UCLA Samueli
Mechanical & Aerospace Engineering

Parametric Study of Reef and Granny Knots

Midterm Progress Report

MAE 259B

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Overall Implementation Steps

1. Framework Discrete Elastic Rod (DER)

2. **Incorporate Implicit Contact Model (IMC)**
    - Provides accurate frictional contact

3. Derive necessary boundary condition
    - Deform an initially straight elastic rod into both a granny and reef knot

4. Conduct a parametric study of both knots (If time permits)

Progress Report Overview

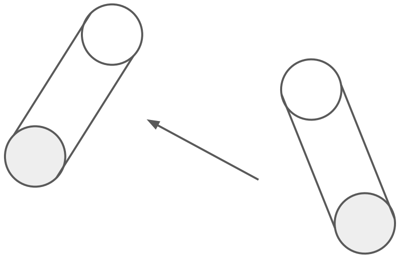
- Still in the process of deriving the necessary boundary condition sequences for granny and reef knots.
- Today, we will go over our frictional contact formulation.
    - Why is contact and friction important in knots?
    - How to model contact and friction.
    - Mathematical formulation.

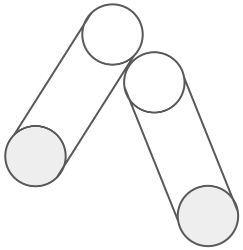
Why is contact and friction important?

- Recall that DER computes the inner elastic energies of a rod.
  - Stretching energy
  - Bending energy
  - Twisting energy

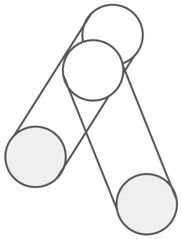
No formulation of contact energy! Penetrations occur when collision arises.

1) Knots are impossible without self-contact.
2) Friction is a major influence on the strength of a knot.





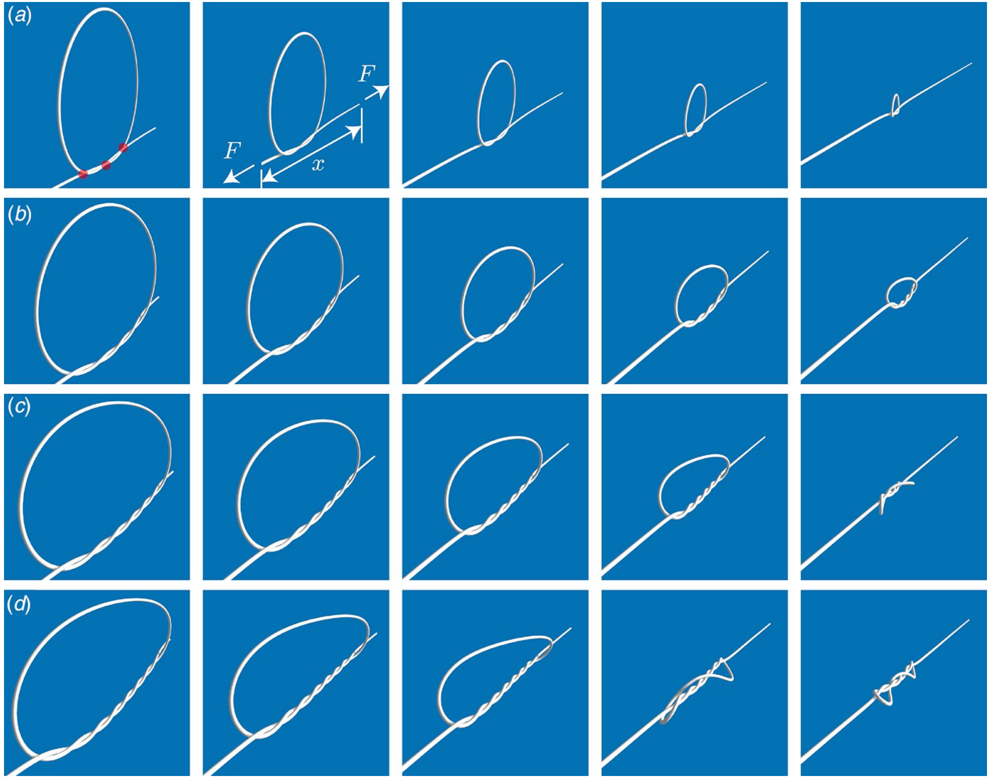
What we want



What happens

Recall: Implicit Contact Model (IMC)

- IMC is a fully implicit frictional contact model that is easily integrated into the DER framework.
- Has shown previous success in accurately simulating overhand knots.



