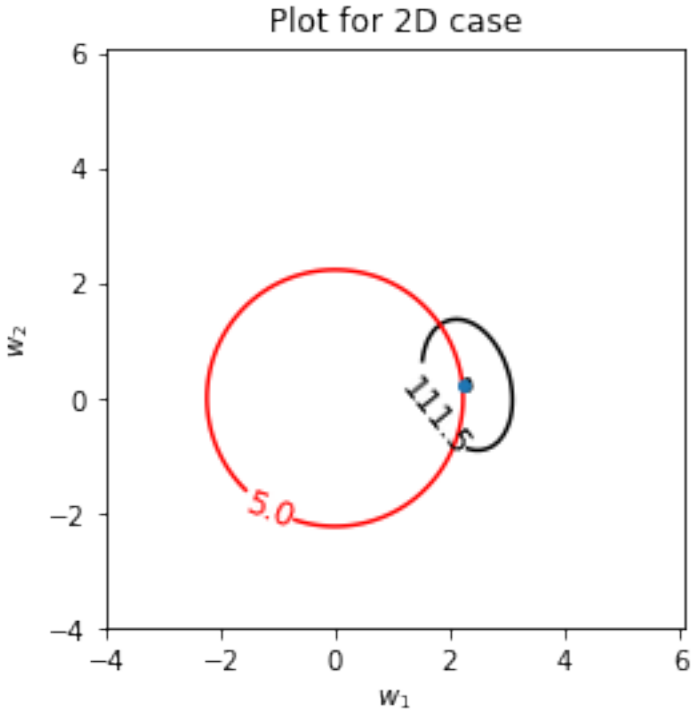
[133]:
```
# D4-l1 results
display(w_d5_lasso,Xtrain_d5,Ytrain_d5,norm='l1')

# D4-l2 results
display(w_d5_ridge,Xtrain_d5,Ytrain_d5,norm='l2')
plt.show()
```



