Lab - 11

0. $\{w \mid w \in \Sigma^*, \text{rev}(w) = w\}$ — Palindrome

$\Sigma = \{a, b\}$ $\quad \left.\begin{array}{l} w_1 = a^i b \\ w_2 = a^j b \end{array}\right\}$, where i & j are NOT equal. $i \neq j$

$w_1 \cdot a^i$ will be accepted but $w_2 \cdot a^i$ will not be accepted.

EX $i = 2$, $j = 3$

$w_1 \cdot a^i = aabaa$, $w_2 \cdot a^j = aaabaa$

This shows that the equivalence classes are NOT finite.

So, the language is not regular.

G_0: $S \to aSa \mid bSb \mid a \mid b \mid \varepsilon$

1. $S \to ABS \mid AB$

$A \to aA \mid a$

$B \to bA$

① aabaab $\Rightarrow$ b without sequential A. Because every b must be followed by a.

② aaaaba $\Rightarrow$ $S \to AB \to aAB \to aaAB \to aaaAB \to aaaaAB \to aaaabA \to aaaaba$

③ aabbaa $\Rightarrow$ Consecutive b's. Because No ways to Produce consecutive b's.

④ abaaba $\Rightarrow$ $S \to ABS \to ABAB \to aBAB \to abAAB \to abaAB \to abaaB \to abaabA \to abaaba$

2. $\Sigma = \{a, b, 0, 1, +, *, (,)\}$

G_2: $S \to 0 \mid 1 \mid a \mid b \mid SS \mid S+S \mid S^* \mid (S)$

$r = ab^*(ba + b + bb)^* + aba + 1$

$S \to S+S \to SS+S+S \to SSS^* + SS + 1 \to aS^*(S)^* + SSS + 1$

$\to ab^*(S+S)^* + aba + 1 \to ab^*(S+S+S)^* + aba + 1 \to$

$ab^*(SS + b + SS)^* + aba + 1 \to \boxed{ab^*(ba + b + bb)^* + aba + 1}$

3. Context free Grammer of $r = ab^*(ba + b + bb)^* + aba + 1$

$S \to aBA \mid aba \mid \varepsilon$

$B \to bB \mid \varepsilon$

$A \to baA \mid bA \mid bbA \mid \varepsilon$

4 $\Sigma = \{a, b\}$, Prove $S \to aSb \mid bSa \mid SS \mid \varepsilon$ generate same number's of a's & b's.

$a : (\Sigma \cup \{S\})^* \to N$
$a(\varepsilon) = 0$
$a(aw) = 1 + a(w)$
$a(bw) = a(w)$
$a(Sw) = a(w)$

$b : (\Sigma \cup \{S\})^* \to N$
$b(\varepsilon) = 0$
$b(aw) = b(w)$
$b(bw) = 1 + b(w)$
$b(Sw) = b(w)$

$w \in \Sigma^*$ has equal a's & b's iff $a(w) = b(w)$

① $\forall w \in (\Sigma \cup \{S\})^*, (S \mapsto^* w) \to a(w) = b(w)$

② $\forall w \in \Sigma^*, (a(w) = b(w)) \to S \mapsto^* w$

* Prove S has same number of a's & b's.
S, when used in the function above, yields 0 a's and 0 b's.
So, S has equal a's and b's as $a(S) = b(S) = 0$

When looking at the rules $S \to aSb$ and $S \to bSa$, an a and a b is added at the same time, so at each step, there will be and same numbers of a's & b's.

Rule 1: $\alpha S \beta \mapsto \alpha a S b \beta$
Assume: $a(\alpha S \beta) = b(\alpha S \beta)$
$\Rightarrow a(\alpha a S b \beta) = b(\alpha a S b \beta)$
$= 1 + a(\alpha S \beta) = 1 + b(\alpha S \beta)$ ✓

Rule 2: $\alpha S \beta \mapsto \alpha b S a \beta$
Assume: $a(\alpha S \beta) = b(\alpha S \beta)$
$a(\alpha b S a \beta) = b(\alpha b S a \beta)$
$1 + a(\alpha S \beta) = 1 + b(\alpha S \beta)$ ✓

Rule 3: $\alpha S \beta \mapsto \alpha S S \beta$
Assume: $a(\alpha S \beta) = b(\alpha S \beta)$
$a(\alpha S S \beta) = b(\alpha S S \beta)$
$0 + a(\alpha S \beta) = 0 + b(\alpha S \beta)$ ✓

Rule 4: $\alpha S \beta \mapsto \alpha \beta$
Assume: $a(\alpha S \beta) = b(\alpha S \beta)$
remove S,
$a(\alpha \beta) = b(\alpha \beta)$ ✓

* Suppose w has same number's of a's & b's and $|w| = n$.
for all strings w_1 where $|w_1| < n$,
$(a(w_1) = b(w_1)) \to S \mapsto^* w_1$ } IH

Case 1: $n=0$ $(w=\varepsilon)$

$S \overset{4}{\longmapsto} \varepsilon$

Case 2: $w \neq \varepsilon$ and has a prefix with equal a's & b's.

