Parametric Study of Granny and Reef Knots

MAE259B Project Proposal

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***Abstract*— Knots are a complex topological pattern of self-contact composed of slender elastic structures. Found all throughout everyday life, they are used for their ability to fasten objects. In this work, we focus on studying two knots in particular: the granny and reef knot. These knots are almost identical in structure and vary by the over/under order of a single crossing. Although these two knots are similar at a glance, one has vastly higher tensile resistance over the other. We propose to implement an accurate physical simulator capable of simulating elasticity, contact, and friction. To study the underlying mechanics behind granny and reef knots, we wish to carry out a parametric study carried out through physical simulation.**

I. Proposal

The study of the mechanics of knots has been receiving increased attention due to their various implications in manufacturing, medicine (knotting of DNA), and more. The deformation and nonlinearity of elastic rods combined with the sheer number of types of knots result in a highly nontrivial problem to study. Previous works have studied the mechanics of overhand knots where Audoly et al. derived a theoretical model relating geometric and mechanical properties for trefoil (single crossing overhand) and cinquefoil (double crossing overhand) knots [1]. The work of Jawed et al. [2] expanded upon this by producing an analytical theoretical model for overhand knots for various crossing numbers. More recently, the Patil et al. produced methods for predicting the mechanical stability of various knots based on topological observables [3]. Regardless, many questions surrounding the mechanics of knots still remain.

In this work, we study granny and reef (square) knots as shown in Fig. 1. The reef knot is known famously as the one used for tying shoelaces. The granny knot is another type of knot that differs from the reef knot by a single over/under crossing order as shown by the circled regions in Fig. 1. Despite this stark similarity in structure, the reef knot possesses significantly higher tensile resistance, i.e. the knot stays fastened from self-friction when the ends are pulled. We propose to study this mechanical phenomenon by utilizing physically accurate simulation frameworks.

II. Implementation Plan

In recent years, the framework Discrete Elastic Rods (DER) [5], [6], has shown remarkable performance in accurately simulating elastic rods. First, we wish to implement DER in the Python programming language. This language is chosen for its ease of use and rapid prototyping capabilities. After a working copy of DER has been created, we will next incorporate Implicit Contact Model (IMC) [4], [7] for

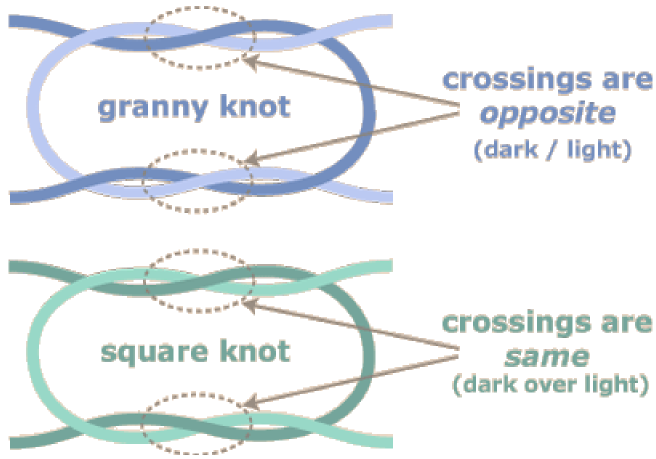

Fig. 1. Visualization of granny and reef (square) knots. The difference between them is simply a result of the circled crossings.

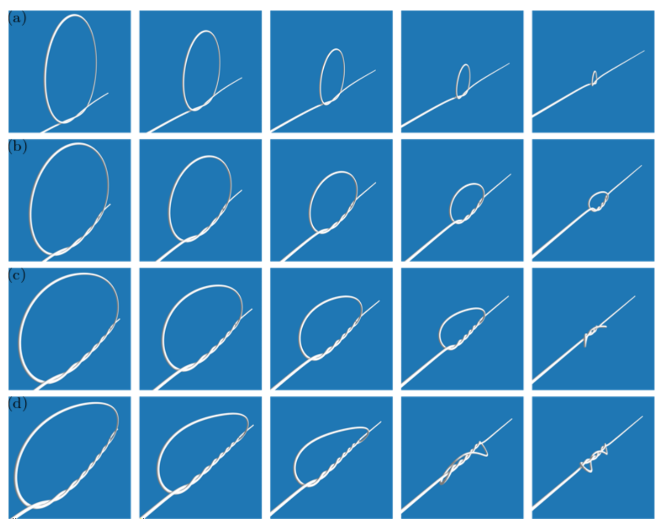

Fig. 2. Snapshots of tightening overhand knots from a simulation framework composed of DER + IMC [4].

accurate frictional contact. This contact model was designed to be easily integrated into the DER framework and the physical realism of the produced forces have been validated through rigorous comparison with the overhand knot theories proposed by [1], [2]. Snapshots of simulated overhand knots produced by DER + IMC can be observed in Fig. 2.

Once a DER + IMC model has been implemented in Python, we will then focus on deriving the necessary boundary conditions to deform an initially straight elastic rod into both a granny and reef knot. With the sequence of boundary conditions discovered, visually realistic results for both these knots can then be delivered. If time permits, we can then use our rapid simulation framework to conduct a parametric study for both knots. Such a parametric study would analyze the effects of parameters such as rod radius h, Young's Modulus E, and friction coefficient μ on the experienced traction forces of each knot during the tightening process.

Overall, the key tasks of our project include

1) Implementing a working version of DER in Python.
2) Incorporating IMC into DER for frictional contact.
3) Figuring out the necessary boundary conditions to tie both granny and reef knots.
4) Conducting a parameteric study of both knots *if time permits*.

We hope that the simulation tool produced by this course project can eventually be used for publication-worthy research in the future.

REFERENCES

[1] B. Audoly, N. Clauvelin, and S. Neukirch, "Elastic knots," *Physical review letters*, vol. 99, p. 164301, 11 2007.

[2] M. K. Jawed, P. Dieleman, B. Audoly, and P. M. Reis, "Untangling the mechanics and topology in the frictional response of long overhand elastic knots," *Phys. Rev. Lett.*, vol. 115, p. 118302, Sep 2015.

[3] V. P. Patil, J. D. Sandt, M. Kolle, and J. Dunkel, "Topological mechanics of knots and tangles," *Science*, vol. 367, no. 6473, pp. 71–75, 2020.

[4] A. Choi, D. Tong, M. K. Jawed, and J. Joo, "Implicit Contact Model for Discrete Elastic Rods in Knot Tying," *Journal of Applied Mechanics*, vol. 88, 03 2021. 051010.

[5] M. Bergou, M. Wardetzky, S. Robinson, B. Audoly, and E. Grinspun, "Discrete elastic rods," in *ACM SIGGRAPH 2008 Papers*, SIGGRAPH '08, (New York, NY, USA), Association for Computing Machinery, 2008.

[6] M. Bergou, B. Audoly, E. Vouga, M. Wardetzky, and E. Grinspun, "Discrete viscous threads," *ACM Trans. Graph.*, vol. 29, jul 2010.

[7] D. Tong, A. Choi, J. Joo, and M. K. Jawed, "A Fully Implicit Method for Robust Frictional Contact Handling in Elastic Rods," 2022.