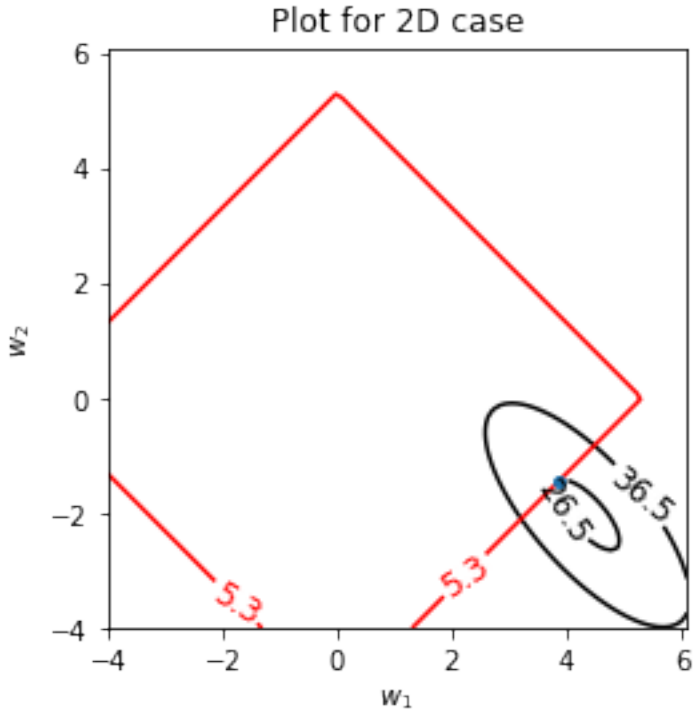


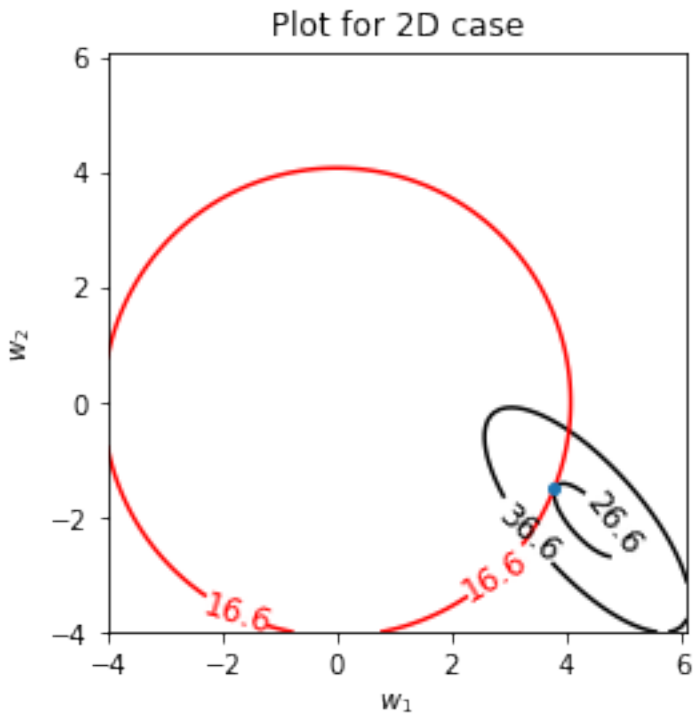
[134]:
```
# D6-l1 results
display(w_d6_lasso,Xtrain_d6,Ytrain_d6,norm='l1')

# D6-l2 results
display(w_d6_ridge,Xtrain_d6,Ytrain_d6,norm='l2')
plt.show()
```


Plot for 2D case
111.4
2.5
w_1
w_2

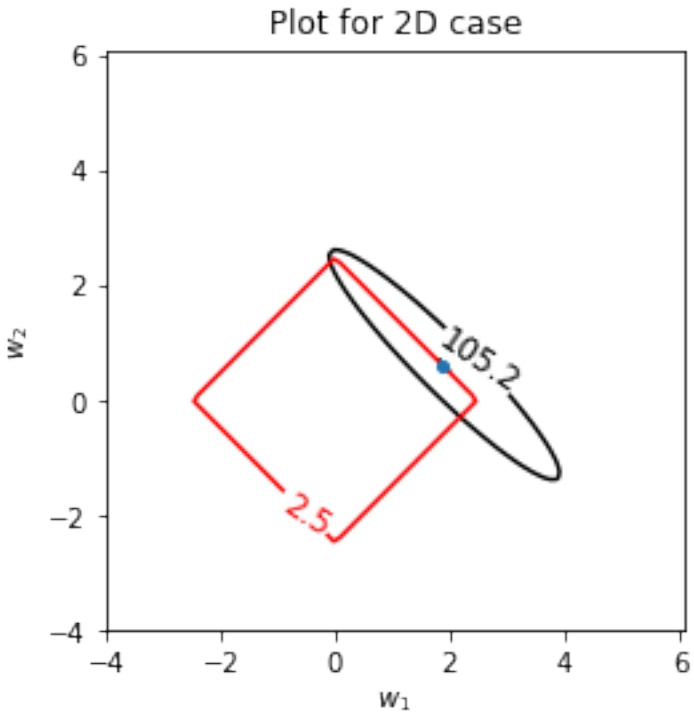

Plot for 2D case
111.5
5.0
w_1
w_2

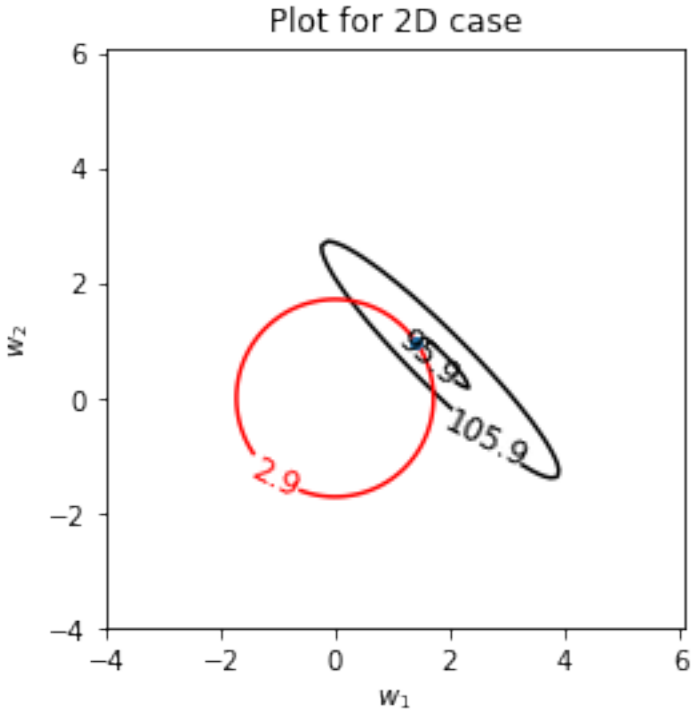
[135]:
```
# D7-l1 results
display(w_d7_lasso,Xtrain_d7,Ytrain_d7,norm='l1')

# D7-l2 results
display(w_d7_ridge,Xtrain_d7,Ytrain_d7,norm='l2')
plt.show()
```