[1] A. Choi, D. Tong, M. K. Jawed, and J. Joo,"Implicit Contact Model for Discrete Elastic Rods in Knot Tying," *Journal of Applied Mechanics*, vol. 88, March 2021.
[2] D. Tong*, A. Choi*, J. Joo, and M. K. Jawed, "A Fully Implicit Method for Robust Frictional Contact Handling in Elastic Rods," arXiv 2022.

How to incorporate IMC's forces?

- DER's degrees of freedom (DOFs) are nodal positions and edge twists.
- Following this, IMC is only dependent on nodal positions.

Recall that $\mathbf{q} = [x_0, y_0, z_0, \theta_0, ..., \theta_{N-2}, x_{N-1}, y_{N-1}, z_{N-1}]^T$

Then IMC's contact force should just be

$$\mathbf{F}_{\text{IMC}} = [F_{x_0}, F_{y_0}, F_{z_0}, 0, ..., 0, F_{x_{N-1}}, F_{y_{N-1}}, F_{z_{N-1}}]^T$$

$$\mathbf{M}\ddot{\mathbf{q}} = \frac{\partial E_{\text{elastic}}}{\partial \mathbf{q}} + \mathbf{F}_{\text{ext}}$$

Contact Energy

- Let us derive a contact energy function $E_{\text{contact}}(\mathbf{q})$

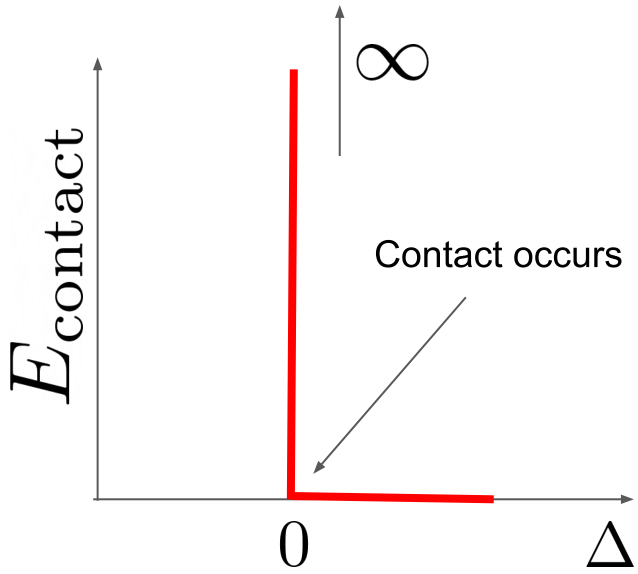
Similar to elastic energies:

$$\mathbf{F}_{\text{contact}} \equiv -\nabla_{\mathbf{q}} E_{\text{contact}}$$
$$\mathbf{J}_{\text{contact}} \equiv -\nabla^2_{\mathbf{q}} E_{\text{contact}}$$

Nonlinearity of Contact Energy

Δ Minimum distance between two bodies

In the real world, contact energy looks like this. Highly nonlinear.

