The transducer synthesis problem

$$T() = \exists H_1, H_2, \dots H_n. \forall (I, O) \in E,$$
$$\exists_{t \in |I|} T_{s,t,v}, T$$