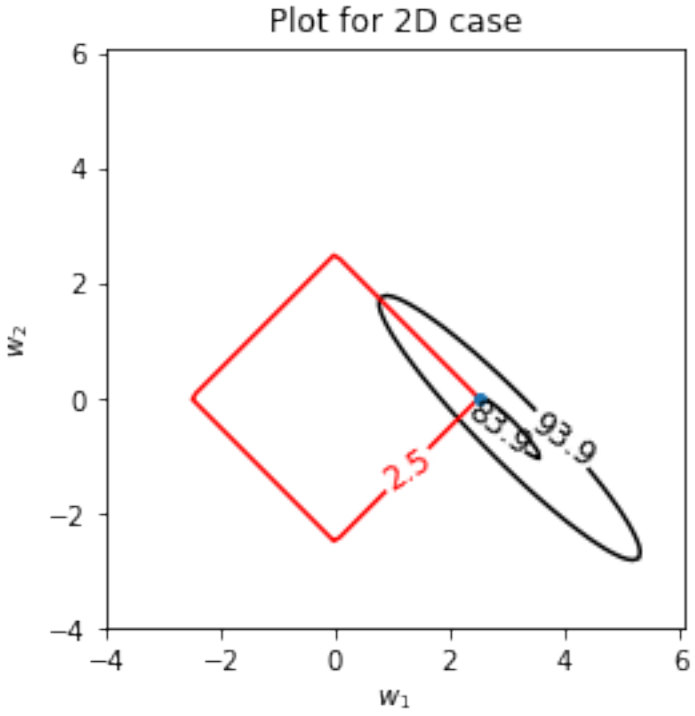


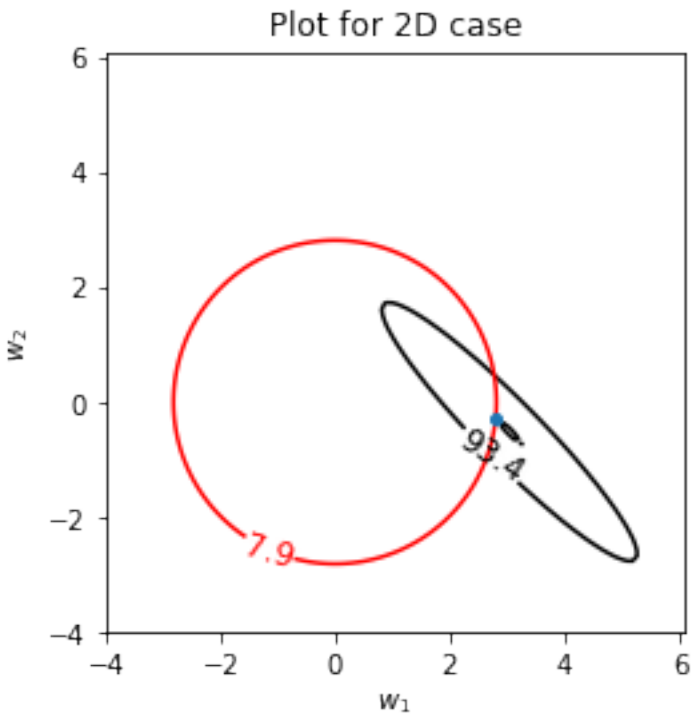
[136]:
```
# D8-l1 results
display(w_d8_lasso,Xtrain_d8,Ytrain_d8,norm='l1')

# D8-l2 results
display(w_d8_ridge,Xtrain_d8,Ytrain_d8,norm='l2')
plt.show()
```