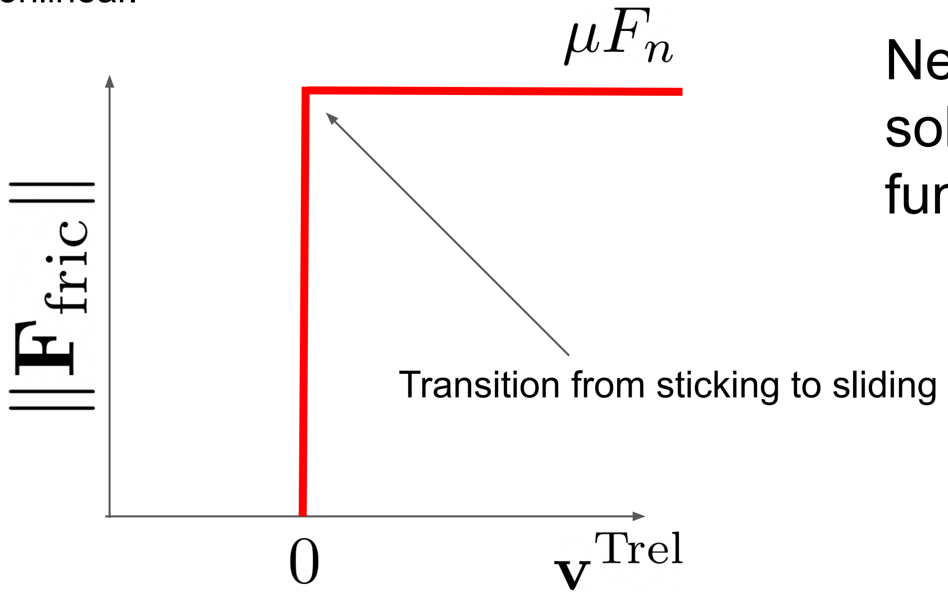


Newton's method fails when solving the roots of nonlinear functions!!!

Nonlinearity of Friction

$\mathbf{v}^{\mathrm{Trel}}$ Tangential relative velocity of contacting bodies at contact point

In the real world, friction is also highly nonlinear.



Newton's method fails when solving the roots of nonlinear functions!!!

Solution: Smoothly Approximate!

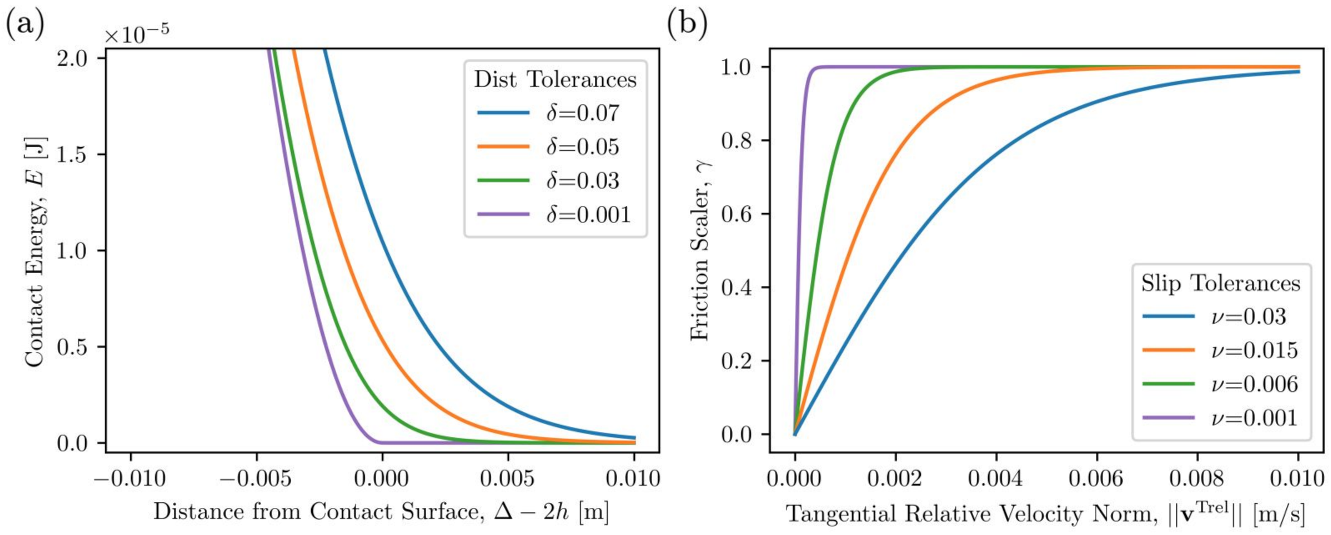


Figure 3: Plots for the approximation functions in (a) Eq. 9 and (b) Eq. 17 with varying tolerance values. Note that some of the tolerances displayed are unrealistically large for clarity.

