UCLA Samueli
Mechanical & Aerospace Engineering

Parametric Study of Reef and Granny Knots

Project Proposal

MAE 259B

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Knots

- Complex topological pattern of self-contact composed of slender elastic structures
- Used all throughout everyday life to fasten objects

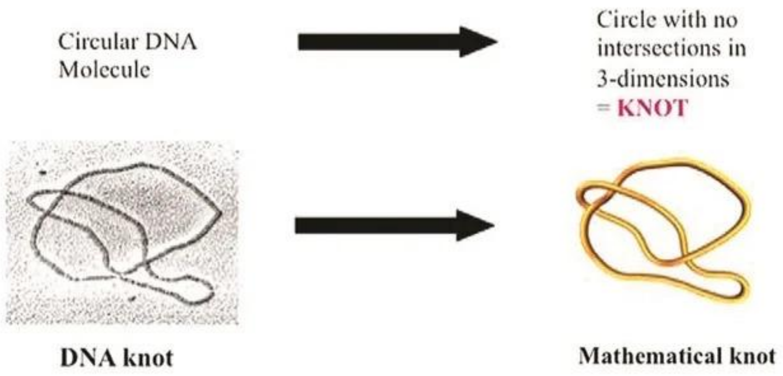
Fig 1. Shoelace

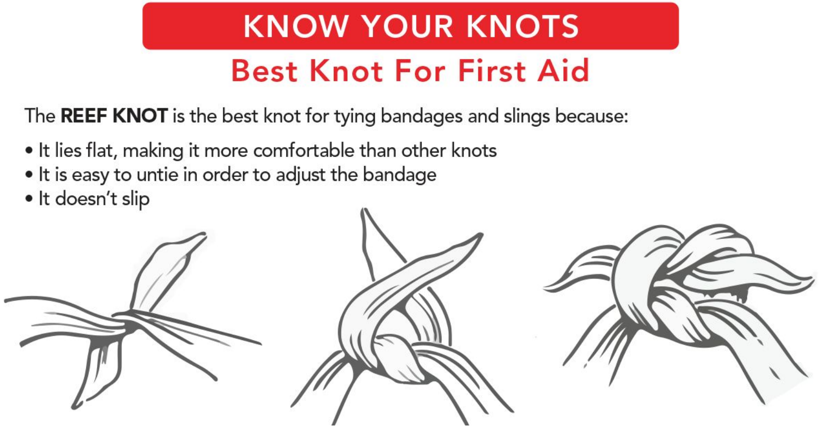
Fig 2. Climbing

Fig 3. Boat berth

Why are knots important?

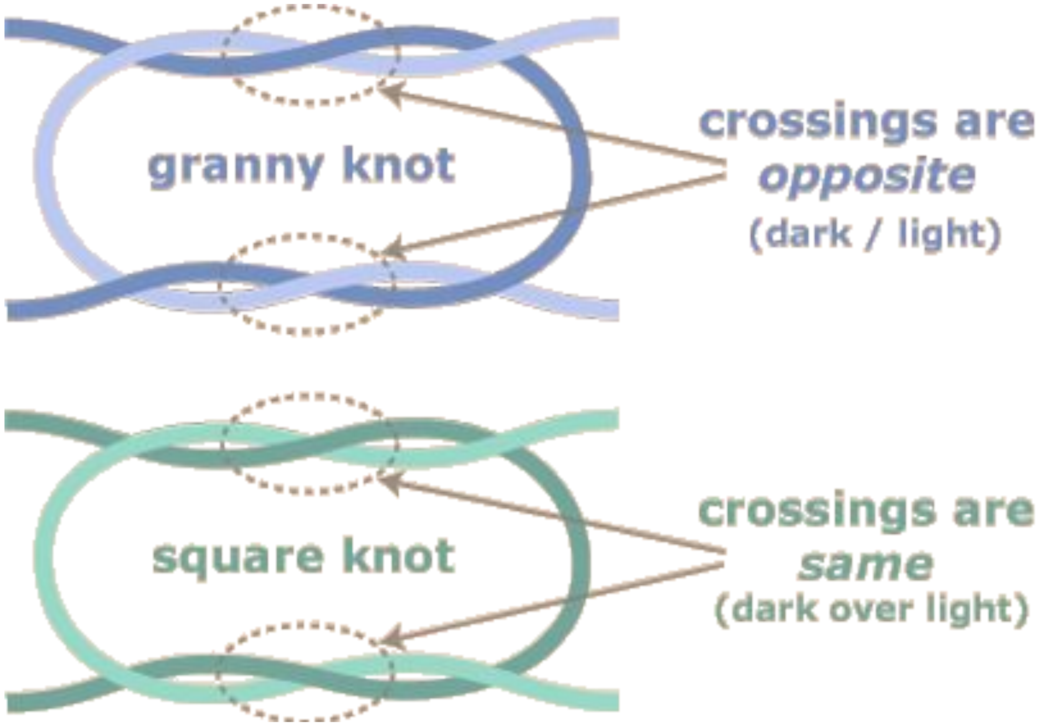
- Obvious everyday utility
  - Tying shoelaces, neckties, etc.
- Medical emergencies
  - First aid bandaging
- Implications in biology
  - Knots occur in DNA and have direct influence on functionality.