Plot for 2D case
105.2
2.5
w_1
w_2


Plot for 2D case
95.9
105.9
2.9
w_1
w_2

[137]:
```
# D9-l1 results
display(w_d9_lasso,Xtrain_d9,Ytrain_d9,norm='l1')

# D9-l2 results
display(w_d9_ridge,Xtrain_d9,Ytrain_d9,norm='l2')
plt.show()
```




Plot for 2D case
6
4
2
0
−2
−4
−4
−2
0
2
4
6
w_1
w_2
9.4
7.9

Q2) a) i)

Dataset1-9 features

<table>
<tr><td rowspan="2"></td><td colspan="3">Model Selection</td><td colspan="2">Performance</td></tr>
<tr><td>Best Param
$Log_2\lambda$</td><td>Mean of MSE</td><td>STD of MSE</td><td>MSE on Train</td><td>MSE ON Test</td></tr>
<tr><td rowspan="3">Least Square</td><td>-</td><td>-</td><td>-</td><td>5.275e-28</td><td>480.897</td></tr>
<tr><td>w</td><td colspan="4">[-7.01477582 3.20265861 -2.01056618 4.61891474 -8.48679639 5.34513234 -1.36854253 -20.00142649 13.2641012 3.11232438]</td></tr>
<tr><td></td><td>L1(w)= 68.425</td><td>L2(w)= 27.802</td><td colspan="2">Spars = 0</td></tr>
<tr><td rowspan="3">LASSO</td><td>2.0</td><td>1067.508</td><td>1492.756</td><td>14.107</td><td>233.383</td></tr>
<tr><td>w</td><td colspan="4">[0.12578696 2.26001059 0. -3.34237423 -0. 5.01163416
0. -5.93509725 -0. 1.43300028]</td></tr>
<tr><td></td><td>L1(w)= 18.107</td><td>L2(w)= 8.870</td><td colspan="2">Spars = 4</td></tr>
<tr><td rowspan="3">RIDGE</td><td>4.0</td><td>-840.630</td><td>628.977</td><td>18.117</td><td>264.804</td></tr>
<tr><td>w</td><td colspan="4">[-0.24139412 2.5665958 -0.28155627 -1.71113314 -1.61199141 2.81003838 2.21325862 -3.03423719 -2.75553818 1.62177076]</td></tr>
<tr><td></td><td>L1(w)= 18.847</td><td>L2(w)= 6.669</td><td colspan="2">Spars = 0</td></tr>
</table>

Dataset2-9 features

	Model Selection			Performance	
	Best Param $Log_2\lambda$	Mean of MSE	STD of MSE	MSE on Train	MSE ON Test
Least Square	-	-	-	86.336	112.651
	w	[0.433, 2.397, 0.568, -3.870, 0.855, 2.250, 2.041, -6.177, -1.804, 1.254]			
		L1(w)= 21.654	L2(w)= 8.613	Spars = 0	
LASSO	0.0	723.813	1269.935	87.635	110.196
	w	[0.43208693 2.33887343 0.43071582 -2.94652499 0. 2.36208926 1.92436118 -6.33525333 -1.61782688 1.14532181]			
		L1(w)= 19.533	L2(w)= 8.238	Spars = 1	
RIDGE	6.0	-121.861	27.571	89.147	111.420
	w	[0.45904623 2.25533004 0.55844399 -2.57037539 -0.32269209 2.23048119 2.05571123 -4.14875114 -3 .77234633 1.17294188]			
		L1(w)= 19.546	L2(w)= 7.371	Spars = 0	