δ Distance tolerance

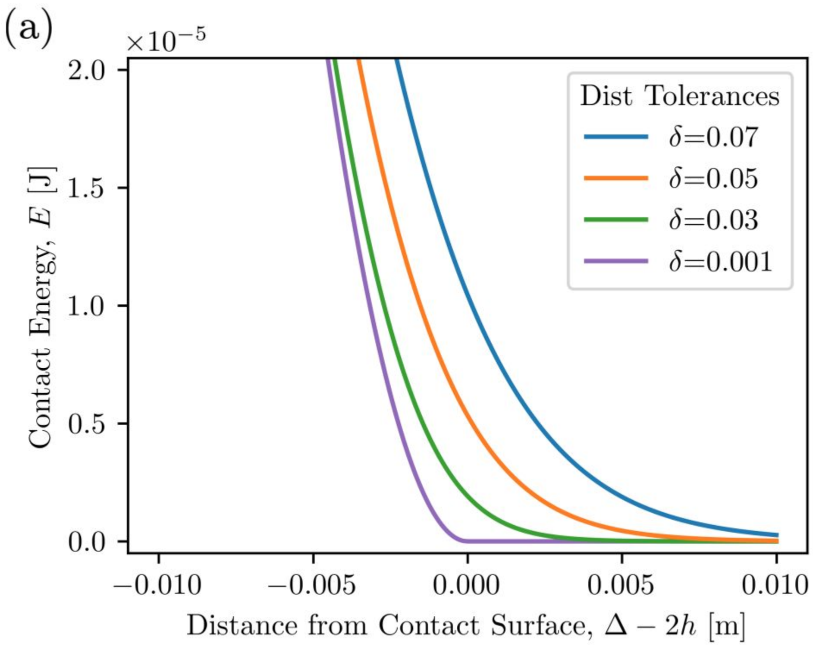
ν Slipping tolerance

Smooth Contact Energy

δ Determines the stiffness of the equation.

$\delta \uparrow$ Physical realism $\uparrow$ Convergence $\downarrow$

$\delta \downarrow$ Physical realism $\downarrow$ Convergence $\uparrow$



$$E(\Delta, \delta) = \begin{cases} (2h - \Delta)^2 & \Delta \in (0, 2h - \delta] \\ \left(\frac{1}{K_1} \log(1 + \exp(K_1(2h - \Delta)))\right)^2 & \Delta \in (2h - \delta, 2h + \delta) \\ 0 & \Delta \geq 2h + \delta, \end{cases}$$

$$K_1 = 15/\delta$$

Friction Formulation

Kinetic friction is simply

$$\mathbf{F}_{\text{fric}} = -\mu \hat{\mathbf{v}}^{\text{Trel}} F_n$$

To model slipping sticking phenomenon, let us add a $\gamma \in [0, 1]$ term.

$$\mathbf{F}_{\text{fric}} = -\gamma(\|\mathbf{v}^{\text{Trel}}\|, \nu)\mu \hat{\mathbf{v}}^{\text{Trel}} F_n$$

Smooth Friction

ν Determines the stiffness of the equation.