$w = w_1 \cdot w_2$ where w_1 and $w_2 \neq \varepsilon$ and w_1 has equal a's & b's.

and Assume w_2 also has equal a's & b's.

For w to have equal a's, b's if w_1 has equal a's & b's.

w_2 would also have to have equal a's & b's.

$S \overset{2}{\longmapsto} SS \longmapsto^* w_1 S \longmapsto^* w_1 w_2 = w \Rightarrow S \longmapsto^* w$ ✓

Case 3: $w \neq \varepsilon$ and has no ~~proper~~ prefix with equal a's & b's. can split w into aw^*b or bw^*a where w^* has equal a's & b's.

$w = aw^* \cdot$ or bw^*

aw^* must end in b otherwise w^* would need to have two b's. to balance this out and would not equal a's & b's.

Similarly, bw^* must end in a, w^* must have equal a's, b's because w has equal a's & b's and there is 1 a and 1 b surrounding it

5. $S \to aS \mid aSbS \mid \varepsilon$ } set generated by frammar:

① $a(w) \geq b(w)$ } not enough

② for every prefix ~~w~~ of w_1, $a(w_1) \geq b(w_1)$

1. Every prefix of S meets the criteria since 0 a's and b's.

Case 1: $\alpha S \beta \longmapsto \alpha a S \beta$ and Assume $a(\alpha S \beta) \geq b(\alpha S \beta)$

Add 1a and 0b, so $a(\alpha a S \beta) \geq b(\alpha a S \beta)$

Prefixes: $\alpha a \mid S \mid \beta$ } additional a so, still true ✓

Case 2: $\alpha S \beta \longmapsto \alpha a S b S \beta$ and Assume: $a(\alpha S \beta) \geq b(\alpha S \beta)$

Add 1a and 1b, so $a(\alpha a S b S \beta)$ is still $\geq b(\alpha a S b S \beta)$

Prefixes: $\alpha a \mid S \mid b \mid S \mid \beta$ } additional a with no b, so, still true ✓

Case 3: $\alpha S \beta \longmapsto \alpha \beta$ and Assume: $a(\alpha S \beta) \geq b(\alpha S \beta)$

since, no elements were added, we can assume to prove that $a(\alpha \beta) \geq b(\alpha \beta)$, and same for prefixes because no elements were added.

2. Suppose $a(w) \geq b(w)$ and $|w| = n$

for all strings w_1 where $|w_1| < n$?

$(a(w_1) \geq b(w_1)) \rightarrow S \vdash^* w_1$ } IH

Case 1: $n=0$, $w=\varepsilon$ so, $S \vdash^{3} \varepsilon$

Case 2: $w \neq \varepsilon$ and has a prefix where a's $\geq$ b's

$w = w_1 \cdot w_2$ where w_1 and $w_2 \neq \varepsilon$ and $a(w_1) \geq b(w_2)$

Since, Assume $a(w) \geq b(w)$ and $a(w_2)$ must be $\geq b(w_2)$.

if w_1 is a prefix of w, then it is also prefix of w.

So, $a(w_1) \geq b(w_1)$.

if w_2 is a prefix of w then $w_1 \cdot w_2$ is a prefix of w.

So $a(w_2) \geq b(w_2)$.

Case 3: $w \neq \varepsilon$ and has no prefix where a's $\geq$ b's can split w into $aw'bS$ or $aSbw'$ where $a(w') \geq b(w')$ in either case, 1 a & 1 b has been added, so if w upholds the rule, the split will as well. S can only be replaced by a's or an a paired with a b. Neither case increases the number of b's while keeping a's the same.

6. CNF for $\{a^i b^j a^i \mid i \geq 0, j \geq 1\}$

$S \rightarrow aSa \mid T$

$T \rightarrow bT \mid b$

CFG

$S \rightarrow XA \mid BT \mid b$

$T \rightarrow BT \mid b$

$A \rightarrow \varepsilon$

$X \rightarrow AS$

$B \rightarrow b$

CNF