Dataset3-9 features

	Model Selection			Performance	
	Best Param $Log_2\lambda$	Mean of MSE	STD of MSE	MSE on Train	MSE ON Test
Least Square	-	-	-	98.213	109.124
	w	[1.715, 1.904, 0.412, -3.172, 0.253, 4.872, -0.252, -8.712, 0.805, 0.891]			
		L1(w)= 22.994	L2(w)= 10.864	Spars = 0	
LASSO	-2.0	674.273	1094.949	98.45	109.072
	w	[1.70180095 1.88836961 0.37895217 -2.91361253 0. 4.60921066 0. -7.90337403 -0. 0.87016587]			
		L1(w)= 20.265	L2(w)= 9.977	Spars = 3	
RIDGE	2.0	-102.137	2.223	98.222	108.986
	w	[1.71554948 1.90414688 0.41169323 -3.16239173 0.24355474 4.83535943 -0.21617519 -8.46183322 0.55511326 0.8910176]			
		L1(w)= 22.396	L2(w)= 10.628	Spars = 0	

Q2) a) ii)

1) Test MSE is best obtained with L1 (LASSO) regularizer. For Dataset 1&2, Test MSE was better for Ridge regularizer compared to no regularizer.
2) Yes, each regularizer lowers the corresponding norm of w. As training data increases, the difference in corresponding norms starts decreasing (with Ntr → 1000+, it's almost insignificant). This is because, with few training samples, our weight vector takes higher values for Linear regression with no regularizer classifier. As the training samples increases, our Linear regression model with no regularizer performs well and takes lower weight values.
3) We obtain sparsity with only LASSO regularizer. As training data increases, our best fit Lambda value (λ) decreases. In general, Sparsity is seen more with lower sized training dataset.

Q2) b) i)

Dataset4-2 features

	Model Selection			Performance	
	Best Param $Log_2 \lambda$	Mean of MSE	STD of MSE	MSE on Train	MSE ON Test
Least Square	-	-	-	95.38019904643618	163.48761227397387
	w	[6.77265711 -2.4928513 7.23801612]			
		L1(w)= 16.503524531665292	L2(w)= 10.221157922129628	Spars=0	
LASSO	2.0	256.25146952709184	249.16970913724862	98.92934420027042	134.48864379047166
	w	[5.53704387 0. 4.00141645]			
		L1(w)= 9.538460320163804	L2(w)= 6.831558271694046	Spars=1	
RIDGE	4.0	-196.337	50.48813742731631	102.92593770240387	127.90734292133521
	w	[4.61680832 1.55139276 2.16748412]			
		L1(w)= 8.335685196859789	L2(w)= 5.331015470702115	Spars=0	

Dataset5-2 features

	Model Selection			Performance	
	Best Param $Log_2 \lambda$	Mean of MSE	STD of MSE	MSE on Train	MSE ON Test
Least Square	-	-	-	87.12261767437649	114.70433167932684
	w	[4.05510307 2.74884213 -0.29784002]			
		L1(w)= 7.101785217832998	L2(w)= 4.908024311927913	Spars=0	
LASSO	2.0	139.29912344057894	75.98257910583324	88.723533705402 13	105.57083378205994
	w	[3.7532937 2.47956376 0.]			
		L1(w)= 6.232857452937665	L2(w)= 4.4983830422430 42	Spars=1	
RIDGE	6.0	-115.603	14.631848794699655	88.86631773419617	106.00172830717882
	w	[3.79650626 2.41954902 -0.18839922]			
		L1(w)= 6.404454500561 9415	L2(w)= 4.50590407379831 1	Spars=0	

Dataset6-2 features

	Model Selection			Performance	
	Best Param $Log_2 \lambda$	Mean of MSE	STD of MSE	MSE on Train	MSE ON Test
Least Square	-	-	-	101.35833777888638	101.44570933707959
	w	[1.21077089 2.30240071 0.23152547]			
		L1(w)= 3.74469706915353	L2(w)= 2.6116315269679835	Spars=0	
LASSO	-10.0	137.055342060517	63.836299120708816	101.35833791468285	101.44595275106839
	w	[1.2107991 2.30236633 0.23141756]			
		L1(w)= 3.744582986648264	L2(w)= 2.6116047303958663	Spars=0	
RIDGE	6.0	-112.144	4.02467732912253 95	101.47660067151448	101.30261474052405
	w	[1.22012634 2.2180724 0.24090883]			
		L1(w)= 3.679107562730 1373	L2(w)= 2.542949176508624	Spars=0	