$\nu \uparrow$ Physical realism $\uparrow$ Convergence $\downarrow$

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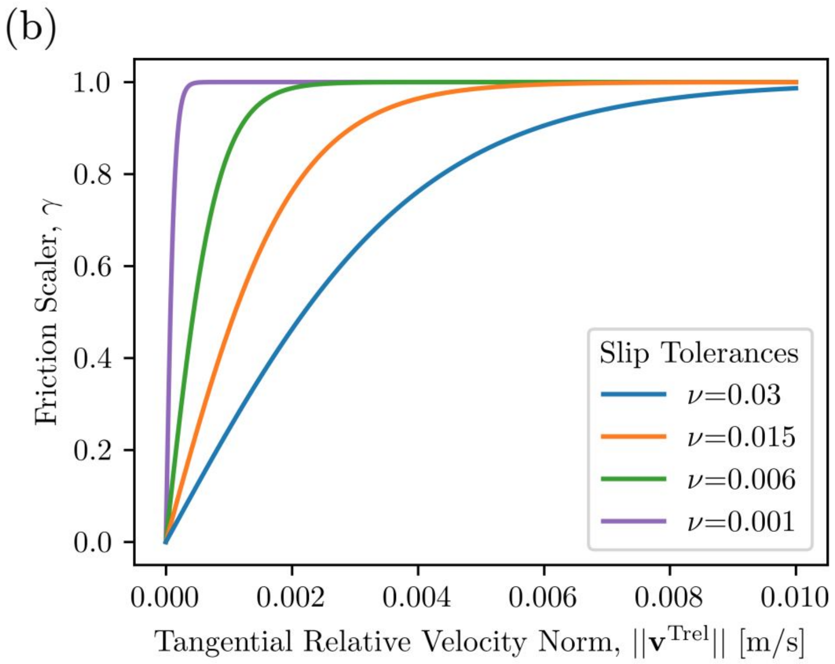
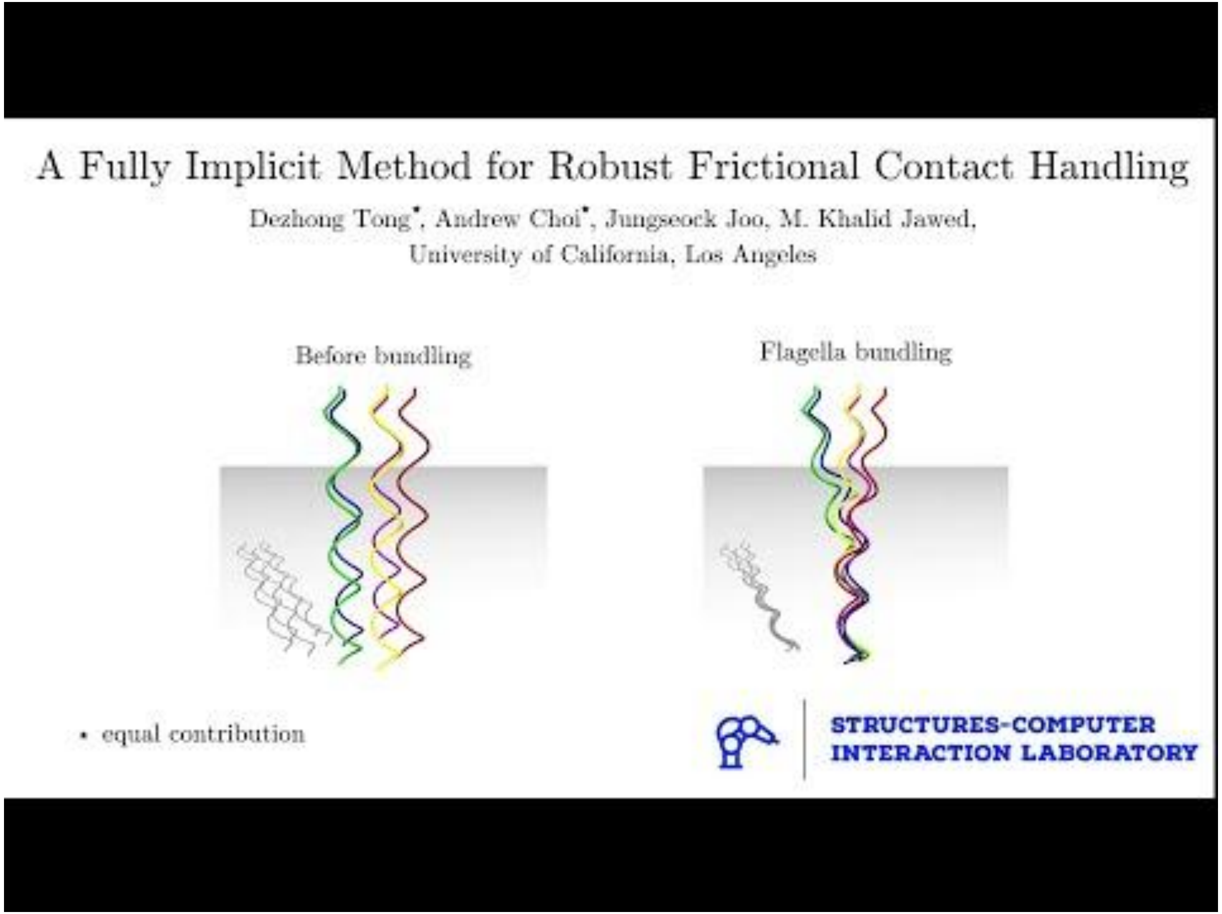


$$\gamma\left(||\mathbf{v}^{\mathrm{Trel}}||, \nu\right) = \frac{2}{1 + \exp\left(-K_2||\mathbf{v}^{\mathrm{Trel}}||\right)} - 1 \qquad K_2 = 15/\nu$$

Visual Demo


A Fully Implicit Method for Robust Frictional Contact Handling
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