Parametric Study of Granny and Reef Knots
MAE259B Project Proposal

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***Abstract*— Knots are a complex topological pattern of self-contact composed of slender elastic structures. Found all throughout everyday life, they are used for their ability to fasten objects. In this work, we focus on studying two knots in particular: the granny and reef knot. These knots are almost identical in structure and vary by the over/under order of a single crossing. Although these two knots are strikingly similar at a glance, one has a vastly higher knot strength over the other. We propose to implement an accurate physical simulator capable of simulating elastic rods along with contact and friction. Using this simulator, we wish to to study the underlying mechanics behind granny and reef knots through a parametric study.**

I. Proposal

The study of the mechanics of knots has been receiving increased attention due to their various implications in manufacturing, medicine (knotting of DNA), and more. The deformation and nonlinearity of elastic rods combined with the sheer number of types of knots result in a highly nontrivial problem to study. Previous works have studied the mechanics of overhand knots where Audoly et al. derived a theoretical model relating geometric and mechanical properties for trefoil (single crossing overhand) and cinquefoil (double crossing overhand) knots [1]. The work of Jawed et al. [2] expanded upon this by producing an analytical theoretical model for overhand knots for all crossing numbers. More recently, Patil et al. produced methods for predicting the mechanical stability of various knots based on topological observables [3]. Regardless, many questions surrounding the mechanics of knots still remain.

In this work, we study granny and reef (square) knots as shown in Fig. 1. The reef knot is known famously as the one used for tying shoelaces. The granny knot differs from the reef knot by a single over/under crossing order as shown by the circled regions in Fig. 1. Despite this stark similarity in structure, the reef knot possesses significantly higher "knot strength", i.e. the knot stays fastened from self-friction when the ends are pulled. While reef knots tend to stay fastened until breaking when pulled on, granny knots will simply slip and unravel. We propose to study and analyze this mechanical phenomenon by utilizing physically accurate simulation frameworks.

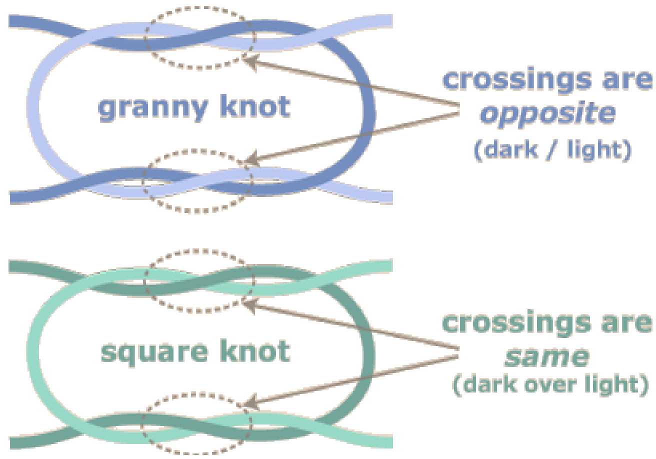

Fig. 1. Visualization of granny and reef (square) knots. The difference between them is simply a result of the circled crossings.

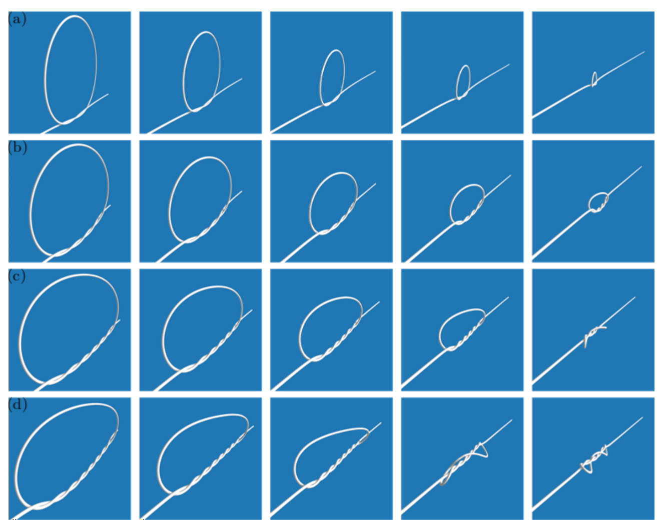

Fig. 2. Snapshots of tightening overhand knots from a simulation framework composed of DER + IMC [4]. Physically accurate friction forces allow the frame to capture buckling for the overhand knots in rows (c) and (d).

II. Implementation Plan

In recent years, the framework Discrete Elastic Rods (DER) [5], [6] has shown remarkable performance in accurately simulating elastic rods. Based off of Kirchhoff rod theory, this framework discretizes an elastic rod into discrete nodes and edges and accurately simulates deformations by computing the stretching, bending, and twisting energies of the rod. The first step of our project is to obtain a working implementation of DER. After a working copy of DER has been created, we will next incorporate Implicit Contact Model (IMC) [4], [7] for accurate frictional contact. This contact model was designed to be easily integrated into the DER framework. As it is a fully implicit formulation, large time steps can be achieved which will later speed up any necessary simulations for the parametric study. Furthermore, the physical realism of IMC's produced forces have been validated through rigorous comparison with the overhand knot theories proposed by [1], [2]. Snapshots of simulated overhand knots produced by DER + IMC can be observed in Fig. 2. Fortunately, we have access to an existing C++ implementation of DER + IMC for overhand knots, which we will modify for this project.

We must then focus on deriving the necessary boundary conditions to deform an initially straight elastic rod into both a granny and reef knot. Instead of hardcoding a sequence of boundary conditions, we would like to develop an intuitive interface that can be used to rapidly test out different time varying boundary condition applications. With the sequence of boundary conditions discovered, visually realistic results for both these knots can then be delivered. We can then use our rapid simulation framework to conduct a parametric study for both knots. Such a parametric study would analyze the effects of parameters such as rod radius h, Young's Modulus E, and friction coefficient μ on the experienced traction forces of each knot during the tightening process.

Overall, the key tasks of our project include

1) Implementing a working version of DER. *done*
2) Incorporating IMC into DER for frictional contact. *done*
3) Developing an intuitive interface for applying boundary conditions.
4) Figuring out the necessary boundary conditions to tie both granny and reef knots.
5) Conducting a parameteric study of both knots.

We hope that the simulation tool produced by this course project can eventually be used for publication-worthy research in the future.

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