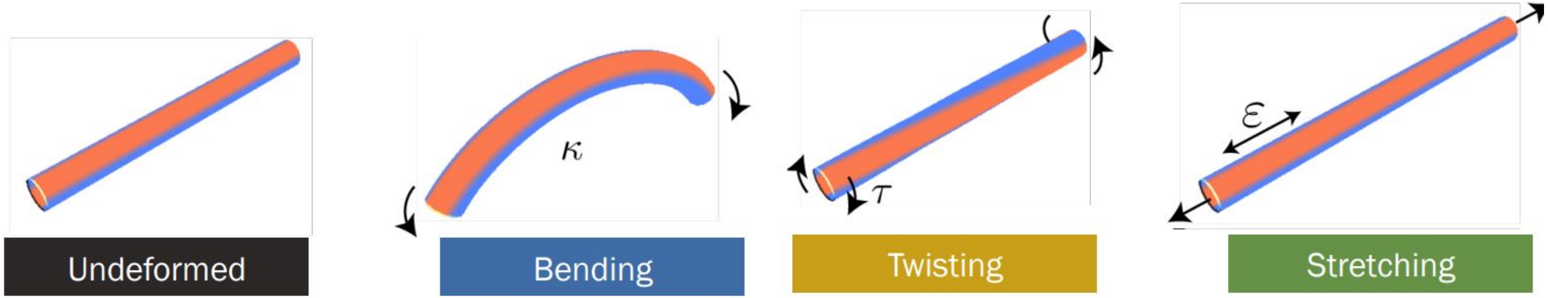


Granny knot / Reef (square) knot

- Granny and reef knots differ in topology by one single crossing order difference.
- Despite this, reef knots are known to possess much higher “knot strength”.
- We wish to study and analyze this mechanical phenomenon through the use of physically accurate simulation.



Step 1: Implement Discrete Elastic Rods (DER)



[3]

- Implement a working version of DER [1, 2]
- At this stage, should be able to accurately simulate elastic rods deforming freely non-contact based external forces such as gravity.
- There should be an intuitive framework for easily applying boundary conditions.