Dataset7-2 features

	Model Selection			Performance	
	Best Param $Log_2 \lambda$	Mean of MSE	STD of MSE	MSE on Train	MSE ON Test
Least Square	-	-	-	25.417551693358597	116.51141337592615
	w	[1.6193184 4.35846137 -2.05316003]			
		L1(w)= 8.030939797861016	L2(w)= 5.082700433707281	Spars=0	
LASSO	0.0	80.56417433998845	88.39221271457387	26.522508540648026	108.51285152596137
	w	[1.49398093 3.84541442 -1.4616377]			
		L1(w)= 6.801033058092814	L2(w)= 4.3767083394342	Spars=0	
RIDGE	2.0	-70.890	13.911391429017938	26.617548498116584	109.29761312532439
	w	[1.59580358 3.78666035 -1.49465134]			
		L1(w)= 6.877115268463562	L2(w)= 4.37256998866517	Spars=0	

Dataset8-2 features

	Model Selection			Performance	
	Best Param $Log_2 \lambda$	Mean of MSE	STD of MSE	MSE on Train	MSE ON Test
Least Square	-	-	-	95.15432277075584	109.24257017878496
	w	[3.58068323 1.91863829 0.60434473]			
		L1(w)= 6.10366625566634	L2(w)= 4.107030295969647	Spars=0	
LASSO	0.0	155.94480304519337	88.25024220759911	95.20146796600213	109.46763164804094
	w	[3.51599517 1.88223308 0.593483]			
		L1(w)= 5.991711258559851	L2(w)= 4.032027469140141	Spars=0	
RIDGE	6.0	-115.911	10.698199618203887	95.90349733909382	110.51198296942115
	w	[3.20005109 1.41174703 0.97725369]			
		L1(w)= 5.589051809257596	L2(w)= 3.6315811189650824	Spars=0	

Dataset9-2 features

	Model Selection			Performance	
	Best Param $Log_2 \lambda$	Mean of MSE	STD of MSE	MSE on Train	MSE ON Test
Least Square	-	-	-	83.3240152571577	111.41165530589785
	w	[4.11404128 3.04009919 -0.51630424]			
		L1(w)= 7.670444709675 4755	L2(w)= 5.141411166840 956	Spars=0	
LASSO	0.0	123.0456877361 9322	57.15983617651 494	83.90836860287 891	109.50398425369 923
	w	[4.10151715 2.50447238 0.]			
		L1(w)= 6.605989527505 749	L2(w)= 4.8057075252116 49	Spars=1	
RIDGE	4.0	-90.854	3.3415442744863 117	83.44263614295 154	110.3600950184 0029
	w	[4.10848705 2.79878652 -0.28354144]			
		L1(w)= 7.190815012245 55	L2(w)= 4.97928384123978 2	Spars=0	

Q2) b) ii) --- plots are in the pdf file containing code part. Please refer over there.

Q2) b) iii)

1) The plots have 2 figures each, MSE AND regularizer constraints drawn over a range of w1 and w2 for a particular value of w0(for the best lambda). We can see that MSE with no regularizer intercepts the constraints contour at greater values of w1 and w2. Also, LASSO (l1 regularizer)

gives us sparsity which can be observed by the intersection of MSE (lowest) with constraints contour at either w1=0 or w2=0.

2) With lower training data size, the regularizer has a larger effect on MSE. Regularized MSE takes a lower value compared with unregularized MSE.

3) With the 'special case' datasets, the LASSO regularization does not give any sparsity with Ntr=10 and Ntr=30 but it does give sparsity for Ntr=100 which is not the case in set of datasets 4,5,6 where we got sparsity for Ntr=10(dataset 4) and Ntr=30(dataset 5) but did not get sparsity for Ntr=100(dataset 6). In general, Sparsity is observed in lower sized training datasets as its easier for model to overfit training data with lesser samples and obtain poor generalization. This was opposite with Datasets 7,8,9 (Special Case).