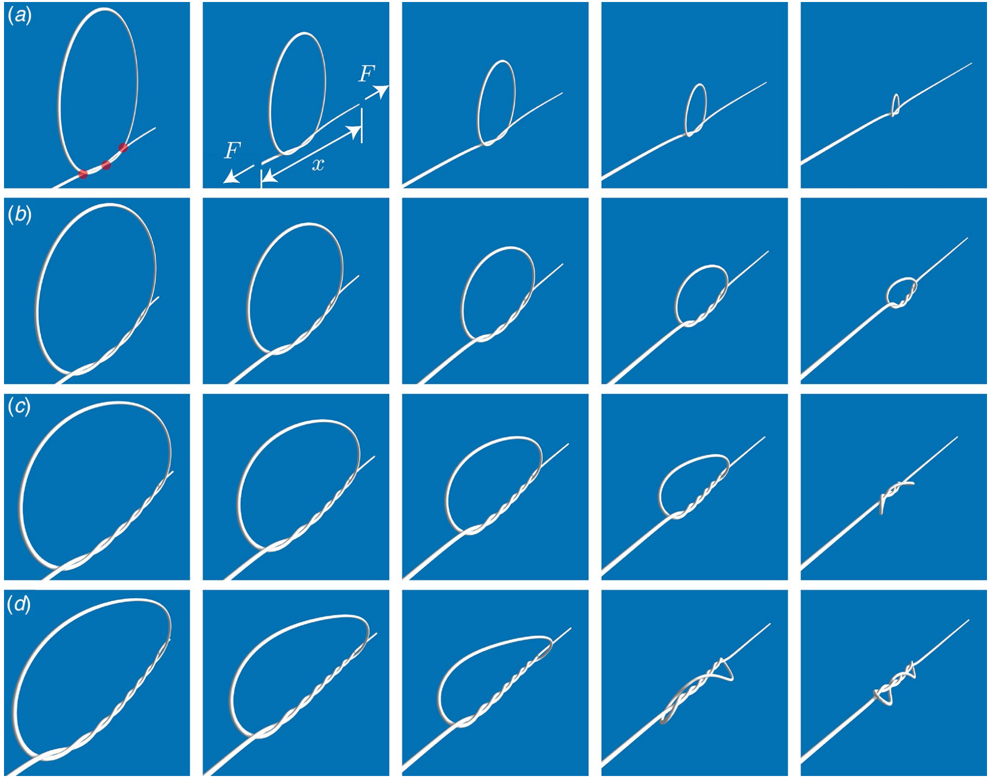
[1] M. Bergou, M. Wardetzky, S. Robinson, B. Audoly, and E. Grinspun, “Discrete elastic rods,” *ACM SIGGRAPH 2008*

[2] M. Bergou, B. Audoly, E. Vouga, M. Wardetzky, and E. Grinspun, “Discrete viscous threads,” *ACM Trans. Graph.* July 2010.

[3] https://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/16311/www/s17/syllabus/ppp/Rods_Mechanics_MK.pdf

Step 2: Incorporate Implicit Contact Model (IMC)

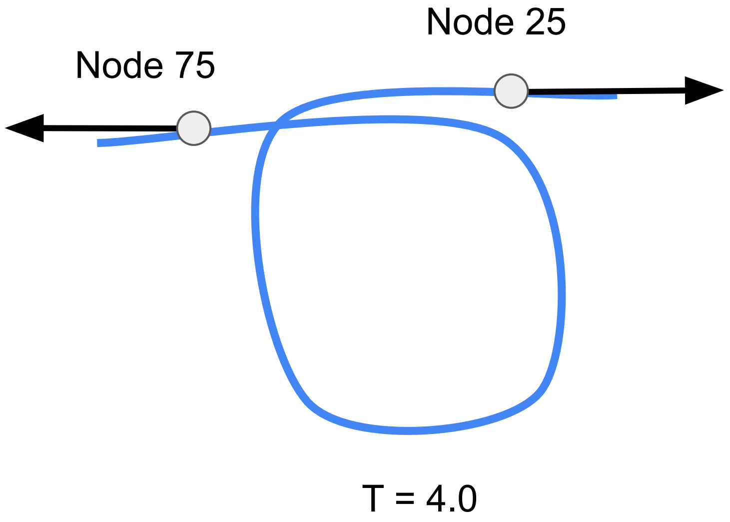
- Implement and incorporate IMC [1, 2] into DER.
- IMC is a fully implicit frictional contact model that is easily integrated into the DER framework.
- Has shown previous success in accurately simulating overhand knots.



[1] A. Choi, D. Tong, M. K. Jawed, and J. Joo,"Implicit Contact Model for Discrete Elastic Rods in Knot Tying," *Journal of Applied Mechanics*, vol. 88, March 2021.
[2] D. Tong*, A. Choi*, J. Joo, and M. K. Jawed, "A Fully Implicit Method for Robust Frictional Contact Handling in Elastic Rods," arXiv 2022.

Step 3: Intuitive Interface for Applying Boundary Conditions

- Develop an intuitive interface for applying time varying boundary conditions.
- These boundary conditions should be limited to straight vector trajectories, i.e. no splines for now.
- Use a text file as an input.

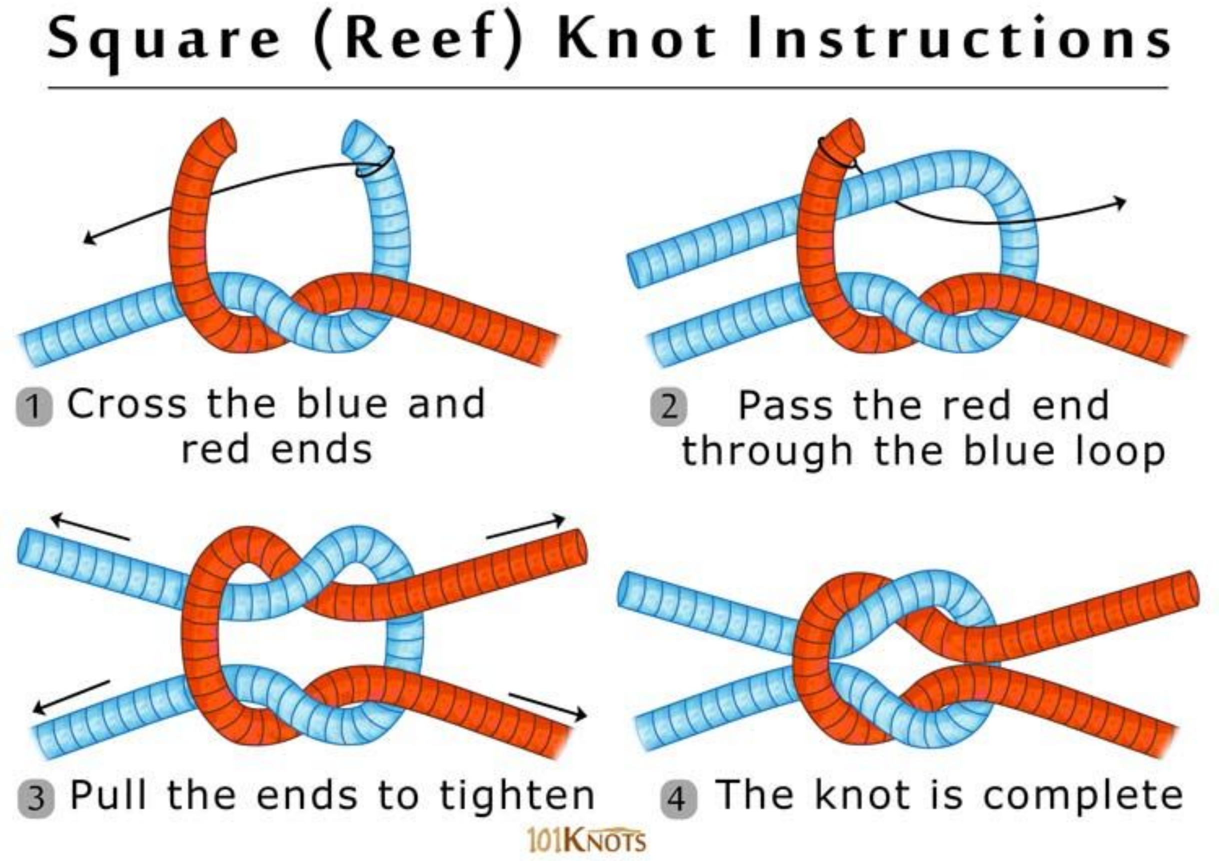


Sample text file

```
at time = 4.0 for 3.0 seconds, pull node 25 with velocity u = [0, 0.005, 0]
at time = 4.0 for 3.0 seconds, pull node 75 with velocity u = [0, -0.005, 0]
```

Step 4: Derive Necessary Boundary Conditions

- Replicate this sequence of steps through our simulation framework.
- Derive a sequence of boundary conditions through an intuitive interface.
- Do the same for both reef and granny knots.



Step 5: Conduct a Parametric Study

- If all goes well and our simulation framework is robust and bug-free, conduct parametric study for both knots.
- Study and analyze the effects of …
  - *h,* rod radius
  - *E,* Young’s modulus
  - mu, coefficient of friction
- Record and plot the effects of these parameters on traction force *Ft* when pulling ends of the knot.

Overall Implementation Steps

1. Framework Discrete Elastic Rod (DER)

2. Incorporate Implicit Contact Model (IMC)
   - Provides accurate frictional contact

3. Develop intuitive boundary condition interface

4. Derive necessary boundary condition
   - Deform an initially straight elastic rod into both a granny and reef knot

5. Conduct a parametric study of both knots